Collider Physics

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Chapt. 1: Introduction
Chapt. 2: Basic formalism
} Lect. 1

Chapt. 3: Kinematics & phase space
Chapt. 4: Particle detection @ colliders
} Lect. 2

Chapt. 5: Lepton colliders Lect. 3

Chapt. 6: Hadron colliders Lect. 4

Four 1.5-hr lectures

Approach:

- Pedagogical
- Self-contained
- Basic concepts & methods
- Avoid technicalities & specific models

References:

arXiv:hep-ph/0508097, TASI lecturer notes, Han;
arXiv:1002.0274, TASI lecture notes, Perelstein;
arXiv:0910.4182, TASI lecture notes, Plehn.

Book:

"Review of Particle Physics", PDG: Chin Phys C40 (2016);
"Collider Physics", Barger & Phillips (1987);
"The Black Book of QCD", Campbell, Huston, Krauss (2017).

Collider Physics

Chapt. 1: Introduction

"energy frontier"

§1.1: High Energy Physics Phenomenology

(A) HEP $\Longleftrightarrow$ Elementary Particle Phys.

$\Uparrow$ $\Uparrow$

the means: $E \simeq pc$ Objects: building blocks!

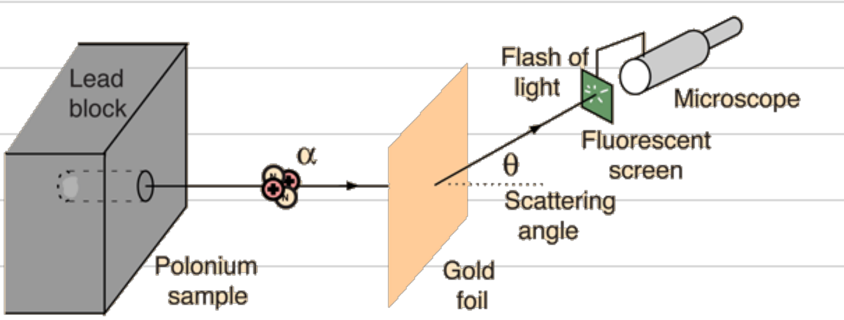
$= h\nu$ (five/classical) elements,

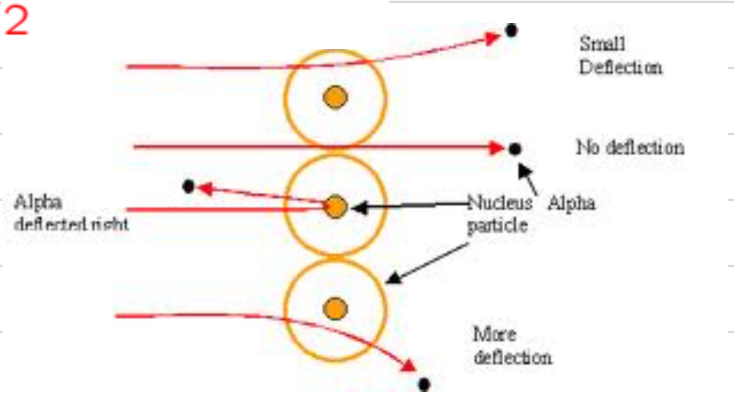
atoms ...

Till Rutherford α-Scattering $\Rightarrow$ Atom: planetary

the point-like nucleus:

$$\frac{d\sigma}{d\Omega} = \frac{(\alpha Z_1 Z_2)^2}{4E^2 \sin^4\theta/2}$$





Higher energies, shorter distances,

$E = h\nu$ deeper probe:

$\Rightarrow$ nucleons, quarks ...

Rutherford's legendary method continues in HEP!

(B) Phenomenology: a term introduced in '60s, empirical approaches to describe data.

Bridge between Theory & Experiments

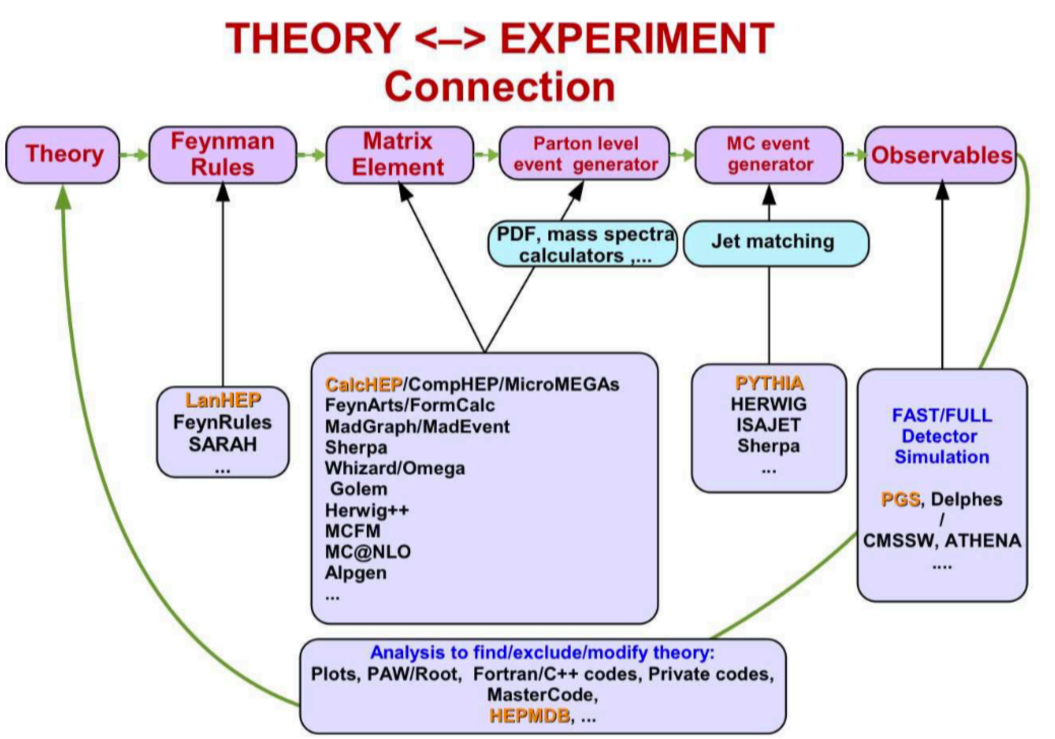
from theory ⟹ explain/predict new phenomena

from exp'ts ⟹ develop new theory.

"Phenomenologists" need to understand both theory & experiments!

(C) Methodology:

Relativistic, quantum-mechanical, guided by symmetry principles, rooted to experimental observables, much sophisticated methods/~~tools~~ developed...

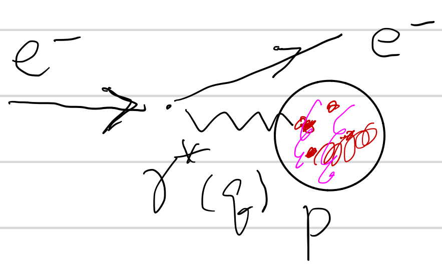


advanced, automated, flexible Codes, but blackboxes.

What's there?

Example 1: From DIS to QCD

Experiment	Phenomenology	Underlying theory
Deeply in-elastic Scattering (60's–70's)	Bjorken's Scaling, Feynman's partons	Quarks, gluons, $SU(3)_c$
$e^- p \to e^- + X$	$\sigma(E, x)$	



discover the point-like structure of the proton:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4\theta/2}\left(\frac{F_1(x,Q^2)}{m_p}\sin^2\frac{\theta}{2} + \frac{F_2(x,Q^2)}{E-E'}\cos^2\frac{\theta}{2}\right)$$

QCD parton model $\Rightarrow 2xF_1(x,Q^2) = F_2(x,Q^2) = \sum_i x f_i(x) e_i^2.$

$Q^2 = -q^2$ (1)

"Structure functions"

$$F_{1,2}(x = Q^2/2p\cdot q) \sim \ln Q^2 \quad (2)$$

Bjorken Scaling / Feynman's parton   DGLAP evol. / QCD prediction

phenomenological work crucial!

Example 2: Discovery of the Higgs boson

Theoretical idea | Phenomenology | Exp't discovery

1964: The Higgs mechanism.

1967: A model for leptons: $SU(2)_L \otimes U(1)_Y$

1972: Renormalization, regularization

1973: Higgs properties phenomenology!

1976 – 90's: full layout:
Production & decays,
Search strategies.

Then on-going experiments:

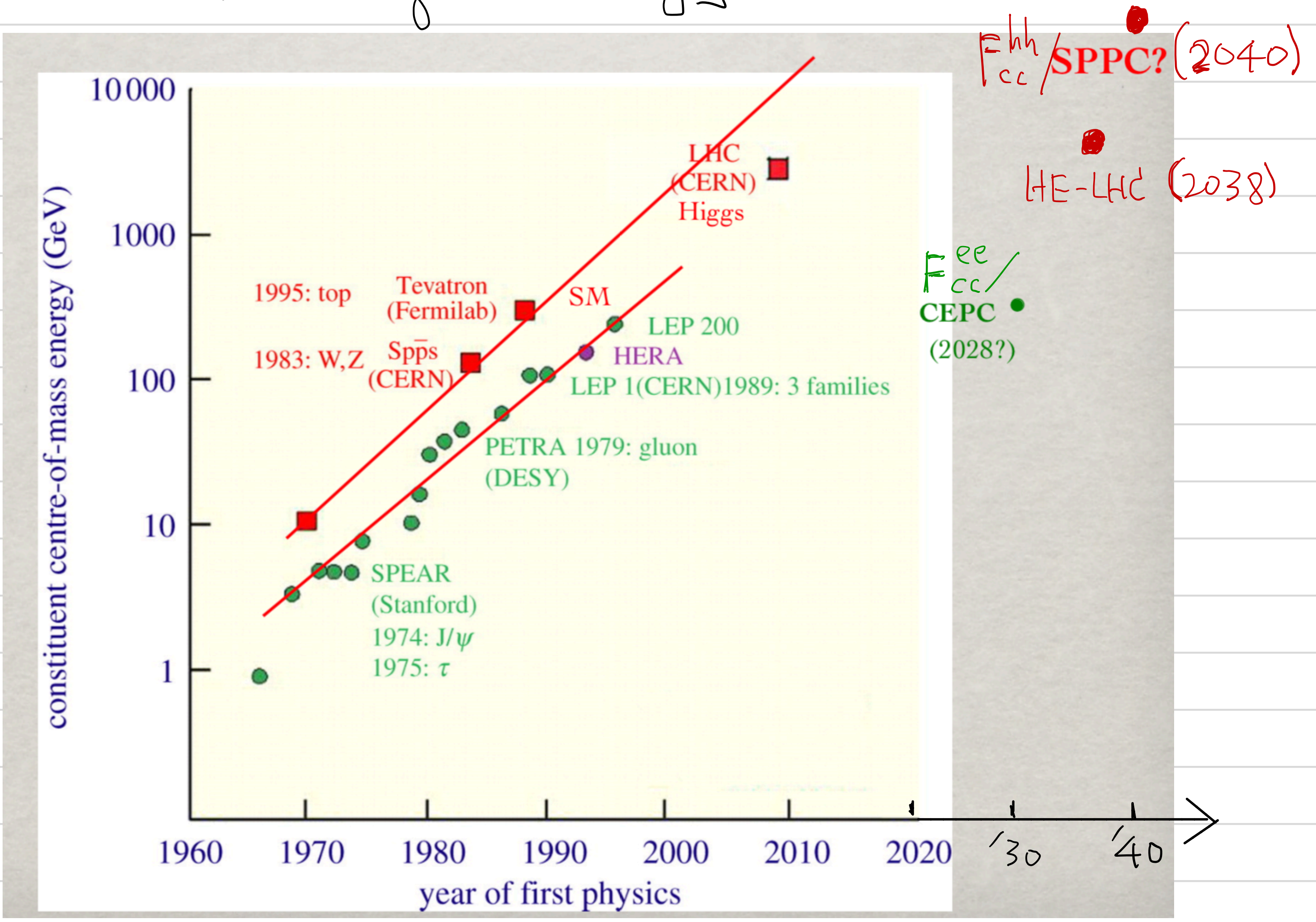
$Sp\bar{p}S$; LEP1,2; Tevatron

2000 – '10s: precision Higgs calculations

July 4, 2012: ATLAS/CMS: Discovery!

a long challenging effort.

§1.2. Some historic perspectives of high-energy colliders



Major discoveries @ Colliders:

50 Years Continuing discovery!

1968: proton structure (DIS @ SLAC)

1974: J/ψ (SPEAR @ SLAC)

1975: τ lepton (...)

1977: b-quark (E288 @ FNAL)

1979: gluon in 3-jets (PETRA @ DESY)

1983: $W^{\pm}/Z^{0}$ (Spp̄s @ CERN)

1989: EW precision / 3 ν's (LEP1,2 @ CERN)

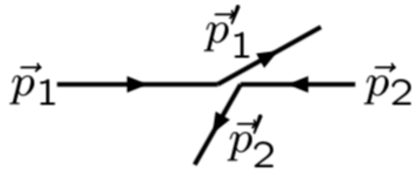
1995: top quark (Tevatron @ FNAL)

2012: Higgs boson (LHC @ CERN) ⇒ bright future!

§1.3:

Two parameters of importance:

1. The energy:



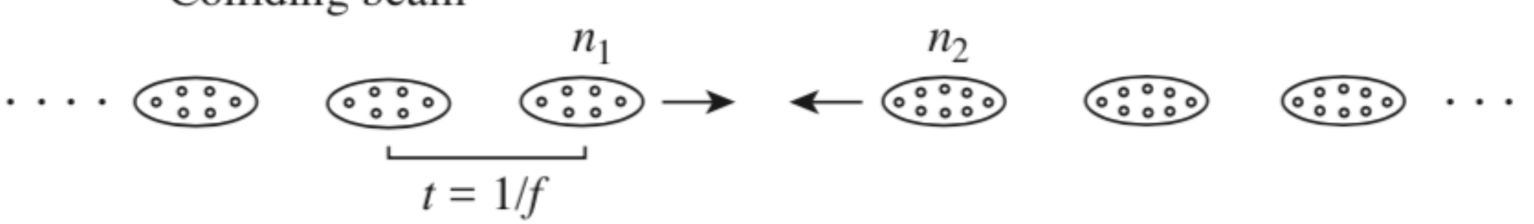
$$s \equiv (p_1+p_2)^2 = \begin{cases} (E_1+E_2)^2 - (\vec{p}_1+\vec{p}_2)^2, \\ m_1^2 + m_2^2 + 2(E_1E_2 - \vec{p}_1\cdot\vec{p}_2). \end{cases}$$

$$E_{cm} \equiv \sqrt{s} \approx \begin{cases} 2E_1 \approx 2E_2 & \text{in the c.m. frame } \vec{p}_1+\vec{p}_2=0, \\ \sqrt{2E_1m_2} & \text{in the fixed target frame } \vec{p}_2 = 0. \end{cases}$$

[HW 1.1]

Colliding particles aren't Continous:

Colliding beam



2. Luminosity:

$$\mathcal{L} \propto f n_1 n_2 / a,$$

(a some beam transverse profile) in units of #particles/cm^2/s

$\Rightarrow 10^{33}$ cm^{-2}s^{-1} $= 1$ nb^{-1} s^{-1} ≈ 10 fb^{-1}/year.

$\pi \cdot 10^7$ s

Now Units:

In relativistic quantum mechanics, the "natural units" are $c = \hbar = 1$:

$m \cdot kg \cdot s \Longrightarrow 1$ independent: GeV.

$10^{-15}\,m$

$$c \simeq 3 \cdot 10^{8}\, m/s \Rightarrow 1\,\mathrm{fm} = 3.4 \cdot 10^{-24}\, s$$

$$\hbar \simeq 6.6 \cdot 10^{-22}\, \mathrm{MeV} \cdot s \Rightarrow \mathrm{GeV}^{-1} \simeq 0.2\,\mathrm{fm}$$

$$\simeq 6.6 \cdot 10^{-25}\, s$$

$$(\hbar c)^2 \simeq 0.389379\, \mathrm{GeV}^2 \cdot \mathrm{mbarn}$$

$$\Rightarrow \mathrm{GeV}^{-2} \simeq 389379\, \mathrm{nb}$$

$$1\,\mathrm{pb} \simeq 2.6 \cdot 10^{-9} / \mathrm{GeV}^2$$

[See HW #1.2]

Chapt. 2: Basic Formalism:

§2.1 Scattering processes:

(A) General description:

An incident particle beam w/ momentum $\vec{p}$

On a local target

$$\psi_i(r) \sim e^{ipz} \qquad \psi_f(r) \underset{r \to \text{large}}{\sim} e^{i\vec{p}\cdot\vec{r}} + \frac{e^{ipr}}{r} f(\theta)$$

Scattering cross section:

$$d\sigma = \frac{\#\text{particles scattered into } d\Omega/\text{unit time}}{\#\text{incident particles}/\text{unit area}/\text{unit time}}$$

Recall the current: $\vec{j} = -\frac{i}{2m}\left(\psi^* \vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)$

$$\Rightarrow j_{inc} = p/m \;; \quad j_{scat} = \frac{p}{m}\frac{|f(\theta)|^2}{r^2}$$

$$\therefore \frac{d\sigma}{d\Omega} = |f(\theta)|^2$$ Scattering amplitude

$$\sigma_{tot} = \int |f(\theta)|^2 d\Omega$$

(B) Partial wave properties

P.W. Expansion:

Partial wave amplitude

$$f(\theta) = \frac{1}{k}\sum_\ell (2\ell+1)\, a_\ell(k)\, P_\ell(\cos\theta)$$

phase shift

$$a_\ell(k) = \frac{e^{2i\delta_\ell}-1}{2i} = e^{i\delta_\ell}\sin\delta_\ell = \frac{1}{\cot\delta_\ell - i}$$

Partial wave cross section

$$\therefore\ \sigma_{tot} = \int |f(\theta)|^2\, d\Omega$$

$$= \frac{4\pi}{k^2}\sum_\ell (2\ell+1)\sin^2\delta_\ell = \sum_\ell \sigma_\ell$$

$$\sigma_\ell = \frac{4\pi}{k^2}(2\ell+1)\sin^2\delta_\ell$$

Optical theorem:

$$\mathrm{Im}\, f(\theta=0) = \frac{1}{k}\sum_\ell (2\ell+1)\,\mathrm{Im}\, a_\ell(k)\, P_\ell(1)$$

$$= \frac{k}{4\pi}\Big[\underbrace{\frac{4\pi}{k^2}\sum_\ell (2\ell+1)\sin^2\delta_\ell}_{\sigma_{tot}}\Big]$$

The imaginary part of the forward scatt. amplitude equals to the tot. cross section!

Properties:

1). Partial wave Unitarity:

$|a_\ell(k)| = |\sin\delta_\ell| \leq 1$ applicable to any ℓ.

$\Rightarrow$ Partial wave amplitude bounded by 1.

$\Rightarrow$ For any fixed angular momentum ℓ, its contribution to the cross section is bounded from above:

$$\sigma_\ell = \frac{4\pi}{k^2}(2\ell+1)\sin^2\delta_\ell \leq \frac{4\pi}{k^2}(2\ell+1)$$

* σ_{tot} can only be (divergently) large if many partial waves contribute.

* Energy dependence:

$$\sigma_\ell \sim \frac{1}{k^2} \longrightarrow \begin{cases} \dfrac{1}{E_{kin}} & (\text{non-relativistic}, \ k^2 = 2mE) \\ \dfrac{1}{E^2} & (\text{relativistic}, \ k^2 = E^2 - m^2) \end{cases}$$

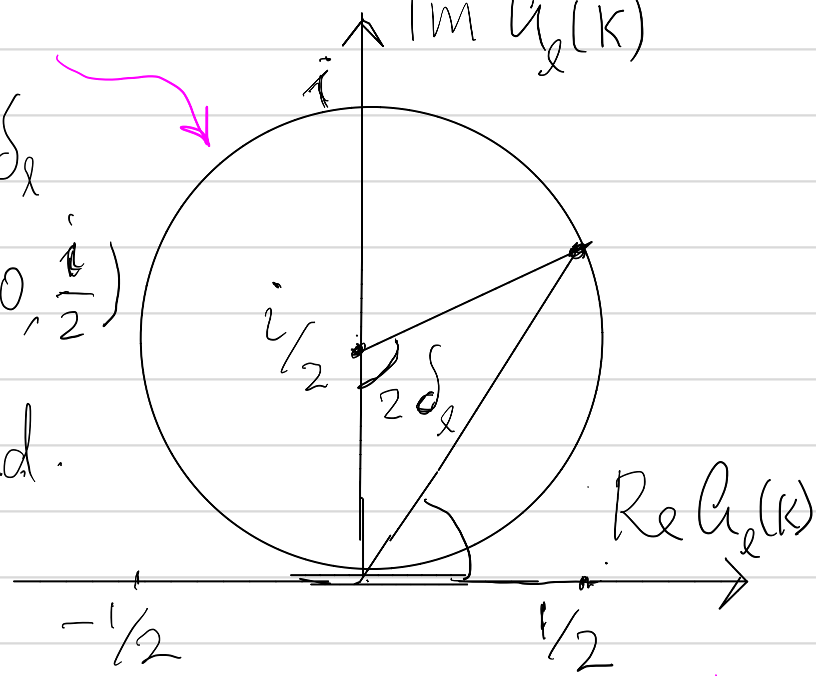
2). Argand diagram: in a complex plane

$$a_\ell(k) = \frac{i}{2} + \frac{1}{2} e^{2i\delta_\ell - i\frac{\pi}{2}}$$

Argand Circle:

$$= e^{i\delta_\ell} \sin\delta_\ell$$

radius $1/2$, centered at $(0, \frac{i}{2})$



* Tighter Unitarity Cond.

[HW 1.3]

$$|\mathrm{Re}\, a_\ell(k)| \le 1/2$$

* Elastic scattering: $|a_\ell(k)|^2 = \mathrm{Im}\, a_\ell$. on the circle.

Inelastic scattering: inside the circle

$k f_\ell(k) < e^{i\delta_\ell} \sin\delta_\ell$, amplitude damped!

* $\delta_\ell \to 0$: $a_\ell \simeq \delta_\ell + i\delta_\ell^2$, real at LO.

or $(\mathrm{Re}\, a_\ell)^2 \simeq \mathrm{Im}\, a_\ell \Rightarrow$ perturbative

$\delta_\ell \to \pi/2$: $a_\ell \to i$. $|a_\ell| = 1$

$\Rightarrow$ amplitude maximal, resonance!

[End of Lect. 1]

(c) Lorentz invariant amplitude:

S-matrix & transition-matrix

$$S = 1 + iT.$$

Lorentz invariant
gauge invariant

$$\langle f | iT | AB \rangle = (2\pi)^4 \delta^4 (p_A + p_B - \sum_f p_f)\, i\mathcal{M}(AB \to f)$$

Feynman Calculus

Partial Wave properties:

Partial wave expansion for $a + b \to 1 + 2$:

$$\mathcal{M}(s,t) = 16\pi \sum_{J=M}^{\infty} (2J+1) a_J(s) d^J_{\mu\mu'}(\cos\theta)$$

$$a_J(s) = \frac{1}{32\pi} \int_{-1}^{1} \mathcal{M}(s,t)\ d^J_{\mu\mu'}(\cos\theta) d\cos\theta.$$

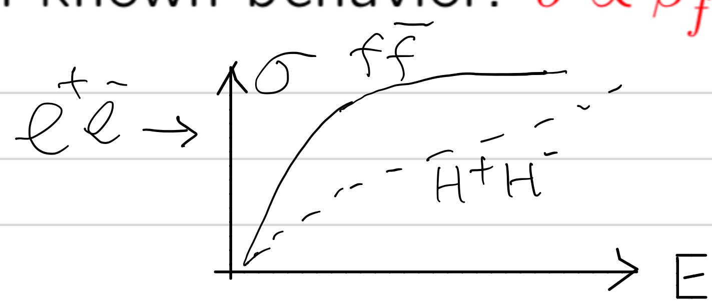
where $\mu = s_a - s_b,\ \mu' = s_1 - s_2,\ M = \max(|\mu|, |\mu'|)$.

(See PDG for the Wigner d-functions)

Speed $\leftarrow \beta = \left[1 - \frac{4m^2}{s}\right]^{1/2}$

kinematical thresholds: $a_J(s) \propto \beta_i^{l_i}\ \beta_f^{l_f} \quad (J = L + S)$.

$\Rightarrow$ well-known behavior: $\sigma \propto \beta_f^{2l_f+1} \Longrightarrow \begin{cases} \beta_f; & l=0, \text{S-wave} \\ \beta_f^3; & l=1, \text{P-wave} \end{cases}$



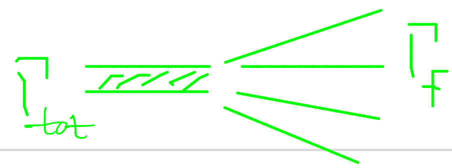
§2.2 Transition rate formalism

Decay rate $1 \to n$:

For a $1 \to n$ decay process, the partial width in the rest frame:

$$\Gamma(a \to 1+2+...n) = \frac{1}{2M_a} \overline{\sum} |\mathcal{M}|^2 \, dPS_n.$$

$$\tau = \Gamma_{tot}^{-1} = (\sum_f \Gamma_f)^{-1}.$$



Scattering Cross Section $2 \to n$:

For a $2 \to n$ scattering process:

$$\sigma(ab \to 1+2+...n) = \frac{1}{2s} \overline{\sum} |\mathcal{M}|^2 \, dPS_n,$$

$$dPS_n \equiv (2\pi)^4 \, \delta^4 \left(P - \sum_{i=1}^{n} p_i\right) \Pi_{i=1}^{n} \frac{1}{(2\pi)^3} \frac{d^3\vec{p}_i}{2E_i},$$

$$s = (p_a + p_b)^2 \equiv P^2 = \left(\sum_{i=1}^{n} p_i\right)^2,$$

① Dimensionality:

where $\overline{\sum}|\mathcal{M}|^2$: $\boxed{\text{dynamics}}$ (dimension $4-2n$);
dPS_n: $\boxed{\text{kinematics}}$ (Lorentz invariant, dimension $2n-4$.)

$\left\{\begin{array}{l} \text{dim'less} \\ n=2; \\ 2 \to n=3 \end{array}\right.$

② Event rate: $\frac{\#}{\text{time}} = \sigma[\text{cm}^2] \cdot L \, [\#/\text{cm}^2 \cdot s]$

[HW 1.4]

$$R(s) = \mathcal{L} \int d\tau \frac{dL(s,\tau)}{d\tau} \sigma(\hat{s}), \quad \tau = \frac{\hat{s}}{s}.$$

$$= \sigma(s)\mathcal{L},$$ for mono-chromatic $\frac{dL}{d\hat{s}} \sim \delta(s-\hat{s})$

*E.Byckling, K. Kajantie: Particle Kinemaitcs (1973).

Chapt.3 : Relativistic Kinematics & phase space treatments

$$dPS_n \equiv (2\pi)^4\ \delta^4\left(P - \sum_{i=1}^{n} p_i\right) \Pi_{i=1}^{n} \frac{1}{(2\pi)^3}\frac{d^3\vec{p}_i}{2E_i},$$

$$s = (p_a + p_b)^2 \equiv P^2 = \left(\sum_{i=1}^{n} p_i\right)^2,$$

§3.1: One-particle Final State $a + b \rightarrow 1$:

[HW. 2.1, 2.2]

$|\vec{p}_1|^2 d|\vec{p}_1| d\Omega$

$$\begin{aligned} dPS_1 &\equiv (2\pi)\ \frac{d^3\vec{p}_1}{2E_1}\delta^4(P - p_1) \\ &\doteq \pi|\vec{p}_1|d\Omega_1\delta^3(\vec{P} - \vec{p}_1) \\ &\doteq 2\pi\ \delta(s - m_1^2). \end{aligned}$$

either

or

where the first and second equal signs made use of the identities:

$$|\vec{p}|d|\vec{p}| = EdE, \quad \frac{d^3\vec{p}}{2E} = \int d^4p\ \delta(\underbrace{p^2 - m^2}_{E^2}).$$

Kinematical relations:

$$\begin{aligned} \vec{P} &\equiv \vec{p}_a + \vec{p}_b = \vec{p}_1, \quad E_1^{cm} = \sqrt{s} \text{ in the c.m. frame,} \\ s &= (p_a + p_b)^2 = m_1^2. \end{aligned}$$

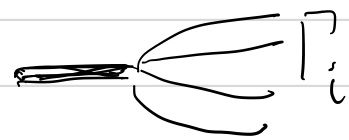
constraints

The "dimensinless phase-space volume" is $s(dPS_1) = 2\pi$.

Un-Stable particle:

After production, an ~~ex~~ exponential decay:

$$N(t) = N_0 e^{-t/\tau}, \quad \tau = 1/\Gamma, \quad \Gamma = \sum_i \Gamma_i$$



The Smaller (larger) Γ is, the longer-lived (shorter) it is.

Breit-Wignar relativistic propagator:

An unstable particle of mass M and total width Γ_V, the propagator is

$$R(s) = \frac{1}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2}.$$

[Non-relativistic $\frac{1}{E - M + i\Gamma/2}$]

Its δ-function representation:

the Narrow Width Approximation

$$\frac{1}{(m_*^2 - M_V^2)^2 + \Gamma_V^2 M_V^2} \approx \frac{\pi}{\Gamma_V M_V}\, \delta(m_*^2 - M_V^2)$$

$\Big|\ \Gamma_V \ll M_V$

When we talk about "on-shell" un-stable particle, it is in this sense $\Gamma \ll M$, a well-defined resonance.

$= d^4 p_2\,\delta(p_2^2-m_2^2)$

$2\sqrt{s}E_1-(s+m_1^2-m_2^2)$

Two-particle Final State $a+b\to 1+2$:

$$dPS_2 \equiv \frac{1}{(2\pi)^2}\,\delta^4(P-p_1-p_2)\,\frac{d^3\vec{p}_1}{2E_1}\frac{d^3\vec{p}_2}{2E_2}$$

$$\doteq \frac{1}{(4\pi)^2}\frac{|\vec{p}_1^{\,cm}|}{\sqrt{s}}\,d\Omega_1 = \frac{1}{(4\pi)^2}\frac{|\vec{p}_1^{\,cm}|}{\sqrt{s}}\,d\cos\theta_1 d\phi_1$$

$$= \frac{1}{4\pi}\frac{1}{2}\,\lambda^{1/2}\left(1,\frac{m_1^2}{s},\frac{m_2^2}{s}\right)dx_1 dx_2,$$ only 2

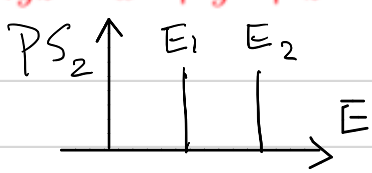
$$d\cos\theta_1 = 2dx_1,\quad d\phi_1 = 2\pi dx_2,\quad 0\le x_{1,2}\le 1,$$ angular Var.

The magnitudes of the energy-momentum of the two particles are fully determined by the four-momentum conservation:

$$|\vec{p}_1^{\,cm}| = |\vec{p}_2^{\,cm}| = \frac{\lambda^{1/2}(s,m_1^2,m_2^2)}{2\sqrt{s}},\quad E_1^{cm} = \frac{s+m_1^2-m_2^2}{2\sqrt{s}},\quad E_2^{cm} = \frac{s+m_2^2-m_1^2}{2\sqrt{s}},$$

$$\lambda(x,y,z) = (x-y-z)^2-4yz = x^2+y^2+z^2-2xy-2xz-2yz.$$

Kinematic function.



mono-Chromatic!

The phase-space volume of the two-body is scaled down with respect to that of the one-particle by a factor

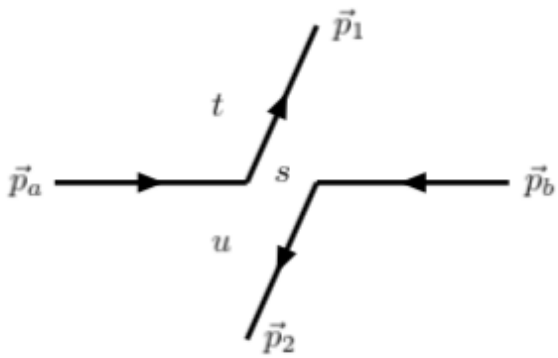
dimensionless $\Rightarrow$ $$\frac{dPS_2}{s\ dPS_1}\approx\frac{1}{(4\pi)^2}.$$

This is of the Same origin as 1-loop suppression.

$$d^4q\,\delta(q^2-m^2) \Longleftrightarrow \frac{d^3\vec{q}}{2E_q}$$

Example $a + b \to 1 + 2$

Consider a $2 \to 2$ scattering process $p_a + p_b \to p_1 + p_2$,



[HW #2,3]

the (Lorentz invariant) Mandelstam variables are defined as

$$
\begin{aligned}
s &= (p_a + p_b)^2 = (p_1 + p_2)^2 = E_{cm}^2, \\
t &= (p_a - p_1)^2 = (p_b - p_2)^2 = m_a^2 + m_1^2 - 2(E_a E_1 - p_a p_1 \cos\theta_{a1}), \\
u &= (p_a - p_2)^2 = (p_b - p_1)^2 = m_a^2 + m_2^2 - 2(E_a E_2 - p_a p_2 \cos\theta_{a2}), \\
& s + t + u = m_a^2 + m_b^2 + m_1^2 + m_2^2.
\end{aligned}
$$

$$dt = 2 p_a p_1 \, d\cos\theta_1 = \sqrt{s}\, \lambda^{\frac{1}{2}}\left(1, \frac{m_a^2}{s}, \frac{m_b^2}{s}\right) |\tilde{p}_1^{cm}| \, d\cos\theta_1^*$$

The two-body phase space can be thus written as

$$dPS_2 = \frac{1}{(4\pi)^2} \frac{dt \; d\phi_1}{s \; \lambda^{1/2}\left(1, m_a^2/s, m_b^2/s\right)}.$$

Lorentz-invariance form convenient.

§3.3: 3-body Kinematics

Three-particle Final State $a + b \to 1 + 2 + 3$:

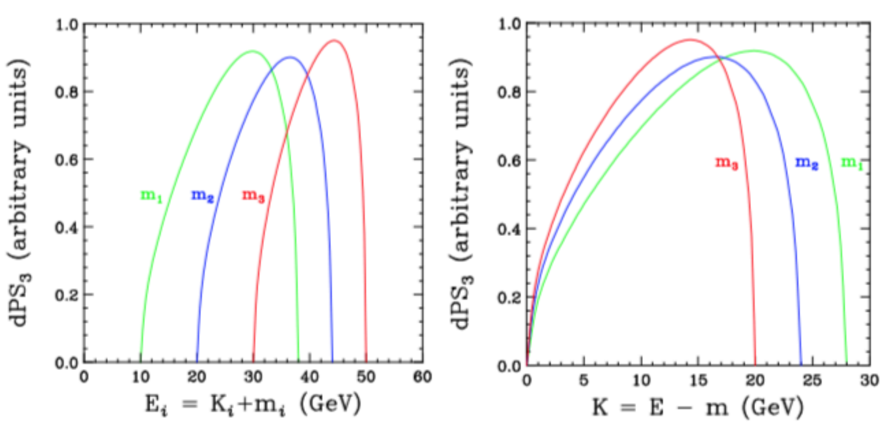
$$
\begin{aligned}
dPS_3 &\equiv \frac{1}{(2\pi)^5}\,\delta^4(P - p_1 - p_2 - p_3)\,\frac{d^3\vec{p}_1}{2E_1}\frac{d^3\vec{p}_2}{2E_2}\frac{d^3\vec{p}_3}{2E_3} \\
&\doteq \frac{|\vec{p}_1|^2\ d|\vec{p}_1|\ d\Omega_1}{(2\pi)^3\ 2E_1}\ \frac{1}{(4\pi)^2}\ \frac{|\vec{p}_2^{(23)}|}{m_{23}}\ d\Omega_2 \\
&= \frac{1}{(4\pi)^3}\ \lambda^{1/2}\left(1, \frac{m_2^2}{m_{23}^2}, \frac{m_3^2}{m_{23}^2}\right)\ 2|\vec{p}_1|\ \overbrace{dE_1\ dx_2 dx_3 dx_4 dx_5}^{9-4=5}.
\end{aligned}
$$

$$
\begin{aligned}
d\cos\theta_{1,2} &= 2dx_{2,4}, \quad d\phi_{1,2} = 2\pi dx_{3,5}, \quad 0 \le x_{2,3,4,5} \le 1, \\
& |\vec{p}_1^{cm}|^2 = |\vec{p}_2^{cm} + \vec{p}_3^{cm}|^2 = (E_1^{cm})^2 - m_1^2, \\
& m_{23}^2 = s - 2\sqrt{s}E_1^{cm} + m_1^2, \quad |\vec{p}_2^{23}| = |\vec{p}_3^{23}| = \frac{\lambda^{1/2}(m_{23}^2, m_2^2, m_3^2)}{2m_{23}},
\end{aligned}
$$

The particle energy spectrum is not monochromatic.
The maximum value (the end-point) for particle 1 in c.m. frame is

$$
\begin{aligned}
E_1^{max} &= \frac{s + m_1^2 - (m_2 + m_3)^2}{2\sqrt{s}}, \quad m_1 \le E_1 \le E_1^{max}, \\
|\vec{p}_1^{max}| &= \frac{\lambda^{1/2}(s, m_1^2, (m_2 + m_3)^2)}{2\sqrt{s}}, \quad 0 \le p_1 \le p_1^{max}.
\end{aligned}
$$

With $m_i = 10,\ 20,\ 30,\ \sqrt{s} = 100$ GeV.

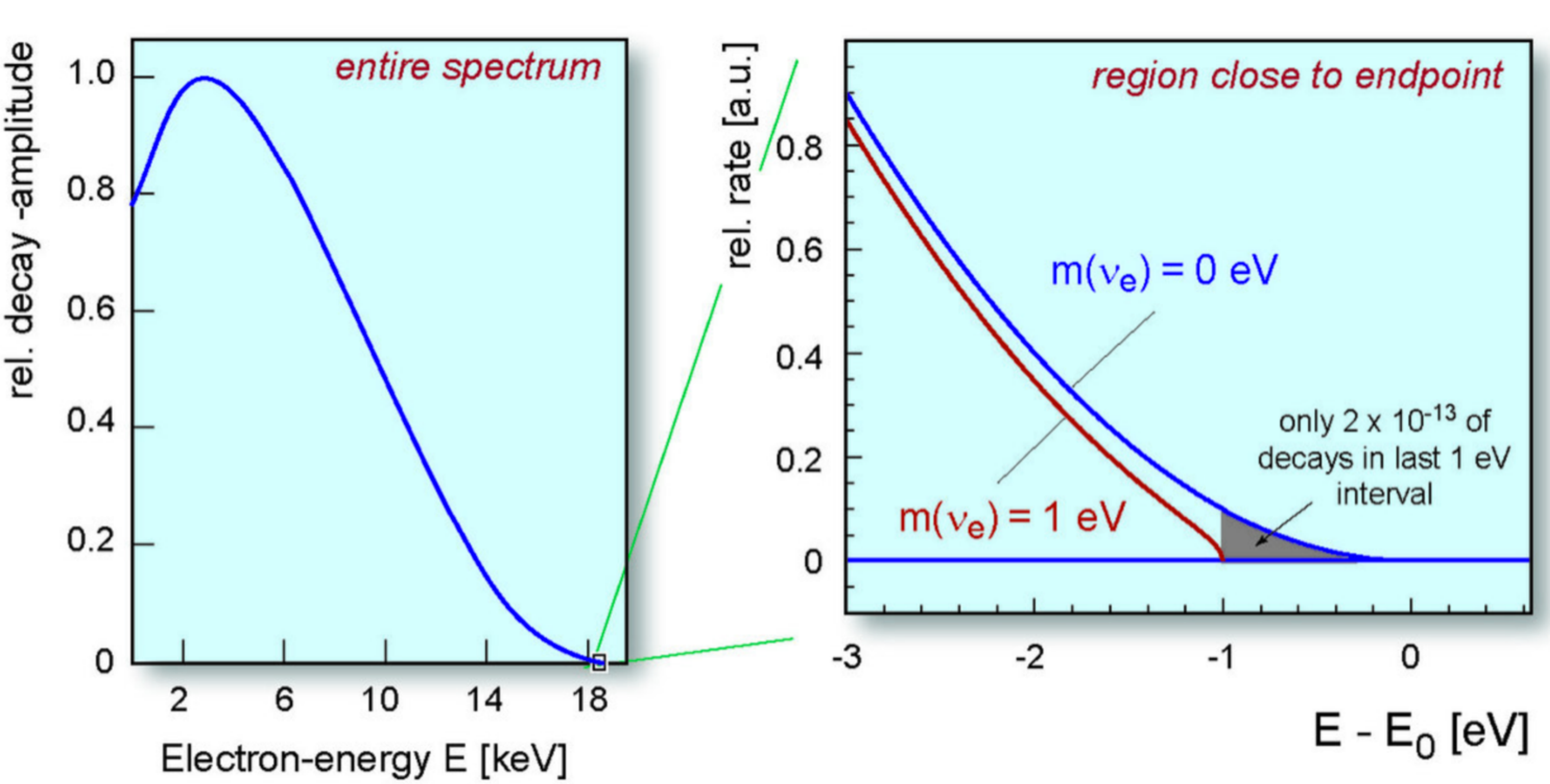


the heavier the particle is { the more tot. energy it carries; the less kinetic energy it takes!

Example 1: $a+b \to 1+2+3$

More intuitive to work out the end-point for the kinetic energy,
– recall the direct neutrino mass bound in β-decay:

$$K_1^{max} = E_1^{max} - m_1 = \frac{(\sqrt{s} - m_1 - m_2 - m_3)(\sqrt{s} - m_1 + m_2 + m_3)}{2\sqrt{s}}.$$



Thus, $K_{e^-}^{max} = \frac{m_n}{2}\left(1 - \frac{m_p}{m_n} - \frac{m_e}{m_n}\right)\left(1 + \frac{m_p}{m_n} - \frac{m_e}{m_n}\right) - \frac{m_p}{m_n} m_\nu$

the electron energy end point.

Example 2: $M \to a+b+c$

One practically useful formula is: For a decay, $d\Omega_1 d\Omega_2 \to d\cos\theta_2$ indpt

Exercise 2.4: A particle of mass M decays to 3 particles $M \to abc$. Show that the phase space element can be expressed as

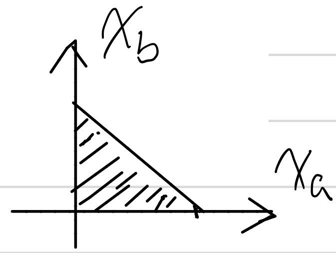
[HW. 2.4]

$$dPS_3 = \frac{1}{2^7\pi^3}\ M^2 dx_a dx_b.$$

$$x_i = \frac{2E_i}{M},\ \ (i = a, b, c,\ \sum_i x_i = 2).$$

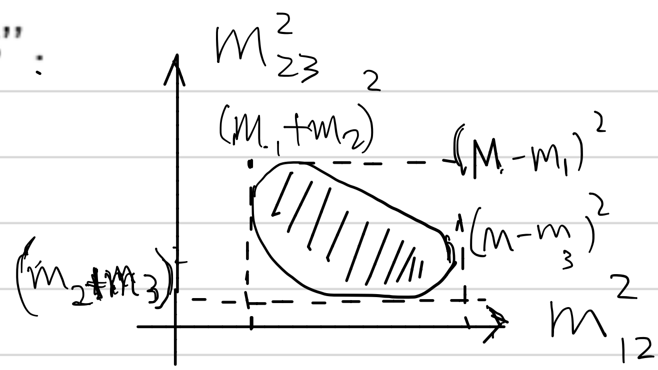
where the integration limits for $m_a = m_b = m_c = 0$ are

$$0 \le x_a \le 1, \quad 1 - x_a \le x_b \le 1.$$



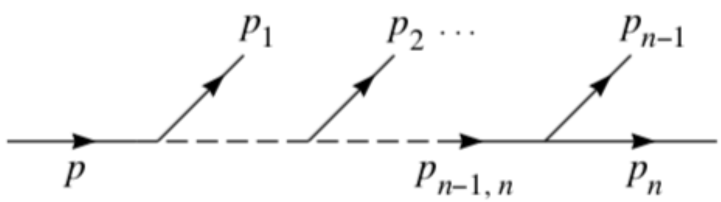
In general, the 3-body phase space boundaries are non-trivial. That leads to the "Dalitz Plots":

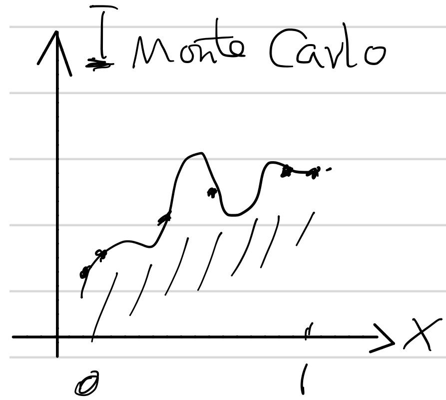
in $m_{12}^2 - m_{23}^2$ plane



n-body final state phase space

Recursion relation $P \to 1 + 2 + 3 \ldots + n$:





$$dPS_n(P; p_1, \ldots, p_n) = dPS_{n-1}(P; p_1, \ldots, p_{n-1,n}) \, dPS_2(p_{n-1,n}; p_{n-1}, p_n) \frac{dm^2_{n-1,n}}{2\pi}.$$

$$dPS_3 = dPS_2(i) \frac{dm^2_{prop}}{2\pi} dPS_2(f).$$

This is generically true, but particularly useful when the diagram has an s-channel particle propagation, See Next page.

Multiple dimensional integration: $3n-4$

(For $n=3$: 5-dim)

$$I = \int_\Omega f(\overline{\mathbf{x}}) \, d\overline{\mathbf{x}}; \qquad V = \int_\Omega d\overline{\mathbf{x}}$$

Monte Carlo method:

$$I \approx Q_N \equiv V \frac{1}{N} \sum_{i=1}^{N} f(\overline{\mathbf{x}}_i) = V \langle f \rangle$$

Advantages: * More efficient for 4-dim: $\varepsilon \sim 1/\sqrt{N}$

* close simulation of real events.

Importance sampling algorithm

Importance sampling provides a very important tool to perform Monte-Carlo integration.[3][8] The main result of importance sampling to this method is that the uniform sampling of $\overline{\mathbf{x}}$ is a particular case of a more generic choice, on which the samples are drawn from any distribution $p(\overline{\mathbf{x}})$. The idea is that $p(\overline{\mathbf{x}})$ can be chosen to decrease the variance of the measurement Q_N.

The VEGAS algorithm takes advantage of the information stored during the sampling, and uses it and importance sampling to efficiently estimate the integral I. It samples points from the probability distribution described by the function $|f|$ so that the points are concentrated in the regions that make the largest contribution to the integral.

Consider an intermediate state V^* in a chain decay:

$$a \to bV^* \to b\ p_1 p_2.$$

By the reduction formula, the resonant integral reads

$$\int_{(m_*^{min})^2=(m_1+m_2)^2}^{(m_*^{max})^2=(m_a-m_b)^2} dm_*^2.$$

Variable change

$$\tan\theta = \frac{m_*^2 - M_V^2}{\Gamma_V M_V},$$

resulting in a flat integrand over θ

$\Gamma_V M_V \sec^2\theta \, d\theta$

$$\int_{(m_*^{min})^2}^{(m_*^{max})^2} \frac{dm_*^2}{(m_*^2 - M_V^2)^2 + \Gamma_V^2 M_V^2} = \int_{\theta^{min}}^{\theta^{max}} \frac{d\theta}{\Gamma_V M_V}.$$

$\Gamma_V^2 M_V^2 \sec^2\theta$

In the limit

$$(m_1+m_2) + \Gamma_V \ll M_V \ll m_a - m_b - \Gamma_V,$$

$$\theta^{min} = \tan^{-1}\frac{(m_1+m_2)^2 - M_V^2}{\Gamma_V M_V} \to -\pi,$$

$$\theta^{max} = \tan^{-1}\frac{(m_a-m_b)^2 - M_V^2}{\Gamma_V M_V} \to 0,$$

then the Narrow Width Approximation

$$\frac{1}{(m_*^2 - M_V^2)^2 + \Gamma_V^2 M_V^2} \approx \frac{\pi}{\Gamma_V M_V}\,\delta(m_*^2 - M_V^2).$$

Try on your own:

~~Exercise~~ Consider a three-body decay of a top quark, $t \to bW^* \to b\ e\nu$. Making use of the phase space recursion relation and the narrow width approximation for the intermediate W boson, show that the partial decay width of the top quark can be expressed as

ignore spin-corr.

$$\Gamma(t \to bW^* \to b\ e\nu) \approx \Gamma(t \to bW) \cdot BR(W \to e\nu).$$

Chapt. 4: Particle Detection @ Colliders

How do we "see" particles?

Rutherford's expt: flash counts by eyes!

What we "see" as particles in the detector: (a few meters)

For a relativistic particle, the travel distance:

$$d = (\beta c\ \tau)\gamma \approx (300\ \mu m)(\frac{\tau}{10^{-12}\ s})\ \gamma$$

[HW # 2,5]

- stable particles directly "seen":

$$p,\ \bar{p},\ e^{\pm},\ \gamma$$

- quasi-stable particles of a life-time $\tau \geq 10^{-10}$ s also directly "seen":

$$n, \Lambda, K_L^0, ...,\ \mu^{\pm},\ \pi^{\pm}, K^{\pm}...$$

- a life-time $\tau \sim 10^{-12}$ s may display a secondary decay vertex, "vertex-tagged particles":

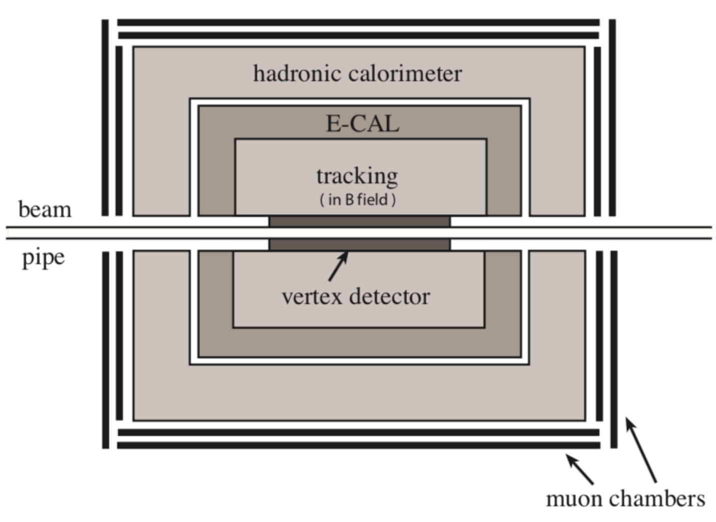
$$B^{0,\pm},\ D^{0,\pm},\ \tau^{\pm}...$$

- short-lived not "directly seen", but "reconstructable":

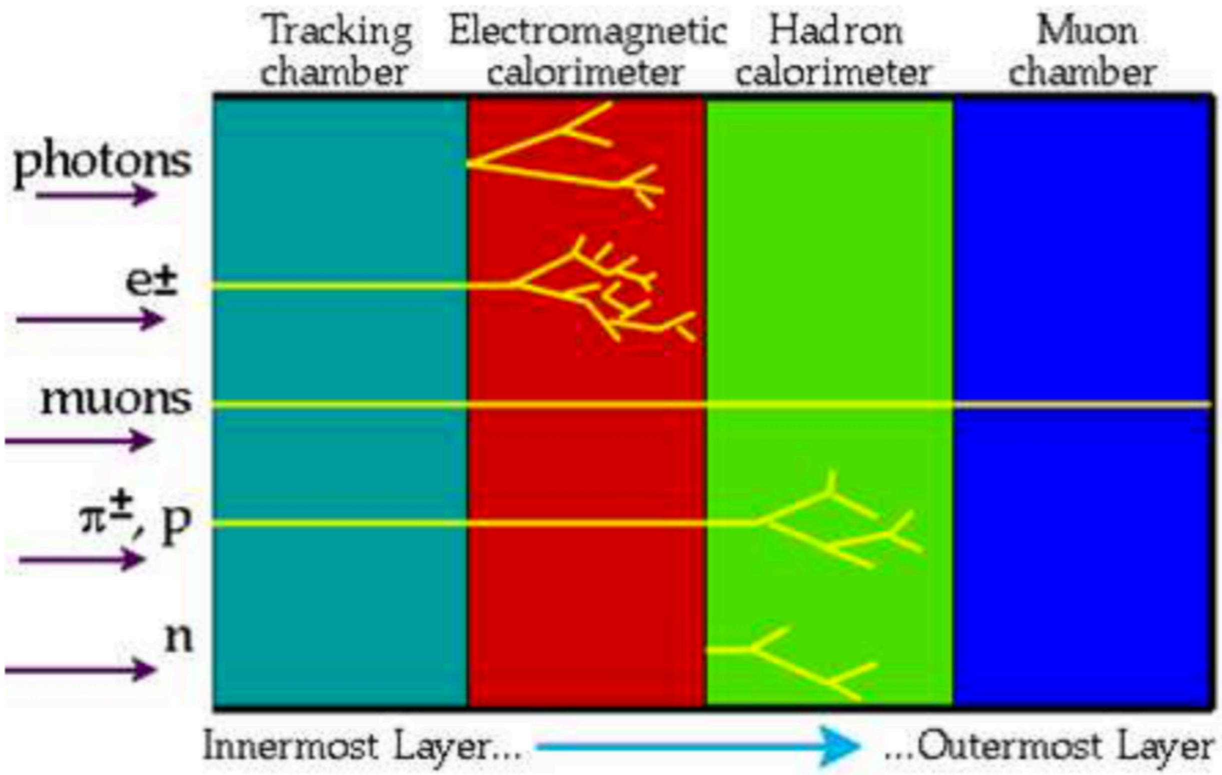
$$\pi^0,\ \rho^{0,\pm}...\ ,\quad Z, W^{\pm}, t, H...$$

- missing particles are weakly-interacting and neutral:

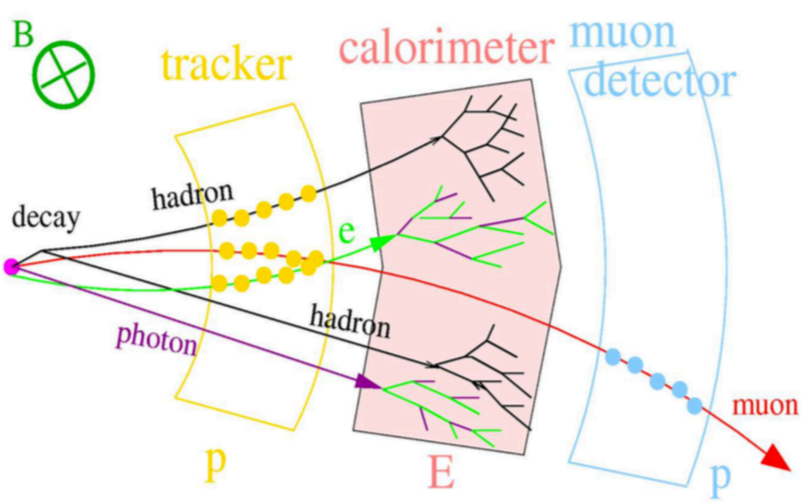
$$\nu,\ \tilde{\chi}^0, G_{KK}...$$



† For stable and quasi-stable particles of a life-time $\tau \geq 10^{-10} - 10^{-12}$ s, they show up as



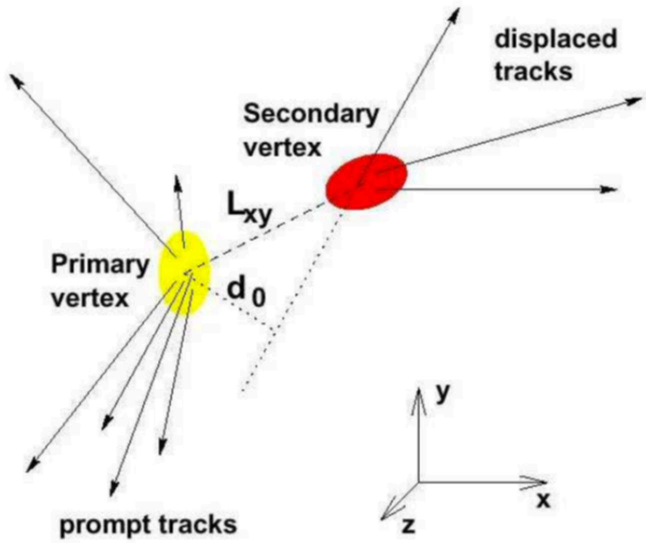
A closer look:



Theorists should know: measured curvature $k \propto \frac{1}{p} \sim \frac{BQ}{p}$

$$\text{For charged tracks}: \quad \Delta p/p \quad \propto \quad p,$$
$$\text{typical resolution}: \quad \sim \quad p/(10^4 \text{ GeV}).$$
$$\text{For calorimetry}: \quad \Delta E/E \quad \propto \quad \frac{1}{\sqrt{E}},$$
$$\text{typical resolution}: \quad \sim \quad (10\%_{ecal},\ 50\%_{hcal})/\sqrt{E/\text{GeV}}$$

† For vertex-tagged particles $\tau \approx 10^{-12}$ s,
heavy flavor tagging: the secondary vertex:



Typical resolution: $d_0 \sim 30 - 50\ \mu$m or so

$\Rightarrow$ Better have two (non-collinear) charged tracks for a secondary vertex;
Or use the "impact parameter" w.r.t. the primary vertex.

For theorists: just multiply a "tagging efficiency":

$$\epsilon_b \sim 70\%; \quad \epsilon_c \sim 40\%; \quad \epsilon_\tau \sim 40\%.$$

† For short-lived particles: $\tau < 10^{-12}$ s or so,
make use of final state kinematics to reconstruct the resonance.

† For missing particles:
make use of energy-momentum conservation to deduce their existence

$$p_1^i + p_2^i = \sum_f^{obs.} p_f \ + \ p_{miss}.$$

But in hadron collisions, the longitudinal momenta unknown,
thus transverse direction only:

$$0 = \sum_f^{obs.} \vec{p}_{f\ T} \ + \ \vec{p}_{miss\ T}.$$

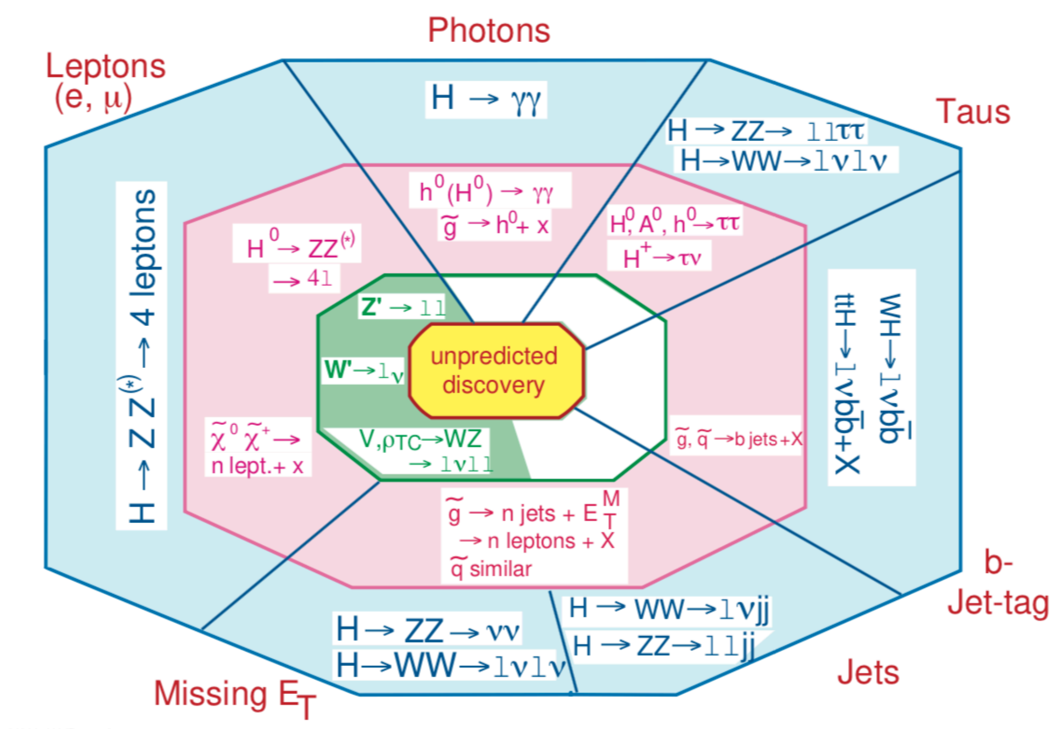
often called "missing p_T" ($\not{p}_T$) or (conventionally) "missing E_T" ($\not{E}_T$).

Note: "missing E_T" (MET) is *conceptually* ill-defined!
It is only sensible for massless particles: $\not{E}_T = \sqrt{\vec{p}^{\,2}_{miss\ T} + m^2}$.

What we "see" for the SM particles (no universality!)

Leptons	Vetexing	Tracking	ECAL	HCAL	Muon Cham.
$e^\pm$	$\times$	$\vec{p}$	E	$\times$	$\times$
$\mu^\pm$	$\times$	$\vec{p}$	$\surd$	$\surd$	$\vec{p}$
$\tau^\pm$	$\surd\times$	$\surd$	$e^\pm$	$h^\pm$; $3h^\pm$	$\mu^\pm$
ν_e, ν_μ, ν_τ	$\times$	$\times$	$\times$	$\times$	$\times$
Quarks					
u, d, s	$\times$	$\surd$	$\surd$	$\surd$	$\times$
$c \to D$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$b \to B$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$t \to bW^\pm$	b	$\surd$	$e^\pm$	$b + 2$ jets	$\mu^\pm$
Gauge bosons					
γ	$\times$	$\times$	E	$\times$	$\times$
g	$\times$	$\surd$	$\surd$	$\surd$	$\times$
$W^\pm \to \ell^\pm \nu$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\to q\bar{q}'$	$\times$	$\surd$	$\surd$	2 jets	$\times$
$Z^0 \to \ell^+\ell^-$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\to q\bar{q}$	$(b\bar{b})$	$\surd$	$\surd$	2 jets	$\times$
the Higgs boson					
$h^0 \to b\bar{b}$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$\to ZZ^*$	$\times$	$\vec{p}$	$e^\pm$	$\surd$	$\mu^\pm$
$\to WW^*$	$\times$	$\vec{p}$	$e^\pm$	$\surd$	$\mu^\pm$

How to search for new particles?



Chapt. 5: Lepton Colliders

A few representative Colliders:

Colliders	$\sqrt{s}$ (GeV) (GeV)	$\mathcal{L}$ ($\mathrm{cm}^{-2}\mathrm{s}^{-1}$)	$\delta E/E$	f (kHz)	polar.	L (km)
LEP I	M_Z	2.4×10^{31}	$\sim 0.1\%$	45	55%	26.7
SLC	~ 100	2.5×10^{30}	0.12%	0.12	80%	2.9
LEP II	~ 210	10^{32}	$\sim 0.1\%$	45		26.7
ILC	0.5–1	2.5×10^{34}	0.1%	3	80, 60%	14 – 33
CEPC	0.25–0.35	2×10^{34}	0.13%			50-100
CLIC	3–5	$\sim 10^{35}$	0.35%	1500	80, 60%	33 – 53

§5.1 e^+e^- Colliders

(A) The collisions between e^- and e^+ have major advantages:

- The system of an electron and a positron has zero charge, zero lepton number etc.,

$\Longrightarrow$ it is suitable to create new particles after e^+e^- annihilation.

- With symmetric beams between the electrons and positrons, the laboratory frame is the same as the c.m. frame,

$\Longrightarrow$ the total c.m. energy is fully exploited to reach the highest possible physics threshold.

- With well-understood beam properties,

$\Longrightarrow$ the scattering kinematics is well-constrained.

- Backgrounds low and well-undercontrol:

For $\sigma \approx 10$ pb $\Rightarrow$ 0.1 Hz at 10^{34} cm^{-2}s^{-1}.

- Linear Collider: possible to achieve high degrees of beam polarizations,

$\Longrightarrow$ chiral couplings and other asymmetries can be effectively explored.

Disadvantages

- Large synchrotron radiation due to acceleration,

$$\Delta E \sim \frac{1}{R}\left(\frac{E}{m_e}\right)^4.$$

Thus, a multi-hundred GeV e^+e^- collider will have to be made a linear accelerator.

- This becomes a major challenge for achieving a high luminosity when a storage ring is not utilized; beamsstrahlung severe.

(B). Particle production:

As for the differential production cross section of two-particle a, b,

$$\frac{d\sigma(e^+e^- \to ab)}{d\cos\theta} = \frac{\beta}{32\pi s}\overline{\sum|\mathcal{M}|^2}$$

where

- $\beta = \lambda^{1/2}(1, m_a^2/s, m_b^2/s)$, is the speed factor for the out-going particles in the c.m. frame, and $p_{cm} = \beta\sqrt{s}/2$,
- $\overline{\sum|\mathcal{M}|^2}$ the squared matrix element, summed and averaged over quantum numbers (like color and spins etc.)
- unpolarized beams so that the azimuthal angle trivially integrated out,

$$\#\text{Events} = \mathcal{L}\int d\tau \frac{dL(s,\tau)}{d\tau}\,\sigma(\hat{s}), \qquad \tau = \frac{\hat{s}}{s}.$$

typically
$$\frac{dL}{d\sqrt{\hat{s}}} = \frac{1}{\sqrt{2\pi}\,\Delta}\exp[\frac{-(\sqrt{\hat{s}}-\sqrt{s})^2}{2\Delta^2}]$$

with $\Delta/\sqrt{\hat{s}} \sim 10^{-3}$, typically

Resonant production: Breit-Wigner formula

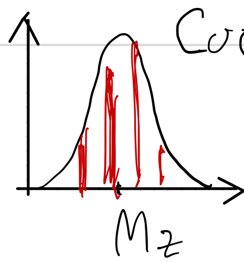
$$\frac{1}{(s-M_V^2)^2+\Gamma_V^2 M_V^2}$$

If the energy spread $\delta\sqrt{s} \ll \Gamma_V$, the line-shape mapped out:

[HW#3.1] $$\sigma(e^+e^- \to V^* \to X) = \frac{4\pi(2j+1)\Gamma(V\to e^+e^-)\Gamma(V\to X)}{(s-M_V^2)^2+\Gamma_V^2 M_V^2}\,\frac{s}{M_V^2}$$

This is like LEP I: $\delta E \approx 10^{-3} E \ll \Gamma_Z \simeq 2.4$ GeV.

Scan out the full line-shape, measure the total width (neutrino counting).



If $\delta\sqrt{s} \gg \Gamma_V$, the narrow-width approximation:

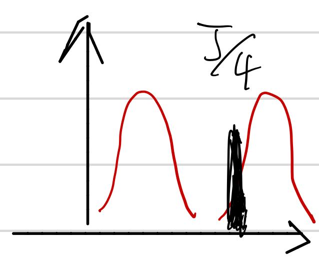
$$\frac{1}{(s-M_V^2)^2+\Gamma_V^2 M_V^2} \to \frac{\pi}{M_V\Gamma_V}\,\delta(s-M_V^2),$$

$$\sigma(e^+e^- \to V^* \to X) = \frac{2\pi^2(2j+1)\Gamma(V\to e^+e^-)BF(V\to X)}{M_V^2}\,\frac{dL(\hat{s}=M_V^2)}{d\sqrt{\hat{s}}}$$

This is like J/ψ production:

$\delta E \simeq 10^{-3} E \gg \Gamma_{J/\psi} \simeq 93$ keV.

(MeV)



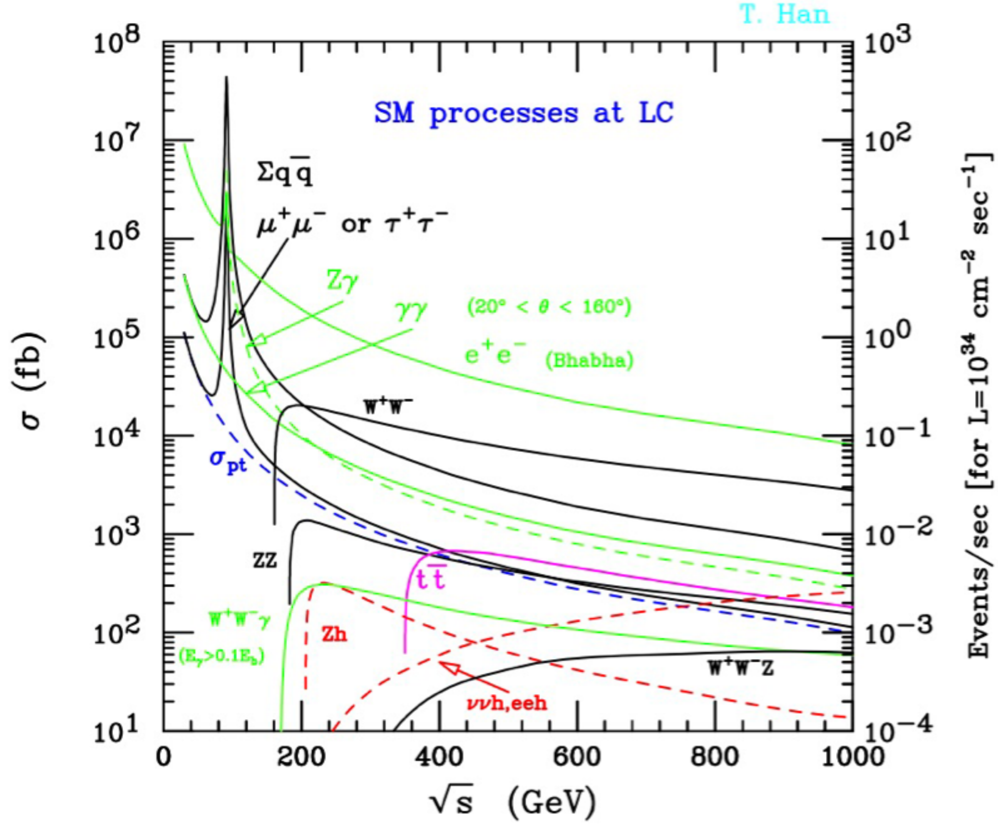
Off resonance:

1). For an s-channel or a finite-angle scattering:

$$\sigma \sim \frac{1}{s}.$$

2). For forward (co-linear) scattering:

$$\sigma \sim \frac{1}{M_V^2} \ln^2 \frac{s}{M_V^2}.$$



- The simplest reaction

$$\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-) \equiv \sigma_{pt} = \frac{4\pi\alpha^2}{3s}.$$

In fact, $\sigma_{pt} \approx 100$ fb$/(\sqrt{s}/\text{TeV})^2$ has become standard units to measure the size of cross sections.

- The Z resonance prominent (or other M_V),
- At the ILC $\sqrt{s} = 500$ GeV,

$$\sigma(e^+e^- \to e^+e^-) \sim 100\sigma_{pt} \sim 40 \text{ pb}.$$

(anglular cut dependent.)

$$\sigma_{pt} \sim \sigma(ZZ) \sim \sigma(t\bar{t}) \sim 400 \text{ fb};$$
$$\sigma(u,d,s) \sim 9\sigma_{pt} \sim 3.6 \text{ pb};$$
$$\sigma(WW) \sim 20\sigma_{pt} \sim 8 \text{ pb}.$$

and

$$\sigma(ZH) \sim \sigma(WW \to H) \sim \sigma_{pt}/4 \sim 100 \text{ fb};$$
$$\sigma(WWZ) \sim 0.1\sigma_{pt} \sim 40 \text{ fb}.$$

(c) Constrained Kinematics

one of the most important features in e^+e^- collisions:

precisely known kinematics.

One of the most important techniques, that distinguishes an e^+e^- collisions from hadronic collisions.

Consider a process:

$$e^+ + e^- \to V + X,$$

where V: a (bunch of) visible particle(s); X: unspecified.

Then:

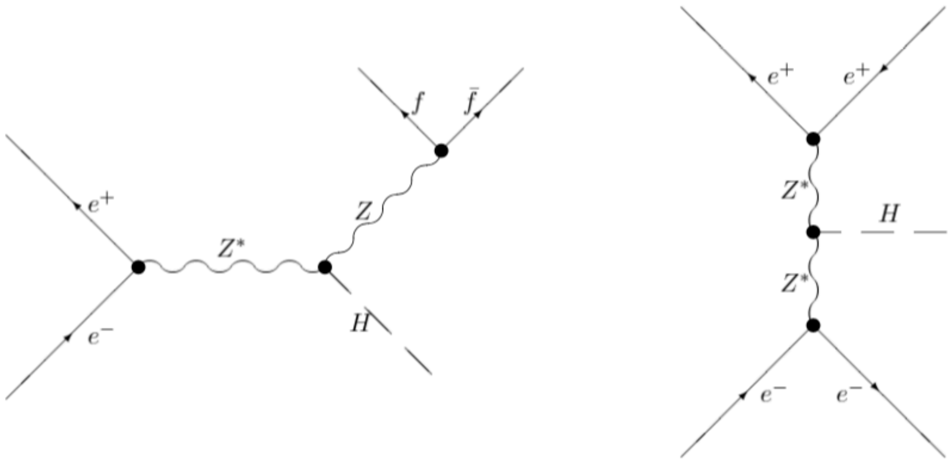
$$p_{e^+} + p_{e^-} = p_V + p_X, \ (p_{e^+} + p_{e^-} - p_V)^2 = p_X^2,$$
$$M_X^2 = (p_{e^+} + p_{e^-} - p_V)^2 = s + M_V^2 - 2\sqrt{s}E_V.$$

One thus obtain the "model-independent" inclusive measurements

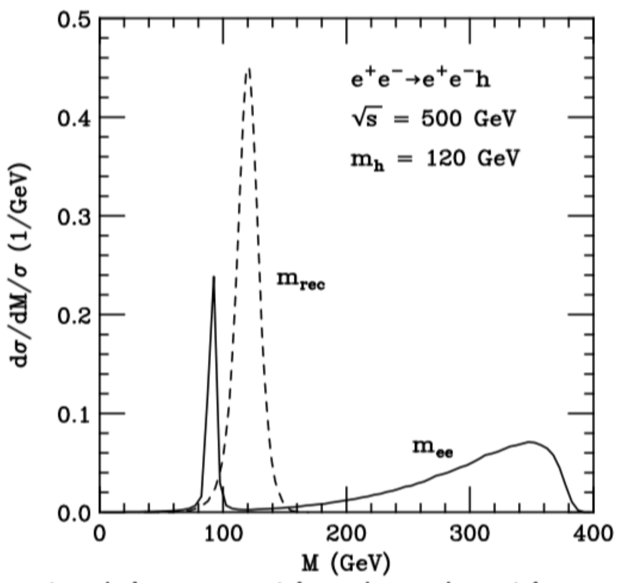
(recoil mass)

Best example:

The key point for a Higgs factory: $e^+ + e^- \to f\bar{f} + h.$



Then: $$M_h^2 = (p_{e^+} + p_{e^-} - p_f - p_{\bar{f}})^2 = s + M_V^2 - 2\sqrt{s}E_{f\bar{f}}.$$



Model-independent, kinematical selection of signal events!

[HW #3,2]

Missing particles

1). Neutrino Counting @ e^+e^- Collider

2). Dark matter mass

Utilizing two-body Kinematics

- Energy end-point and mass edges:

  utilizing the "two-body kinematics"

Consider a simple case:

$$e^+e^- \to \tilde{\mu}_R^+ \; \tilde{\mu}_R^-$$
$$\text{with two} - \text{body decays}: \quad \tilde{\mu}_R^+ \to \mu^+\tilde{\chi}_0, \quad \tilde{\mu}_R^- \to \mu^-\tilde{\chi}_0.$$

In the $\tilde{\mu}_R^+$-rest frame: $E_\mu^0 = \frac{M_{\tilde{\mu}_R}^2 - m_\chi^2}{2M_{\tilde{\mu}_R}}$.

In the Lab-frame:

$$(1-\beta)\gamma E_\mu^0 \le E_\mu^{lab} \le (1+\beta)\gamma E_\mu^0$$
$$\text{with} \quad \beta = \left(1 - 4M_{\tilde{\mu}_R}^2/s\right)^{1/2}, \quad \gamma = (1-\beta)^{-1/2}.$$

Energy end-point: $E_\mu^{lab} \Rightarrow M_{\tilde{\mu}_R}^2 - m_\chi^2$.

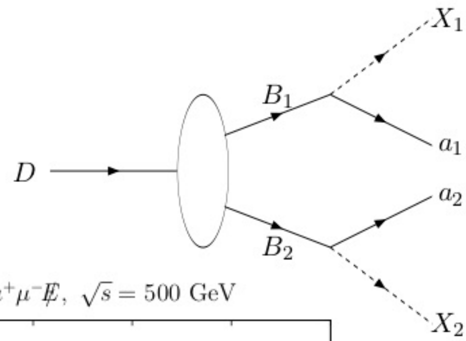
Mass edge: $m_{\mu^+\mu^-}^{max} = \sqrt{s} - 2m_\chi$.

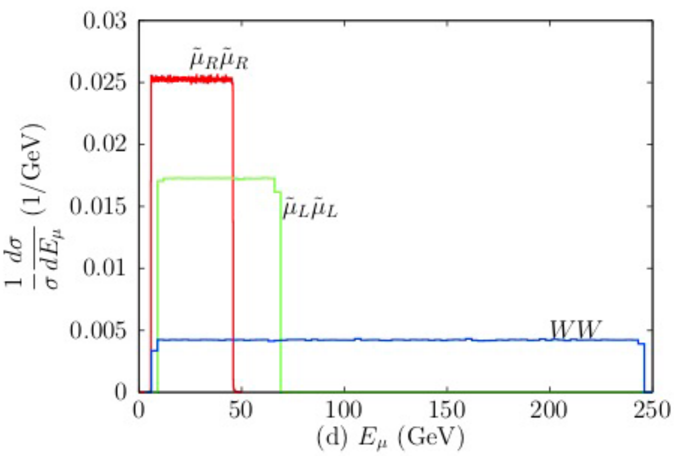
Same idea can be applied to hadron colliders ...

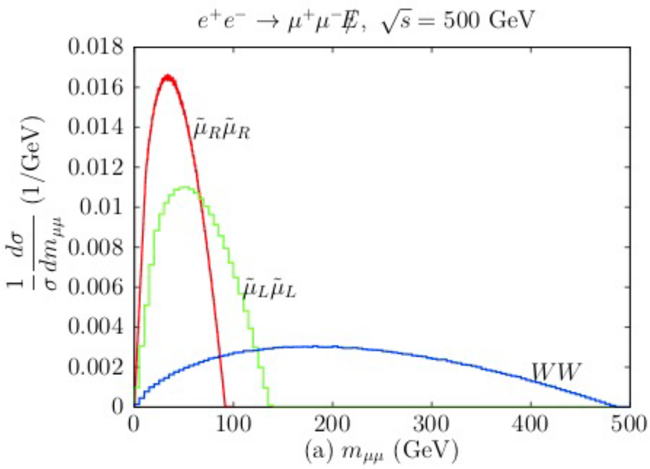
More observables: Cusps

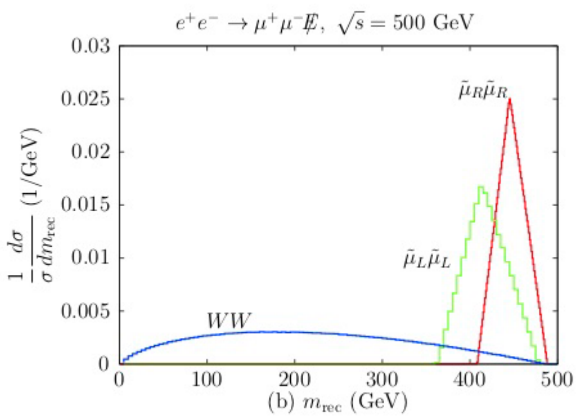
$$e^+e^- \to B_1 + B_2,$$

$$B_1 \to a_1 + X_1, \quad B_2 \to a_2 + X_2$$



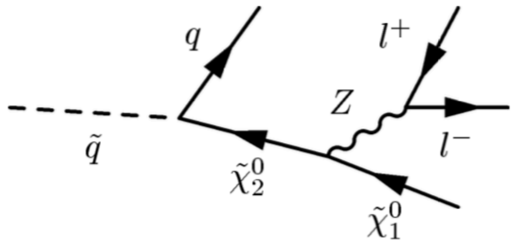






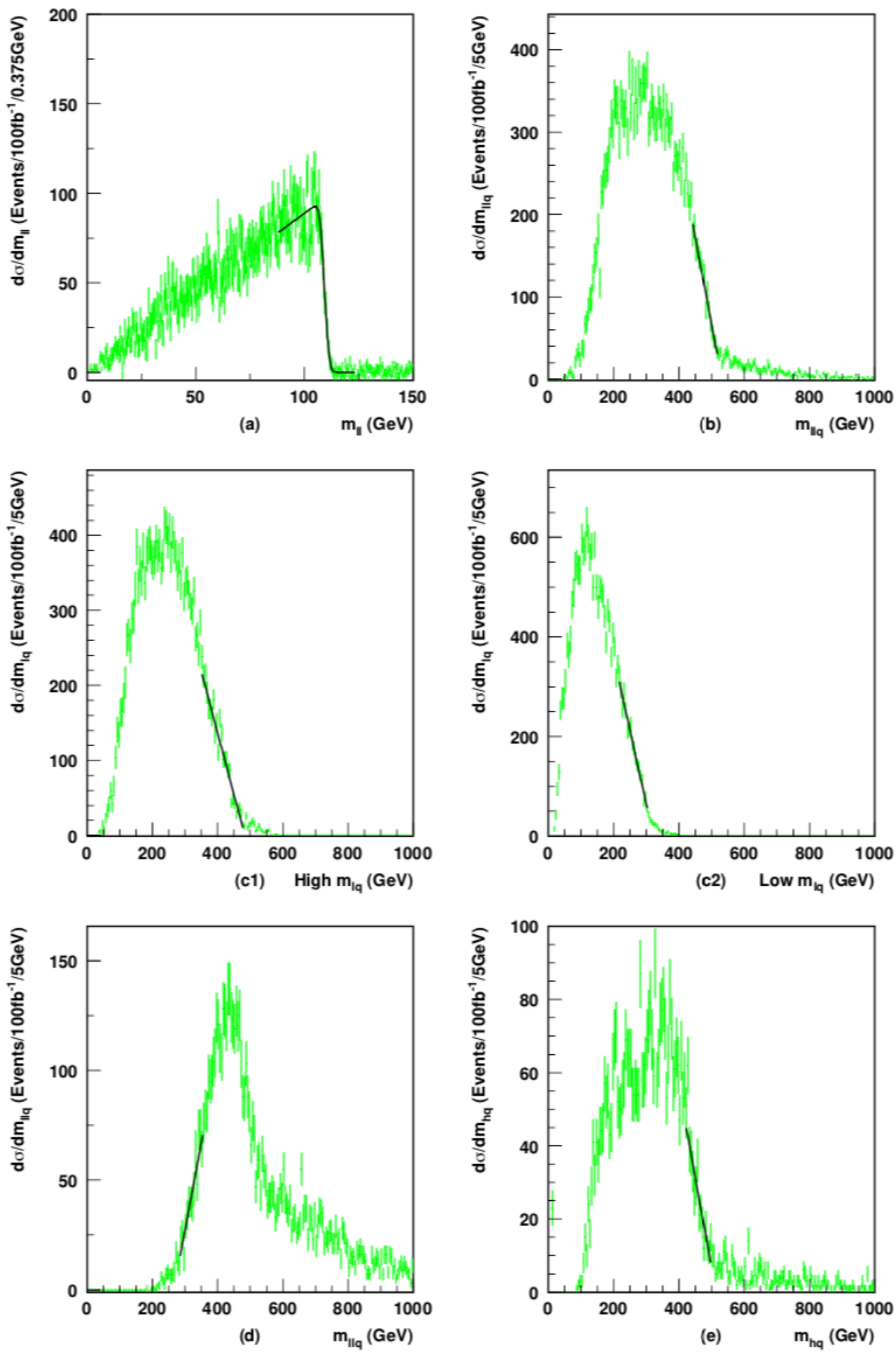
Utilizing two-body Kinematics

Consider a squark cascade decay:



1st edge : $M^{max}(\ell\ell) = M_{\chi_2^0} - M_{\chi_1^0}$;

2nd edge : $M^{max}(\ell\ell j) = M_{\tilde{q}} - M_{\chi_1^0}$.



[redacted] Charge forward-backward asymmetry A_{FB}:

The coupling vertex of a vector boson V_μ to an arbitrary fermion pair f

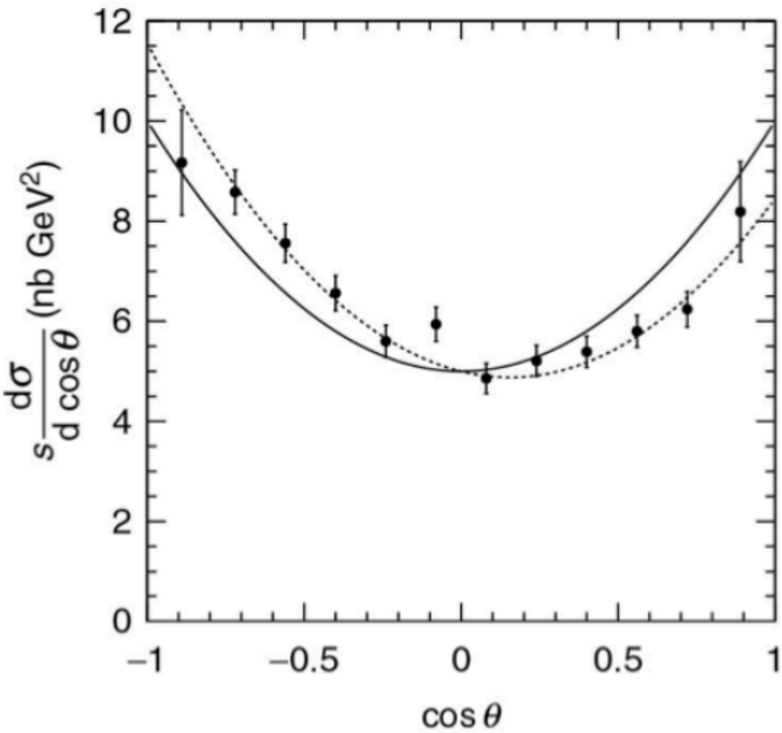
$$i\sum_{\tau}^{L,R} g_\tau^f\ \gamma^\mu\ P_\tau \qquad \rightarrow \qquad \text{crucial to probe chiral structures.}$$

The parton-level forward-backward asymmetry is defined as

$$A_{FB}^{i,f} \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3}{4}\mathcal{A}_i\mathcal{A}_f,$$

$$\mathcal{A}_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}.$$

where N_F (N_B) is the number of events in the forward (backward) direction defined in the parton c.m. frame relative to the initial-state fermion $\vec{p}_i$.



Sensitive to the Chiral interactions of the underlying physics @ given $\sqrt{s}$.

one of the most important features in e^+e^- collisions:

Beam polarization:

One of the merits for an e^+e^- linear collider is the possible high polarization for both beams.
Consider first the longitudinal polarization along the beam line direction. Denote the average $e^{\pm}$ beam polarization by $P^L_{\pm}$, with $P^L_{\pm} = -1$ purely left-handed and $+1$ purely right-handed.

The polarized squared matrix element, based on the helicity amplitudes $\mathcal{M}_{\sigma_{e^-}\sigma_{e^+}}$:

$$\overline{\sum}|\mathcal{M}|^2 = \frac{1}{4}[(1-P^L_-)(1-P^L_+)|\mathcal{M}_{--}|^2 + (1-P^L_-)(1+P^L_+)|\mathcal{M}_{-+}|^2 \\ +(1+P^L_-(1-P^L_+)|\mathcal{M}_{+-}|^2 + (1+P^L_-)(1+P^L_+)|\mathcal{M}_{++}|^2].$$

Major benefit for e^+e^- linear Collider!

Furthermore, it is possible to produce transversely polarized beams with the help of a spin-rotator.
If the beams present average polarizations with respect to a specific direction perpendicular to the beam line direction, $-1 < P^T_{\pm} < 1$, then there will be one additional term in the limit $m_e \to 0$,

$$\frac{1}{4}\, 2\, P^T_- P^T_+ \, \mathrm{Re}(\mathcal{M}_{-+}\mathcal{M}^*_{+-}).$$

Common processes: $e^- e^+ \to f\bar{f}$.
For most of the situations, the scattering matrix element can be casted into a $V \pm A$ chiral structure of the form (sometimes with the help of Fierz transformations)

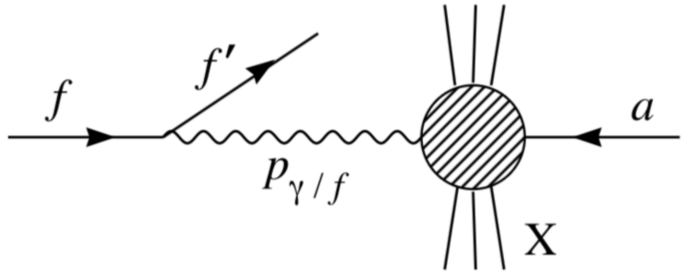
$$\mathcal{M} = \frac{e^2}{s} Q_{\alpha\beta}\ [\bar{v}_{e^+}(p_2)\gamma^\mu P_\alpha u_{e^-}(p_1)]\ [\bar{\psi}_f(q_1)\gamma_\mu P_\beta \psi'_{\bar{f}}(q_2)],$$

where $P_\mp = (1 \mp \gamma_5)/2$ are the L, R chirality projection operators, and $Q_{\alpha\beta}$ are the bilinear couplings governed by the underlying physics of the interactions with the intermediate propagating fields.
With this structure, the scattering matrix element squared:

$$\begin{aligned}\overline{\sum}|\mathcal{M}|^2 &= \frac{e^4}{s^2}\left[(|Q_{LL}|^2 + |Q_{RR}|^2)\ u_i u_j + (|Q_{LL}|^2 + |Q_{RL}|^2)\ t_i t_j\right. \\ &+ \left. 2Re(Q^*_{LL}Q_{LR} + Q^*_{RR}Q_{RL}) m_f m_{\bar{f}} s\right],\end{aligned}$$

where $t_i = t - m_i^2 = (p_1 - q_1)^2 - m_i^2$ and $u_i = u - m_i^2 = (p_1 - q_2)^2 - m_i^2$.

§5.2 $e\gamma$ collider



The simplest case is the photon radiation off an electron, like:

$$e^+e^- \to e^+, \ \gamma^* e^- \to e^+e^-.$$

The dominant features are due to the result of a t-channel singularity, induced by the collinear photon splitting:

$$\sigma(e^- a \to e^- X) \approx \int dx \ P_{\gamma/e}(x) \ \sigma(\gamma a \to X).$$

The so called the effective photon approximation.

For an electron of energy E, the probability of finding a collinear photon of energy xE is given by

$$P_{\gamma/e}(x) = \frac{\alpha}{2\pi} \frac{1 + (1-x)^2}{x} \ln \frac{E^2}{m_e^2},$$

[HW#3,3]

known as the Weizsäcker-Williams spectrum.

We see that:

- m_e enters the log to regularize the collinear singularity;
- $1/x$ leads to the infrared behavior of the photon;
- This picture of the photon probability distribution is also valid for other photon spectrum:

Based on the back-scattering laser technique, it has been proposed to produce much harder photon spectrum, to construct a "photon collider"...

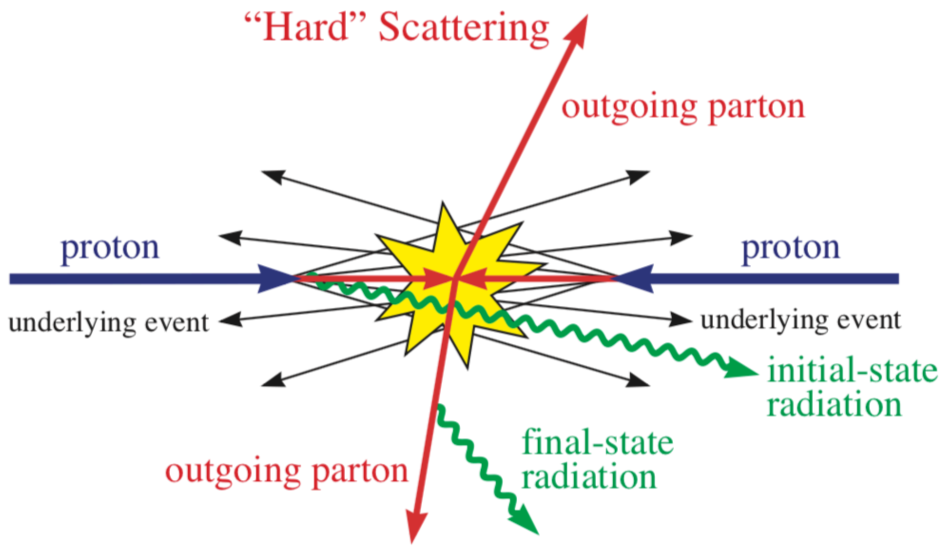
Chapt. 6: Hadron-hadron Colliders

A few representative Colliders:

§6.1: Hadron Colliders: Pros & Cons

Colliders	$\sqrt{s}$ (TeV)	$\mathcal{L}$ ($\mathrm{cm}^{-2}\mathrm{s}^{-1}$)	$\delta E/E$	f (MHz)	#/bunch (10^{10})	L (km)
Tevatron	1.96	2.1×10^{32}	9×10^{-5}	2.5	p: 27, $\bar{p}$: 7.5	6.28
HERA	314	1.4×10^{31}	$0.1, 0.02\%$	10	e: 3, p: 7	6.34
LHC	14	10^{34}	0.01%	40	10.5	26.66
LHC Run (I) II	(7,8) 13	$(10^{32})\ 10^{33}$	0.01%	40	10.5	26.66
HL-LHC	14	7×10^{34}	0.013%	40	22	26.66
FCC_{hh} (SppC)	100	1.2×10^{35}	0.01%	40	10	100

with Strong interaction:



(a) Total hadronic cross section: Non-perturbative.
The order of magnitude estimate:

$$\sigma_{pp} = \pi r_{eff}^2 \approx \pi/m_\pi^2 \sim 120 \text{ mb}.$$

Energy-dependence?

$$\sigma(pp) \begin{cases} \approx 21.7\ (\frac{s}{\text{GeV}^2})^{0.0808} \text{ mb}, & \text{Empirical relation} \\ < \frac{\pi}{m_\pi^2}\ \ln^2 \frac{s}{s_0}, & \text{Froissart bound.} \end{cases}$$

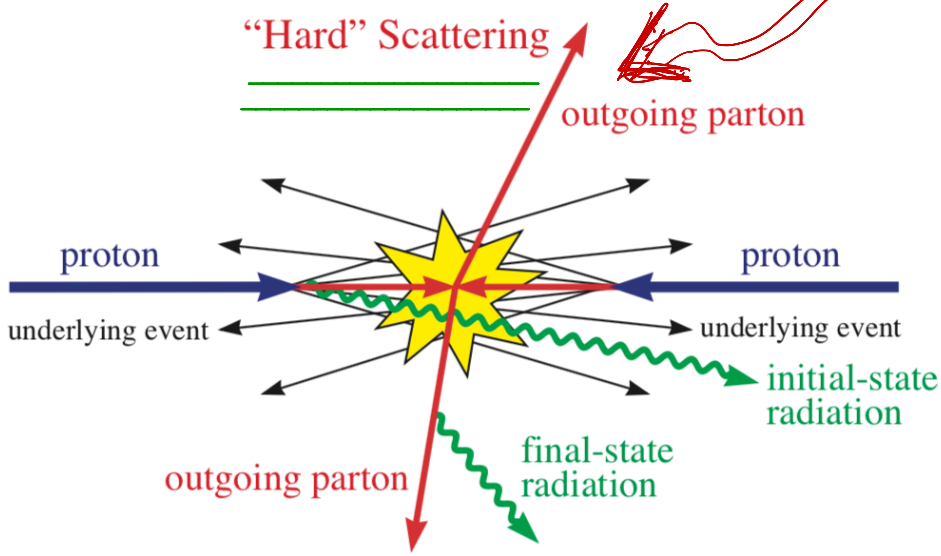
(b) Perturbative hadronic cross section:

$$\sigma_{pp}(S) = \int dx_1 dx_2 P_1(x_1, Q^2) P_2(x_2, Q^2)\ \hat{\sigma}_{parton}(s).$$

- Accurate (higher orders) partonic cross sections $\hat{\sigma}_{parton}(s)$.
- Parton distribution functions to the extreme (density):

$$Q^2 \sim (a\ few\ TeV)^2, \quad x \sim 10^{-3} - 10^{-6}.$$

What we are interested in:



Advantages

- Higher c.m. energy, thus higher energy threshold:

$\sqrt{S} = 14$ TeV: $\quad M_{new}^2 \sim s = x_1 x_2 S \quad \Rightarrow \quad M_{new} \sim 0.3\sqrt{S} \sim 4$ TeV.

- Higher luminosity: $10^{34}/\text{cm}^2/\text{s} \Rightarrow 100\ \text{fb}^{-1}/\text{yr}$.

  Annual yield: 10B $W^\pm$; 100M $t\bar{t}$; 10M W^+W^-; 1M H^0...

- Multiple (strong, electroweak) channels:

  $q\bar{q}'$, gg, qg, $b\bar{b} \to$ colored; $Q = 0, \pm 1$; $J = 0, 1, 2$ states;

  WW, WZ, ZZ, $\gamma\gamma \to I_W = 0, 1, 2$; $Q = 0, \pm 1, \pm 2$; $J = 0, 1, 2$ states.

Disadvantages

- Initial state unknown:

  colliding partons unknown on event-by-event basis;

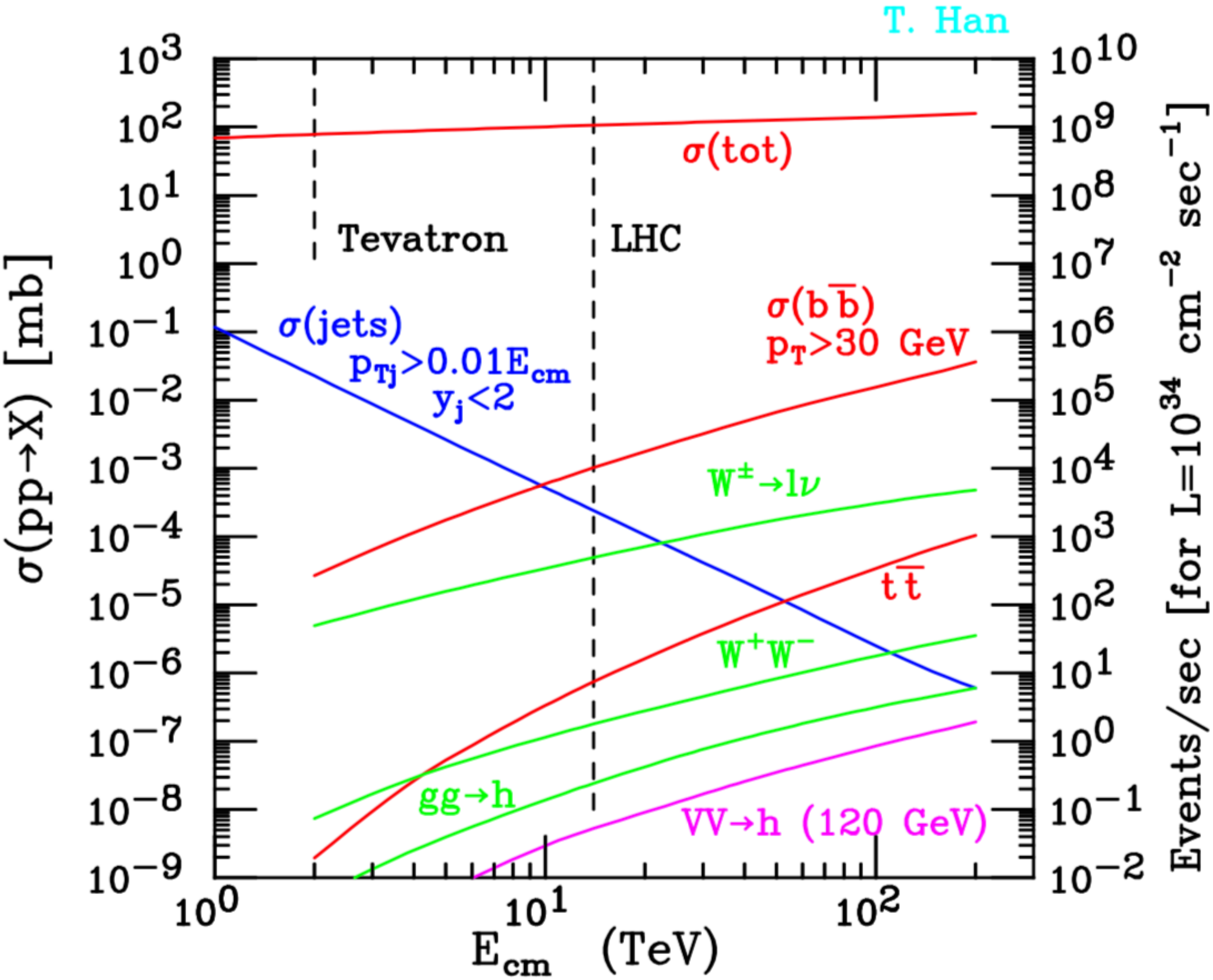
  parton c.m. energy unknown: $E_{cm}^2 \equiv s = x_1 x_2 S$;

  parton c.m. frame unknown.

  $\Rightarrow$ largely rely on final state reconstruction.

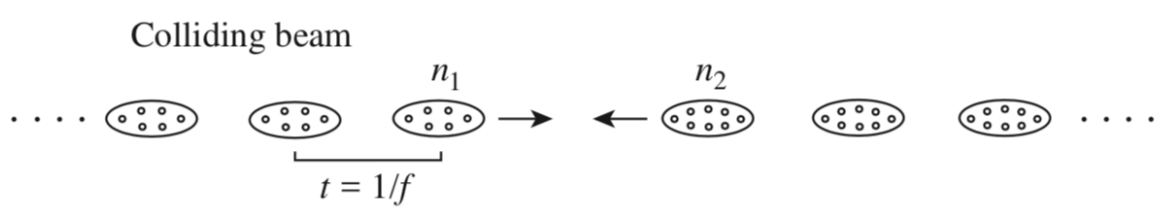
- The large rate turns to a hostile environment:

  $\Rightarrow$ Severe backgrounds!



Experimental challenges:

- The large rate turns to a hostile environment:
  - ≈ 1 billion event/sec: impossible read-off !
  - ≈ 1 interesting event per $1,000,000$: selection (triggering).
  - ≈ 25 overlapping events/bunch crossing:



$\Rightarrow$ Severe backgrounds!

§6.2. Event selection:

(A) "Triggers"

one is unable to tape most events...

Triggering thresholds:

	ATLAS	
Objects	η	p_T (GeV)
μ inclusive	2.4	6 (20)
e/photon inclusive	2.5	17 (26)
Two e's or two photons	2.5	12 (15)
1-jet inclusive	3.2	180 (290)
3 jets	3.2	75 (130)
4 jets	3.2	55 (90)
τ/hadrons	2.5	43 (65)
$\not{E}_T$	4.9	100
Jets+$\not{E}_T$	3.2, 4.9	50,50 (100,100)

($\eta = 2.5 \Rightarrow 10°$; $\eta = 5 \Rightarrow 0.8°$.)

With optimal triggering and kinematical selections:

$p_T \geq 30 - 100$ GeV, $|\eta| \leq 3 - 5$; $\not{E}_T \geq 100$ GeV.

Would like to measure:

Energy momentum observables $\Longrightarrow$ mass parameters

Angular observables $\Longrightarrow$ nature of couplings;

Production rates, decay branchings/lifetimes $\Longrightarrow$ interaction strengths.

(B) Special kinematics for hadronic collisions

Hadron momenta: $P_A = (E_A, 0, 0, p_A), \quad P_B = (E_A, 0, 0, -p_A),$
The parton momenta: $p_1 = x_1 P_A, \quad p_2 = x_2 P_B.$

$$P_{cm} = [(x_1 + x_2)E_A,\ 0,\ 0,\ (x_1 - x_2)p_A] \quad (E_A \approx p_A),$$

Then the parton c.m. frame moves randomly, even by event:

$$\beta_{cm} = \frac{x_1 - x_2}{x_1 + x_2}, \quad \text{or :}$$
$$y_{cm} = \frac{1}{2}\ln\frac{1+\beta_{cm}}{1-\beta_{cm}} = \frac{1}{2}\ln\frac{x_1}{x_2}, \quad (-\infty < y_{cm} < \infty).$$

The four-momentum vector transforms as

$$\begin{pmatrix} E' \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\ \beta_{cm} \\ -\gamma\ \beta_{cm} & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix} = \begin{pmatrix} \cosh y_{cm} & -\sinh y_{cm} \\ -\sinh y_{cm} & \cosh y_{cm} \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix}.$$

This is often called the "boost".

Denote the total hadronic c.m. energy by $S = 4E_A^2$ and the partonic c.m. energy by s, we have

$$s \equiv \tau S, \quad \tau = x_1 x_2 = \frac{s}{S}. \tag{8.3}$$

The parton energy fractions are thus given by

$$x_{1,2} = \sqrt{\tau}\ e^{\pm y_{cm}}. \tag{8.4}$$

One always encounters the integration over the energy fractions as in Eq. (8.15). With this variable change, one has

$$\int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 = \int_{\tau_0}^{1} d\tau \int_{\frac{1}{2}\ln\tau}^{-\frac{1}{2}\ln\tau} dy_{cm}. \tag{8.5}$$

One wishes to design final-state kinematics invariant under the boost:
For a four-momentum $p \equiv p^\mu = (E, \vec{p})$,

$$E_T = \sqrt{p_T^2 + m^2}, \quad y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}, \qquad \text{[HW \#4.1]}$$
$$p^\mu = (E_T \cosh y,\ p_T \sin\phi,\ p_T \cos\phi,\ E_T \sinh y),$$
$$\frac{d^3\vec{p}}{E} = p_T dp_T d\phi\ dy = E_T dE_T d\phi\ dy.$$

$(E_T \simeq p_T$ when $m \to 0)$

Due to random boost between Lab-frame/c.m. frame event-by-event,

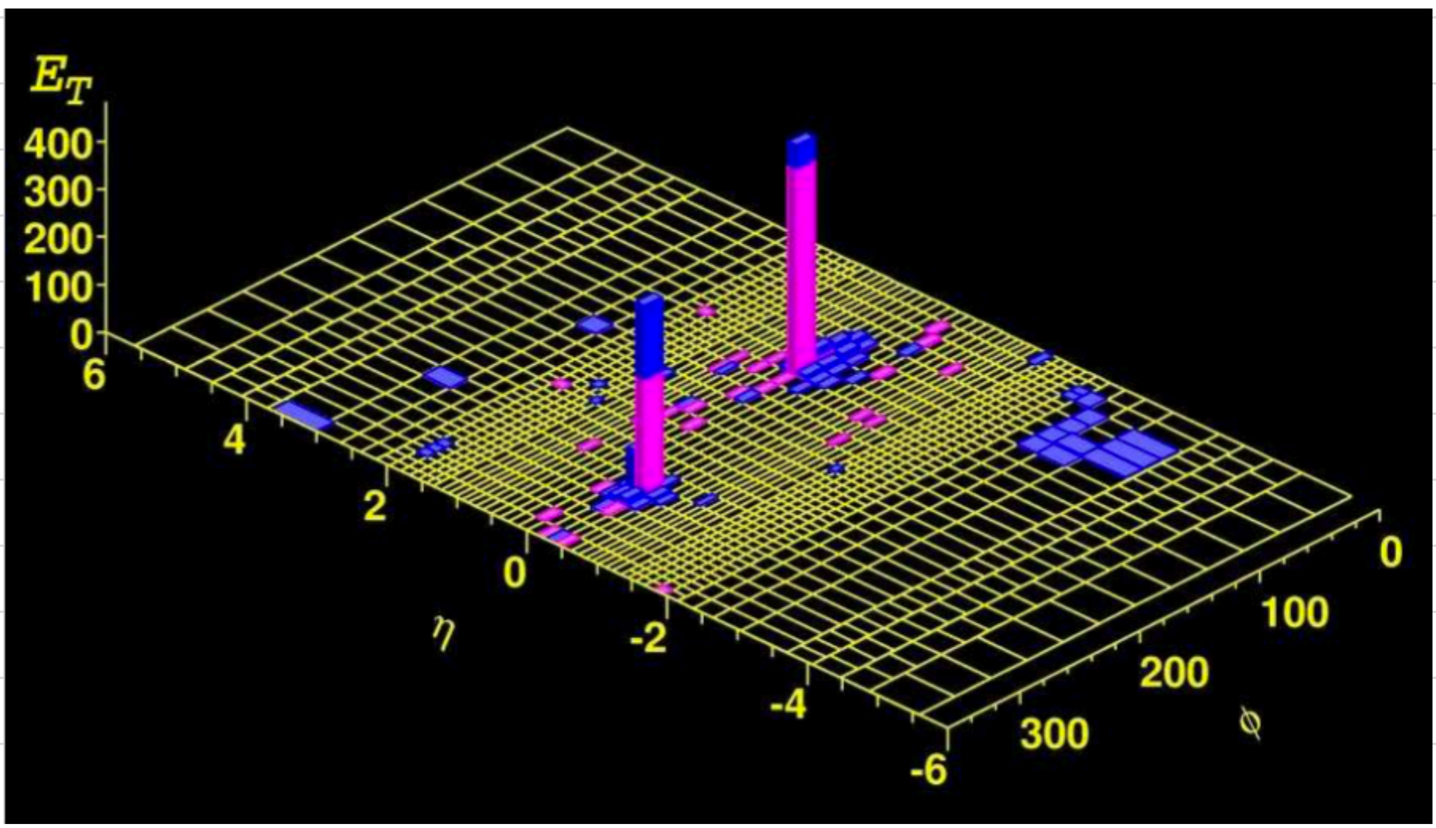
$$y' = \frac{1}{2} \ln \frac{E' + p'_z}{E' - p'_z} = \frac{1}{2} \ln \frac{(1-\beta_{cm})(E+p_z)}{(1+\beta_{cm})(E-p_z)} = y - y_{cm}.$$

In the massless limit, rapidity $\to$ pseudo-rapidity:

$$y \to \eta = \frac{1}{2} \ln \frac{1+\cos\theta}{1-\cos\theta} = \ln \cot \frac{\theta}{2}.$$

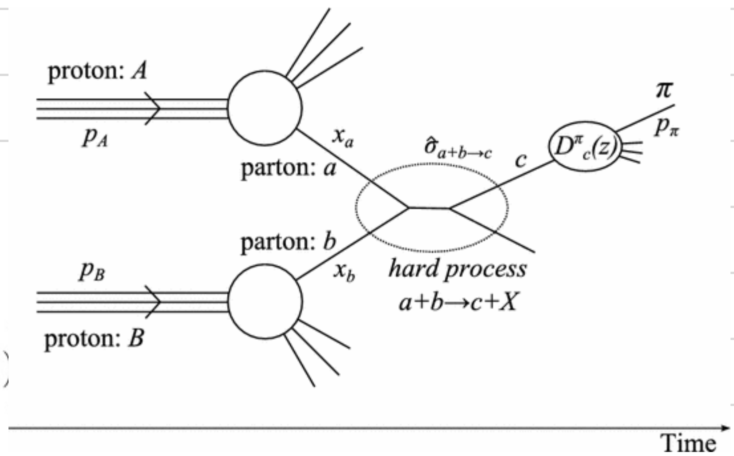
$\phi, \Delta y = y_2 - y_1$ is boost-invariant.

Thus the "separation" between two particles in an event $\Delta R = \sqrt{\Delta\phi^2 + \Delta y^2}$ is boost-invariant, and lead to the "cone definition" of a jet.



§6.3 Partonic Luminosity:

Hadron Collider: partonic collisions have a wide spectrum!



$$\sigma(pp \to X + \text{anything}) = \int_{\tau_0}^{1} d\tau \sum_{ij} \frac{d\mathcal{L}_{ij}}{d\tau}\, \hat{\sigma}(ij \to X),$$

$$\frac{d\mathcal{L}_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}} \int_{\tau}^{1} \frac{d\xi}{\xi} \left[f_{i/p}(\xi, Q_f^2) f_{j/p}\left(\frac{\tau}{\xi}, Q_f^2\right) + (i \leftrightarrow j)\right.$$

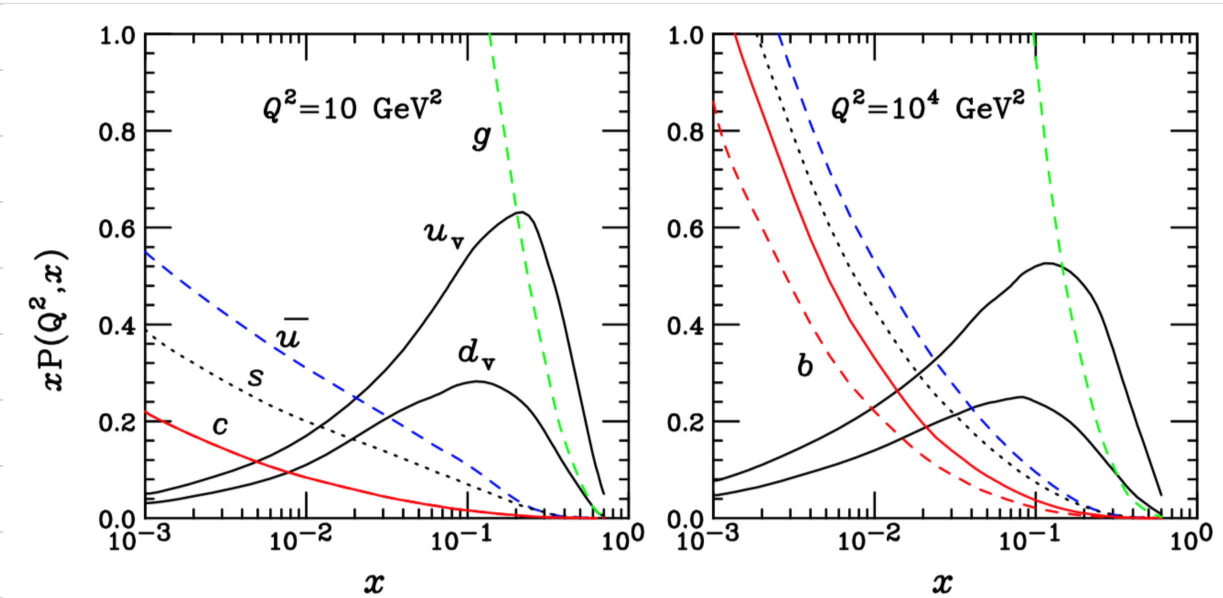


Figure 8.2: Parton momentum distributions versus their energy fraction x at two different factorization scales, from CTEQ-5.

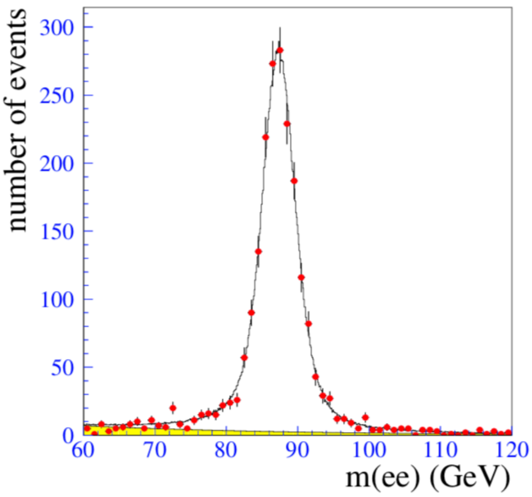
* Valence quarks (u, d) peak at 0.2 – 0.3;
* gluons drive sea quarks, all at small x; gluons lumi large, and larger at high E.

§6.4. S-channel features:

- invariant mass of two-body $R \to ab$: $m_{ab}^2 = (p_a + p_b)^2 = M_R^2$.

combined with the two-body Jacobian peak in transverse momentum:

$$\propto \frac{\Gamma_Z M_Z}{(m_{ee}^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}$$



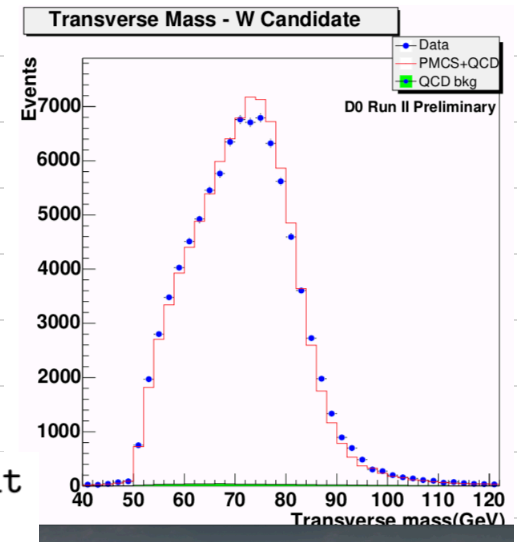
$Z \to e^+e^-$

With a "missing" neutrino, $M_{e\nu}$ not there:

"transverse" mass of two-body $W^- \to e^- \bar{\nu}_e$:

$$\begin{aligned} m_{e\nu\ T}^2 &= (E_{eT} + E_{\nu T})^2 - (\vec{p}_{eT} + \vec{p}_{\nu T})^2 \\ &= 2E_{eT}E_T^{miss}(1 - \cos\phi) \le m_{e\nu}^2. \end{aligned}$$

If $p_T(W) = 0$, then $m_{e\nu\ T} = 2E_{eT} = 2E_T^{miss}$.



In fact, $M_{e\nu,T}$ & P_{eT} related:

For a two-body final state kinematics, show that

$$\frac{d\hat{\sigma}}{dp_{eT}} = \frac{4p_{eT}}{s\sqrt{1 - 4p_{eT}^2/s}} \frac{d\hat{\sigma}}{d\cos\theta^*}.$$

[HW #4.2]

1 - Missing Particle:

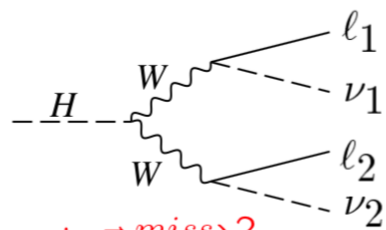
- $H^0 \to W^+W^- \to j_1 j_2\ e^- \bar{\nu}_e$:

_ cluster transverse mass (I):

$$m^2_{WW\ T} = (E_{W_1T} + E_{W_2T})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2$$
$$= (\sqrt{p^2_{jjT} + M_W^2} + \sqrt{p^2_{e\nu T} + M_W^2})^2 - (\vec{p}_{jjT} + \vec{p}_{eT} + \vec{p}_T^{\,miss})^2 \le M_H^2.$$

where $\vec{p}_T^{\,miss} \equiv \vec{\not{p}}_T = -\sum_{obs} \vec{p}_T^{\,obs}$.

2-missing particles:



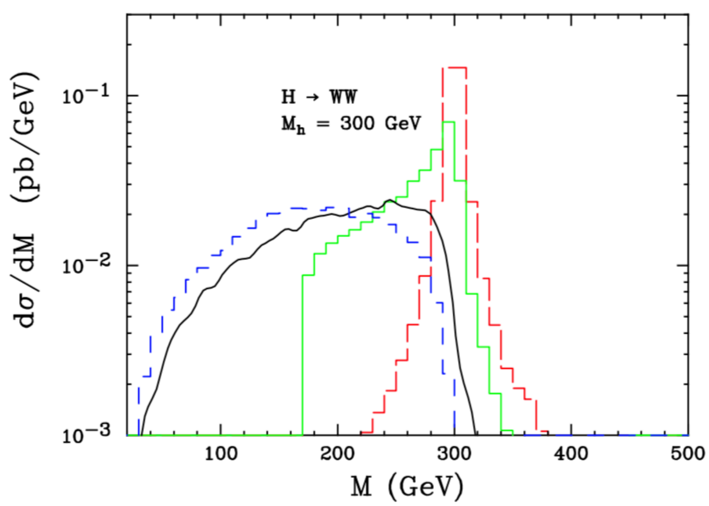
$$m^2_{eff\ T} = (E_{e1T} + E_{e2T} + E_T^{miss})^2 - (\vec{p}_{e1T} + \vec{p}_{e2T} + \vec{p}_T^{\,miss})^2$$
$$m_{eff\ T} \approx E_{e1T} + E_{e2T} + E_T^{miss}$$

this is a Common Variable

cluster transverse mass (II):

$$m^2_{WW\ C} = \left(\sqrt{p^2_{T,\ell\ell} + M^2_{\ell\ell}} + \not{p}_T\right)^2 - (\vec{p}_{T,\ell\ell} + \vec{\not{p}}_T)^2$$
$$m_{WW\ C} \approx \sqrt{p^2_{T,\ell\ell} + M^2_{\ell\ell}} + \not{p}_T$$

It all boils down to the choice of transverse energy!



M_{WW} invariant mass (WW fully reconstructable): - - - - - - - -

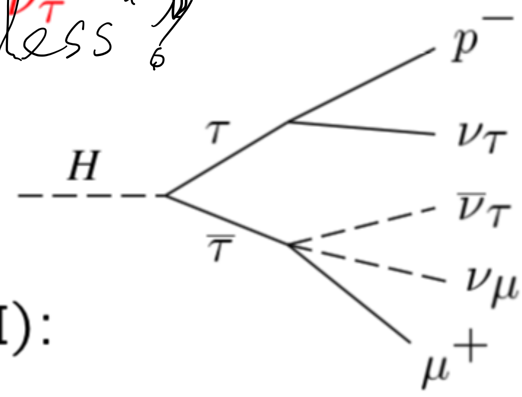
$M_{WW,\ T}$ transverse mass (one missing particle ν): ————

$M_{eff,\ T}$ effetive trans. mass (two missing particles): - - - - - - -

$M_{WW,\ C}$ cluster trans. mass (two missing particles): ————

Many missing particles hopeless? ν_τ

Not necessarily! $\Rightarrow$



- cluster transverse mass (III):

$$H^0 \to \tau^+\tau^- \to \mu^+\ \bar{\nu}_\tau\ \nu_\mu, \quad \rho^-\ \nu_\tau$$

$\tau^+\tau^-$ ultra-relativistic, the final states from a τ decay highly collimated:

$$\theta \approx \gamma_\tau^{-1} = m_\tau/E_\tau = 2m_\tau/m_H \approx 1.5^\circ \quad (m_H = 120 \text{ GeV}).$$

We can thus take

$$\begin{aligned} \vec{p}_{\tau^+} &= \vec{p}_{\mu^+} + \vec{p}_+^{\,\nu's}, \quad \vec{p}_+^{\,\nu's} \approx c_+ \vec{p}_{\mu^+}. \\ \vec{p}_{\tau^-} &= \vec{p}_{\rho^-} + \vec{p}_-^{\,\nu's}, \quad \vec{p}_-^{\,\nu's} \approx c_- \vec{p}_{\rho^-}. \end{aligned}$$

where $c_\pm$ are proportionality constants, to be determined.

Experimental measurements: $p_{\rho^-},\ p_{\mu^+},\ \not{p}_T$:

$$\begin{aligned} c_+(p_{\mu^+})_x + c_-(p_{\rho^-})_x &= (\not{p}_T)_x, \\ c_+(p_{\mu^+})_y + c_-(p_{\rho^-})_y &= (\not{p}_T)_y. \end{aligned}$$

Unique solutions for $c_\pm$ exist if

$$(p_{\mu^+})_x/(p_{\mu^+})_y \neq (p_{\rho^-})_x/(p_{\rho^-})_y.$$

Physically, the τ^+ and τ^- should form a finite angle, or the Higgs should have a non-zero transverse momentum.

This is applicable to any highly-boosted objects: $b \to c\ell\nu$, $t \to b\ell\nu$ ...

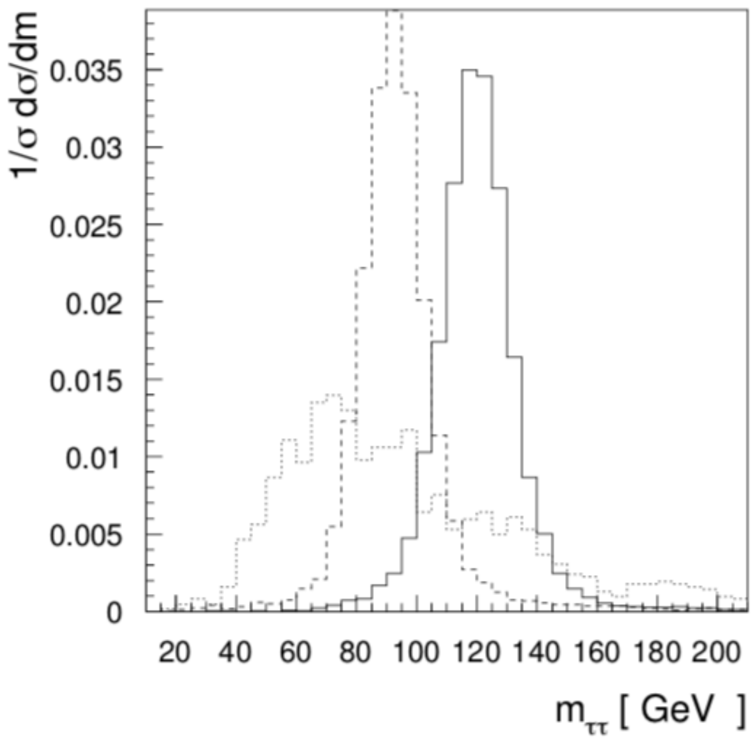
Experimental measurements: $p_{\rho^-},\ p_{\mu^+},\ \not{p}_T$:

$$c_+(p_{\mu^+})_x + c_-(p_{\rho^-})_x = (\not{p}_T)_x,$$
$$c_+(p_{\mu^+})_y + c_-(p_{\rho^-})_y = (\not{p}_T)_y.$$

Unique solutions for $c_\pm$ exist if

$$(p_{\mu^+})_x/(p_{\mu^+})_y \neq (p_{\rho^-})_x/(p_{\rho^-})_y.$$

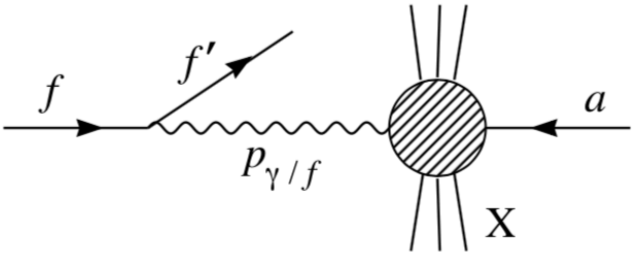
Physically, the τ^+ and τ^- should form a finite angle,
or the Higgs should have a non-zero transverse momentum.



§6.5: t-channel features

splitting functions

The familiar Weizsäcker-Williams approximation

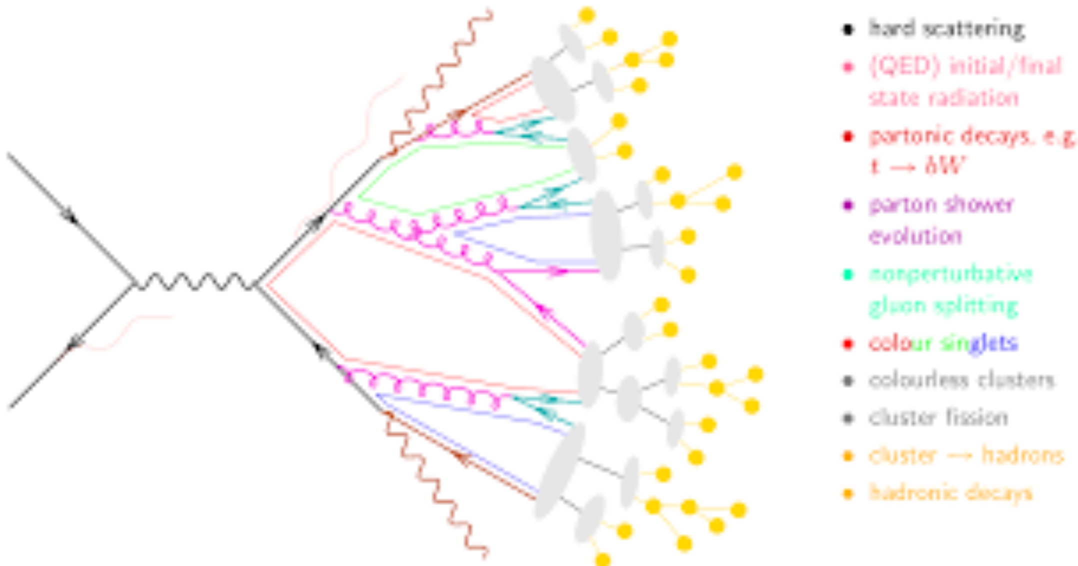


$$\sigma(fa \to f'X) \approx \int dx\, dp_T^2\, P_{\gamma/f}(x, p_T^2)\, \sigma(\gamma a \to X),$$

$$P_{\gamma/e}(x, p_T^2) = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\left(\frac{1}{p_T^2}\right)\Big|_{m_e}^{E}.$$

† The kernel is the same as $q \to qg^*$ $\Rightarrow$ generic for parton splitting;
† The form $dp_T^2/p_T^2 \to \ln(E^2/m_e^2)$ reflects the collinear behavior.

QCD dominates hadron collider physics:



A similar picture may be envisioned for the electroweak massive gauge bosons, $V = W^\pm, Z$.

Consider a fermion f of energy E, the probability of finding a (nearly) collinear gauge boson V of energy xE and transverse momentum p_T (with respect to $\vec{p}_f$) is approximated by

$$
\begin{aligned}
P^T_{V/f}(x, p_T^2) &= \frac{g_V^2 + g_A^2}{8\pi^2} \, \frac{1 + (1-x)^2}{x} \, \frac{p_T^2}{(p_T^2 + (1-x)M_V^2)^2}, \\
P^L_{V/f}(x, p_T^2) &= \frac{g_V^2 + g_A^2}{4\pi^2} \, \frac{1-x}{x} \, \frac{(1-x)M_V^2}{(p_T^2 + (1-x)M_V^2)^2}.
\end{aligned}
$$

Although the collinear scattering would not be a good approximation until reaching very high energies $\sqrt{s} \gg M_V$, it is instructive to consider the qualitative features.

For the accompanying jets,

At low-p_{jT},

$$
\left.\begin{array}{l} p_{jT}^2 \approx (1-x)M_V^2 \\ E_j \sim (1-x)E_q \end{array}\right\} \textit{forward jet tagging}
$$

At high-p_{jT},

$$
\left.\begin{array}{l} \dfrac{d\sigma(V_T)}{dp_{jT}^2} \propto 1/p_{jT}^2 \\ \dfrac{d\sigma(V_L)}{dp_{jT}^2} \propto 1/p_{jT}^4 \end{array}\right\} \textit{central jet vetoing}
$$

has become important tools for Higgs searches, single-top signal etc.

More relevant at higher energies, perhaps beyond the LHC.

§6.6:

(A). Charge forward-backward asymmetry A_{FB}:

The coupling vertex of a vector boson V_μ to an arbitrary fermion pair f

$$i \sum_{\tau}^{L,R} g_\tau^f \, \gamma^\mu \, P_\tau \qquad \rightarrow \qquad \text{crucial to probe chiral structures.}$$

The parton-level forward-backward asymmetry is defined as

all ready shown ...

$$A_{FB}^{i,f} \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3}{4}\mathcal{A}_i \mathcal{A}_f,$$

$$\mathcal{A}_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}.$$

where N_F (N_B) is the number of events in the forward (backward) direction defined in the parton c.m. frame relative to the initial-state fermion $\vec{p}_i$.

At hadronic level:

$$A_{FB}^{\text{LHC}} = \frac{\int dx_1 \sum_q A_{FB}^{q,f} \left(P_q(x_1) P_{\bar{q}}(x_2) - P_{\bar{q}}(x_1) P_q(x_2) \right) \text{sign}(x_1 - x_2)}{\int dx_1 \sum_q \left(P_q(x_1) P_{\bar{q}}(x_2) + P_{\bar{q}}(x_1) P_q(x_2) \right)}.$$

Perfectly fine for Z/Z'-type:

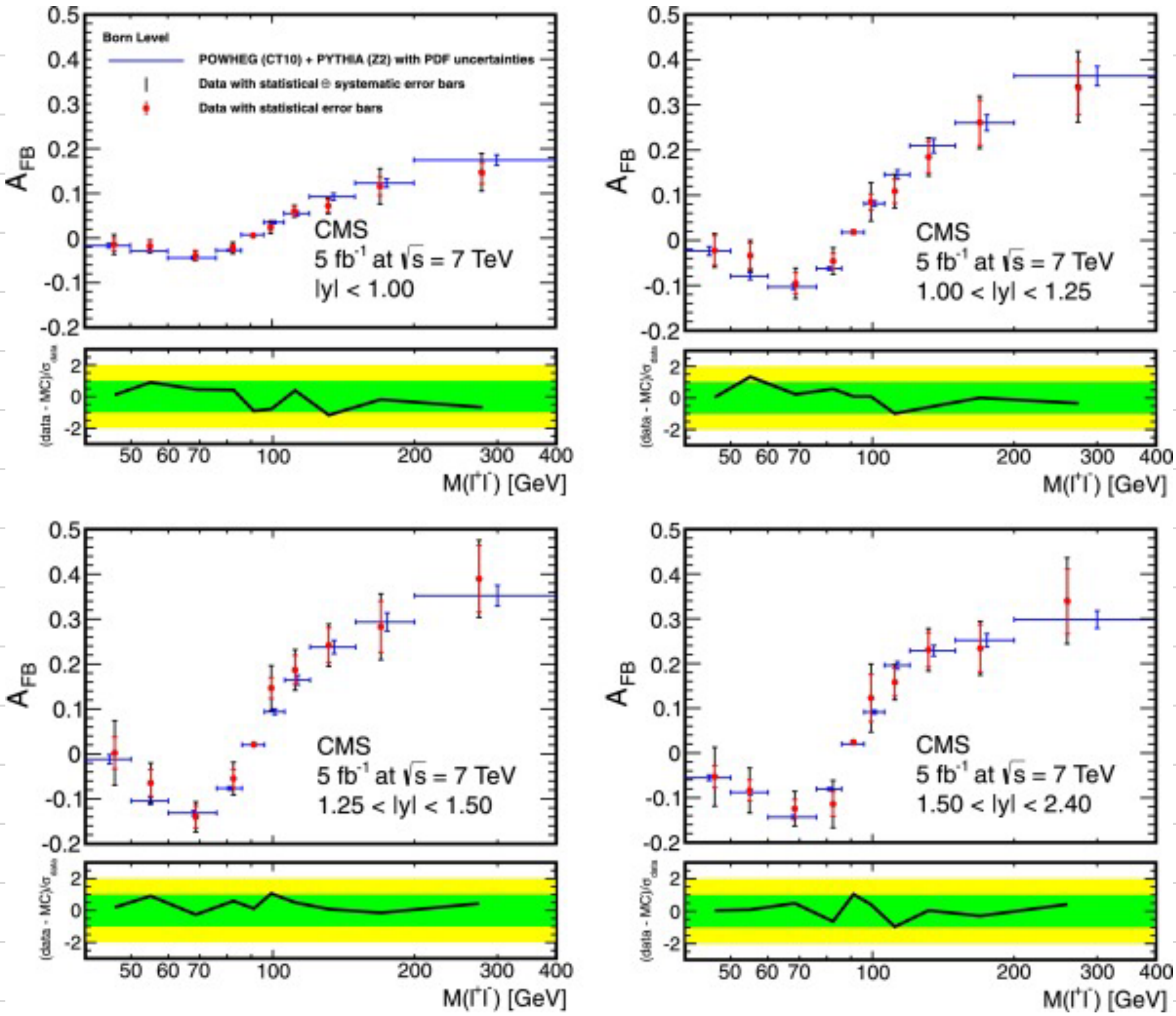
In $p\bar{p}$ collisions, $\vec{p}_{proton}$ is the direction of $\vec{p}_{quark}$.

In pp collisions, however, what is the direction of $\vec{p}_{quark}$?

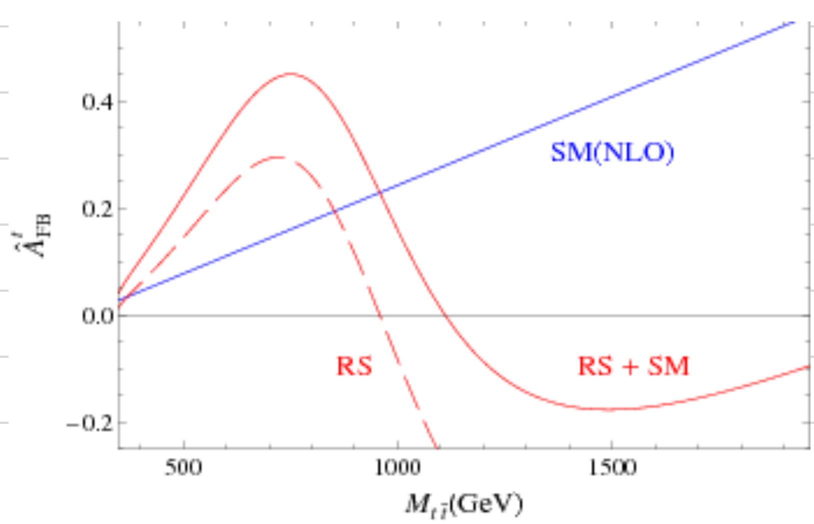
It is the boost-direction of $\ell^+\ell^-$.

LHC $\ell^+\ell^-$ production, CMS:

Sensitive to chiral interactions @ $m(\ell^+\bar{\ell})$.



e.g. Randall-Sundrum $Z' \to t\bar{t}$:



(B).

Definition: A_{CP} vanishes if CP-violation interactions do not exist (for the relevant particles involved).

This is meant to be in contrast to an observable: that'd be *modified* by the presence of CP-violation, but is *not zero* when CP-violation is absent.

e.g. $M_{(\chi^\pm\ \chi^0)},\quad \sigma(H^0,\ A^0),\ldots$

Two ways:

a). Compare the rates between a process and its CP-conjugate process:

$$\frac{R(i\to f)-R(\bar{i}\to\bar{f})}{R(i\to f)+R(\bar{i}\to\bar{f})},\qquad \text{e.g.}\qquad \frac{\Gamma(t\to W^+q)-\Gamma(\bar{t}\to W^-\bar{q})}{\Gamma(t\to W^+q)+\Gamma(\bar{t}\to W^-\bar{q})}.$$

b). Construct a CP-odd kinematical variable for an initially CP-eigenstate:

$$\mathcal{M}\sim M_1+M_2\sin\theta,$$
$$A_{CP}=\sigma^F-\sigma^B=\int_0^1\frac{d\sigma}{d\cos\theta}d\cos\theta-\int_{-1}^0\frac{d\sigma}{d\cos\theta}d\cos\theta$$

Very challenging in hadronic environment!

[HW#4.3]