Supersymmetric Field Theories
MITP Summer School 2018
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Homework 2

Question 1 Show that with

$$\delta_\xi \phi_i = \sqrt{2}\xi^T \epsilon \psi_i \tag{1}$$
$$\delta_\xi \psi_i = \sqrt{2} i \sigma^\mu \epsilon \xi^* \partial_\mu \phi_i + \sqrt{2}\xi F_i \tag{2}$$
$$\delta_\xi F_i = -\sqrt{2} i \xi^\dagger \bar{\sigma}^\mu \partial_\mu \psi_i \quad i = +, - \tag{3}$$

the SUSY algebra closes off-shell.

Question 2 Consider a number of chiral smultiplets (ϕ_i, ψ_i, F_i) with a superpotential $W(\phi_i)$. Show that the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm int} \tag{4}$$

with

$$\mathcal{L}_{\rm kin} = \partial^\mu \phi_i^* \partial_\mu \phi_i + \psi_i^\dagger i \bar{\sigma}^\mu \partial_\mu \psi_i + F_i^* F_i \tag{5}$$
$$\mathcal{L}_{\rm int} = \frac{\partial W}{\partial \phi_i} F_i - \frac{1}{2}\frac{\partial^2 W}{\partial \phi_i \partial_j} \psi_i^T \epsilon \psi_j + \text{ h.c.} \tag{6}$$

is supersymmetric.

Question 3

a. Show that $W = m\phi_+\phi_-$ gives the first example we studied (note that if we have a $U(1)$ under which $\phi_\pm$ has charge ± 1, this is the only allowed superpotential.)

b. $W = h\phi_+\phi_-$ gives our second example.

c. $W = \phi(\phi_1^2 - f) + m\phi_1\phi_2$ gives the O'Raifeartaigh potential. Work out the spectrum at $\phi_1 = \phi_2 = 0$.

Question 4 (lower priority) Show that a general superfield satisfies

$$A'(x^\mu, \theta, \bar\theta) = A(x^\mu - i\xi\sigma^\mu\bar\theta + i\theta\sigma^\mu\bar\xi, \theta + \xi, \bar\theta + \bar\xi) \tag{7}$$

(do this just for a few components).

Question 5 For the chiral superfield

$$\Phi(x) = \phi(x) + \sqrt{2}\theta\psi(x) + i\theta\sigma^\mu\bar\theta\partial_\mu\phi(x) - \frac{i}{\sqrt{2}}\theta^2\bar\theta\sigma^\mu\partial_\mu\psi(x) + \theta^2 F(x) + \frac{1}{4}\theta^2\bar\theta^2(x) \tag{8}$$

evaluate the supersymmetric transformation

$$\delta_x i\Phi = (\xi^\alpha Q_\alpha + \bar\xi_{\dot\alpha}\bar Q^{\dot\alpha})\phi(x)\ . \tag{9}$$

Convince yourself that this reproduces the transformation we wrote before for the component fields.