# Naturalness and Top-Down BSM
# Lecture 4

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*Suggestions, clarifications, and comments are welcome.*

# Contents

# 1 New Hierarchy Solutions

Let's conclude by discussing more recent solutions to the electroweak hierarchy problem, largely motivated by the lack of evidence for more conventional solutions at the LHC.

## 1.1 Twin Higgs / Neutral naturalness

One interesting direction is to retain the symmetry-based approach but expand the scope of possible symmetries. The natural possibility is to work with discrete symmetries, rather than continuous ones. The idea is that the new particles required by a discrete symmetry need not carry the same Standard Model quantum numbers, and so are less strongly constrained by data from the LHC.

There are by now many different examples of neutral naturalness, but the simplest is the original: the Twin Higgs [1]. The idea is to introduce a mirror copy of the Standard Model along with a $\mathbb{Z}_2$ symmetry exchanging each field with its mirror counterpart. On top of this, one needs to assume an approximate global symmetry in the Higgs sector, which may be $U(4)$ or $O(8)$ depending on one's level of ambition. This global symmetry need not be exact, and is violated by all SM yukawa and gauge couplings, but should be an approximate symmetry of the Higgs potential.

For simplicity, we will consider the perturbative case where it suffices to work in terms of a $U(4) \simeq SU(4) \times U(1)$ approximate global symmetry, gauged by the Standard Model and twin electroweak interactions (i.e., gauging the $SU(2) \times SU(2) \times U(1)$ subgroup of $SU(4)$ and the additional $U(1)$). We can assemble the Higgs doublets $H_A$ and $H_B$ into a fundamental of $SU(4)$,

$$H = \begin{pmatrix} H_A \\ H_B \end{pmatrix} \tag{1}$$

and, under the assumption that the Higgs sector potential is approximately $SU(4)$ symmetric, write down a potential of the form

$$V(H) = m^2 H^\dagger H + \lambda (H^\dagger H)^2 \tag{2}$$

For $m^2 < 0$, $H$ acquires a vev and the $SU(4) \times U(1)$ is spontaneously broken to $SU(3) \times U(1)$, yielding seven goldstones. Depending on the vacuum alignment, all goldstones will be eaten, but it's also possible to align the vev entirely in the $A$ sector or $B$ sector by judicious adjustment of the potential. This adjustment is accomplished by terms that softly break the $U(4)$, and so ultimately will induce finite corrections to the goldstone mass through radiative corrections.

This theory accumulates radiative corrections from the usual couplings, for example top yukawas of the form

$$\lambda_A H_A Q_A t_A + \lambda_B H_B Q_B t_B \tag{3}$$

This gives the usual quadratic divergence,

$$\delta m^2 = -\frac{6}{16\pi^2}\Lambda^2\left(\lambda_A^2|H_A|^2 + \lambda_B^2|H_B|^2\right) \tag{4}$$

but the $\mathbb{Z}_2$ symmetry enforces $\lambda_A = \lambda_B = \lambda$, so that

$$\delta m^2 = -\frac{6\lambda^2}{16\pi^2}\Lambda^2\left(|H_A|^2 + |H_B|^2\right) \tag{5}$$

Now we can see the magic of the discrete symmetry. At the level of mass terms, the quadratic divergences respect the $U(4)$ symmetry. Thus the goldstones of the spontaneous breaking of the $U(4)$ symmetry will be protected against UV contributions.

We could continue to study the linear model (see, e.g., [2]), but it's convenient to focus on the low-energy theory of the goldstones [3]. In the limit where the vacuum expectation value lies entirely in the $B$ sector, in $B$-sector unitary gauge we have

$$H = \begin{pmatrix} H_A \\ H_B \end{pmatrix} = \exp\left[\frac{i}{f}\left(\begin{array}{ccc|c} & & & h_1 \\ & & & h_2 \\ & & & 0 \\ \hline h_1^* & h_2^* & 0 & 0 \end{array}\right)\right]\begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix} \equiv e^{i\pi/f}H_0 \tag{6}$$

Expanding out the exponential, we then get (up to a phase on $h$)

$$H = \begin{pmatrix} h\frac{if}{\sqrt{h^\dagger h}}\sin\left(\frac{\sqrt{h^\dagger h}}{f}\right) \\ 0 \\ f\cos\left(\frac{\sqrt{h^\dagger h}}{f}\right) \end{pmatrix} \tag{7}$$

where $h = (h_1, h_2)^T$. Then we can immediately expand out $H_A$ and $H_B$ in terms of the goldstone modes, obtaining

$$H_A = h\frac{f}{\sqrt{h^\dagger h}}\sin\left(\frac{\sqrt{h^\dagger h}}{f}\right) = h + \ldots \tag{8}$$

$$H_B = \begin{pmatrix} 0 \\ f\cos\left(\frac{\sqrt{h^\dagger h}}{f}\right) \end{pmatrix} = \begin{pmatrix} 0 \\ f - \frac{1}{2}\frac{h^\dagger h}{f} + \ldots \end{pmatrix} \tag{9}$$

The goldstones inherit Yukawa couplings (which break the $U(4)$)

$$\lambda_A H_A Q_A t_A + \lambda_B H_B Q_B t_B \to \lambda_A h Q_A t_A + \lambda_B \left( f - \frac{h^\dagger h}{2f} \right) Q_B t_B + \dots \tag{10}$$

and now we can see in detail the cancellation of quadratic divergences, in exact analogy with the case of a continuous global symmetry: As in our other symmetry

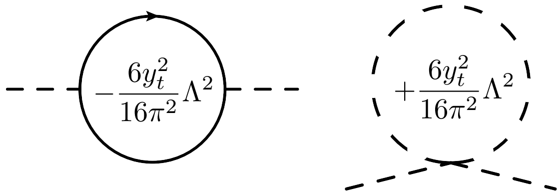


Figure 1: Quadratic divergence cancellation in the discrete global symmetry case.

examples, the UV sensitivity is replaced by finite corrections coming from the mass of the SM-enutral top partners, which violate the accidental $U(4)$ through the soft breaking terms in the potential.

But in contrast to older continuous symmetry approaches, now there are no direct constraints on the partner particles, and so no tension with null results in direct searches at the LHC.

However, that's not to say that the theory is unconstrained. There are three salient points worth discussing. First, there is a question of vacuum alignment, why $v \ll f$, or equivalently why the Higgs is light compared to the radial mode of approximate $SU(4)$ breaking.

In the simplest twin Higgs model, this hierarchy requires a tuning. Trivially, if $\mathbb{Z}_2$ is an exact symmetry, then $v = f/\sqrt{2}$; nothing distinguishes $H_A$ from $H_B$. So we must break the discrete symmetry, ideally with a soft mass. Even then, tuning is necessary to obtain $v \ll f$. This is easy to see if you write down the most general twin Higgs potential allowing for soft $\mathbb{Z}_2$ breaking:

$$V = \lambda(|H_A|^2 + |H_B|^2 - f^2)^2 + \kappa(|H_A|^4 + |H_B|^4) + \sigma f^2 |H_A|^2 \tag{11}$$

Here I've just traded the $SU(4)$-symmetric mass for the overall scale $f$ of symmetry breaking, and written the soft $\mathbb{Z}_2$ breaking parameter in terms of a dimensionless parameter $\sigma$.

Now in the limit $\lambda \ll 1$, the $SU(4)$ symmetric quartic ridigly fixes $|H_B|^2 = f^2 - |H_A|^2$, the goldstone comes from $H_A$, and the potential for $H_A$ is just

$$V \to 2\kappa|H_A|^4 + (\sigma - 2\kappa)f^2|H_A|^2 = \lambda_{SM}|H_A|^4 - 2\lambda_{SM}v^2|H_A|^2 \tag{12}$$

From this we can read off the relation between $v$ and $f$,

$$\frac{2v^2}{f^2} = \frac{2\kappa - \sigma}{2\kappa} \tag{13}$$

We must tune $\kappa$ against $\sigma$ to make this ratio small, and the tuning is precisely of order $f^2/2v^2$.

I should mention that a similar tuning arises in the global symmetry models considered in last lecture, albeit for different reasons having to do with how the Higgs potential is generated.

Second, also as in the global symmetry case, there are Higgs coupling deviations coming from the goldstone nature of the Higgs – you can again think of this as due to higher-dimensional operators in the NLSM parameterization, or just mixing in the linear sigma model. These currently provide the leading constraint on the model.

Third, also as in the global symmetry case, something must protect the scale $f$. It could be compositeness or supersymmetry or turtles, but the discrete symmetry alone is not sufficient to stabilize the Higgs all the way up to the Planck scale.

There are various generalizations of this idea. One can trivially construct $\mathbb{Z}_N$ models, which generalize naturally to multiple sectors. Alternately, one can construct "fraternal" models where the $\mathbb{Z}_2$ symmetry is only a good symmetry for the states most relevant to the Higgs potential [2]. One can also construct more elaborate symmetry structures using orbifold projections [4]. The signatures are rich and interesting and worth looking for enthusiastically in the remaining lifetime of the LHC.

## 1.2 Relaxion

In some sense, neutral naturalness is the most conservative "new" idea, retaining an old mechanism (symmetry protection) and pushing the specific realization in a new direction. But an even more exciting thing about the modern era is that we are now beginning to see radically new ideas that don't fit into traditional paradigms. The most exciting recent ideas involve dynamics to select the Higgs mass from a range of values consistent with a cutoff well above the weak scale.

### 1.2.1 QCD/QCD′ Relaxion

The simplest and original incarnation [5], inspired by the Abbotte model, is that of a QCD axion-like particle $\phi$ coupled to the Standard Model, with an additional inflationary sector whose properties will turn out to be somewhat special. Here we emphasize *axion-like* because the axion-like field will not be manifestly compact, but rather possess only a shift symmetry. This shift symmetry will be broken by a small, dimensionful coupling to the Higgs. We will circle back to these features, and their potential relation to technical naturalness arguments, towards the end.

We envision enlarging the Standard Model with the following terms:

$$\delta\mathcal{L} = (-M^2 + g\phi)|H|^2 + V(g\phi) + \frac{1}{32\pi^2}\frac{\phi}{f}\tilde{G}^{\mu\nu}G_{\mu\nu} \tag{14}$$

where $M$ is of the order of the cutoff of the SM Higgs sector, $H$ is the Higgs doublet, $g$ is the dimensionful coupling that breaks the shift symmetry, and

$$V(g\phi) \sim gM^2\phi + g^2\phi^2 + \ldots$$

parameterizes the non-derivative terms solely involving $\phi$. We will be interested in field values of $\phi$ that greatly exceed $f$, so we should understand it as a non-compact field. Now clearly when $g/M \to 0$ the Lagrangian has a shift symmetry $\phi \to \phi + 2\pi f$, and $g$ can be treated as a spurion for breaking of the shift symmetry.

Below the QCD confinement scale, the coupling between $\phi$ and the gluon field strength gives rise to the familiar periodic axion potential

$$\frac{1}{32\pi^2}\frac{\phi}{f}\tilde{G}^{\mu\nu}G_{\mu\nu} \to \Lambda^4\cos(\phi/f) \tag{15}$$

For values of the Higgs vev near the Standard Model value, the height of the cosine potential is

$$\Lambda^4 \sim f_\pi^2 m_\pi^2 \sim yvf_\pi^3 \tag{16}$$

where $m_\pi^2$ changes linearly with the quark masses, and so the barrier height is linearly proportional to the Higgs vev (at least roughly speaking; there are of course logarithmic corrections from the contributions to QCD running).

Now the idea is clear: starting at values of $\phi$ such that the total Higgs mass is large and positive, and assuming the slope of the $\phi$ potential causes it to evolve in a direction that lowers the Higgs mass, the $\phi$ potential will initially be completely dominated by the $g\phi$ potential terms, until the point at which the total Higgs mass-squared goes from positive to negative and the Higgs acquires a vacuum expectation value. At this point the wiggles due to the quark masses grow linearly in the Higgs vev, and generically $\phi$ will stop when the slope of the QCD-induced wiggles matches the slope of $V(\phi)$. This classical stopping point occurs when the maximum slope of the cosine potential is of the same order as the linear tilt,

$$g \sim \frac{yvf_\pi^3}{M^2 f} \tag{17}$$

This allows for a light Higgs (i.e., a small total Higgs mass-squared and small electroweak scale) relative to a cutoff $M$ provided $g/M \ll 1$. For example, with a QCD axion decay constant $f = 10^9$ GeV and $M \sim 10^7$ GeV we have $g/M \sim 10^{-30}$.

So far we have only accounted for the parametrics of the potential, neglecting the actual dynamical process. In the minimal realization of the relaxion mechanism, $\phi$ is made to roll slowly by imagining that its evolution occurs during a period of inflation, such that Hubble friction provides efficient dissipation of kinetic energy in $\phi$.

Now there are various considerations that must be taken into account. They are:

1. In order to sensibly yield a Higgs mass much smaller than the cutoff, $\phi$ must scan over a sufficiently large range such that $m_H^2$ varies from $\mathcal{O}(M^2)$ to $\mathcal{O}(0)$. Thus the field range of interest is $\Delta\phi \sim M^2/g$. Inflation must endure for the entirety of this scanning. During $N$ e-folds of inflation, the field rolls by an amount

$$\Delta\phi \sim N\dot{\phi}/H \sim NV'_\phi/H^2 \sim NgM^2/H^2 \tag{18}$$

   where the first expression just relates the displacement to the velocity of the slow-rolling field and the duration of inflation, the second uses slow-roll conditions for $\phi$, and the third uses the leading form of $V_\phi$. Requiring that this cover a change of order $M^2/g$ implies the number of e-folds of inflation is at least

$$N \gtrsim \frac{H^2}{g^2} \tag{19}$$

2. The scanning of $\phi$ results in a change in vacuum energy of order $M^4$. We require the vacuum energy during inflation to exceed this change so that the dynamics is dominated by inflation throughout the evolution of $\phi$. This amounts to requiring

$$H > \frac{M^2}{M_{Pl}} \tag{20}$$

3. During the inflationary epoch, the evolution of $\phi$ involves both classical rolling and quantum fluctuations. If this were not the case, different patches of the universe could end up in different electroweak vacua. Classical rolling beats quantum fluctuations.

$$H < \frac{V'_\phi}{H^2} \Rightarrow H < (gM^2)^{1/3} \tag{21}$$

4. Finally, it should be the case that the barriers from QCD are higher than the Hubble scale during inflation, so that the barriers are sufficient to stop scanning. This amounts to

$$H < \Lambda_{QCD} \tag{22}$$

which in general is superseded by the previous requirement.

Putting everything together, we can see that the cutoff of the theory is at most

$$M \lesssim \left(\frac{\Lambda^4 M_{Pl}^3}{f}\right)^{1/6} \sim 10^7 \text{ GeV} \times \left(\frac{10^9 \text{ GeV}}{f}\right)^{1/6} \tag{23}$$

It is worth pausing to work out the numerical consequences. Maximizing the cutoff, we have $g \simeq 10^{-23}$ GeV, so $H < 1$ MeV, $N = 10^{40}$, and the field range is $\Delta\phi = 10^{47}$ GeV. While the first two problems are aesthetic in nature, the third is more severe. It requires the relaxion potential to be valid over field ranges vastly in excess of the Planck scale. In general it is difficult to protect a potential over trans-Planckian field ranges, and – as we will discuss more shortly – particularly so in this case.

Unfortunately, even if all of these criteria are satisfied, there is an observational problem with this simplest scenario. The field $\phi$ stops not at the minimum of the QCD cosine potential (for which the effective $\theta$ angle is zero), but is rather displaced by an amount proportional to the slope of $\phi$. This amounts to $\theta \sim 1$, which is excluded by bounds on the neutron EDM that constrain $\theta \lesssim 10^{-11}$. So the mechanism is ruled out by a natural prediction, though it is certainly no fault of the mechanism.

A simple solution is to repeat all of the same ingredients, but make the relaxion a non-compact axion of another gauge group for which constraints on the $\theta$ parameter are weaker or nonexistent. This scenario should involve quarks of a new gauge group that are also charged under the electroweak gauge group. For example, consider adding vector-like lepton doublets $L, L^c, N, N^c$ with charges

| Field | $SU(3)_N$ | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|---|
| $L$ | $\square$ | $-$ | $\square$ | $-1/2$ |
| $L^c$ | $\overline{\square}$ | $-$ | $\square$ | $+1/2$ |
| $N$ | $\square$ | $-$ | $-$ | $+1$ |
| $N^c$ | $\overline{\square}$ | $-$ | $-$ | $+1$ |

This model is now subject to a variety of additional constraints, namely

1. The quarks of the new gauge group must get most of their mass from the Higgs:
$$\mathcal{L} \supset m_L L L^c + m_N N N^c + y H L N^c + y' H^\dagger L^c N \tag{24}$$
2. The new gauge group must confine with light flavor,
$$\Lambda^4 \simeq 4\pi f_{\pi'}^3 m_N \tag{25}$$
3. The natural size of the smallest mass from the see-saw (assuming a heavy $L$) is
$$m_N \geq y y' v^2 / m_L \tag{26}$$
4. The see-saw mass is at least as large as the radiative Dirac mass
$$m_N \geq \frac{y y'}{16\pi^2} m_L \log(M/m_L) \tag{27}$$
5. The wiggles in the potential due to EWSB exceed the wiggles due to confinement alone
$$m_N \geq y y' f_{\pi'}^2 / m_L \tag{28}$$

Taken together, these bounds imply $f_{\pi'} < v$ and
$$m_L < \frac{4\pi v}{\sqrt{\log(M/m_L)}} \tag{29}$$
That is to say, although the mechanism lives in a sector distinct from the Standard Model, the scale of new physics still lies near the weak scale.

The details of the inflationary scenario are similar, though now the axion is not a QCD axion so the constraints on the PQ scale are not as stringent. Taking it to be of the same order a the cutoff, the cutoff in this case is pushed to
$$M < 2 \times 10^8\,\mathrm{GeV} \left(\frac{f_{\pi'}}{30\,\mathrm{GeV}}\right)^{4/7} \left(\frac{M}{f}\right)^{1/7} \tag{30}$$

**Caveats** At this point it is worth mentioning several caveats to these scenarios that may compromise or spoil the mechanism. To be clear, the mechanism is brilliant, and the problems are modest compared to the originality of the mechanism. In any event, the first caveat relates to the cosmological constant. In symmetry solutions to the hierarchy problem, one can effectively factorize the solution to the CC problem from the solution to the hierarchy problem – because there is one value for the weak scale, one tuning (or other mechanism) can then set the CC to the observed value. In the relaxion scenario, the cosmological constant changes by large amounts from minimum to minimum over which the changes to the Higgs mass are negligible. From one minimum to another, we have $\Delta\phi \sim f$ and thus $\Delta V \sim gfM^2 \sim \Lambda^4$, while the change in the Higgs mass-squared is infinitesimally small, $\Delta m_H^2 \sim gf \sim \Lambda^4/M^2$. So while there are many vacua with Higgs masses-squared at the electroweak scale, the changes in the cosmological constant from vacuum to vacuum are all vastly larger than the observed cosmological constant.

From this it is tempting to argue that one needs enough vacua to scan the full range of the CC for each viable electroweak minimum. This would require an even larger tuning than one would require to tune the CC in a theory with a unique electroweak vacuum. On the other hand, these arguments are not necessarily well-defined.

The second issue relates to the technical naturalness of the scenario, or the lack thereof. As constructed, $\phi$ possesses a non-compact shift symmetry. While the parameter $g$ breaks the shift symmetry, when $g \to 0$ no compact global symmetry is restored. Thus the theory does not satisfy the typical considerations of technical naturalness.

Let's pull this apart a bit more. If we have a theory with an exact global symmetry that is spontaneously broken, then the effective action of the theory has a continuous symmetry under which

$$\phi \to \phi + \alpha f \tag{31}$$

for any real $\alpha$. But a subgroup of this is gauged, in the sense that

$$\phi \to \phi + 2\pi k f \tag{32}$$

is a gauge symmetry for $k \in \mathbb{Z}$ because $\phi$ is really an angle, and no local operator can break the angular periodicity. In practice, in QCD this means that quark masses and anomaly couplings break the continuous shift symmetry but preserve the discrete one.

Concretely, one can think of a $U(1)$ global symmetry spontaneously broken by the vev of a complex scalar $\Phi$, expressed via a non-linear mapping $\Phi \to \rho e^{i\phi/f}$. This parameterization has a clear invariance under $\phi \to \phi + 2\pi k f$ in the sense that it maps $\Phi$ back to itself. Explicit breaking of the symmetry in terms of operators involving $\Phi$ will still have this invariance purely from the mapping between $\Phi$ and $\phi$.

As far as the relaxion goes, the coupling $g$ breaks both the global symmetry and the gauge symmetry. If the relaxion were to be a genuine goldstone (or, more restrictively, expressly the QCD axion), then the potential – and the Higgs mass – would need to be a periodic in $2\pi f$ if it's to come from a local QFT. Since the theory requires a non-periodic field excursion of order $\phi \sim M^2/g$, this would imply $f > M^2/g$. This ultimately forces the cutoff of the theory to live down at the weak scale, giving no parametric improvement [6].

Thus we are forced to conclude that the relaxion is not an axion, and the shift symmetry does not arise from a compact global symmetry. So what if the relaxion is *not* an axion? In this case, there is no compact global symmetry, and no mechanism to protect the shift symmetry over field excursions beyond the Planck scale. One expects quantum gravity effects to alter the picture significantly, preventing the large field excursions required for the mechanism to operate. There have been attempts to model-build a relaxion from multiple compact fields arranged to give larger effective periods, but it is not obvious that these attempts are successful.

### 1.2.2 Interactive Relaxion

Given the challenges facing the original relaxion mechanism, it is worth asking if there are other mechanisms that might work along similar lines. Indeed, there are several, of which one is worth briefly sketching here. Whereas the initial realization of the relaxion has an omnipresent source of dissipation and a potential that turns on near $m_H^2 = 0$, this alternative has an omnipresent potential and a source of dissipation that turns on near $m_H^2 = 0$ [7].

The basic idea is to start with a relaxion of the familiar form, for an Abelian Higgs toy model

$$\delta\mathcal{L} = (-M^2 + g\phi)|H|^2 + V(g\phi) + \Lambda^4 \cos\frac{\phi}{f'} + \frac{\phi}{4f} F\tilde{F} \tag{33}$$

where the cosine potential is an axion potential generated from the confinement of some non-SM gauge group – so that $\Lambda$ is not related to the Higgs vev – and $F\tilde{F}$ is some abelian gauge group (we'll get to the SM version momentarily). The coupling between the relaxion and the $U(1)$ gauge field will be a source of particle production, which will provide dissipation.

The essential idea is for $\phi$ to start at some large field value, $\phi \sim M^2/g$ with some nonzero velocity, and from the direction in which the Higgs mass-squared is large and negative. In this case, the abelian gauge group is Higgsed, with correspondingly large masses. For $\dot{\phi} \gtrsim \Lambda^2$, $\phi$ then rolls down its potential without slowing on the cosine bumps, such that the Higgs mass-squared decreases in magnitude. Eventually, the vev becomes small enough that $\phi$ can dissipate kinetic energy through production of gauge bosons.

For simplicity, we will consider the process at zero temperature. The equations of motion for the transverse modes of the gauge field $A$ – call them $A_\pm$ – in unitary gauge ($\partial_\mu A^\mu = 0$) are

$$\ddot{A}_\pm + \left(k^2 + m_A^2 \pm \frac{k\dot{\phi}}{f}\right) A_\pm = 0 \tag{34}$$

Neglecting backreaction on $\phi$, and treating $\dot{\phi}$ as constant, the solutions are

$$A_\pm(k) \propto e^{i\omega_\pm t} \tag{35}$$

$$\omega_\pm^2 = k^2 + m_A^2 \pm \frac{k\dot{\phi}}{f} \tag{36}$$

There is a tachyonic growing mode for imaginary frequencies, corresponding to

$$\omega_\pm^2 = k^2 + m_A^2 \pm \frac{k\dot{\phi}}{f} < 0 \Rightarrow |\dot{\phi}| \gtrsim 2 f m_A \tag{37}$$

The tachyonically growing mode drains the kinetic energy of $\phi$ exponentially quickly, as the growing mode backreacts on $\phi$.

Of course, for a fully accurate picture the analysis must be repeated at finite temperature. While the qualitative picture persists, some subtleties arise, including the fact that exponential growth only occurs for abelian gauge fields at finite temperature.

To implement this mechanism in the Standard Model, $\phi$ must couple to a linear combination of electroweak gauge bosons, but cannot couple to pairs of photons. If it coupled to pairs of photons, it could dissipate energy into photon pair production irrespective of the value of the Higgs vev. Rather, we require it to dissipate energy only to gauge fields acquiring mass through the Higgs mechanism. The natural candidate is thus

$$\mathcal{L} \supset \frac{\phi}{f}\left(\alpha_Y B\tilde{B} - \alpha_2 W\tilde{W}\right) \tag{38}$$

where this linear combination contains all appropriate pairs of electroweak gauge bosons except $\gamma\gamma$ Such a coupling might look like a fine-tuning, but can be protected in a UV model for the axion where the SM electroweak group is embedded in an $SU(2)_L \times SU(2)_R$ gauge theory. In such a theory there is a PQ symmetry under which

$$\phi \to \phi + \alpha \tag{39}$$

$$\theta_L \to \theta_L - \alpha \tag{40}$$

$$\theta_R \to \theta_R + \alpha \tag{41}$$

where $\theta_{L,R}$ are the $\theta$ angles of $SU(2)_{L,R}$ respectively. This forces $\phi$ to couple to $W_L\tilde{W}_L - W_R\tilde{W}_R$. The combination $\gamma\tilde{\gamma}$ is invariant under the PQ symmetry, and so can only appear in the combination $\propto (\theta_L + \theta_R)\gamma\tilde{\gamma}$, i.e., cannot couple to $\phi$. In this way we can forbid the $\gamma\tilde{\gamma}$ coupling with symmetries.

There are various other subtleties in this scenario, too many to enumerate here, but hopefully we have articulated the sense in which there are multiple possible realizations of the essential relaxion mechanism.

## 1.3 NNaturalness

An alternative that proceeds from similar inspiration is to put many copies of the Standard Model in the same universe, but explain why one copy acquires the dominant energy density [8].

The idea is to envision $N$ sectors which are mutually decoupled. For simplicity, we could take it to be $N$ copies of the Standard Model, though this is not an important restriction. From copy to copy, we imagine the Higgs mass parameters are distributed in some range from $-\Lambda_H^2$ to $\Lambda_H^2$ according to some probability distribution. For a wide range of distributions, the generic expectation is that some sectors have accidentally small Higgs masses, $m_H^2 \sim \Lambda_H^2/N$. For large enough $N$, this implies that there is a sector whose electroweak scale is well below the cutoff, which we might identify with "our" Standard Model.

Reversing the argument, this implies that the cutoff of the theory should be

$$\Lambda_H \sim \sqrt{N}|m_H|$$

E.g. a cutoff of 10 TeV corresponds to $N = 10^4$, whereas a cutoff of $10^{10}$ GeV requires $N = 10^{16}$.

There is another factor in play when $N$ is large. While the naive scale of quantum gravity is $M_{Pl}$, in the presence of a large number of species the scale at which gravity becomes strongly coupled is lowered,

$$\Lambda_G^2 \sim M_{Pl}^2/N$$

You can think of this as just coming from wavefunction renormalization of the graviton by $N$ fields whose contributions are dominated by the scale $N$. This implies the effective Planck scale should be at least $M_{Pl}^2 \sim N\Lambda_G^2$. Solving the entire hierarchy problem this way would entail $N = 10^{32}$. However, this lowers the cutoff of quantum gravity to the weak scale, and gives us the usual problems associated with a low cutoff.

But we would naturally have one sector with the observed value of the weak scale and a Higgs cutoff associated with the cutoff of quantum gravity for $N = 10^{16}$, for which $\Lambda_H = \Lambda_G = 10^{10}$ GeV. Alternately, we could preserve a notion of grand unification for $N = 10^4$, for which quantum gravity grows strong at $10^{16}$ GeV, and something like supersymmetry enters at $\Lambda_H = 10$ TeV to cut off the Higgs sector.

The question, then, is to explain why this sector with "our" Standard Model is populated, while all of the other sectors are not. As with the relaxion, this is accomplished through cosmology. In a universe with many sectors, the universe is populated by whatever sectors are abundant. If all sectors had a thermal abundance, there would be an enormous contribution to the energy density of the universe, and we would not have any ability to understand why we are the sector with the smallest scales. Thus we can imagine a cosmological mechanism that preferentially reheats sectors with smaller scales.

The simplest way to accomplish this is to imagine an inflationary epoch, followed by reheating due to the decay of some reheaton. To avoid tuning, this reheaton should couple universally to all sectors. The Standard Model can be preferentially reheated (i.e., absorb most of the energy from the reheaton decays) if the branching ratio of the reheaton to each sector scales like an inverse power of the (absolute value of the) Higgs mass-squared in each sector.

The simplest example is of a scalar $\phi$ with couplings

$$\mathcal{L} \supset -a\phi \sum_i |H_i|^2 - \frac{1}{2} m^2 \phi^2 \tag{42}$$

The branching ratios of $\phi$ to each sector depend on its mass and whether or not electroweak symmetry is broken in each sector (in general, it will be broken in half and unbroken in the other half). If we imagine that $m_\phi \ll |m_H|$ in all the sectors, then we can work out the branching ratios by integrating out the Higgses and gauge bosons (when massive) in each sector.

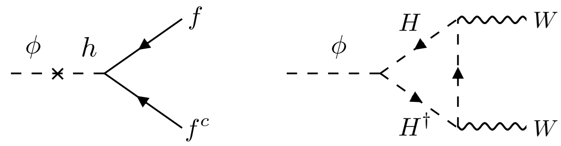


Figure 2: Dominant decays when $\langle H \rangle \neq 0$ (left) and $\langle H \rangle = 0$ (right)

For sectors where electroweak symmetry is broken, the dominant decay is into fermions, via

$$\mathcal{L} \supset ay\frac{v}{m_h^2}\phi q q^c \tag{43}$$

whereas when electroweak symmetry is unbroken the dominant decay is into gauge bosons, via

$$\mathcal{L} \supset a\frac{g^2}{16\pi^2}\frac{1}{m_H^2}\phi W_{\mu\nu}W^{\mu\nu} \tag{44}$$

Thus the decay rate into broken-phase sectors scales as $1/m_h^2$, while the decay into unbroken-phase sectors scales as $1/m_H^4$. Reheaton decays prefer a sector with broken electroweak symmetry and the smallest possible value of $m_h$.

The resulting energy density of each sector is proportional to the decay width,

$$\frac{\rho_i}{\rho_{us}} \simeq \frac{\Gamma_i}{\Gamma_{us}} \tag{45}$$

This leads to some energy density in the sectors nearest to ours in mass, with attendant predictions for dark radiation within the reach of future CMB experiments.

# 2 Rampant Speculation

## 2.1 UV/IR mixing

Let's end with an excursion into radically different territory, which marks a sharp departure from the types of solutions considered thus far. One way to frame the hierarchy problem is as a separation of UV physics from IR physics in effective field theory: the theory in the far UV knows nothing about the theory in the far IR. From this perspective, one might hope to work around the hierarchy problem by linking the far UV and the far IR. This would represent a sharp departure from effective field theory, and the challenge is to make the departure well posed.

As we have already seen during our discussion of the cosmological constant problem, we might hope to expect a theory of quantum gravity to feature UV/IR mixing. There are two ways we could try to put this to work. The first is to leverage conjectured constraints on EFT parameters, as we did in discussing the CC. The second is to look for theories where UV/IR mixing manifests itself directly. Let's consider each in turn.

#### 2.1.1 Indirect UV/IR mixing

An illustrative effort along these lines was made by Cheung & Remmen '14 [9]. They wished to make use of one conjectured constraint on EFT parameters imposed by a consistent theory of quantum gravity, namely the (electric) Weak Gravity Conjecture [10].

In its simplest form, the WGC posits that an abelian gauge theory coupled to gravity must contain a state of charge $q$ (in units of the gauge coupling $g$) and mass $m$ satisfying

$$q > \frac{m}{M_{Pl}} \tag{46}$$

which amounts to the statement that gravity is the weakest force, since this implies the gauge force between two charges exceeds the gravitational one.

There are a variety of arguments for this bound, but perhaps the simplest can be made purely from GR + charge/energy conservation + minimal assumptions about the theory of quantum gravity. Consider a black hole of charge $Q$ and mass $M$ decaying solely to some number of the charged particle in question. Charge conservation tells us

$$N_{\text{particles}} = \frac{Q}{q} \tag{47}$$

Conservation of energy requires that the rest mass of the final state be less than that of the black hole,

$$N_{\text{particles}} m = \frac{Q}{q} m < M \tag{48}$$

From these two statements we can conclude that the charge-to-mass ratio of the particle $z$ must exceed that of the black hole $Z$, which we can appropriately nondimensionalize with $M_{Pl}$:

$$z = q\frac{M_{Pl}}{m} > Z = Q\frac{M_{Pl}}{M} \tag{49}$$

An extremal black hole corresponds to $Z = 1$ and is stable unless there exists a particle for which $z > 1$. If extremal black holes are stable, then the spectrum of the theory contains a large number of stable black hole remnants, which are in tension with holographic grounds and imply various thermodynamic catastrophes.

Avoiding stability of extremal black holes implies

$$z = q\frac{M_{Pl}}{m} > 1 \Rightarrow q > \frac{m}{M_{Pl}} \tag{50}$$

as desired. Now the idea of Cheung & Remmen was to extend the Standard Model to include an unbroken $U(1)$ and some particle charged under it whose mass satisfies the WGC and is controlled by electroweak symmetry breaking. A natural candidate is gauging $U(1)_{B-L}$, which can be rendered anomaly-free by adding a right-handed neutrino $\nu_R$. Current bounds on $U(1)_{B-L}$ require $q \lesssim 10^{-24}$.

Now neutrino masses arise from a yukawa coupling to the Higgs, giving a Dirac mass

$$y_\nu v \bar{\nu}_L \nu_R + \text{h.c.} \tag{51}$$

The lightest neutrino has the largest charge-to-mass ratio. Let's say its mass is $m_\nu \sim 0.1$ eV. Then if

$$q \sim \frac{m_\nu}{M_{Pl}} \sim 10^{-29} \tag{52}$$

consistent with current bounds, then the WGC is just barely satisfied. If the values of the yukawa coupling $y_\nu$ and $U(1)_{B-L}$ coupling-times-charge $q$ are held fixed, then higher values of the Higgs vev $v$ would violate the WGC. So one could imagine that consistency of quantum gravity bounds $v$.

Of course, there are many outs – there could be lighter states charged under $U(1)_{B-L}$ that satisfy the WGC. Keeping $y_\nu$ and $q$ fixed is an arbitrary restriction, much as in the application of the atomic principle to anthropic reasoning. But it still illustrates an interesting way of entraining the weak scale to conjectured properties of quantum gravity.

Unfortunately, even taking the premises to be true, the argument itself fails due to another conjecture. So far we have discussed the electric form of the WGC, but there is also a magnetic form, which can be justified on grounds of allowing magnetically charged black holes to decay (or simply requiring that magnetically charged black holes be made of some lower-entropy particle states). The magnetic WGC posits that the cutoff $\Lambda$ of a purely electric description of an Abelian gauge theory with charged states must satisfy

$$\Lambda \lesssim q M_{Pl} \tag{53}$$

where here the cutoff could correspond to e.g. the scale of monopoles in the theory or some other breakdown of the purely electric description. This would imply the above construction breaks down at the scale of neutrino masses, and additional degrees of freedom associated with $\Lambda$ would appear well before the scale $v$. So there is no free lunch.

### 2.1.2 Direct UV/IR mixing

Another possibility is to grab the proverbial bull by the horns and look for field theories directly manifesting UV/IR mixing. Thankfully, we have a well-posed example in the guise of quantum field theory on noncommutative backgrounds [11].

The starting point is to imagine a nonvanishing commutator between coordinates on $\mathbb{R}^4$,

$$[x^\mu, x^\nu] = i\Theta^{\mu\nu}$$

where $\Theta$ is a constant, real, antisymmetric noncommutativity matrix. The algebra of functions on this noncommutative space can be viewed as an algebra of ordinary functions on the usual $\mathbb{R}^4$ with the product deformed to the noncommutative, associative star product,

$$(\phi_1 \star \phi_2)(x) = e^{\frac{i}{2}\Theta^{\mu\nu}\partial_\mu^y \partial_\nu^z} \phi_1(y)\phi_2(z)\Big|_{y=z=x} \tag{54}$$

So we are studying theories whose fields are functions on ordinary $\mathbb{R}^4$ with ordinary actions, except that products of fields are replaced by the star product.

To see evidence for UV/IR mixing, it suffices to consider the appropriate generalization of $\phi^4$ theory. This is a theory with a mass gap and quadratic divergences in the commutative version. The non-commutative version is simply

$$S = \int d^4x \left( \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda \phi \star \phi \star \phi \star \phi \right) \tag{55}$$

where the star product in the quadratic pieces of the action reduces to the normal commutative products up to total derivatives; only the interactions are modified.

This amounts to modifying the Feynman rules so that the interaction vertex has an additional phase factor of the form

$$e^{-\frac{i}{2}\sum_{i<j} k_i \times k_j}$$

where $k_i$ is the momentum flowing into the vertex through the $i$th field and the "cross product" is

$$k_i \times k_j \equiv k_{i\mu}\Theta^{\mu\nu}k_{j\nu}$$

This phase factor is invariant under cyclic permutations, but not arbitrary permutations. In a Feynman diagram with fixed external legs, there are then "planar" graphs, where propagators don't cross on their way to external states, and "non-planar" graphs where propagators cross.

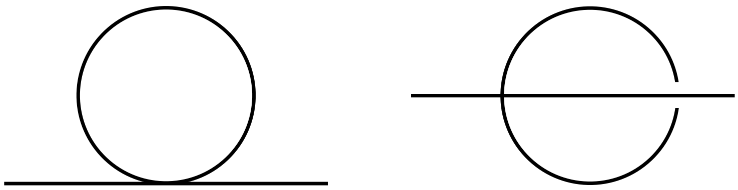

Figure 3: Planar and nonplanar diagrams in non-commutative $\phi^4$

At one loop, the two-point function receives corrections from one planar graph and one non-planar graph, shown in in Figure 3. The two diagrams give

$$\mathrm{Planar} \sim \frac{\lambda}{3}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+m^2} \tag{56}$$

$$\mathrm{Non-planar} \sim \frac{\lambda}{6}\int \frac{d^4k}{(2\pi)^4}\frac{e^{ik\times p}}{k^2+m^2} \tag{57}$$

$$\tag{58}$$

where the planar one is just the usual quadratically divergent graph, and the non-planar one picks up a phase factor from the crossing of an internal line. To see the effect of the phase factor, we can re-write the propagators in terms of Schwinger parameters

$$\frac{1}{k^2+m^2} = \int_0^\infty d\alpha e^{-\alpha(k^2+m^2)} \tag{59}$$

to get gaussian integrals

$$\text{Planar} \sim \frac{\lambda}{48\pi^2}\int \frac{d\alpha}{\alpha^2} e^{-\alpha m^2} \tag{60}$$

$$\text{Non}-\text{planar} \sim \frac{\lambda}{96\pi^2}\int \frac{d\alpha}{\alpha^2} e^{-\alpha m^2 - p\circ p/\alpha} \tag{61}$$

$$\tag{62}$$

where $p\circ q = -p_\mu \Theta^2_{\mu\nu} q_\nu$ has dimensions of $1/\text{mass}^2$. These integrals are divergent, which we regulate by multiplying the integrand by a smooth cutoff $e^{-1/(\Lambda^2\alpha)}$. Then we find the graphs give the following contributions

$$\text{Planar} \sim \frac{\lambda}{48\pi^2}\left(\Lambda^2 - m^2\log(\Lambda^2/m^2) + \dots\right) \tag{63}$$

$$\text{Non}-\text{planar} \sim \frac{\lambda}{96\pi^2}\left(\Lambda^2_{eff} - m^2\log(\Lambda^2_{eff}/m^2) + \dots\right) \tag{64}$$

$$\tag{65}$$

where

$$\Lambda^2_{eff} = \frac{1}{1/\Lambda^2 + p\circ p}$$

In this latter case, taking $\Lambda \to \infty$ gives $\Lambda_{eff} = \frac{1}{p\circ p}$. Taking $p \to 0$ then gives $\Lambda_{eff} \to \infty$. That is to say, the non-planar diagram generates an IR divergence from what we normally think of as a UV divergence. In addition, the limits $\Lambda \to \infty$ and $p \to 0$ do not commute; there is UV/IR mixing.

The 1-loop 1PI quadratic effective action now takes the form

$$\frac{1}{2}\left(p^2 + M^2 + \frac{g^2}{96\pi^2(p\circ p + 1/\Lambda^2)} + \dots\right)\phi(p)\phi(-p) \tag{66}$$

where here $M^2$ is the "renormalized" mass parameter $M^2 = m^2 + g^2\Lambda^2/48\pi^2 + \dots$. For the moment, imagine just taking $\Lambda \to \infty$. Now there are two poles in the effective action:

- The usual one at $p^2 + m^2 \simeq \mathcal{O}(g^2)$
- A new one at $p\circ p \simeq -\frac{g^2}{96\pi^2}\frac{1}{p_c^2+m^2}$

where $p_c^2$ is the projection of the momentum onto the commutative subspace. You can think of this second pole as signalling the existence of a new light particle that is, in some sense, "dual" to the high-momentum modes of $\phi$.

There is another interesting feature, which is the specific breakdown of Wilsonian EFT when the theory is considered at finite cutoff. Normally, a renormalizable Wilsonian action must have well-defined correlation functions as $\Lambda \to \infty$, and additionally correlation functions at fiite $\Lambda$ differ from their limiting values by $\mathcal{O}(1/\Lambda)$ at all values of the external momenta. This second condition is badly violated here by the cutoff-dependence in the 1PI effective action.

A sensible Wilsonian interpretation could be restored precisely by introducing a new particle with action of the form

$$\delta S = \int d^4x \left( \frac{1}{2}\partial\chi \circ \partial\chi + \frac{1}{2}\Lambda^2(\partial \circ \partial\chi)^2 + \frac{i\sqrt{\lambda}}{\sqrt{96\pi^2}}\chi\phi \right) \tag{67}$$

When integrated out, this would cancel off the $g^2/(p \circ p + 1/\Lambda^2)$ term in the effective action and replace it with a pure $g^2/p \circ p$ one, restoring the consistency of Wilsonian EFT.

But whether you think of this from the perspective of Wilsonian EFT or at infinite cutoff, there are clearly surprising departures from effective field theory, ones which suggestively hint at an approach to the electroweak hierarchy problem. Of course, this is a *long* way from solving the hierarchy problem. The field $\chi$ doesn't look anything like a standard propagating degree of freedom in Lorentzian signature, much less the higgs. But it points to a qualitatively interest direction in which to probe the hierarchy problem, one which is unlike any we have encountered before. If the hierarchy problem is solved by radically new ideas in quantum field theory, UV/IR mixing seems like a promising direction.

# 3 Conclusion

This brings us to an end. Hopefully I have conveyed the essential character of naturalness arguments as a motivator for physics beyond the Standard Model, as well as a variety of proposed solutions and their observable consequences. Although the naturalness problems of the Standard Model appear in diverse contexts and at diverse scaling dimensions, they have surprisingly much in common. So, too, do the solutions we have investigated thus far: discrete and continuous symmetries, dynamical evolution, anthropics, and UV/IR mixing each make frequent appearances. Indeed, if you're in the mood to look for new approaches, you may find the following table helpful. I have no idea how UV/IR mixing could solve the Strong CP problem, but it couldn't hurt to try.

| | Strong CP Problem | CC Problem | Hierarchy Problem |
|---|---|---|---|
| Cts. symmetry | $U(1)_{PQ}$ | SUSY | SUSY, global |
| Disc. symmetry | P/CP | $E \to -E$ | $\mathbb{Z}_2$ |
| Dynamical field | $U(1)_{PQ}$ | Abbott | Relaxion |
| Anthropics | ? | Structure formation | Atomic principle |
| UV/IR mixing | ? | Holography | WGC/NCQFT/... |

We are in an exciting time, where tests of some naturalness problems (the electroweak hierarchy problem) are at hand, and other (strong CP) are on the horizon. Current tests of the electroweak hierarchy problem make things quite interesting, given the onward march of null results. There are old solutions which are compelling but in tension with data, and new solutions which are born of necessity and take us in wildly new directions. Time considerations have prevented us from exploring the full set of new directions, and some of my favorites which you may wish to investigate further include approaches using conformal symmetry [12] and ones using disorder [13].

If nothing else, hopefully is far from clear to you that we have systematically studied all such solutions. Many new directions remain, some of which proceed along avenues sketched here, and some which have yet to be imagined. Null results in conventional channels free us to break new ground. This is where you come in!

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