Naturalness and Top-Down BSM
Lecture 3

MITP Summer School 2018

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Suggestions, clarifications, and comments are welcome.

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1 The Electroweak Hierarchy Problem

Now we turn to the electroweak hierarchy problem. There are various levels to the problem, but the essential issue is that the observed Higgs mass is some seventeen orders of magnitude smaller than the apparent cutoff of the Standard Model EFT associated with the scale of quantum gravity,

$$\frac{m_H^2}{M_{Pl}^2} \sim 10^{-34} \tag{1}$$

While this would not be a concern if the mass parameter were technically natural in the Standard Model, we are not so fortunate, and so we are faced with a striking violation of our notions of naturalness.

Of course, not all mass parameters need be problematic. Consider, for example, the mass of a Dirac fermion Ψ with a mass term of the form

$$m\bar{\Psi}\Psi \,. \tag{2}$$

As we have already discussed, this mass term is invariant under a vector-like $U(1)$ global symmetry under which $\Psi \to e^{i\alpha}\Psi$, but in the limit $m \to 0$ there is an additional symmetry, namely axial transformations of the form $\Psi \to e^{i\alpha\gamma_5}\Psi$. We could equivalently think of the symmetries in the massless limit as the two $U(1)$ symmetries of two free Weyl fermions.

The same does not in general hold for the mass terms for scalar fields. In particular, in the Standard Model the mass term

$$m^2 H^\dagger H \tag{3}$$

is in general a complete invariant under any gauge or global symmetry acting on H, and no symmetry is enhanced when the mass is zero. Thus we are without any argument to justify the stability of the Higgs mass parameter against radiative corrections. Indeed, we find in any theory with multiple mass scales that the Higgs accumulates radiative corrections from every scale with which it interacts, proportional to those scales.

Thus our naturalness expectation is that $m_H^2 \sim \Lambda^2$. This is enforced by radiative corrections: if we consider the Standard Model as an effective field theory up to some cutoff Λ, computing loop corrections from SM fields to the Higgs mass gives us the famous quadratic divergence,

$$\delta m_H^2 = \frac{\Lambda^2}{16\pi^2}\left[6\lambda + \frac{9}{4}g_2^2 + \frac{3}{4}g_Y^2 - 6y_t^2 + \dots\right] \tag{4}$$

There is a great deal of confusion about quadratic divergences and their significance, so it is worth parsing this result very carefully.

The first question is whether we need to treat the Standard Model as an EFT in the first place. In general, this is a sensible thing to do – even if it were not for the apparent cutoff imposed by strong gravity at the scale M_{Pl}, if the Standard Model were run up to arbitrarily high energies, it would hit a Landau pole in the hypercharge gauge coupling around 10^{41} GeV. More precisely, given the measured value of the hypercharge coupling at the Z pole, and the beta function

$$\frac{\partial \alpha_Y}{\partial \ln \mu} = \frac{41}{10} \frac{\alpha_Y^2}{2\pi} + \dots \tag{5}$$

the hypercharge coupling is fated to diverge around 10^{41} GeV. If this were to occur, then Standard Model fermions would form non-zero vacuum condensates in the UV, which is inconsistent with the long-range degrees of freedom in the IR. So the Standard Model is genuinely an effective field theory with cutoff Λ whether or not one is concerned about the implications of quantum gravity.

The second question is what to think of the quadratic divergence itself. We learn at an early age how to deal with divergent results in quantum field theory – we introduce counterterms and fix their coefficients according to some renormalization scheme, and then use this scheme to make finite predictions for observables at other scales. So at first glance, one might not be too troubled by the quadratic divergence. But even if one doesn't ascribe physical significance to the quadratic divergence alone, it signals the existence of sensitivity to UV physics.

From the Wilsonian perspective, the quadratic divergence is really all there is. The underlying idea is that the fundamental theory is finite, and divergences in the EFT are physical (e.g. cutoff = lattice spacing, or mass scale of particles rendering the Higgs mass finite), and counterterms just manifest fine-tuning.

A less ambitious reading, but one that is much clearer to interpret than musings about cutoffs, is that the quadratic divergence is just a placeholder for physical thresholds. The detailed relationship between the cutoff and the mass of new physical particles is a bit subtle, but as an order of magnitude relationship, it typically holds true. And, indeed, when we know what those thresholds are, we can go ahead and compute explicitly to see what's going on. To see this, it helps to construct a toy model.

1.1 A toy model

Concretely, consider as a toy model a real scalar coupled to a Dirac fermion,

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \bar{\Psi}i\not{\partial}\Psi - M\bar{\Psi}\Psi + y\phi\bar{\Psi}\Psi \tag{6}$$

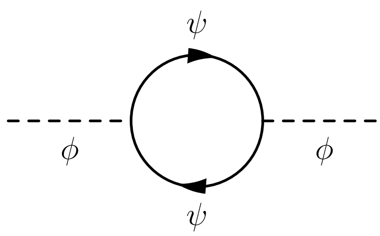
The yukawa coupling of this particular toy model breaks the continuous chiral symmetry we discussed earlier, but retains a discrete chiral symmetry under which

$$\Psi \to \gamma_5\Psi \qquad \phi \to -\phi \tag{7}$$

Under this symmetry $\bar{\Psi}\Psi \to -\bar{\Psi}\Psi$, so the fermion mass M is rendered technically natural. But there is no additional symmetry that is manifest when $m \to 0$, so we expect to see a hierarchy problem.

We would like to imagine that we keep the scalar much lighter than the fermion, and to consider matching between the full theory and an effective theory in which the fermion has been integrated out. To avoid any confusion about quadratic divergences, we will work in terms of a mass-independent renormalization scheme, dimensional regularization with minimal subtraction ($\overline{MS}$). In this scheme, the mass parameters of the theory can be thought of as Lagrangian parameters that evolve as a function of scale. We deform the theory by non-integer dimension (e.g. $d = 4 - \varepsilon$) to tame divergences, and the divergences are parameterized by $1/\varepsilon$ poles. The renormalization prescription is to choose our counterterms to cancel those poles plus some superfluous factors of 4π and γ.

We would like to carry out a matching procedure between the full theory and the effective field theory, matched at the scale M. To do so, we match the scalar two-point function in the EFT to the scalar two-point function in the full theory, at whatever order we care to compute. At one loop, the matching involves tree-level diagrams plus a one-loop diagram



which evaluates to a contribution to the scalar self-energy of the form

$$\Sigma_2(p^2) = \frac{4y^2}{16\pi^2}\left[\left(\frac{3}{\bar{\epsilon}} + 1 + 3\log(\mu^2/M^2)\right)\left(M^2 - \frac{p^2}{6}\right) + \frac{p^2}{2} - \frac{p^2}{20M^2} + \dots\right] \tag{8}$$

where $\frac{1}{\bar{\epsilon}} = \frac{1}{\epsilon} - \gamma + \log(4\pi)$. Note that there are no logarithms involving m^2 or p^2, as these diagrams match on to an EFT that contains only a free scalar field at tree level, so there are no loop diagrams that could reproduce the logarithm.

Now we renormalize by adding counterterms to cancel the $1/\bar{\epsilon}$ pole and match at the scale $\mu = M$. The matched Lagrangian in the scalar theory is thus

$$\mathcal{L} = \left(1 - \frac{4}{3}\frac{y^2}{16\pi^2}\right) \cdot \frac{1}{2}(\partial\phi)^2 - \left(m^2 - \frac{4y^2}{16\pi^2}M^2\right) \cdot \frac{1}{2}\phi^2 + \dots \tag{9}$$

where ... includes higher-derivative terms and interactions.

It's clear that the mass in the effective field theory contains a threshold correction relative to the UV theory proportional to $\frac{4y^2}{16\pi^2}M^2$. We could have also calculated the above loop diagram with a hard momentum cutoff, and found a quadratically divergent contribution to the mass-squared

$$\delta m^2 \supset \frac{3\lambda^2}{4\pi^2}\Lambda^2 \tag{10}$$

In this sense, the quadratic divergence is just a stand-in for the finite threshold corrections. If we were infinitely powerful, we could compute everything explicitly and see the finite effects. But if we are not, and are only working from the bottom up, the quadratic divegences are a handy way to estimate the effects of new physics.

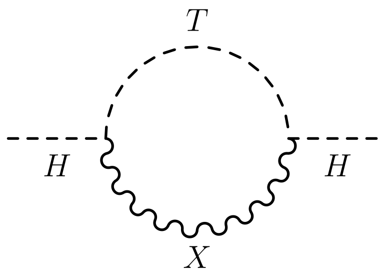
We can also see technical naturalness at play by reversing the setup, and considering a theory in which the fermion is light while the scalar is heavy. In this version, the threshold correction to the fermion mass is proportional to the fermion mass, rather than the scalar mass, a manifestation of the technical naturalness of the discrete chiral symmetry.

In any event, now we can extract the appropriate lesson from the naive quadratic divergence in the Standard Model. If physics enters to render the Higgs mass finite and calculable, then it will of course give contributions of this form. Indeed, this occurs for every theory in which the Higgs mass is rendered calculable, where the finite contributions are precisely from whatever new degrees of freedom render the Higgs mass finite. We will see such contributions in explicit examples.

But even if the physics in the far UV is mysterious and behaves differently from our expectations, it's also clear that there are finite contributions from other degrees of freedom entirely unrelated to the finiteness of the Higgs mass. For example,

Unification One of the first concrete settings in which the hierarchy problem became apparent was that of grand unification. In grand unified theories there are heavy gauge bosons associated with the scale of unification that interact with the Higgs boson.

Details depend on the precise model of unification, and the representation into which the Higgs is embedded. For example, in $SU(5)$ unification the SM gauge bosons are embedded into the 24 of $SU(5)$, which decomposes into the SM gauge bosons plus X gauge bosons transforming in the $(3,2)_{-5/6}$ + conjugate representation. Moreover, the Higgs is embedded in a $\bar{5}$ of $SU(5)$. In this case there are loops involving a triplet scalar Higgs and X boson of the form



In general, these loops of heavy bosons give corrections of order

$$\delta m_H^2 \sim \frac{\alpha_{GUT}}{4\pi} M_{GUT}^2 \tag{11}$$

The original apparent scale of unification in nonsupersymmetric theories was $\mathcal{O}(10^{15})$ GeV, while bounds on proton decay now imply $M_{GUT} \gtrsim 10^{16}$ GeV. So grand unification implies a huge hierarchy problem.

Neutrino masses Now we can have a perfectly consistent universe without new electroweak fermions, but there are scenarios that favor the existence of new fermions. For example, the generation of neutrino masses may strictly be due to a dimension-five operator,

$$\mathcal{L} \supset \frac{(L^i H)(L^j H))}{M} + \text{h.c.} \tag{12}$$

without further ado. However, we expect that if the theory is genuinely renormalizable, this interaction arose from integrating out heavier states with mass $\sim M$. In particular, the Type-I seesaw entails right-handed neutrinos N with couplings

$$\mathcal{L} \supset -\frac{M_R^{ij}}{2} N_i N_j - y_{ij} L^i N^j H + \text{h.c.} \tag{13}$$

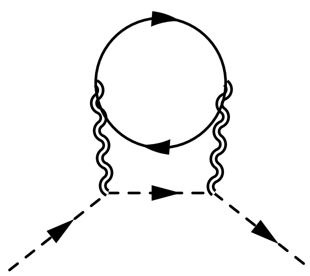
This provides a very concrete example of new fermions coupling to the Higgs. The leading one-loop correction to the Higgs mass is

$$\delta m_H^2 = -\frac{1}{4\pi^2} \sum_{ij} |y_{ij}|^2 M_j^2 \tag{14}$$

If all the RH neutrinos have a common mass M, the bound will be dominated by the combination of yukawas giving the heaviest SM neutrinos. In this case the naturalness bound is $M \lesssim 10^4$ TeV. This has amusing implications because thermal leptogenesis requires much higher values of M, on the order of $M \gtrsim 10^6$ TeV. So in this case naturalness would rule out thermal leptogenesis in a Type 1 see-saw.

Gravity Even giving up on these things, some UV completion is forced upon us. We have already encountered the physics of quantum gravity at a scale $M_P \sim 10^{19}$ GeV. Do not have a complete theory of quantum gravity, although it is likely that the answer lies in string theory. We are not yet able to compute the mass of the Higgs in a complete string theory, but the expectation is that string theory contains heavy states whose masses are close to the Planck scale that would give corrections to the Higgs mass.

It's clear that this is a problem, but we can make it even more apparent. Even new states coupling to the Higgs through loops of perturbative gravitons give a large threshold correction. For example, imagine there is some massive Dirac fermion Ψ with mass m_Ψ and it coupled to the Standard Model only gravitationally. Then as long as we are at energies $E \ll M_{Pl}$ we can compute loop diagrams including gravitons. The correction to the Higgs mass in this case arises at two loops,

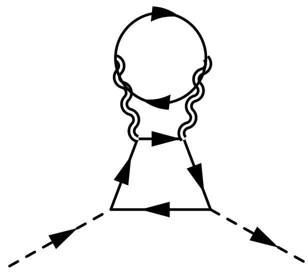


and gives a correction parametrically of order

$$\delta m_H^2 \sim \frac{m_H^2}{(16\pi^2)^2}\frac{m_\Psi^4}{M_{Pl}^4}$$

This correction is small because the graviton coupling to a massless, on-shell particle at zero momentum vanishes, and so the result is proportional to m_H.

However, we could also have a three-loop diagram where the graviton couples to a loop of top quarks,



The correction from this diagram is parametrically of the form

$$\delta m_H^2 \sim \frac{6y_t^2}{(16\pi^2)^3}\frac{m_\Psi^6}{M_{Pl}^4}$$

and is much larger because now the gravitons are coupling to off-shell states.

If $m_\Psi \sim M_{Pl}$, correction is $\sim \frac{6y_t^2}{16\pi^2}\frac{M_{Pl}^2}{(16\pi^2)^2}$. Of course at this point we doubt the validity of our gravity EFT, but this parametrically validates our naive expectation from the cutoff argument, now with $\Lambda \sim M_{Pl}/16\pi^2$. So even gravitational physics is sufficient to feed through threshold corrections to the Higgs mass.

The conclusion is that if there are *any* other states out there, even ones that only couple to the Higgs gravitationally, they give a threshold correction to the Higgs mass that is proportional to the mass scale of the new states. We can see these corrections in $\overline{MS}$ or any other scheme; they are physical threshold corrections and have unambiguous value. The result using a hard cutoff was merely a placeholder for threshold corrections, which we could only see in $\overline{MS}$ if we had actual physical states in the theory.

1.2 The naturalness strategy

Now we can convert the UV sensitivity of the Higgs mass into a strategy for new physics. We imagine that the Higgs mass is natural because the theory changes not far from the weak scale. Even without specifying the details, we can first estimate where the change must occur. Considering the "quadratic divergence",

$$\delta m_H^2 = \frac{\Lambda^2}{16\pi^2}\left(-6y_t^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda\right) \tag{15}$$

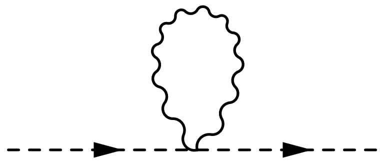
we imagine that the scale Λ is such that contributions from the cutoff are of the natural size of the Higgs mass itself. If so, this implies $\Lambda \lesssim 500$ GeV. Higher cutoffs imply contributions larger than the observed mass, and a correspondingly increased tuning – for example, tuning at the percent level would correspond to a cutoff of 5 TeV.

This is a strategy for new physics, not a necessity. Nothing fails in the field theory if the expectation is violated; it is simply difficult to understand from the perspective of naturalness. Of course, one might wonder whether this strategy is justified – after all, nature does not care much about our level of puzzlement.

It turns out that there are many instances of naturalness in nature, including naturalness at the level of mass parameters. One of my favorite is the mass splitting between the charged and neutral pions, which differ by about 5 MeV. These states are all goldstones of the spontaneously broken chiral symmetries of QCD, and these symmetry arguments lead one to expect the pions to be nearly degenerate. The answer is that we have radiative corrections from the explicit breaking of chiral symmetries by QED. The charge matrix for three flavors is

$$\begin{pmatrix} +2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}$$

This matrix breaks the chiral symmetry associated with the generators of the charged pions and kaons (i.e., it only commutes with the generators associated with the neutral pions – i.e., $[Q, T^a\pi^a] \sim f(\pi^\pm, K^\pm)$). So the charged pions and kaons can get a mass contribution from electromagnetic loops. If we compute the photon loop that would give a mass correction, using a hard cutoff to estimate the threshold correction we get



$$\sim \frac{3e^2}{16\pi^2}\Lambda^2 = \delta m^2 \tag{16}$$

Given the size of the charged-neutral meson splittings, $m_{\pi^\pm}^2 - m_{\pi_0}^2 \sim (35.5\,\text{MeV})^2$, we expect the loop should be cut off around 850 MeV if electromagnetic loops explain the mass difference. In fact, the ρ meson enters at 770 MeV, which provides a cutoff for the effective theory. Here the ρ meson is a proxy for compositeness, as it is the first QCD bound state outside of the chiral lagrangian. Thus there is perfect agreement between the size of the mass correction based on cutoff-based arguments and the scale at which new physics enters.

Another beautiful example is the mass difference between the K_L^0 and K_S^0 states. Computed in the effective theory at the scale of the kaons, the splitting is

$$\frac{M_{K_L^0} - M_{K_S^0}}{M_{K_L^0}} = \frac{G_F^2 f_K^2}{6\pi^2}\sin^2\theta_c \Lambda^2 \tag{17}$$

where $f_K = 114$ MeV is the kaon decay constant and $\sin\theta_C = 0.22$ is the Cabibbo angle. Requiring this correction to be smaller than the measured value $(M_{K_L^0} - M_{K_S^0})/M_{K_L^0} = 7\times 10^{-15}$ gives $\Lambda < 2$ GeV. And lo, the charm quark enters with mass $m_c \sim 1.2$ GeV to modify the short-distance behavior of the theory by implementing the GIM mechanism. Moreover, this is not merely rationalization; this was the actual argument used by Gaillard and Lee to compute the mass of the charm quark before its discovery.

2 Old Hierarchy Solutions

Inspecting the cartoon of the hierarchy problem, there are more or less three obvious things to try, which we can illustrate with their own cartoons, as in Figure 1.

2.1 Lowered cutoff

The first thing one is tempted to do when confronted by the hierarchy problem is to erase the apparent hierarchy itself, bringing down the cutoff of the Higgs sector or

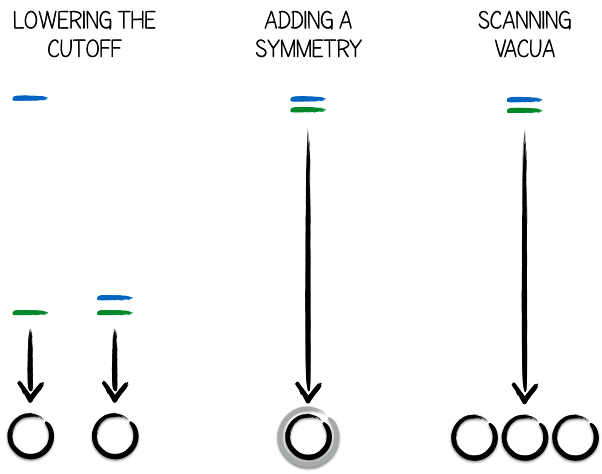


Figure 1: Ways to solve the hierarchy problem. Here the blue and green bars represent the cutoffs of the entire Standard Model and the Higgs boson, respectively. The vertical arrow indicates the large energy range separating these cutoffs from the electroweak scale, and the circle(s) denote the electroweak vacua.

the entire Standard Model. Indeed, this was the nature of the first attempted solution to the hierarchy problem, *technicolor* (due to Weinberg '76 [1] and Susskind '79 [2]), which attempted to replicate the success of the proton mass prediction by imagining that electroweak symmetry was broken by the vacuum condensate of a strongly coupled group. The five-dimensional holographic duals of technicolor are Randall-Sundrum models [3, 4], specifically ones on a finite interval with branes at either end. In these cases, the Higgs is not an elementary degree of freedom, and the cutoff is provided by compositeness of the Higgs itself.

Alternately, we could imagine leaving the Higgs alone and lowering the scale of quantum gravity, so that all field theoretic physics reaches an end at the cutoff. This is the nature of solutions such as large extra dimensions [5, 6].

The problem with pure lowered-cutoff solutions is that they generically do not allow a small bare mass term for the Higgs. That is to say, the natural expectation of the Higgs mass is of order

$$m_H^2 = c\Lambda^2 + \delta m_H^2 \tag{18}$$

As such, the cutoff of the theory must be close to the Higgs mass, rather than parametrically separated. Such theories then predict a host of particles near in mass to the Higgs, as well as a host of higher-dimensional operators suppressed by a low cutoff. The nonobservation of new particles close in mass to the Higgs, as well as strong bounds on dimension-6 operators, suggests that this mechanism is not operative on its own. This brings us to...

2.2 Symmetries

The idea behind symmetry solutions is to enlarge the Standard Model so that the Higgs mass becomes a technically natural parameter. What possible symmetries can we use? Coleman-Mandula theorem constrains options in four dimensions:

The Coleman-Mandula theorem (1967): *in a theory with non-trivial interactions (scattering) in more than 1+1 dimensions, the only possible conserved quantities that transform as tensors under the Lorentz group are the energy-momentum vector P_μ, the generators of Lorentz transformations $M_{\mu\nu}$, and possible scalar symmetry charges Z_i corresponding to internal symmetries, which commute with both P_μ and $M_{\mu\nu}$.* For theories with only massless particles, this can be extended to include generators of conformal transformations.

The Coleman-Mandula theorem can be generalized to include spinorial symmetry charges, giving rise to supersymmetry. First identified by Golfand and Likhtman, the full set of possible generalizations were identified by Haag, Sohnius, and Lopuszanski.

So possible options seem to be: Spinorial internal symmetry (supersymmetry); scalar internal symmetry (global symmetry); and potentially conformal symmetry.

2.2.1 Supersymmetry

Here I will assume you have some familiarity with SUSY, and focus on the essential aspects for the hierarchy problem. The idea is to extend Poincare algebra to include conserved supercharges minimally four supercharges $Q_\alpha, \tilde{Q}_{\dot{\alpha}}$ in four dimensions. As a Weyl spinor, the transformation properties of Q_α with respect to the Poincare group are known, namely

$$
\begin{aligned}
[P_\mu, Q_\alpha] &= [P_\mu, \tilde{Q}^{\dot{\alpha}}] = 0 \\
[M^{\mu\nu}, Q_\alpha] &= i(\sigma^{\mu\nu})_\alpha^\beta Q_\beta \\
[M^{\mu\nu}, Q^{\dot{\alpha}}] &= i(\bar{\sigma}^{\mu\nu})_{\dot{\beta}}^{\dot{\alpha}} \tilde{Q}^{\dot{\beta}}
\end{aligned}
$$

We also need anticommutators $\{Q, \tilde{Q}\}$ and $\{Q, Q\}$ to close the algebra. The only option is for $\{Q, \tilde{Q}\}$ to be proportional to $P_{\alpha\dot{\beta}}$, since this is the only conserved operator with the appropriate index structure. The choice of normalization gives us

$$\{Q_\alpha, \tilde{Q}_{\dot{\beta}}\} = 2P_\mu(\sigma^\mu)_{\alpha\dot{\beta}}$$

Finally, the consistent choice for $\{Q_\alpha, Q_\beta\}$ is

$$\{Q_\alpha, Q_\beta\} = \{\tilde{Q}_{\dot{\alpha}}, \tilde{Q}_{\dot{\beta}}\} = 0$$

though of course nonzero values of the anti-commutator are possible given a larger number of supercharges.

Fields will be arranged into supermultiplets, transforming as irreducible representations of super-Poincare. For example, the chiral multiplet contains scalar and fermion related by infinitesimal SUSY rotation,

$$\phi \to \phi + \delta\phi \qquad \psi \to \psi + \delta\psi$$

where

$$\delta\phi = \epsilon^\alpha \psi_\alpha \tag{19}$$

$$\delta\psi_\alpha = -i(\sigma^\nu \epsilon^\dagger)_\alpha \partial_\nu \phi \tag{20}$$

where ϵ_α is a Grassmann variable that you can think of as an infinitestimal parameter multiplying a SUSY generator; it has mass dimension $[\epsilon] = -1/2$.

We see clearly that supersymmetry relates a scalar to a fermion, and so relates a scalar mass to a fermion mass protected by chiral symmetry. This already suggests the sense in which supersymmetry will solve the hierarchy problem: by making the mass of a scalar (the Higgs) proportional to that of a fermion (the appropriately defined superpartner thereof).

The salient properties of supermultiplets are straightforward to work out:

1. Computing the expectation value of the Witten index within a supermultiplet, we have tr $\left[(-1)^{N_f}\right] = 0 \to n_F = n_B$, i.e., supermultiplets contain the same number of bosonic and fermionic degrees of freedom.

2. From $[P^2, Q_\alpha] = [P^2, \tilde{Q}_{\dot{\alpha}}] = 0$ we see that the components of a supermultiplet all have the same mass.

3. There is at most one $U(1)$ global symmetry that does not commute with the supercharges,
$$[R, Q_\alpha] = -Q_\alpha \qquad [R, \tilde{Q}_{\dot\alpha}] = \tilde{Q}_{\dot\alpha} \tag{21}$$
which implies that components of a supermultiplet have the same gauge and global quantum numbers apart from their $U(1)_R$ charges.

The supersymmetric extension of the Standard Model is fairly straightforward, entailing the incorporation of all Standard Model fields into corresponding supermultiplets, with the addition of a second Higgs multiplet. This is necessary on account of both anomalies and holomorphy.

Names		spin 0	spin 1/2	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \;\; \widetilde{d}_L)$	$(u_L \;\; d_L)$	$(\,\mathbf{3},\, \mathbf{2}\,,\, \frac{1}{6})$
($\times$3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\, \mathbf{1},\, -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\, \mathbf{1},\, \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu} \;\; \widetilde{e}_L)$	$(\nu \;\; e_L)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, -\frac{1}{2})$
($\times$3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\, \mathbf{1},\, 1)$
Higgs, higgsinos	H_u	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, +\frac{1}{2})$
	H_d	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, -\frac{1}{2})$

Names	spin 1/2	spin 1	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\,\mathbf{8},\, \mathbf{1}\,,\, 0)$
winos, W bosons	$\widetilde{W}^\pm \;\; \widetilde{W}^0$	$W^\pm \;\; W^0$	$(\,\mathbf{1},\, \mathbf{3}\,,\, 0)$
bino, B boson	$\widetilde{B}^0$	B^0	$(\,\mathbf{1},\, \mathbf{1}\,,\, 0)$

Of course, supersymmetry cannot be an exact symmetry of nature, otherwise we would have seen selectrons degenerate with electrons. So in general we must include soft terms, which can be worked out using the appropriate generalization of spurion techniques to superfields; in the case of the MSSM these take the form

$$\begin{aligned}
\mathcal{L}_{\rm soft}^{\rm MSSM} \;=\;& -\frac{1}{2}\Big(M_3 \widetilde{g}\widetilde{g} + M_2 \widetilde{W}\widetilde{W} + M_1 \widetilde{B}\widetilde{B} + \text{h.c.}\Big) \\
& - \Big(\widetilde{\overline{u}}\, \mathbf{a_u}\, \widetilde{Q} H_u - \widetilde{\overline{d}}\, \mathbf{a_d}\, \widetilde{Q} H_d - \widetilde{\overline{e}}\, \mathbf{a_e}\, \widetilde{L} H_d + \text{c.c.}\Big) \\
& - \widetilde{Q}^\dagger\, \mathbf{m^2_Q}\, \widetilde{Q} - \widetilde{L}^\dagger\, \mathbf{m^2_L}\, \widetilde{L} - \widetilde{\overline{u}}\, \mathbf{m^2_{\overline{u}}}\, \widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\, \mathbf{m^2_{\overline{d}}}\, \widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\, \mathbf{m^2_{\overline{e}}}\, \widetilde{\overline{e}}^\dagger \\
& - m^2_{H_u} H_u^* H_u - m^2_{H_d} H_d^* H_d - (b H_u H_d + \text{c.c.})\,.
\end{aligned} \tag{22}$$

It is straightforward to check that when supersymmetry is broken by these dimensionful soft terms, corrections due to breaking are proportional to these terms.

Now the Higgs mass is calculable by the introduction of supersymmetry. There are lots of ways to see it, but perhaps the simplest is constructive: we know particle content of MSSM, and can again work from an effective field theory perspective where we allow unknown new physics at a cutoff scale Λ. We assume new physics at the cutoff respects the symmetry, and then can compute loops up to cutoff as way of parameterizing our ignorance.

Relative to SM, there are now cancellations between loops of opposite statistics, e.g. top-stop loop

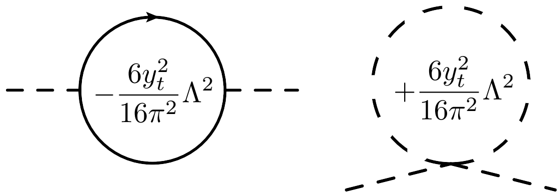


Figure 2: Quadratic divergence cancellation in the top sector of the MSSM.

Carrying out the calculation, we find

$$\delta m_{H_u}^2 = -\frac{6y_t^2}{16\pi^2}\Lambda^2 + \frac{6y_t^2}{16\pi^2}\Lambda^2 - \frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln\left(\Lambda/m_{\tilde{t}}\right) + \ldots \tag{23}$$

The quadratic pieces cancel. There is no longer UV sensitivity! The key assumption is that Λ is same for both loops, true for UV physics respecting supersymmetry. Obviously if supersymmetry were broken by a large amount in another sector, this would spoil the cancellation. In addition to the elimination of UV sensitivity, we are left only with physical threshold corrections (which we can compute in any scheme) from hew heavy states. At most there is logarithmic sensitivity to the cutoff Λ, and even this can be fixed by writing down explicit theory to break SUSY.

Now that mass is finite, can use naturalness argument to determine where the new particles should enter. Now we see the hierarchy problem very explicitly. We have rendered the Higgs mass calculable; now depends on masses of new partner particles, which cannot be too large without increasing fine-tuning.

There are two direct sources of concern, corresponding to tree-level contributions and loop-level contributions. Both play a role primarily through the relation between the weak scale and soft parameters, viz.

$$m_h^2 = -2(m_{H_u}^2 + |\mu|^2) + \ldots \tag{24}$$

Then corrections to Higgs mass come from three places:

- The first is the tree-level potential, which involves certain combinations of soft masses that set the weak scale vev. At tree-level the naturalness of the weak scale implies something about the soft parameters $m_{H_u}^2$ and μ, which itself controls the higgsino masses. Higgsinos should be light! Naturalness suggests $\mu \lesssim 200$ GeV and correspondingly light Higgsinos.

- The second is immediate loop-level corrections. The soft mass parameter $m_{H_u}^2$ accumulates one-loop corrections from other soft parameters. By far the largest is due to the stops, since the top chiral superfields couple most strongly to H_u, with correction of order

$$\delta m_{H_u}^2 = -\frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln\left(\Lambda/m_{\tilde{t}}\right) \tag{25}$$

  Naturalness requires stops ~ 400 GeV with a cutoff $\Lambda \sim 10$ TeV. Other particles are also tied to naturalness, though less directly. After the SM top loop, the gauge and Higgs loops drive the mass corrections, so unsurprisingly the wino and higgsino corrections play a role, with

$$\delta m_{H_u}^2 = -\frac{3g^2}{8\pi^2}(m_{\tilde{W}}^2 + m_{\tilde{h}}^2) \ln\left(\Lambda/m_{\tilde{W}}\right) \tag{26}$$

  Having already bounded Higgsinos, for winos this translates to $m_{\tilde{W}} \lesssim$ TeV. Note that sbottoms need not be directly connected to naturalness, but since the left-handed sbottom gauge eigenstate transforms in the same $SU(2)$ multiplet as the left-handed stop gauge eigenstate, at least one sbottom is typically found in the same mass range as the stops.

- The third is two-loop corrections, due to the naturalness of other sparticles. The stop mass is corrected by the gluino mass due to the size of g_3, so it is hard to separate the gluino substantially from the stops, with

$$\delta m_{\tilde{t}}^2 = \frac{2g_s^2}{3\pi^2} m_{\tilde{g}}^2 \ln\left(\Lambda/m_{\tilde{g}}\right) \tag{27}$$

  which ties $m_{\tilde{g}} \lesssim 2m_{\tilde{t}}$. Indeed, these corrections typically tie the masses of the gluino and all squark flavors quite tightly given even a modest amount of running.

The problem, of course, is that we have yet to observe any evidence for supersymmetry, with bounds exceeding the TeV scale. The implication is that tuning is approaching the percent level in supersymmetric scenarios.

2.2.2 Global symmetry

Let's now turn to the alternate symmetry possibility. The general idea is that the Higgs will be a (pseudo-) goldstone boson of a spontaneously broken global symmetry, which will render the Higgs mass technically natural.

While there are many possible global symmetry structures that lead to the Standard Model at low energies, for simplicity we will focus on a simple example. If we would like to see how a spontaneously broken global symmetry might protect the Higgs, it suffices to consider the simple toy model of $SU(3) \to SU(2)$, where the breaking comes about via the vev of some fundamental scalar ϕ. In this case there are 5 real scalar goldstones.

We can expand ϕ in terms of the goldstones via

$$\phi = e^{i\pi/f}\phi_0 = \exp\left[\frac{i}{f}\left(\begin{array}{cc|c} -\eta/2 & 0 & H_1 \\ 0 & -\eta/2 & H_2 \\ \hline H_1^* & H_2^* & \eta \end{array}\right)\right]\left(\begin{array}{c} 0 \\ 0 \\ f \end{array}\right) \tag{28}$$

where the H_i arrange themselves naturally to form a doublet of the unbroken $SU(2)$ and η is a real singlet scalar that we will largely ignore.

You can see that the H_i transform as doublets by doing an $SU(2)$ transformation corresponding to the unbroken generators on ϕ. In this basis, the unbroken generators correspond to

$$U_2 = \left(\begin{array}{cc} \hat{U}_2 & 0 \\ 0 & 1 \end{array}\right) \tag{29}$$

and so the transformation is

$$\phi \to U_2\phi = (U_2 e^{i\pi/f} U_2^\dagger)U_2\phi_0 = e^{\frac{i}{f}(U_2\pi U_2^\dagger)}\phi_0 \tag{30}$$

By inspection, we can see that the goldstones transform as

$$\left(\begin{array}{c|c} & \vec{H} \\ \hline \vec{H}^\dagger & \eta \end{array}\right) \to U_2 \left(\begin{array}{c|c} & \vec{H} \\ \hline \vec{H}^\dagger & \eta \end{array}\right) U_2^\dagger = \left(\begin{array}{c|c} & \hat{U}_2\vec{H} \\ \hline \vec{H}^\dagger\hat{U}_2^\dagger & \eta \end{array}\right) \tag{31}$$

as promised. Under the broken generators, the goldstones transform with a shift, at least to linear order:

$$\vec{H} \to \vec{H} + \vec{\alpha} \tag{32}$$

where $\vec{\alpha}$ is the parameter of the rotation in the direction of the broken generators.

The statement that the Higgs is a goldstone has lots of fascinating implications. If H is indeed a goldstone, it naturally inherits a series of irrelevant interactions as an expansion in f, namely

$$f^2|\partial_\mu \phi|^2 = |\partial_\mu H|^2 - \frac{H^\dagger H|\partial_\mu H|^2}{2f^2} + \ldots \tag{33}$$

This has two striking implications:

1. This immediately implies that this goldstone Higgs will have coupling deviations relative to the Standard Model, encoded by the higher-dimensional operators and unavoidable if the Higgs is a goldstone.

2. These terms also point to the cutoff of our goldstone EFT. The loop expansion parameter in this theory is of the form $\frac{1}{f^2}\frac{\Lambda^2}{16\pi^2}$, so that a well-defined loop expansion implies a cutoff $\Lambda \lesssim 4\pi f$. Physics at Λ could be strongly coupled, as in Composite Higgs models, or weakly coupled, as in SUSY UV completions of a linear sigma model.

Of course, our discussion thus far has been a little trivial, since it neglects interactions. Consider the consequences of turning on the top yukawa coupling, which takes the form

$$\mathcal{L} \supset -\lambda_t \bar{t}_R \tilde{H} Q_{3L} + \text{h.c.} \tag{34}$$

where $\tilde{H} = (i\sigma_2 H)^\dagger$ and $Q_{3L} = (t_L, b_L)$. This represents an explicit breaking of the global symmetry, and correspondingly gives rise to the usual quadratic divergence in the mass of the H. This is not surprising: the yukawas and gauge couplings of the Standard Model all violate the $SU(3)$ symmetry, and so the global symmetry offers no protection to UV physics entering through SM couplings.

Of course, this does not mean that all is lost. The global symmetry does explain why the Higgs mass is not of order $m_H^2 \sim \Lambda^2$. Assuming that Standard Model couplings are the only things that explicitly violate the global symmetry, then our notion of technical naturalness dictates that contributions to the Higgs mass coming through other Standard Model fields arise at loop level, rather than tree level. And, to a certain extent, some radiative contributions are unavoidable. After all, if the goldstone Higgs is to break electroweak symmetry, it must accumulate some quartic and quadratic terms.

However, given the lightness of the Higgs mass this is somewhat unsatisfying, as we have seen no evidence for the existence of new states beneath a TeV consistent with the requisite low cutoff. As such, it's compelling to extend the Standard Model to make the $SU(3)$ a good global symmetry, at least of the largest couplings. At the level of the top yukawa, this can be accomplished by extending the $SU(2)$ doublet quark Q_3 into a triplet of a global $SU(3)$ via $Q_{3L} \to Q'_{3L} = (\sigma_2 Q_{3L}, T_L)$ (see, e.g. [7]). In order to marry up all degrees of freedom appropriately, we can also extend the $SU(2)$-singlet quark t_R via $t_R \to t'_R + T'_R$. Now the top Yukawa can originate from an $SU(3)$ symmetric coupling in the UV of the form

$$\mathcal{L} \supset -\lambda_1 \bar{t}'_R \phi^\dagger Q'_{3L} - \lambda_2 f \bar{T}'_R T_L + \text{h.c.} \tag{35}$$

Note that this second term breaks the $SU(3)$. We need to break $SU(3)$ in order to make the new top states heavier than the top (since we don't see degenerate top partners in nature). But this breaking occurs in a way that does not talk directly to the Higgs; the Higgs will only see this in loops proportional to both λ_1 and λ_2. As we will see, the breaking is soft, as it depends only on dimensionful parameters, so this will not reintroduce a quadratic divergence, but instead a logarithmic one.

After ϕ acquires a vacuum expectation value, this leads to goldstone couplings of the form

$$\mathcal{L} = -f(\lambda_1 \bar{t}'_R + \lambda_2 \bar{T}'_R) T_L - \lambda_1 \bar{t}'_R \tilde{H} Q_{3L} + \frac{\lambda_1}{2f}(H^\dagger H)\bar{t}'_R T_L + \text{h.c.} + \ldots \tag{36}$$

The Higgs can acquire radiative corrections proportional to this soft breaking, much as in supersymmetric theories.

The approximate mass eigenstates are T_L, t_L, and the linear combinations

$$t_R = \frac{\lambda_2 t'_R - \lambda_1 T'_R}{\sqrt{\lambda_1^2 + \lambda_2^2}} \tag{37}$$

$$T_R = \frac{\hat{t}'_R + \lambda_2 T'_R}{\sqrt{\lambda_1^2 + \lambda_2^2}} \tag{38}$$

In terms of the approximate mass eigenstates, we see the emergence of what we recognize as the usual top yukawa coupling, as well as an additional yukawa involving the new singlet fermions and irrelevant operators dictated by the vacuum manifold. These take the form

$$\mathcal{L} = -\lambda_t \bar{t}_R \tilde{H} Q_{3L} - \lambda_T \bar{T}_R \tilde{H} Q_{3L} + \frac{\lambda_1^2}{m_T}(H^\dagger H)\bar{T}_R T_L + \text{h.c.} + \ldots \tag{39}$$

where $m_T = \sqrt{\lambda_1^2 + \lambda_2^2} f$ and the yukawa couplings are related to the original interactions by

$$\lambda_t = \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} \tag{40}$$

$$\lambda_T = \frac{\lambda_1^2}{\sqrt{\lambda_1^2 + \lambda_2^2}} \tag{41}$$

which satisfy $\lambda_1^2 = \lambda_t^2 + \lambda_T^2$. Having promoted the top yukawa to an $SU(3)$-symmetric form, we know that the goldstone Higgs will be protected from radiative corrections through the top yukawa. But the explicit cancellation mechanism is a bit amusing: it amounts to a cancellation between the normal corrections coming from the yukawas, and the irrelevant interaction enforced by the goldstone nature of the Higgs:

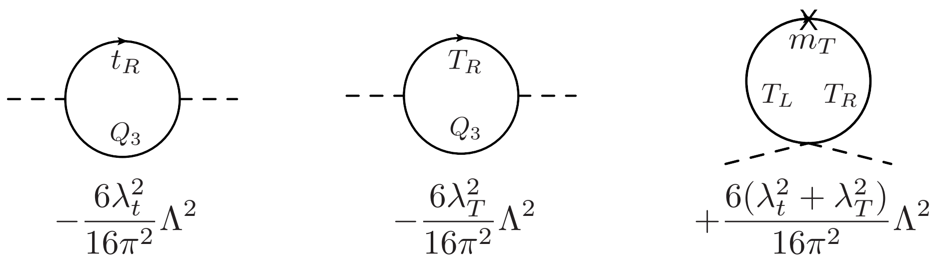


Figure 3: Quadratic divergence cancellation in the global symmetry case with light top partners.

Again the UV sensitivity is removed, and replaced with finite corrections proportional to the masses of the new states that restore the $SU(3)$ symmetry, exactly in analogy with SUSY. Of course, the non-observation of new physics then puts these models on the same footing as supersymmetry in terms of fine-tuning.

2.3 Vacuum selection

A final possibility is that nothing protects the Higgs mass, but rather there are many vacua of the Standard Model over which the Higgs mass varies according to some statistical distribution. If there is then a mechanism for selecting from the tail of the distribution with smaller Higgs masses, one has an explanation for the observed Higgs mass that does not rely on symmetries or a low cutoff.

For example, you can imagine an anthropic pressure fixing the weak scale in a universe where the dimensionful parameters of the Standard Model (i.e., the Higgs mass, or equivalently the vacuum expectation value) vary, but the dimensionless quantities are held fixed. In this case, v is bounded from above to be near its observed value by an argument known as the Atomic Principle [8].

Recall that for $v = v_{SM}$ the lightest baryons are the proton and neutron, of which the proton is lighter because the splitting due to quark masses exceeds the electromagnetic energy splitting:

$$m_n - m_p = (3v/v_{SM} - 1.7)\,\mathrm{MeV}$$

So free neutrons decay into protons, with a reaction energy

$$Q = m_n - m_p - m_e = (2.5v/v_{SM} - 1.7)\,\mathrm{MeV}$$

But in nuclei there is a binding energy that stabilizes the nuclei. The details are a bit complicated. The long-range part of the nucleon-nucleon potential is due to single pion exchange, with a range of $\sim 1/m_\pi$. For small u, d masses $m_\pi \propto ((m_u + m_d) f_\pi)^{1/2}$, so (neglecting the weak dependence of Λ_{QCD} on v) we have $m_\pi \sim v^{1/2}$.

We can mock up the binding energy in deuterons, the most weakly bound system, as a square well with a hard core to mimick short-range repulsion, which accounting for the dependence of the potential on v via m_π gives

$$B_d \simeq \left[2.2 - 5.5\left(\frac{v - v_{SM}}{v_{SM}}\right)\right]\,\mathrm{MeV}$$

for small $v - v_{SM}$.

Now we see that as we increase v, we will eventually reach the point where $B_d < Q$ and the neutron is no longer stabilized by nuclear binding energy. This occurs for

$$v/v_{SM} \gtrsim 1.2$$

which is a tight bound, indeed! The deuteron is fairly important, since all primordial and stellar nucleosynthesis begins with deuterium. But this is not an airtight bound, as nuclei could form in violent astrophysical processes. The binding energies for heavier nuclei are larger, but for

$$v/v_{SM} \gtrsim 5$$

typical nuclei no longer stabilize the neutron against decay.

Assuming that stable protons and complex atoms are required for observers to form, this provides an anthropic pressure that favors $v \lesssim v_{SM}$. But it is clear that a robust constraint only exists if dimensionless couplings are held fixed; variation of the yukawas allows these constraints to be naturally evaded.

Indeed, it is possible to imagine a "weak-less" universe where the gauge group of the Standard Model is $SU(3)_c \times U(1)_{EM}$, and fermions appear in vector-like representations. It has been argued that such a universe undergoes big-bang nucleosynthesis, matter domination, structure formation, and star formation – i.e., sufficient stages of development to produce some form of observers. Of course, truly demonstrating that such a theory is capable of reproducing the physics necessary for forming observers is beyond the scope of a handful of theorists, but suffices to indicate that anthropic reasoning applied to the weak scale is sufficiently permeable.

References

[1] S. Weinberg, "Implications of Dynamical Symmetry Breaking," *Phys. Rev.* **D13** (1976) 974–996. [Addendum: Phys. Rev.D19,1277(1979)].

[2] L. Susskind, "Dynamics of Spontaneous Symmetry Breaking in the Weinberg-Salam Theory," *Phys. Rev.* **D20** (1979) 2619–2625.

[3] L. Randall and R. Sundrum, "A Large mass hierarchy from a small extra dimension," *Phys. Rev. Lett.* **83** (1999) 3370–3373, `arXiv:hep-ph/9905221 [hep-ph]`.

[4] L. Randall and R. Sundrum, "An Alternative to compactification," *Phys. Rev. Lett.* **83** (1999) 4690–4693, `arXiv:hep-th/9906064 [hep-th]`.

[5] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, "The Hierarchy problem and new dimensions at a millimeter," *Phys. Lett.* **B429** (1998) 263–272, `arXiv:hep-ph/9803315 [hep-ph]`.

[6] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, "New dimensions at a millimeter to a Fermi and superstrings at a TeV," *Phys. Lett.* **B436** (1998) 257–263, `arXiv:hep-ph/9804398 [hep-ph]`.

[7] M. Perelstein, M. E. Peskin, and A. Pierce, "Top quarks and electroweak symmetry breaking in little Higgs models," *Phys. Rev.* **D69** (2004) 075002, `arXiv:hep-ph/0310039 [hep-ph]`.

[8] V. Agrawal, S. M. Barr, J. F. Donoghue, and D. Seckel, "The Anthropic principle and the mass scale of the standard model," *Phys. Rev.* **D57** (1998) 5480–5492, `arXiv:hep-ph/9707380 [hep-ph]`.