Supersymmetric Field Theories
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Yael Shadmi

Homework 1

Question 1 For the free theory with one Dirac fermion and 2 complex scalars, show that supersymmetry is indeed a symmetry.

$$\delta_\xi \phi_+ = \sqrt{2}\xi^T \epsilon \psi_+ \tag{1}$$

$$\delta_\xi \psi = \sqrt{2} i \sigma^\mu \epsilon \xi^* \partial_\mu \phi_+ - \sqrt{2} m \xi \phi_-^* \tag{2}$$

(and similarly for $+ \leftrightarrow -$). Note that for $m = 0$ this splits into the $+$, $-$ parts.

Question 2 Show that

$$[\delta_\xi, \delta_\eta]\, \phi_+ = a^\mu \partial_\mu \phi_+ \ . \tag{3}$$

You will have to use the equations of motion. (You can repeat this for the fermion.)

Question 3 The O'Raifeartaigh potential should read

$$V = |\phi_1^2 - f|^2 + m^2 |\phi_1|^2 + |2\phi_1 \phi + m\phi_2|^2 \tag{4}$$

(I had a typo!) ϕ_1, ϕ_2, ϕ are scalar fields, f is a constant of dim 2, m is a constant of dim 1.

a. For $f < \frac{1}{2}m^2$, show that the minimum is at $\phi_1 = \phi_2 = 0$, and ϕ is arbitrary (this is generic for SUSY breaking models.)
b. Find V at the minimum.
c. Calculate the spectrum at the ground state.

Question 4 If you're really motivated: Take the model with

$$\text{complex scalars} \quad h,\ \phi_+,\ \phi_- \tag{5}$$

$$\text{Weyl fermions} \quad \psi_+,\ \psi_-\ \tilde{h} \tag{6}$$

and

$$\mathcal{L}_{\text{int}} = -y \left[h \psi_+^T \epsilon \psi_- + \phi_+ \tilde{h}^T \epsilon \psi_- + \phi_- \tilde{h}^T \epsilon \phi_+ + \text{ c.c.} \right] \tag{7}$$

$$- |y|^2 \left[|\phi_+|^2 |\phi_-|^2 + |h|^2 |\phi_+|^2 + |h|^2 |\phi_-|^2 \right] \ . \tag{8}$$

a. Assuming all fields are massless, calculate the 1-loop correction to the h mass. Show that the quadratic divergence cancels.
b. Repeat for h, $\tilde{h}$ massless, ψ_+, ψ_- with mass m, and the mass squareds for ϕ_+ and ϕ_- as $m^2 + \tilde{m}_1^2$ and $m^2 + \tilde{m}_2^2$, respectively.
c. Convince yourself that this is spoiled for hard breaking, *i.e.* changing some of the dimensionless couplings.