Naturalness and Top-Down BSM
Lecture 2

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Suggestions, clarifications, and comments are welcome.

Contents

1 Strong CP Problem, continued

Last time, we left off with the strong CP problem, namely why the CP-violating phase

$$\mathcal{L} \supset -\theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu} \tag{1}$$

is at most $\theta \sim 10^{-10}$, whereas naturalness expectations dictate it should be $\mathcal{O}(1)$.

We also noted that, thanks to the axial anomaly, we could move this term back and forth between $G\tilde{G}$ and the quark masses. That is, under

$$q \to q' = e^{i\alpha\gamma_5/2} q \tag{2}$$

$$\bar{q} \to \bar{q}' = \bar{q} e^{i\alpha\gamma_5/2} \tag{3}$$

we have

$$-\bar{q}_R M q_L + \text{h.c.} \to -\bar{q}_R M e^{i\alpha} q_L + \text{h.c.} + \alpha \frac{g_s^2}{32\pi^2} G^a \tilde{G}^a \tag{4}$$

where the first term arises because the mass term violates the axial symmetry at the classical level, and the second term is from the axial anomaly. (Here and henceforth we'll work in terms of the left- and right-handed components of the Dirac fermion, which will make things clearer when we study the Standard Model.)

We then discussed explaining the smallness of θ by expanding the Standard Model to include a symmetry that forbids it, namely a generalized P symmetry.

1.1 Axions

We then turned to the θ dependence of the QCD vacuum energy. Loosely speaking, the QCD Lagrangian in the UV for the lightest quarks has the form

$$-m_u \bar{u}_R u_L - m_d \bar{d}_R d_L + \text{h.c.} \tag{5}$$

Under an axial rotation that moves θ into the quark masses, we have

$$-m_u e^{i\theta} \bar{u}_R u_L - m_d e^{i\theta} \bar{d}_R d_L + \text{h.c} \tag{6}$$

When QCD confines, we get a bilinear expectation value for the quarks, giving a contribution to the vacuum energy. Ignoring isospin violation (i.e. taking $m_u = m_d$) and working in terms of low-energy parameters,

$$E(\theta) \sim m_\pi^2 f_\pi^2 \cos(\theta) \tag{7}$$

We then suggested that introducing a pseudoscalar field a, which we'll call an axion, that couples to $G\tilde{G}$ like θ, i.e.,

$$\delta\mathcal{L} = -\frac{g^2}{32\pi^2}\left(\theta + \frac{a}{f_a}\right)G\tilde{G} \tag{8}$$

would give a vacuum energy

$$E(a,\theta) = -m_\pi^2 f_\pi^2 \cos(\theta + a/f_a) \tag{9}$$

with a minimum at $\langle a\rangle = -\theta f_a$ where the total effective CP violating angle is set to zero, solving the strong CP problem.

This might seem like a big ask, but it really isn't. The basic idea is to make $U(1)_A$ a good classical symmetry of the action by introducing an appropriate set of Higgs scalars that give masses to quarks, though it remains anomalous. We call such a symmetry a Peccei-Quinn symmetry, $U(1)_{PQ}$, for its discoverers [1]. When the scalars acquire a vev, the $U(1)_{PQ}$ is spontaneously broken, giving rise to a goldstone boson, identified with the axion a. Just consider a toy model with a single Dirac fermion coupled to a scalar via

$$\mathcal{L} \supset -\Phi\bar{Q}_R Q_L + \text{h.c} \tag{10}$$

Now the $U(1)_A$ is a good classical symmetry provided

$$Q_L \to e^{i\alpha}Q_L \qquad Q_R \to e^{-i\alpha}Q_R \qquad \Phi \to e^{-2i\alpha}\Phi \tag{11}$$

but remains anomalous in the sense that rephasings of the Q can adjust the θ angle. Now if Φ acquires a vev, the $U(1)_A$ is spontaneously broken and there is a goldstone mode. If we focus on the goldstone, $\Phi(x) \to \frac{f_a}{\sqrt{2}}e^{ia(x)/f_a}$, then this couples as

$$\mathcal{L} \supset -\frac{f_a}{\sqrt{2}}e^{ia(x)/f_a}\bar{Q}_R Q_L + \text{h.c.} \tag{12}$$

Performing a spacetime-dependent axial rotation on the quark, $Q \to e^{ia(x)\gamma_5/2f_a}Q$, we have

$$\mathcal{L} \supset i\frac{\partial_\mu a}{2f_a}\bar{Q}\gamma^\mu\gamma_5 Q - \frac{f_a}{\sqrt{2}}\bar{Q}_R Q_L + \text{h.c.} - \frac{g^2}{32\pi^2}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \tag{13}$$

precisely as desired. Note that we also pick up derivative couplings to the fermions, consistent with the shift symmetry.

This doesn't work in the Standard Model with one Higgs doublet, because both H and H^* are involved in Yukawa couplings, so there is no $U(1)_A$ charge assignment we could make to the Higgs to restore the axial symmetry in the yukawas. So there are two things you can try:

1.1.1 Only SM quarks: Weinberg-Wilczek and DFSZ

The first is to introduce a second Higgs doublet and consider the axial symmetry under which

$$q_L \to e^{i\alpha} q_L \qquad q_R \to e^{-i\alpha} q_R \qquad H_1 \to e^{2i\alpha} H_1 \qquad H_2 \to e^{-2i\alpha} H_2 \tag{14}$$

in conventions where both H_1 and H_2 have the same SM quantum numbers. This allows us to write down Yukawa couplings and a Higgs potential of the form

$$Y_u \bar{Q}_L H_1 u_R + Y_d \bar{Q}_L H_2^* d_R + \text{h.c} + V(|H_1|^2, |H_2|^2, |H_1^* H_2|^2) \tag{15}$$

Note that this forbids certain terms in the potential allowed by the gauge symmetry, namely $H_1^* H_2 + \text{h.c.}$ or $(H_1^* H_2)^2 + \text{h.c.}$. When these Higgses acquire vevs, electroweak symmetry and $U(1)_{PQ}$ are spontaneously broken, and the axion is the goldstone

$$a \equiv \frac{1}{v}(v_1 \text{Im} H_1^0 - v_2 \text{Im} H_2^0) \tag{16}$$

This model, the Weinberg-Wilczek model [2, 3], is super predictive! It tells us

$$m_a \simeq \frac{f_\pi m_\pi}{v} \simeq 100\,\text{KeV} \tag{17}$$

and the axion decay constant is $1/v$, which is ruled out by direct searches.

Of course, the idea can be easily rescued by adding a singlet complex scalar S also transforming under $U(1)_{PQ}$ as $S \to e^{2i\alpha} S$, which allows additional potential terms that are functions of $|S|^2$ and $H_1^* H_2 S^2$. Now in general H_1, H_2 and S all acquire vevs, breaking the PQ symmetry, and the axion is

$$a \equiv \frac{1}{f}(v_1 \text{Im} H_1^0 - v_2 \text{Im} H_2^0 + v_S \text{Im} S) \tag{18}$$

where $f^2 \equiv v_1^2 + v_2^2 + v_S^2$. So if $v_S \gg v_1, v_2$, the axion is lighter and more weakly coupled, potentially beyond the reach of current searches. This is the DFSZ (Dine-Fischler-Srednicki-Zhitnisky) axion [4, 5]. Its couplings to other SM fields are fixed by the quantum numbers of SM fields transforming under $U(1)_{PQ}$.

1.1.2 New quarks: KSVZ

The other option is to outsource the anomaly to new fermions charged under QCD. Introduce a complex scalar Φ and a Dirac fermion Q in the fundamental representation of $SU(3)$, with couplings

$$\Phi \bar{Q}_R Q_L + \text{h.c} \tag{19}$$

This is just like the toy model we sketched earlier. Now Φ can be given a vev, and its modulus is the axion field, $a = f \mathrm{Arg}\Phi$. One can imagine introducing more than one extra quark, etc., giving more flexibility in the axion couplings. This is the KSVZ (Kim-Shifman-Vainstein-Zakharov) axion [6, 7], which post-dates the WW axion but pre-dates the DFSZ one.

The essential difference between the KSVZ and DFSZ models are the predictions for couplings to things other than $G\tilde{G}$. For one thing, the axial anomaly also induces couplings to photons that depend on the electromagnetic charges of the relevant fermions. In the KSVZ model, the fermions can be electromagnetically neutral, so this contribution is absent, while in the DFSZ it's fixed by the electromagnetic charges of SM fermions. However, there is also a contribution to the two-photon coupling coming from mixing between the axion and pseudoscalar mesons, so ultimately both couple. In addition, the DFSZ features derivative couplings of the axion to SM fermions.

A historical aside: as I have come to understand it from one of the authors of DFSZ, they came upon the model while working on supersymmetric techicolor, and realized that the essential features could be distilled out from the larger framework. At the time they didn't think it would be tremendously exciting, because it was not wildly different from the KSVZ model, but of course the field felt otherwise. One lesson from this is that elaborate model-building can be useful as a way of discovering simpler structures.

2 The Cosmological Constant Problem

Next we move to the other end of the dimensional spectrum, to the cosmological constant, which we typically discuss in terms of a vacuum energy density,

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}M_{Pl}^2 R - \rho_{vac} + \dots\right] \tag{20}$$

which we sometimes write as $\rho_{vac} = M_{Pl}^2 \lambda$.

The initial evidence was accumulated in 1998 from observations of distance-redshift relations for Type 1a supernovae, and further solidified by CMB meaurements. The most recent CMB result from the Planck 2015 analysis gives

$$\rho_{vac} = (2.26 \times 10^{-3}\,\mathrm{eV})^4 \tag{21}$$

(With an equation of state very near to $w = -1$, $w = -1.006 \pm 0.045$ in Planck 2015 data, for all practical purposes it's a cosmological constant rather than some other exotica, and in what follows I'll treat it as such.)

Now, what's the problem from the perspective of naturalness? Well, Lorentz invariance tells us that in the vacuum the energy-momentum tensor must take the form

$$\langle T_{\mu\nu} \rangle = -\langle \rho \rangle g_{\mu\nu} \tag{22}$$

Plugging this into Einstein's equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = \frac{1}{M_{Pl}^2} T_{\mu\nu} \tag{23}$$

tells us that vacuum energy density contributes to the effective cosmological constant,

$$\rho_{vac} = \langle \rho \rangle + M_{Pl}^2 \lambda \tag{24}$$

Purely on dimensional grounds, in a field theory with a cutoff Λ coupled to gravity we expect $\rho_{vac} \sim \Lambda^4$. Radiatively, vacuum bubbles of a field of mass m in an EFT with cutoff Λ contribute via diagrams of the form

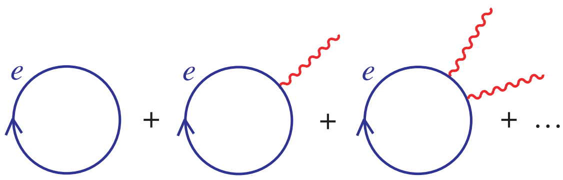


Figure 1: Vacuum loop contributions from, say, an electron, to the effective CC. Red lines are gravitons; contributions sum to $\rho_{vac}\sqrt{-g}$. From [8].

giving

$$\rho_{vac} = \int_0^{\Lambda} \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2} \simeq \frac{\Lambda^4}{16\pi^2} + \frac{m^2 \Lambda^2}{32\pi^2} + \frac{m^4 \ln(\Lambda/m)}{32\pi^2} + \ldots \tag{25}$$

In the Standard Model, if we take $\Lambda \sim M_{Pl}$, then we expect

$$\rho_{vac} \sim 10^{120} \rho_{vac,obs},$$

an enormous violation of naturalness expectations.

There are two things that might bother you about this. The first is the cutoff dependence – we learn in QFT that cutoff dependence should be absorbed by our regularization procedure. We'll discuss this more when we get to the electroweak hierarchy problem, but for now it suffices to note that we have an enormous problem even if we discard the power-law dependence on the cutoff and keep only the finite and log-dependent parts, we have contributions of $\mathcal{O}(m^4)$. Including only the top quark means we already have

$$\rho_{vac} \sim 10^{53} \rho_{vac,obs},$$

and if there are any fields of Planck-scale mass, we are back to our original estimate.

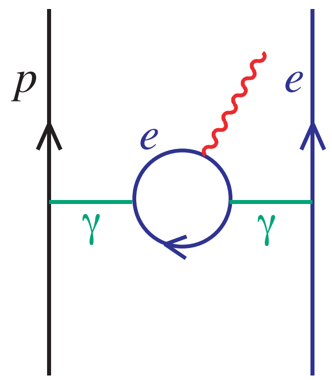


Figure 2: Vacuum loop polarization correction to the Lamb shift coupled to gravity. From [8].

The second thing you might worry about is whether we misunderstand how QFT couples to gravity – perhaps our estimate of radiative contributions to the CC is flawed on the grounds that we misunderstand how gravity couples to virtual particles. But this can't truly be so. For example, we know that a virtual electron contributes to the vacuum polarization correction to the Lamb shift, and by the equivalence principle this must couple to gravity. Loops are real.

On top of this, phase transitions contribute to the cosmological constant at tree level – as we have already seen, we should expect a contribution of order Λ_{QCD}^4 from the QCD phase transition alone, much less contributions from the electroweak phase transition or potential earlier phase transitions associated with unification or sectors other than our own. So there are classical contributions to the CC that are equally problematic.

In addition to the fact that the observed value of the CC differs dramatically from our EFT expectation, there is a second cosmological constant problem: the fact that it is remarkably close to the matter density in the universe, so that it is only now beginning to dominate. This poses an additional obstruction to models that appear capable of tackling the first CC problem.

2.1 Anthropics

The explanation that is both most popular and most controversial among high-energy physicists is related to the formation of observers. For observers to be present in order to *see* a universe with a small CC, the CC must be small enough that sufficiently large gravitationally bound systems can form. By sufficiently large, we have in mind something that forms stars and planets, which requires heavy elements – so the structures of interest are galaxies or globular clusters.

The anthropic argument for the CC is often credited to Weinberg '87 [9], but a general sketch of the argument was made by Banks in 1985 [10], and a qualitative bound along the lines of Weinberg's was made by Barrow & Tipler in '86 [11]. In any event, let us go with Weinberg's argument. Weinberg's argument in '87 was detailed, but a simpler version suffices for illustration: We known that in our universe gravitational condensation had already begun at redshift $z_c \geq 4$ (from the redshifts of the oldest quasars), when the energy density was greater than the present mass density ρ_{M_0} by a factor $(1+z_c)^3$. A CC has little effect as long as the non-vacuum energy density is larger than ρ_{vac}, so this implies

$$\rho_{vac} \leq (1+z_c)^3 \rho_{M_0} \tag{26}$$

The detailed form of the argument gives

$$\rho_{vac} \leq \frac{\pi^2}{3}(1+z_c)^3 \rho_{M_0} \simeq 410 \rho_{M_0} \tag{27}$$

We know in reality $\rho_{vac} \sim 3\rho_{M_0}$, so this bound within two orders of magnitude of the observed value. At this stage one can apply more detailed statistical reasoning to obtain a typical value closer to the observed value.

For this to be truly explanatory, we should envision a landscape of vacua over which the CC varies, all of which can be realized, but only a small number of which produce observers to witness them. Thus the landscape.

When I was first taught about the anthropic argument for the CC, I was told that (1) it was a true *prediction* and (2) that, unlike other anthropic arguments in physics (which we will soon encounter), it did not involve making assumptions about what quantities could or could not vary across an anthropic landscape. Both of these points would seem to make the anthropic argument for the CC unavoidably appealing! But neither are true.

At the time of Weinberg's anthropic argument, Loh and Spillar '86 [12] had set a limit $\rho_{vac}/\rho_{M_0} = 0.1^{-0.4}_{+0.2}$ from surveys of galaxies as a function of redshift (and other arguments for $\rho_{vac} \sim 0.7\rho_c$ were being kicked around to resolve some cosmological tensions). Weinberg's assessment of this result at the time was

> This is more than 3 orders of magnitude below the anthropic upper bound discussed earlier. If the effective cosmological constant is really this small, then we would have to conclude that the anthropic principle does not explain why it is so small.

before going on to discuss possible problems with the experimental result. Of course, we know this bound was off by an order of magnitude of the true value, but it is far from obvious that two orders of magnitude is better than three. So is the anthropic prediction of the CC a success or not? And given that Weinberg had a bound (or a range) in hand, does it count as a prediction? One could argue that Weinberg was unique in positing a mechanism for a small but nonvanishing CC, but as we will see, he was not the first.

Another loophole is that this is not a one-parameter argument. If gravitational condensation occurred at much higher redshift, the bound would be much weaker. This is possible if the amplitude of primordial density perturbations $\delta\rho/\rho \sim 10^{-5}$ were allowed to increase, which could indeed be increased by at least an order of magnitude before impacting anthropic viability, and significantly impacts the anthropic bound.

2.2 Abbott

Abbott's idea, introduced in 1985 [13], is to introduce a new confining sector coupled to an axion-like particle with a classical shift symmetry $\varphi \to \varphi + c$ (not necessarily that of a Goldstone from a compact symmetry group) and the usual coupling

$$\frac{g^2}{32\pi^2}\frac{\varphi}{f_\varphi}F^a\tilde{F}^a \tag{28}$$

Non-perturbative effects give an axion potential

$$V_1 = -\Lambda_\varphi^4 \cos(\varphi/f_\varphi) \tag{29}$$

which break the classical shift symmetry to the discrete subgroup $\varphi \to \varphi + 2\pi N f_\varphi$. We will need Λ_φ to be quite small, $\Lambda_\varphi \leq 10^{-34}$ eV, but this is not so hard to engineer by virtue of dimensional transmutation; it corresponds to $\alpha(M_{Pl}) \leq 0.006$ for a copy of QCD, within an order of magnitude of the QCD coupling at the same scale, or for an $SU(2)$ theory with six quarks, no worse than $\alpha(M_{Pl}) \leq 0.01$. The symmetry breaking scale is taken to be large, perhap $f_\varphi \sim M_{Pl}$.

In addition, a tilt is given to the cosine via a second term,

$$V_2 = \varepsilon \frac{\varphi}{2\pi f_\varphi} \tag{30}$$

where $\varepsilon < \Lambda_\varphi^4$. Here we have taken a linear perturbation, but various other deformations would also work, as long as they don't introduce additional minima over the field range we'll discuss. Since ε breaks the discrete symmetry, its smallness can be technically natural, and all radiative corrections to ε are guaranteed to be proportional to it.

Then we have

$$\rho_{vac} = -\Lambda_\varphi^4 \cos(\varphi/f_\varphi) + \varepsilon \frac{\varphi}{2\pi f_\varphi} + \ldots \tag{31}$$

The minima are at $\varphi_n \approx 2\pi n f_\varphi$ for small ε, and in these minima $\rho_{vac} \approx n\varepsilon - \Lambda_\varphi^4 + \ldots$. Now by assumption, $\varepsilon < (10^{-34}\,\mathrm{eV})^4$, so we are guaranteed there is always a minimum where the total energy density is $\sim \varepsilon$, which we can make arbitrarily small.

To account for the CC, we must explain why the universe is in one of the states with a small CC, instead of another one. If we imagine starting at some arbitrary point on the potential with large, positive CC, we are in a de Sitter spacetime and over time φ will evolve down the potential, decreasing the vacuum energy density at each step. Initially, when $\rho_{vac} > M_{Pl}^2 \Lambda_\varphi^2$ the barriers are irrelevant because of the non-zero Hawking temperature in de Sitter space, $T_H^2 = \frac{2}{3\pi} \frac{\rho_{vac}}{M_{Pl}^2}$, so the field can undergo thermal fluctuations over the barriers (and instantons generating the barriers are moreover suppressed). Eventually, we will hit

$$\rho_{vac} < M_{Pl}^2 \Lambda_\varphi^2 \leq (10^{-3}\,\mathrm{eV})^4 \tag{32}$$

(This is the reason for our Λ_φ, and hence $\varepsilon^{1/4}$, to be much smaller than ρ_{vac} – it's not the step size that matters, but the point at which the barriers switch on.) At this point the barriers become relevant, and field evolution proceeds via tunneling, i.e., bubble nucleation. For $\rho_{vac} \ll M_{Pl}^2 \Lambda_\varphi^2$ the tunneling rate per unit volume is

$$\Gamma/V \sim \Lambda_\varphi^4 e^{-\frac{3}{8} M_{Pl}^4/\rho_{vac}} \tag{33}$$

and eventually the evolution becomes quite slow.

This all takes a long time, 10^{450} years for $\rho_{vac} \sim M_{Pl}^4$ to get reduced to the observed value. However, once we get there, we remain in a series of states with acceptable CC for a far longer time, $10^{10^{248}}$ years. Eventually we tunnel to a state with small, negative vacuum energy, but this is expected to undergo gravitational collapse and the game's over. In the meantime, we have a doubly exponentially long time in a realistic vacuum.

The problem is that the universe *only* contains vacuum energy. Any initial matter density is rapidly inflated away, and any matter density generated during a tunneling event is inflated away while awaiting the next transition. The last transition to the current vacuum can't reheat above $T_{RH} \sim \varepsilon^{1/4}$, and even matter created from this is unlikely to be isotropic because the energy released by the tunneling event is primarily stored in the bubble wall. Even if you imagine raising the scales so that the step size is of order $\rho_{vac}^{1/4}$, you are still impossibly far away from getting a realistic universe. Recently, attempts have been made to develop constructions inspired by the Abbott model (e.g. Creminelli et al. '16 [14]) that solve the reheating problem by more radical means, e.g. violation of the null energy condition.

2.3 Discrete Symmetry

The idea, which originates with Linde '88 [15] but was fleshed out further by Kaplan and Sundrum '05 [16], is to introduce parity partners of all normal fields with opposite-sign Lagrangian density,

$$\mathcal{L} = \sqrt{-g}\left[M_{Pl}^2 R - \rho_{vac} + \mathcal{L}_{matter}(\psi, D_\mu) - \mathcal{L}_{matter}(\hat{\psi}, D_\mu) + \dots\right] \qquad (34)$$

where … denotes visible-ghost couplings that are taken to be small, and possibly gravitational higher-derivative terms. The radiative contributions from the normal matter sector and its wrong-sign partner to the CC cancel, leaving only the bare contribution. We can think of this as arising from a $\mathbb{Z}_2$ energy-parity symmetry P that *anticommutes* with the Hamiltonian, $\{H, P\} = 0$, so that an energy eigenstate ($H|E\rangle = E|E\rangle$) is transformed into one with opposite energy, $HP|E\rangle = -EP|E\rangle$.

The obvious problem is that a Minkowski vacuum is unstable to the pair production of positive- and negative-energy states. The idea is that if the two can be completely decoupled, this pair production process is suppressed and the Minkowski vacuum is effectively stable. If there is a Poincare-invariant state that is P invariant, $P|0\rangle = |0\rangle$, then $\langle 0|\{H, P\}|0\rangle = 2\langle 0|H|0\rangle = 0$, corresponding to vanishing CC.

At the level of fields, the transformation under energy-parity is

$$g_{\mu\nu}(x) \to g_{\mu\nu}(x) \quad (35)$$

$$\psi(x) \leftrightarrow \hat{\psi}(x) \quad (36)$$

$$H \to -H \quad (37)$$

so that the matter action respects energy-parity, but the gravitational action violates it.

Since gravity violates the parity, there will be a gravitational contribution to the CC of order Λ_{grav}, a scale corresponding to the cutoff of graviton momenta – so the scale at which a quantized EFT of Einstein gravity must break down. To reproduce the observed CC, this implies $\Lambda_{grav} \lesssim 2 \times 10^{-3}$ eV, or a length scale of ~ 100 microns, which is in tension with current short-distance tests. One should also worry about vacuum decay induced by gravitational interactions.

2.4 UV/IR Mixing

Another possibility is that there is a breakdown in effective field theory, corresponding to some mixing between UV and IR physics. This is not at all absurd: we have examples of UV/IR mixing. Perhaps the most famous is in quantum gravity. We can imagine accelerating two protons to Planckian energies and smashing them together to create Planck-length-sized black holes. You might then hope to probe distances shorter than the Planck length by increasing the energy of the two protons above the Planck energy. But when you do so, you create larger and larger black holes. More energetic protons mean more massive black holes, which have larger radii. Instead of probing shorter distances, you produce large black holes which resolve only longer distances – exciting the theory in the UV really probes the physics of the IR. A more precise version of the same thing happens with T duality, which relates string theories propagating on some circle of radius R and $1/R$.

Various ideas about UV/IR mixing and the CC have been put forward, most notably by Banks '96 [17] and Horava '97 [18]. Here I'd like to sketch an accessible proposal by Cohen, Kaplan, and Nelson '98 [19]. If there is UV/IR mixing present in the theory of quantum gravity, one might hope to put it to work by inferring long-distance properties that might be felt at lower energies. One such avenue is via entropy bounds.

Normally, an EFT in a box of size L (an IR cutoff) with UV cutoff Λ has extensive entropy, $S \sim L^3\Lambda^3$. Inspired by black hole thermodynamics, Bekenstein ('73-'93) made a series of conjectures about entropy in field theory, namely that the entropy in a box of volume L^3 only grows as the area of the box. Any EFT would violate this bound in a sufficiently large box, so if the bound is true, it implies that conventional field theories vastly overcount degrees of freedom.

One way to reconcile these would be if there is a connection imposed between the UV and IR cutoffs of an EFT by requiring it to satisfy the conjectured bound. This would mean

$$L^3\Lambda^3 \lesssim S_{BH} = \pi L^2 M_{Pl}^2 \Rightarrow L \lesssim \frac{M_{Pl}^2}{\Lambda^3} \tag{38}$$

We can actually make a more refined condition. Note that an EFT satisfying this bound contains many states with Schwarzschild radius larger than the box, which should probably not be described by a local QFT. We can exclude those by requiring the Schwarzschild radius of the maximum energy configuration (corresponding to an energy $L^3\Lambda^4$) not to exceed the size of the box, i.e.,

$$L_s \sim \frac{L^3\Lambda^4}{M_{Pl}^2} \lesssim L \Rightarrow L \lesssim \frac{M_{Pl}}{\Lambda^2} \tag{39}$$

This would imply that any EFT with a cutoff Λ has a correlated IR cutoff L.

The conjectured application to the CC is as follows: if the IR cutoff of the Standard Model (and everything else) is taken to be comparable to the current horizon size, the corresponding UV cutoff is $\Lambda \sim 10^{-2.5}$ eV, surprisingly close to the observed value of the CC. Now, this is not wholly satisfying – as effective field theorists we therefore expect to see something at the cutoff, which we do not. But it illustrates how conjectured properties of a theory of quantum gravity might be brought to bear to constrain otherwise-independent parameters of an EFT.

3 The Electroweak Hierarchy Problem

Now we turn to the electroweak hierarchy problem. There are various levels to the problem, but the essential issue is that the observed Higgs mass is some seventeen orders of magnitude smaller than the apparent cutoff of the Standard Model EFT associated with the scale of quantum gravity,

$$\frac{m_H^2}{M_{Pl}^2} \sim 10^{-34} \tag{40}$$

While this would not be a concern if the mass parameter were technically natural in the Standard Model, we are not so fortunate, and so we are faced with a striking violation of our notions of naturalness.

Of course, not all mass parameters need be problematic. Consider, for example, the mass of a Dirac fermion Ψ with a mass term of the form

$$m\bar{\Psi}\Psi\,. \tag{41}$$

As we have already discussed, this mass term is invariant under a vector-like $U(1)$ global symmetry under which $\Psi \to e^{i\alpha}\Psi$, but in the limit $m \to 0$ there is an additional symmetry, namely axial transformations of the form $\Psi \to e^{i\alpha\gamma_5}\Psi$. We could equivalently think of the symmetries in the massless limit as the two $U(1)$ symmetries of two free Weyl fermions.

Quantum corrections respect the symmetries of the quantum action, so provided that this axial symmetry is a good symmetry of the quantum theory (i.e., is not anomalous), when when $m = 0$ this implies that quantum corrections will not generate a mass term. Moreover, when the chiral symmetry is broken by $m \neq 0$, quantum corrections will be proportional to the symmetry-breaking term. Thus a large hierarchy between fermion masses is a curiosity, but not a deeply troubling one. If the fundamental theory of the universe generates fermions with very different masses, quantum corrections need not disturb the hierarchy.

The same does not in general hold for the mass terms for scalar fields. In particular, in the Standard Model the mass term

$$m^2 H^\dagger H \tag{42}$$

is in general a complete invariant under any gauge or global symmetry acting on H, and no symmetry is enhanced when the mass is zero. Thus we are without any argument to justify the stability of the Higgs mass parameter against radiative corrections. Indeed, we find in any theory with multiple mass scales that the Higgs accumulates radiative corrections from every scale with which it interacts, proportional to those scales. Unlike the case of spin-1/2 or spin-1, we do not have $\delta m^2 \propto m^2$, but rather $\delta m^2 \propto \Lambda^2$, where Λ stands for all other scales probed by the Higgs.

The hierarchy problem is often framed in the language of quadratic divergences. The idea is to consider the Standard Model as an effective field theory up to some cutoff Λ. One can infer the sensitivity of the Higgs mass parameter to the cutoff by computing one-loop radiative corrections up to the scale Λ, which give the famous quadratic divergence,

$$\delta m_H^2 = \frac{\Lambda^2}{16\pi^2}\left[6\lambda + \frac{9}{4}g_2^2 + \frac{3}{4}g_Y^2 - 6y_t^2 + \dots\right] \tag{43}$$

On top of this, one should include a bare term, so that the expectation for the Higgs mass in the Standard Model EFT is

$$m_H^2 = c\Lambda^2 + \delta m_H^2 \tag{44}$$

There is a great deal of confusion about quadratic divergences and their significance, so it is worth parsing this result very carefully.

The first question is whether we need to treat the Standard Model as an EFT in the first place. In general, this is a sensible thing to do – even if it were not for the apparent cutoff imposed by strong gravity at the scale M_{Pl}, if the Standard Model were run up to arbitrarily high energies, it would hit a Landau pole in the hypercharge gauge coupling around 10^{41} GeV. More precisely, given the measured value of the hypercharge coupling at the Z pole, and the beta function

$$\frac{\partial \alpha_Y}{\partial \ln \mu} = \frac{41}{10}\frac{\alpha_Y^2}{2\pi} + \dots \tag{45}$$

the hypercharge coupling is fated to diverge around 10^{41} GeV. If this were to occur, then Standard Model fermions would form non-zero vacuum condensates in the UV, which is inconsistent with the long-range degrees of freedom in the IR. So the Standard Model is genuinely an effective field theory with cutoff Λ whether or not one is concerned about the implications of quantum gravity.

The second question is what to think of the quadratic divergence itself. We learn at an early age how to deal with divergent results in quantum field theory – we introduce counterterms and fix their coefficients according to some renormalization scheme, and then use this scheme to make finite predictions for observables at other scales. So at first glance, one might not be too troubled by the quadratic divergence. But even if one doesn't ascribe physical significance to the quadratic divergence alone, it signals the existence of sensitivity to UV physics.

From the Wilsonian perspective, the quadratic divergence is really all there is. The underlying idea is that the fundamental theory is finite, and divergences in the EFT are physical (e.g. cutoff = lattice spacing, or mass scale of particles rendering the Higgs mass finite), and counterterms just manifest fine-tuning.

A less ambitious reading, but one that is much clearer to interpret than musings about cutoffs, is that the quadratic divergence is just a placeholder for physical thresholds. The detailed relationship between the cutoff and the mass of new physical particles is a bit subtle, but as an order of magnitude relationship, it typically holds true. And, indeed, when we know what those thresholds are, we can go ahead and compute explicitly to see what's going on. To see this, it helps to construct a toy model.

3.1 A toy model

Concretely, consider as a toy model a real scalar coupled to a Dirac fermion,

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \bar{\Psi}i\not{\partial}\Psi - M\bar{\Psi}\Psi + y\phi\bar{\Psi}\Psi \tag{46}$$

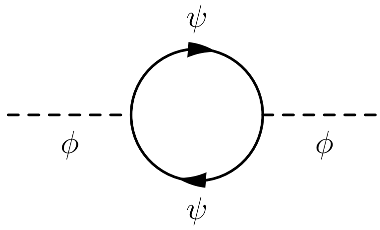
The yukawa coupling of this particular toy model breaks the continuous chiral symmetry we discussed earlier, but retains a discrete chiral symmetry under which

$$\Psi \to \gamma_5\Psi \qquad \phi \to -\phi \tag{47}$$

Under this symmetry $\bar{\Psi}\Psi \to -\bar{\Psi}\Psi$, so the fermion mass M is rendered technically natural. But there is no additional symmetry that is manifest when $m \to 0$, so we expect to see a hierarchy problem.

We would like to imagine that we keep the scalar much lighter than the fermion, and to consider matching between the full theory and an effective theory in which the fermion has been integrated out. To avoid any confusion about quadratic divergences, we will work in terms of a mass-independent renormalization scheme, dimensional regularization with minimal subtraction ($\overline{MS}$). In this scheme, the mass parameters of the theory can be thought of as Lagrangian parameters that evolve as a function of scale. We deform the theory by non-integer dimension (e.g. $d = 4 - \varepsilon$) to tame divergences, and the divergences are parameterized by $1/\varepsilon$ poles. The renormalization prescription is to choose our counterterms to cancel those poles plus some superfluous factors of 4π and γ.

We would like to carry out a matching procedure between the full theory and the effective field theory, matched at the scale M. To do so, we match the scalar two-point function in the EFT to the scalar two-point function in the full theory, at whatever order we care to compute. At one loop, the matching involves tree-level diagrams plus a one-loop diagram



which evaluates to a contribution to the scalar self-energy of the form

$$\Sigma_2(p^2) = \frac{4y^2}{16\pi^2}\left[\left(\frac{3}{\bar{\epsilon}} + 1 + 3\log(\mu^2/M^2)\right)\left(M^2 - \frac{p^2}{6}\right) + \frac{p^2}{2} - \frac{p^2}{20M^2} + \dots\right] \tag{48}$$

where $\frac{1}{\bar{\epsilon}} = \frac{1}{\epsilon} - \gamma + \log(4\pi)$. Note that there are no logarithms involving m^2 or p^2, as these diagrams match on to an EFT that contains only a free scalar field at tree level, so there are no loop diagrams that could reproduce the logarithm.

Now we renormalize by adding counterterms to cancel the $1/\bar{\epsilon}$ pole and match at the scale $\mu = M$. The matched Lagrangian in the scalar theory is thus

$$\mathcal{L} = \left(1 - \frac{4}{3}\frac{y^2}{16\pi^2}\right) \cdot \frac{1}{2}(\partial\phi)^2 - \left(m^2 - \frac{4y^2}{16\pi^2}M^2\right) \cdot \frac{1}{2}\phi^2 + \dots \tag{49}$$

where $\dots$ includes higher-derivative terms and interactions.

It's clear that the mass in the effective field theory contains a threshold correction relative to the UV theory proportional to $\frac{4y^2}{16\pi^2}M^2$. We could have also calculated the above loop diagram with a hard momentum cutoff, and found a quadratically divergent contribution to the mass-squared

$$\delta m^2 \supset \frac{3\lambda^2}{4\pi^2}\Lambda^2 \tag{50}$$

In this sense, the quadratic divergence is just a stand-in for the finite threshold corrections. If we were infinitely powerful, we could compute everything explicitly and see the finite effects. But if we are not, and are only working from the bottom up, the quadratic divegences are a handy way to estimate the effects of new physics.

We can also see technical naturalness at play by reversing the setup, and considering a theory in which the fermion is light while the scalar is heavy. In this version, the threshold correction to the fermion mass is proportional to the fermion mass, rather than the scalar mass, a manifestation of the technical naturalness of the discrete chiral symmetry.

In any event, now we can extract the appropriate lesson from the naive quadratic divergence in the Standard Model. If physics enters to render the Higgs mass finite and calculable, then it will of course give contributions of this form. Indeed, this occurs for every theory in which the Higgs mass is rendered calculable, where the finite contributions are precisely from whatever new degrees of freedom render the Higgs mass finite. We will see such contributions in explicit examples.

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