The Standard Model and the Higgs Boson

3. Properties of the Standard Model Higgs Boson

M. E. Peskin
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This lecture presents the predictions of the Standard Model for the properties of the Higgs boson.

The basics of the theory are extremely simple. A general Higgs field configuration can be simplified by a gauge transformation to the form

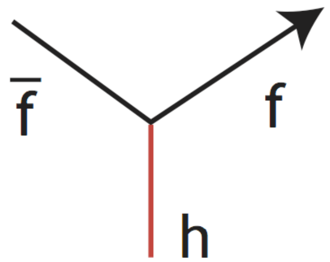
$$\varphi(x) = \exp[-i\alpha^a(x)\sigma^a/2] \begin{pmatrix} 0 \\ (v + h(x))/\sqrt{2} \end{pmatrix}$$

Here v is the vacuum expectation value of the field. From m_W and g, we extract

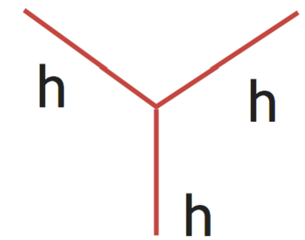
$$v = 250 \text{ GeV}$$

The dynamical part of the field is a single scalar field $h(x)$. The vertices of $h(x)$ are given by shifting v . Thus, the vertices of $h(x)$ are completely determined by known information from the Standard Model.

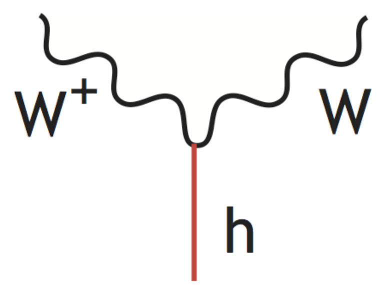
So,



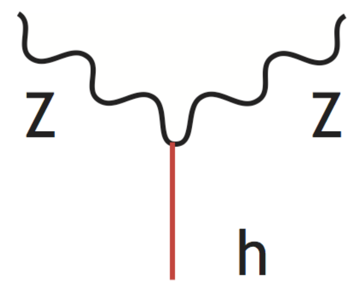
$$= -i\frac{m_f}{v}$$



$$= -3i\frac{m_h^2}{v}$$



$$= 2i\frac{m_W^2}{v}g_{\mu\nu}$$



$$= 2i\frac{m_Z^2}{v}g_{\mu\nu}$$

within the Standard Model, there is no freedom. The decay widths of the Higgs boson will depend on the Higgs boson mass, but, once this is known, these widths can be computed precisely.

It is remarkable that the Higgs field has no flavor-changing interactions. This is a deep property of the Standard Model with 1 Higgs doublet.

So far, we have written the couplings of the Higgs field as between specific flavors, e.g.,

$$y_b \overline{Q} \cdot \varphi b_R$$

However, this is not necessary. Can we allow the most general coupling of Higgs fields and quark and lepton fields? This is

$$\Delta\mathcal{L} = -Y_\ell^{ij} \overline{L}_i \cdot \varphi e_{Rj} - Y_d^{ij} \overline{Q}_i \cdot \varphi d_{Rj} - Y_u^{ij} \overline{Q}_{ia} \epsilon_{ab} \varphi_b u_{Rj} - h.c.$$

where the Y are general complex 3x3 matrices, with i,j running over 3 generations. This manifestly has large flavor and CP violation through Higgs boson couplings.

However, we can change our basis for quarks and leptons, and redefine flavor and CP, to improve this situation.

A general 3x3 complex matrix can be represented as

$$Y = U_L y U_R^\dagger$$

with two (in general, different) unitary matrices and a real positive diagonal matrix. In the lepton sector, we can redefine

$$L \to U_L L \ , \quad e_R \to U_R e_R$$

The U matrices cancel out of the kinetic terms and gauge couplings and disappear without a trace. The resulting Higgs coupling is

$$\mathcal{L} = -y_i \overline{L}_i \cdot \varphi e_{Ri} \quad i = e, \mu, \tau$$

The SM, then, has NO lepton flavor violation. Neutrino mass terms will induce small flavor violations.

In the quark sector, the story is not quite as simple. We need separate redefinitions,

$$d_L \to U_L^{(d)} d_L \ , \quad d_R \to U_R^{(d)} d_R$$
$$u_L \to U_L^{(u)} u_L \ , \quad d_R \to U_R^{(u)} u_R$$

This reduces the Higgs couplings to a diagonal form

$$\mathcal{L} = -y_{di} \overline{L}_i \cdot \varphi d_{Ri} - y_{ui} \overline{Q}_{ia} \epsilon_{ab} u_{Ri}$$

The U matrices cancel out of the kinetic terms and the Z and γ gauge couplings. The the W couplings are modified by a matrix

$$U_L^{(u)\dagger} U_L^{(d)} = V_{CKM}$$

This is the CKM matrix. In the SM, the ONLY source of flavor and CP violation the quark sector is the CKM matrix.

One qualification is necessary for the story on the previous slide. The overall phase of the quark mass matrix cannot be removed, because a needed symmetry is broken by the axial vector anomaly. To remove this phase, which contributes to the neutron electric dipole moment, additional particles or symmetries need to be added to the SM. The simplest solution is to add a very light, weakly coupled particle, the axion.

Aside from this qualification, the Standard Model with one Higgs doublet is a closed theory. It is the most general renormalizable theory of the known elementary particles that is SU(2)xU(1) invariant.

Now let’s use these couplings to predict the decay rates and branching ratios of the Higgs boson.

These couplings imply that a heavy Higgs boson will decay dominantly by

$$h \to W^+W^- \ , \quad h \to ZZ \ , \quad h \to t\bar{t}$$

The theory of these Higgs boson decays is very simple.

However, by now you all know that the LHC experiments exclude a Standard Model Higgs boson in the mass range where decay to these particles would be permitted. The Higgs resonance found at the LHC has a mass of 125 GeV.

Therefore, all of the actual decays of the Higgs boson are suppressed in some way, by factors

$$\frac{m_f^2}{m_W^2} \ , \quad \frac{\alpha_w}{4\pi} \ , \quad (\frac{\alpha_s}{4\pi})^2$$

However, this means that the theory of Higgs boson decays is very rich, with a large number of decay modes accessible.

Begin with the decays to fermions. The matrix element for Higgs decay to a light fermion is

$$i\mathcal{M}(h \to f_R \overline{f}_R) = -i\frac{m_f}{v} u_R^\dagger v_R = -i\frac{m_f}{v}(2E)$$

Summing over final fermion helicities and integrating over phase space

$$\Gamma(h \to f\overline{f}) = \frac{1}{2m_h}\frac{1}{8\pi}\frac{m_f^2 m_h^2}{v^2} \cdot 2$$

or, using $v^2 = 4m_W^2/g^2$

$$\Gamma(h \to f\overline{f}) = \frac{\alpha_w}{8} m_h \frac{m_f^2}{m_W^2}$$

For final leptons, we can immediately evaluate this:

$$\Gamma(h \to \tau^+\tau^-) = 260 \text{ keV} \qquad \Gamma(h \to \mu^+\mu^-) = 9 \text{ keV}$$

for $m_h = 125 \text{ GeV}$.

For quarks, a few more details must be added.

The mass in this formula should be the $\overline{MS}$ mass evaluated at $Q = m_h$. This is related to the quark mass as usually quoted by

$$m_f(m_h) = m_f(m_f)\left[\frac{\alpha_s(m_h)}{\alpha_s(m_f)}\right]^{4/b_0}(1 + \mathcal{O}(\alpha_s))$$

The appropriate values of quark masses (in MeV) are

m_u	m_d	m_s	m_c	m_b
1.5	3	60	700	2800

Also, there is a QCD correction that is larger than the one for e^+e^- annihilation:

$$3(1 + \frac{17}{3\pi}\alpha_s(m_h) + \cdots) = 3 \cdot 1.24$$

Then, for example,

$$\Gamma(h \to b\bar{b}) = \frac{\alpha_w m_h}{8} (\frac{2.8}{m_W})^2 \cdot 3 \cdot (1.24) = 2.4 \text{ MeV}$$

This will turn out to correspond to a BR of 56%. So the total width of the Higgs is about 4 MeV, and the other fermion BRs are

$$\begin{array}{cccc} \tau^+\tau^- & c\bar{c} & s\bar{s} & \mu^+\mu^- \\ 6.3\% & 3\% & 0.03\% & 0.02\% \end{array}$$

(Did you expect that $BR(\tau^+\tau^-) > BR(c\bar{c})$ despite the color factor 3 ?)

For a heavy Higgs that can decay to W and Z bosons on shell, the decay amplitudes would be

$$i\mathcal{M}(h \to W^+W^-) = i\frac{2m_W^2}{v}\ \epsilon_+^* \cdot \epsilon_-^*$$
$$i\mathcal{M}(h \to ZZ) = i\frac{2m_Z^2}{v}\ \epsilon_1^* \cdot \epsilon_2^*$$

For a very heavy Higgs, there is a further enhancement for the longitudinal polarization states

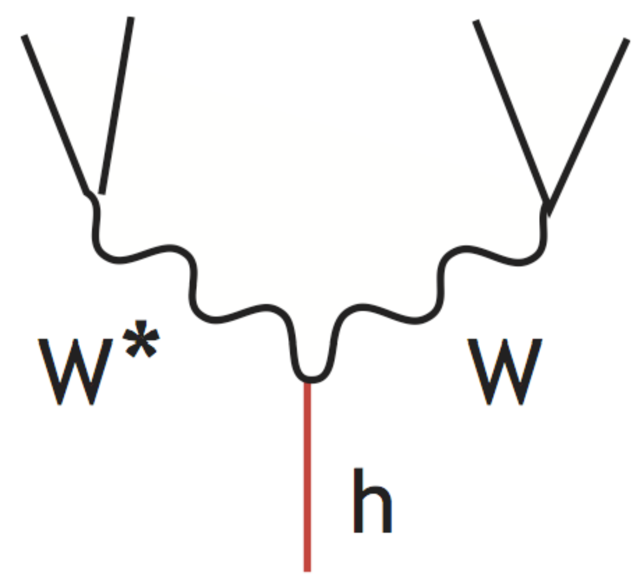
$$\epsilon_1^* \cdot \epsilon_2^* = \frac{k_1 \cdot k_2}{m_Z^2} = \frac{m_h^2}{2m_Z^2}$$

This factor is just

$$\lambda/(g^2 + g'^2)$$

so that the longitudinal Z and W couple like (heavy) Higgs bosons rather than gauge bosons, as predicted by the GBET.

For the actual situation of a 125 GeV Higgs boson, one or both of the Ws or Zs must be off shell. Then the decay is best described as a $h \to 4\ \mathrm{fermion}$ process

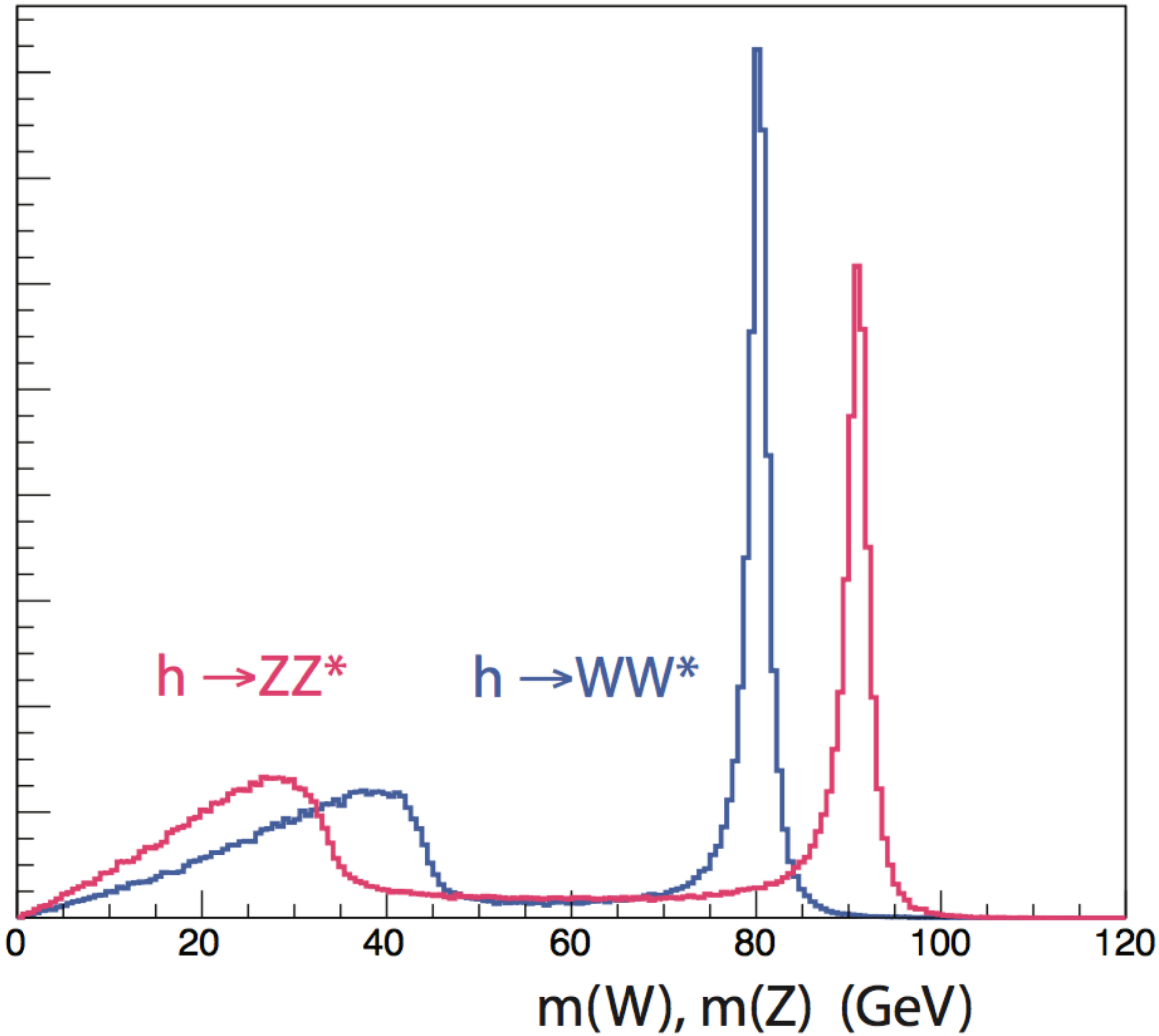


The rate is suppressed by a factor of α_w and by the off-shell W or Z propagator. The result is that the rate is competitive with $b\bar{b}$ for W and a factor 10 smaller for Z.

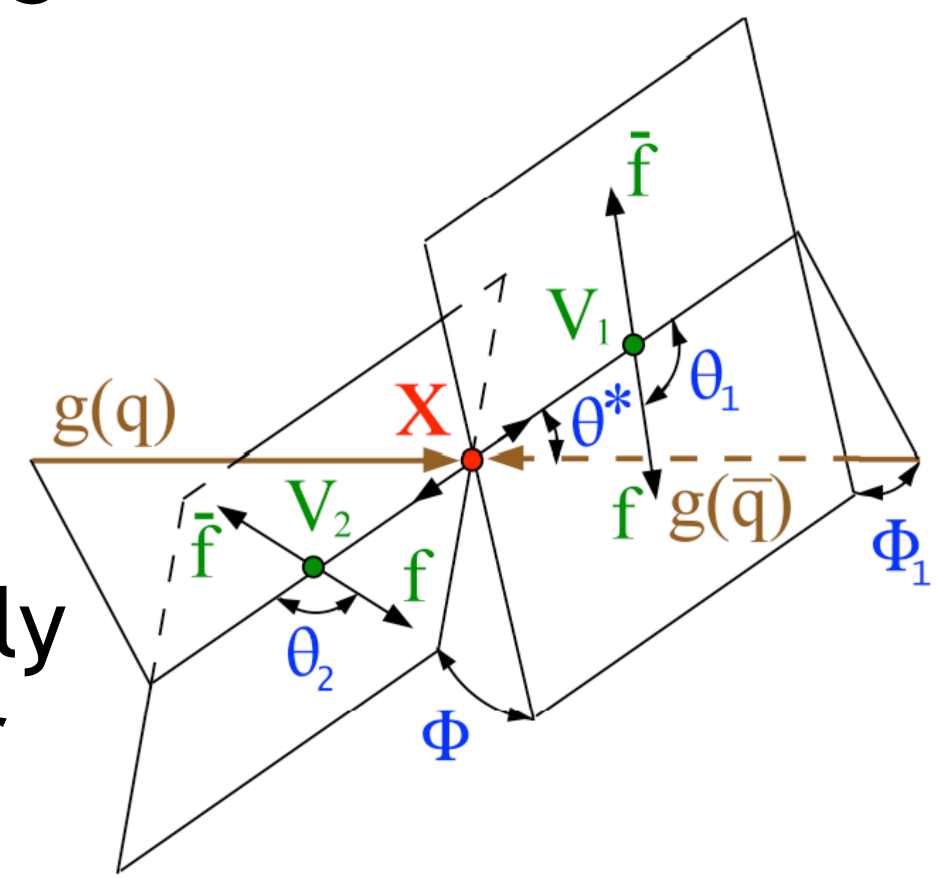
The Standard Model branching fractions are

$$BR(h \to WW^*) = 22\% \qquad BR(h \to ZZ^*) = 2.7\%$$

2-jet mass distributions in h→WW*, ZZ* decays
m(h) = 125 GeV



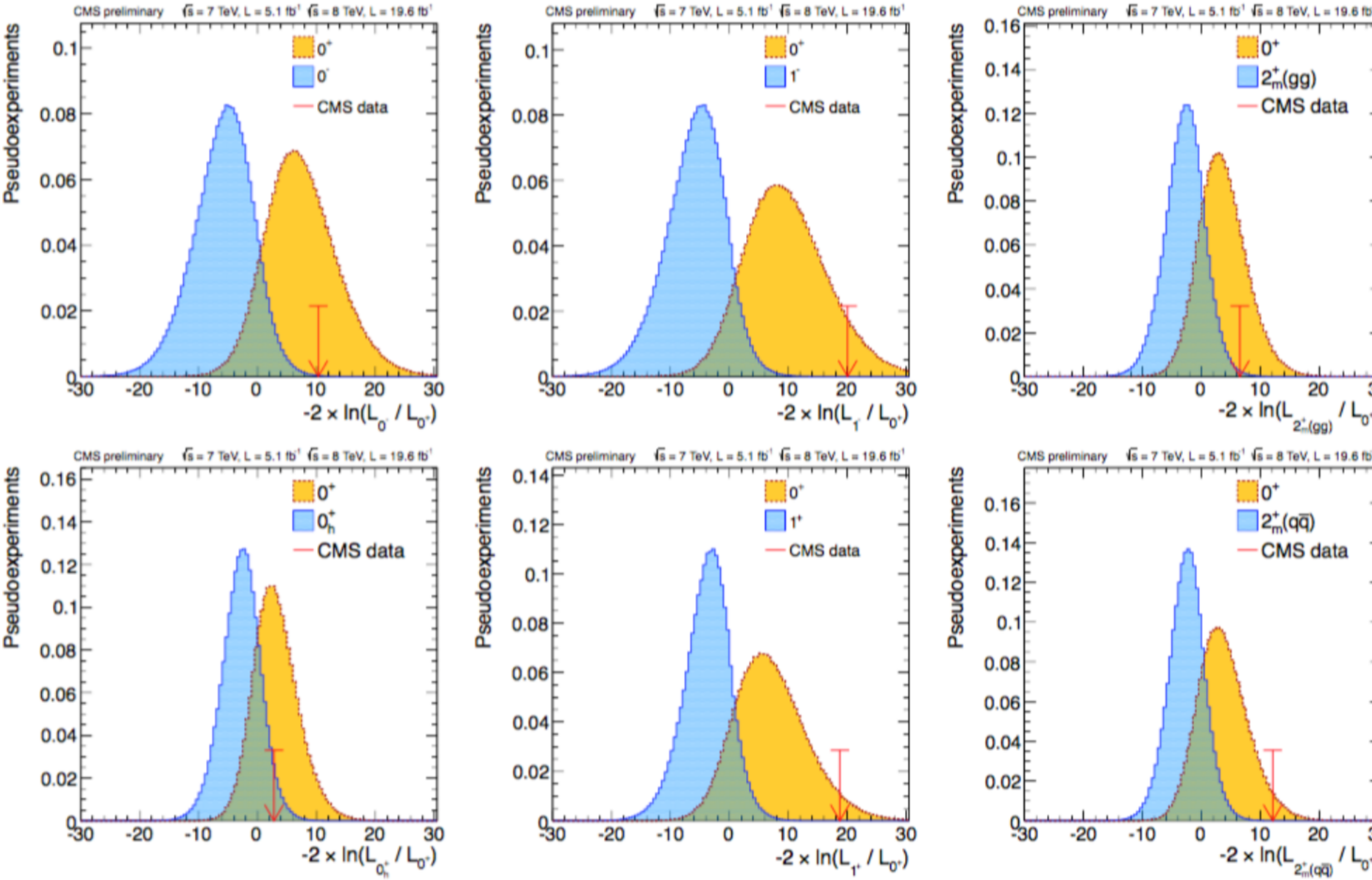
The Higgs decay to ZZ* is exceptionally interesting because it is completely reconstructable when both Zs decay to charged leptons.   The angular distribution of the leptons permits a spin analysis.



For the Standard Model amplitude, the two Zs are preferentially longitudinally polarized, and their decay planes are preferentially parallel.   This contrasts with other possible assignments

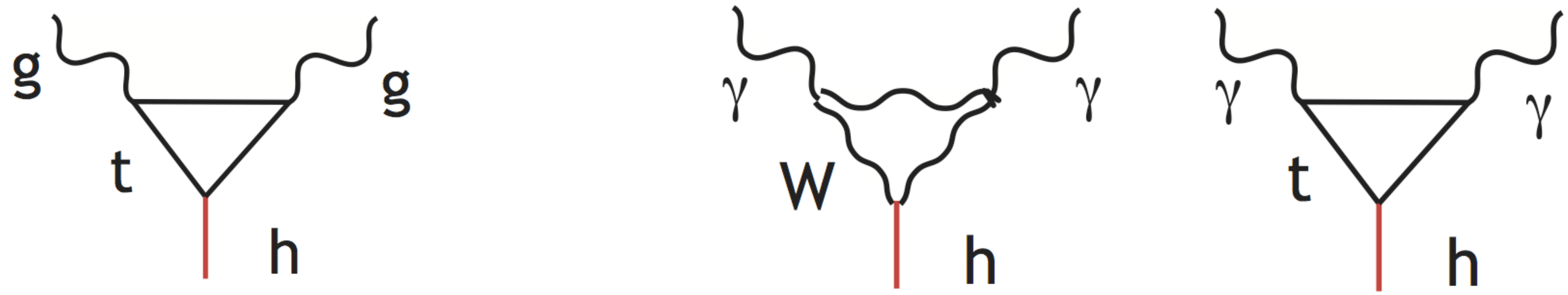
0^- $\quad h\ \epsilon^{\mu\nu\lambda\sigma} Z_{\mu\nu} Z^{\lambda\sigma}$ $\qquad 0_h^+$ $\quad h\ Z_{\mu\nu} Z^{\mu\nu}$

or assignments to spin 2.



CMS preliminary
$\sqrt{s}$ = 7 TeV, L = 5.1 fb^{-1} $\sqrt{s}$ = 8 TeV, L = 19.6 fb^{-1}
Pseudoexperiments
0^+
0^-
CMS data
$-2 \times \ln(L_{0^-} / L_{0^+})$
0^+
1^-
CMS data
$-2 \times \ln(L_{1^-} / L_{0^+})$
0^+
$2^+_m(gg)$
CMS data
$-2 \times \ln(L_{2^+_m(gg)} / L_{0^+})$
0^+
0^+_h
CMS data
$-2 \times \ln(L_{0^+_h} / L_{0^+})$
0^+
1^+
CMS data
$-2 \times \ln(L_{1^+} / L_{0^+})$
0^+
$2^+_m(q\bar{q})$
CMS data
$-2 \times \ln(L_{2^+_m(q\bar{q})} / L_{0^+})$

Finally, there are loop processes that allow the Higgs to decay to massless vector boson states gg and γγ, and to Zγ.



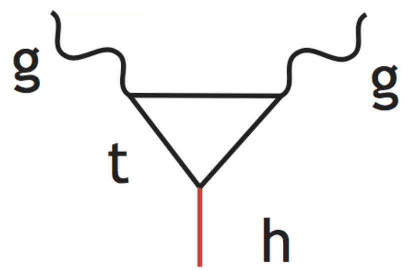
Begin with the hgg vertex. Integrating out the top quark loop gives an effective operator

$$\delta\mathcal{L} = \frac{1}{4} A\ h\ F^a_{\mu\nu} F^{\mu\nu a}$$

where $F^a_{\mu\nu}$ is the QCD field strength and A has dimension $(\mathrm{GeV})^{-1}$. This operator yields the vertex

$$-iA\delta^{ab}(k_1 \cdot k_2 g^{\mu\nu} - k_1^\nu k_2^\mu)$$

For a quark of mass m_f, we might estimate the size of the diagram as



$$\sim \frac{\alpha_s m_f}{v} \cdot \frac{1}{M}$$

where M is the momentum that flows in the loop

$$M \sim \max(m_h, 2m_f)$$

There are two cases: For $2m_f < m_h$, the diagram is suppressed by a factor $2m_f/m_h$. For $2m_f > m_h$, the factors of m_f cancel, and the diagram is at full strength no matter how large m_f is.

So, this diagrams gets large contributions only from those quarks that are too heavy to be decay products of the Higgs. In the Standard Model, this is uniquely the top quark.

To compute the diagram for the top quark, we can start from the top quark QCD vacuum polarization, which has the value

$$i(k^2 g^{\mu\nu} - k^\mu k^\nu)\ \mathrm{tr}[t^a t^b]\ \frac{\alpha_s}{3\pi}\ \log\frac{\Lambda^2}{m_t^2}$$
$$= i(k^2 g^{\mu\nu} - k^\mu k^\nu)\ \delta^{ab}\ \frac{\alpha_s}{6\pi}\ \log\frac{\Lambda^2}{m_t^2}$$

Now introduce a zero momentum Higgs boson by shifting $v \to (v+h)$ where v appears in this expression through

$$m_t^2 = y_t^2 v^2/2$$

The hgg vertex is then

$$i(k^2 g^{\mu\nu} - k^\mu k^\nu)\delta^{ab}\ \frac{\alpha_s}{3\pi}\frac{1}{v}$$

Comparing to our previous expression, we find

$$A = \frac{\alpha_s}{3\pi v} = \frac{g\alpha_s}{6\pi m_W}$$

From this expression, we can compute the partial width $\Gamma(h \to gg)$ in the limit $m_h \ll 4m_t^2$

$$\Gamma(h \to gg) = \frac{\alpha_w \alpha_s^2}{72\pi^2} \frac{m_h^3}{m_W^2}$$

The full expression is

$$\Gamma(h \to gg) = \frac{\alpha_w \alpha_s^2}{72\pi^2} \frac{m_h^3}{m_W^2} \cdot \left| \frac{3}{2}\tau(1 - (\tau - 1)(\sin^{-1} \frac{1}{\sqrt{\tau}})^2) \right|^2$$

where $\tau = 4m_t^2/m_h^2$.

An interesting feature of the argument I have given is that we have related the Higgs coupling to gg to the top quark contribution to the QCD β function. We can use a similar idea to obtain the Higgs coupling to γγ, from the t and W contributions to the QED coupling constant renormalization.

Write the photon vacuum polarization amplitude due to W bosons and top quarks

$$i(k^2 g^{\mu\nu} - k^\mu k^\nu)\frac{\alpha}{4\pi}\left[\overset{W}{-\frac{22}{3}} + \overset{\pi}{\frac{1}{3}} + \overset{t}{\frac{4}{3}\cdot 3 \cdot \left(\frac{2}{3}\right)^2}\right] \log\frac{\Lambda^2}{v^2}$$

$$= -i(k^2 g^{\mu\nu} - k^\mu k^\nu)\frac{\alpha}{3\pi}\left[\frac{21}{4} - \frac{4}{3}\right] \log\frac{\Lambda^2}{v^2}$$

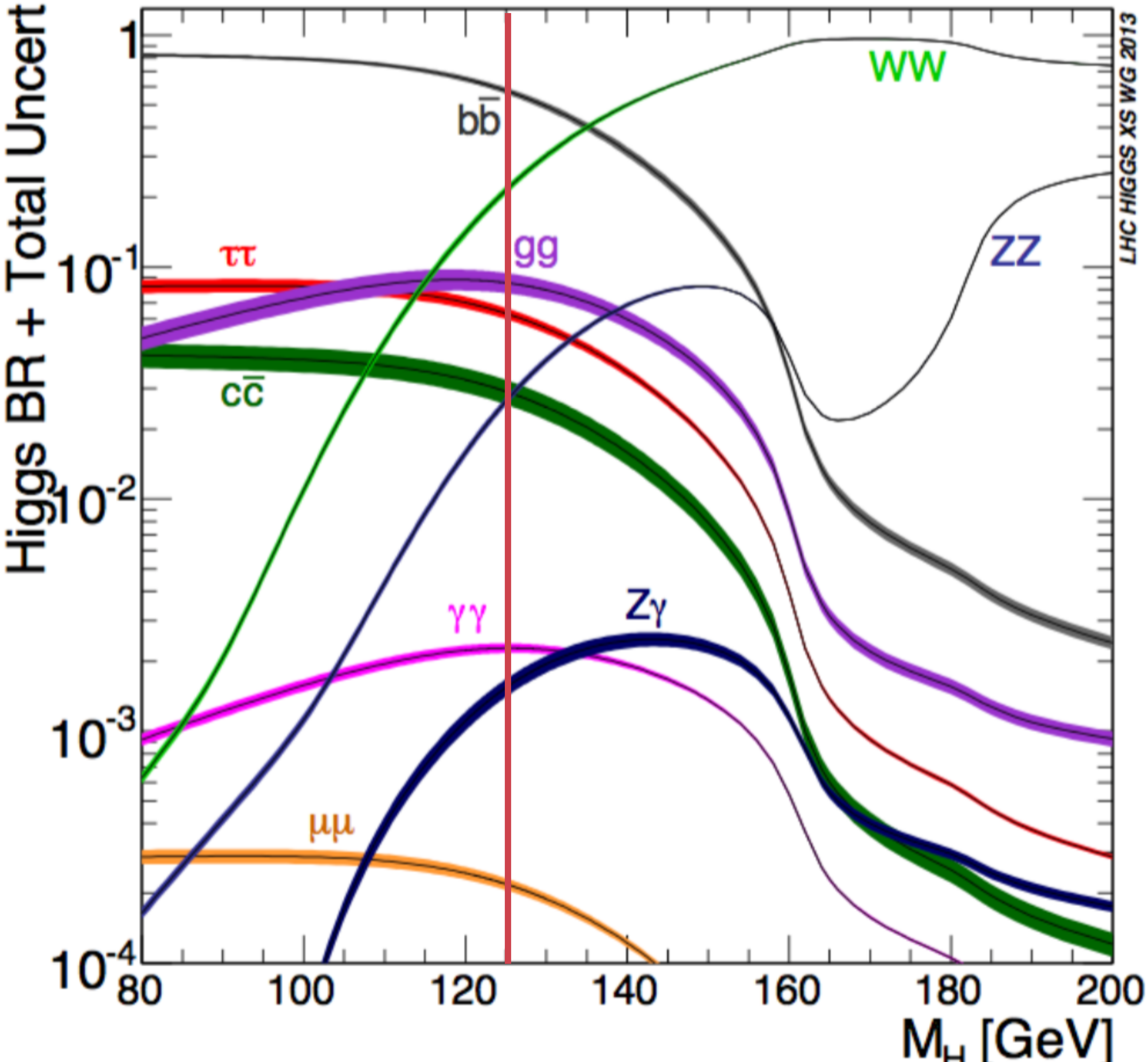
Then, following the same logic, we find in the limit $m_h \ll (2m_W, 2m_t)$

$$\Gamma(h \to \gamma\gamma) = \frac{\alpha_w \alpha^2}{144\pi^2}\frac{m_h^3}{m_W^2}\left|\frac{21}{4} - \frac{4}{3}\right|^2$$

Careful evaluation, including QCD corrections to the gluon width, gives the branching ratios

$$BR(h \to gg) = 8.6\% \qquad BR(h \to \gamma\gamma) = 0.23\%$$

We can now put all of the pieces together and graph the Standard Model predictions for the various branching ratios of the Higgs as a function of the Higgs mass.



LHC Higgs
XS WG

For a Higgs boson of mass 125 GeV, the prediction for the total width is $\Gamma_h = 4.1 \text{ MeV}$.

The branching fractions are predicted to be

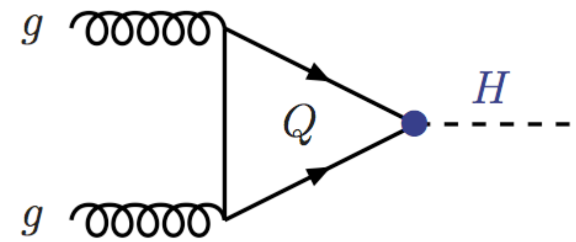
$b\bar{b}$	56%	$\tau^+\tau^-$	6.2%	$\gamma\gamma$	0.23%
WW^*	23%	ZZ^*	2.9%	γZ	0.16%
gg	8.5%	$c\bar{c}$	2.8%	$\mu^+\mu^-$	0.02%

F. Gianotti: “Thank you, Nature.”

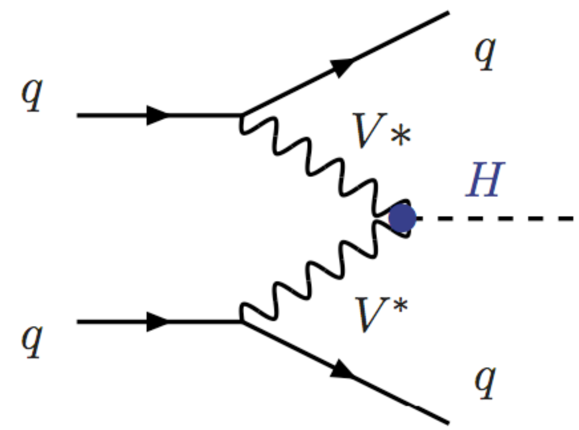
With this introduction to the Standard Model Higgs properties, I can very briefly discuss the study of the Higgs boson at the LHC.

The important production modes for the Higgs boson at hadron colliders are:

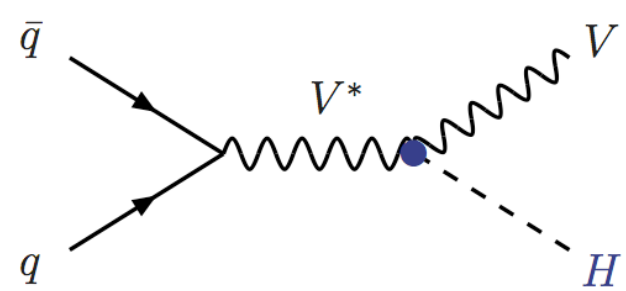
gluon-gluon fusion



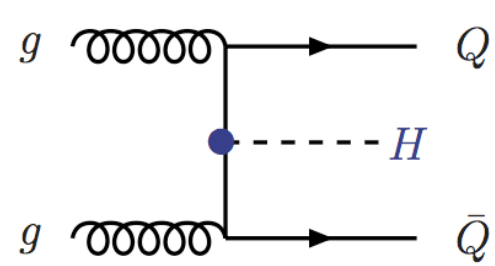
vector boson fusion

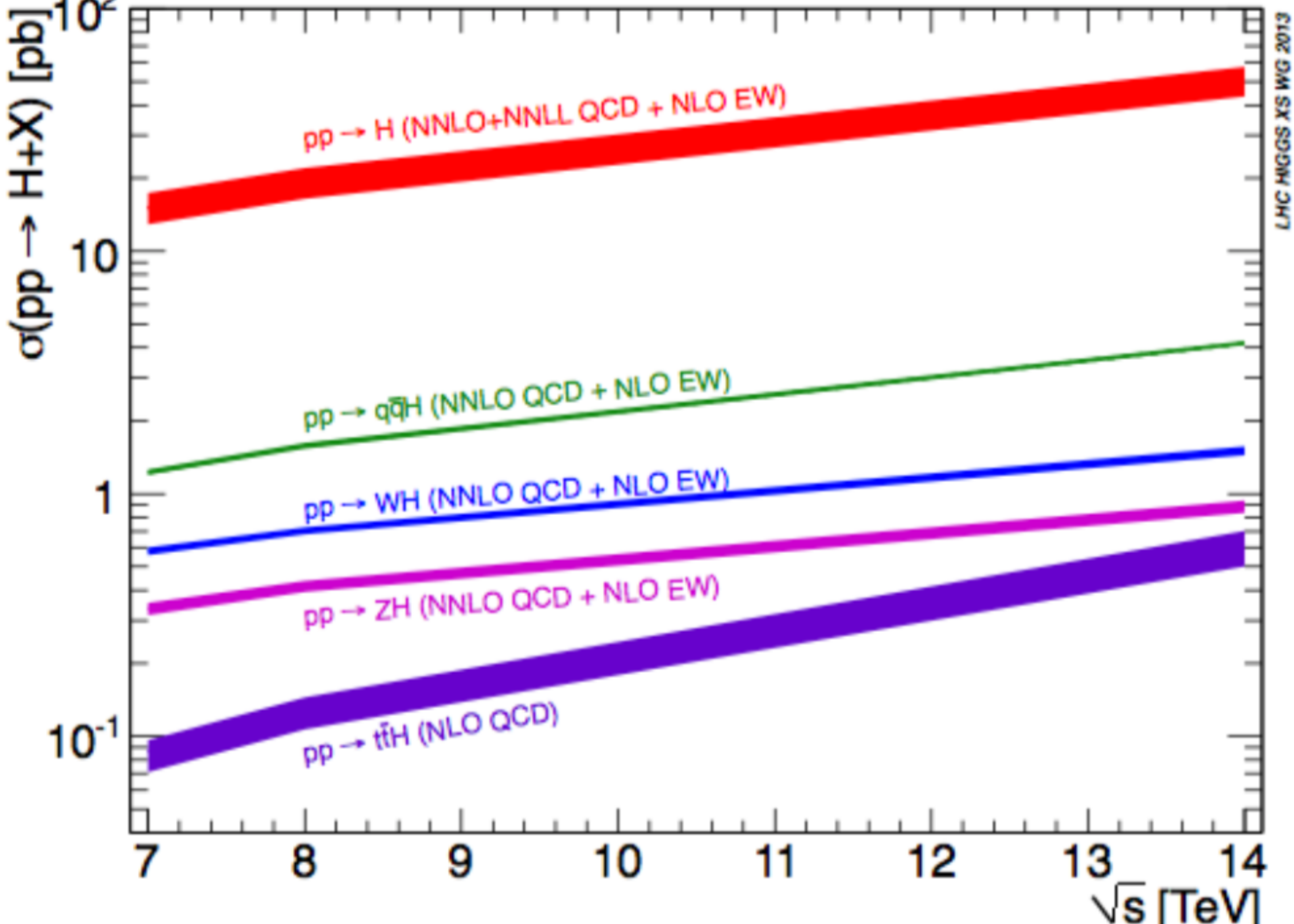


“Higgsstrahlung” associated production w. W, Z



associated production with top





σ(pp → H+X) [pb]
10²
10
1
10⁻¹
7
8
9
10
11
12
13
14
√s [TeV]
pp → H (NNLO+NNLL QCD + NLO EW)
pp → qq̄H (NNLO QCD + NLO EW)
pp → WH (NNLO QCD + NLO EW)
pp → ZH (NNLO QCD + NLO EW)
pp → tt̄H (NLO QCD)
LHC HIGGS XS WG 2013

These four reactions have different advantages for the precision study of Higgs decays:

gluon-gluon fusion:
highest cross section, access to rare decays

WW fusion:
tagged Higgs decays, access to invisible and exotic modes
smallest theoretical error on production cross section

Higgsstrahlung:
tagged Higgs decays
boosted Higgs, for the study of $b\bar{b}$ decay

associated production with top:
access to the Higgs coupling to top

The original strategy for observing the Higgs boson at the LHC used the characteristic decay modes in which the Higgs could be reconstructed as a resonance,

$$h \to \gamma\gamma \qquad\qquad h \to ZZ^* \to \ell^+\ell^-\ell'^+\ell'^-$$

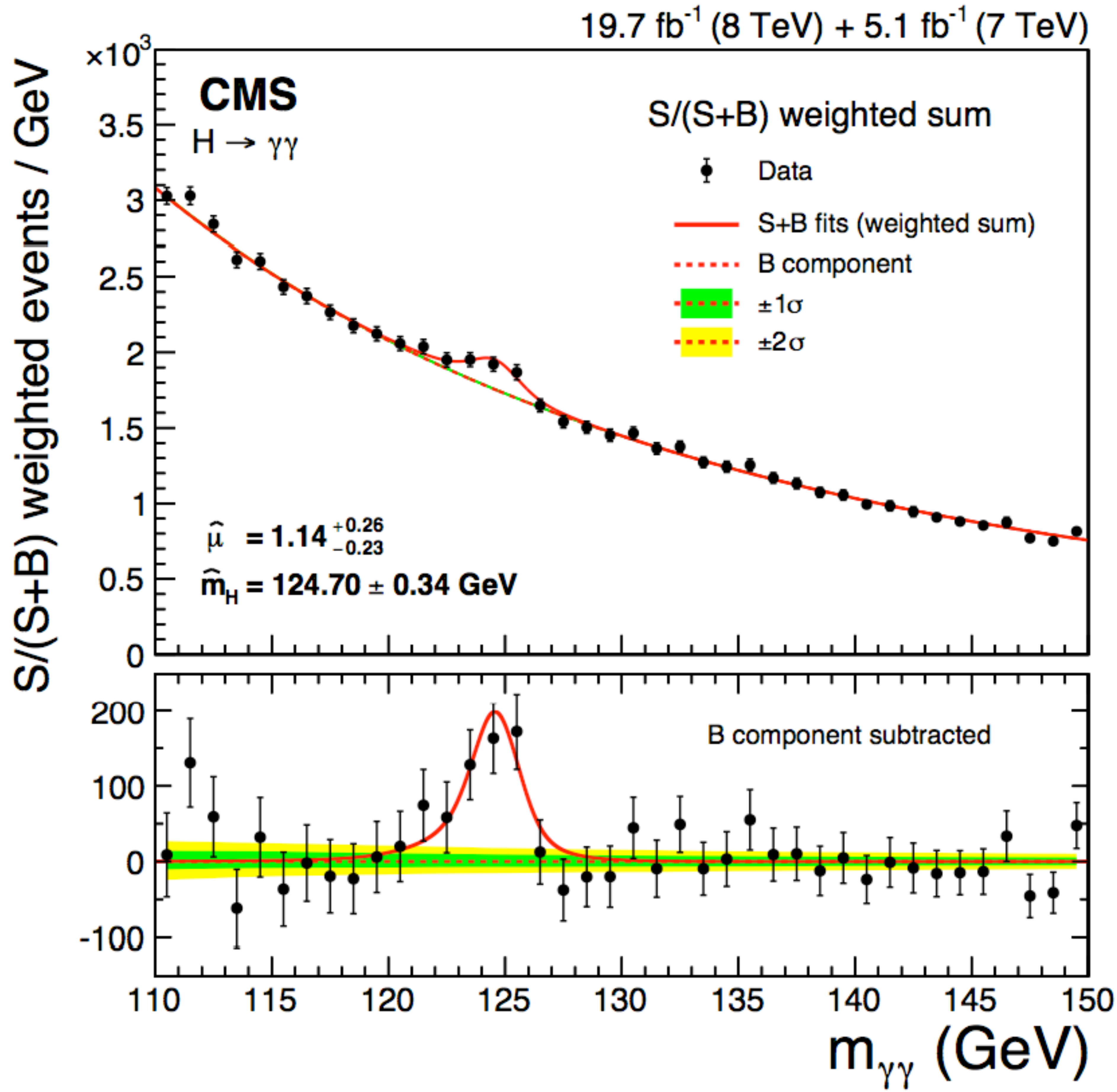
Note that these modes correspond to branching ratios of

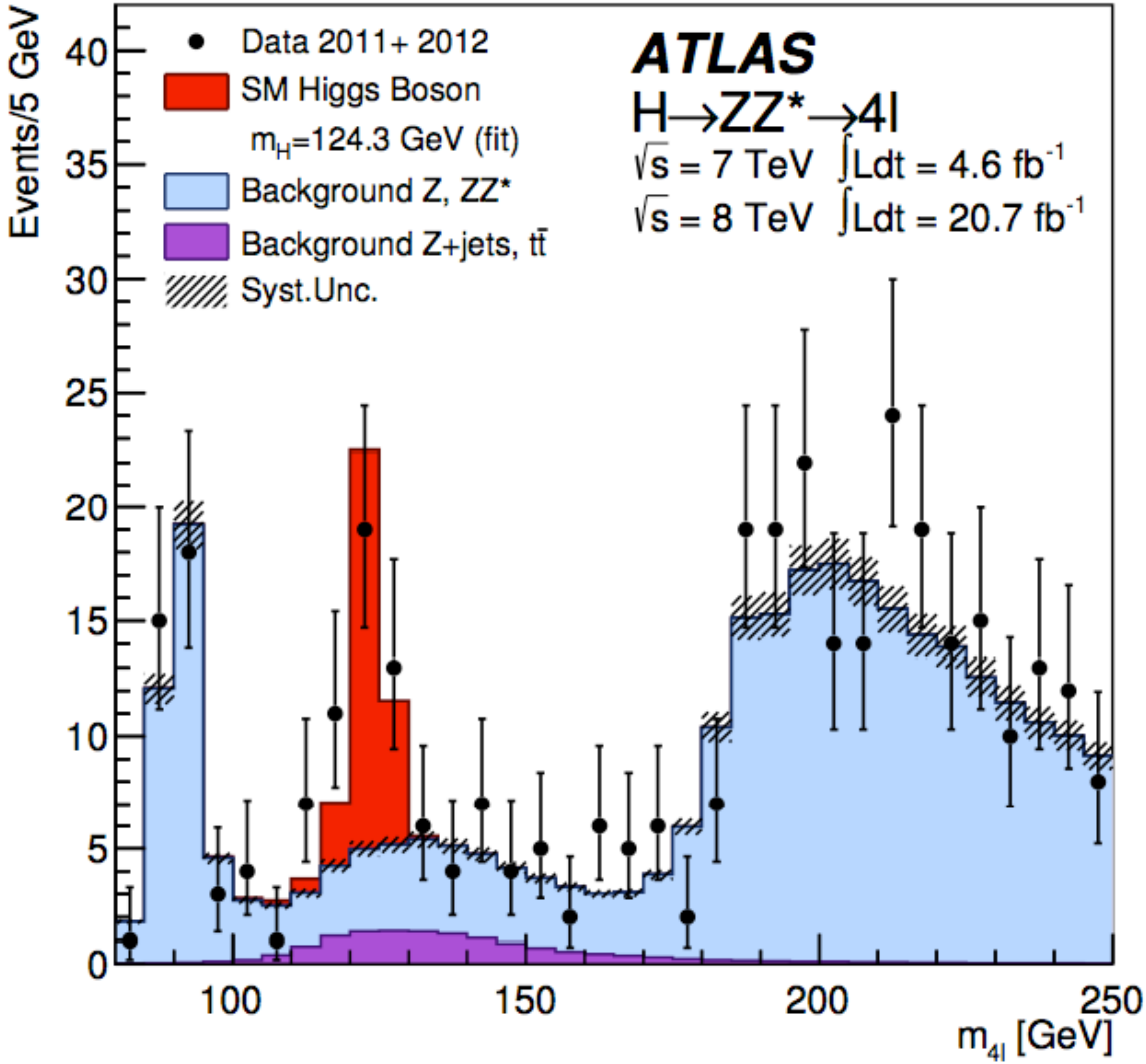
0.23% and 0.012%

respectively. With a production cross section of about 20 pb, these processes have rates

4×10^{-13} and 2×10^{-14}

of the pp total cross section.


19.7 fb-1 (8 TeV) + 5.1 fb-1 (7 TeV)
×10³
CMS
H → γγ
S/(S+B) weighted sum
Data
S+B fits (weighted sum)
B component
±1σ
±2σ
μ̂ = 1.14 +0.26 −0.23
m̂H = 124.70 ± 0.34 GeV
S/(S+B) weighted events / GeV
B component subtracted
mγγ (GeV)



Events/5 GeV
Data 2011+ 2012
SM Higgs Boson
m_H=124.3 GeV (fit)
Background Z, ZZ*
Background Z+jets, $t\bar{t}$
Syst.Unc.
ATLAS
$H \rightarrow ZZ^* \rightarrow 4l$
$\sqrt{s}$ = 7 TeV $\int Ldt$ = 4.6 fb^{-1}
$\sqrt{s}$ = 8 TeV $\int Ldt$ = 20.7 fb^{-1}
m_{4l} [GeV]

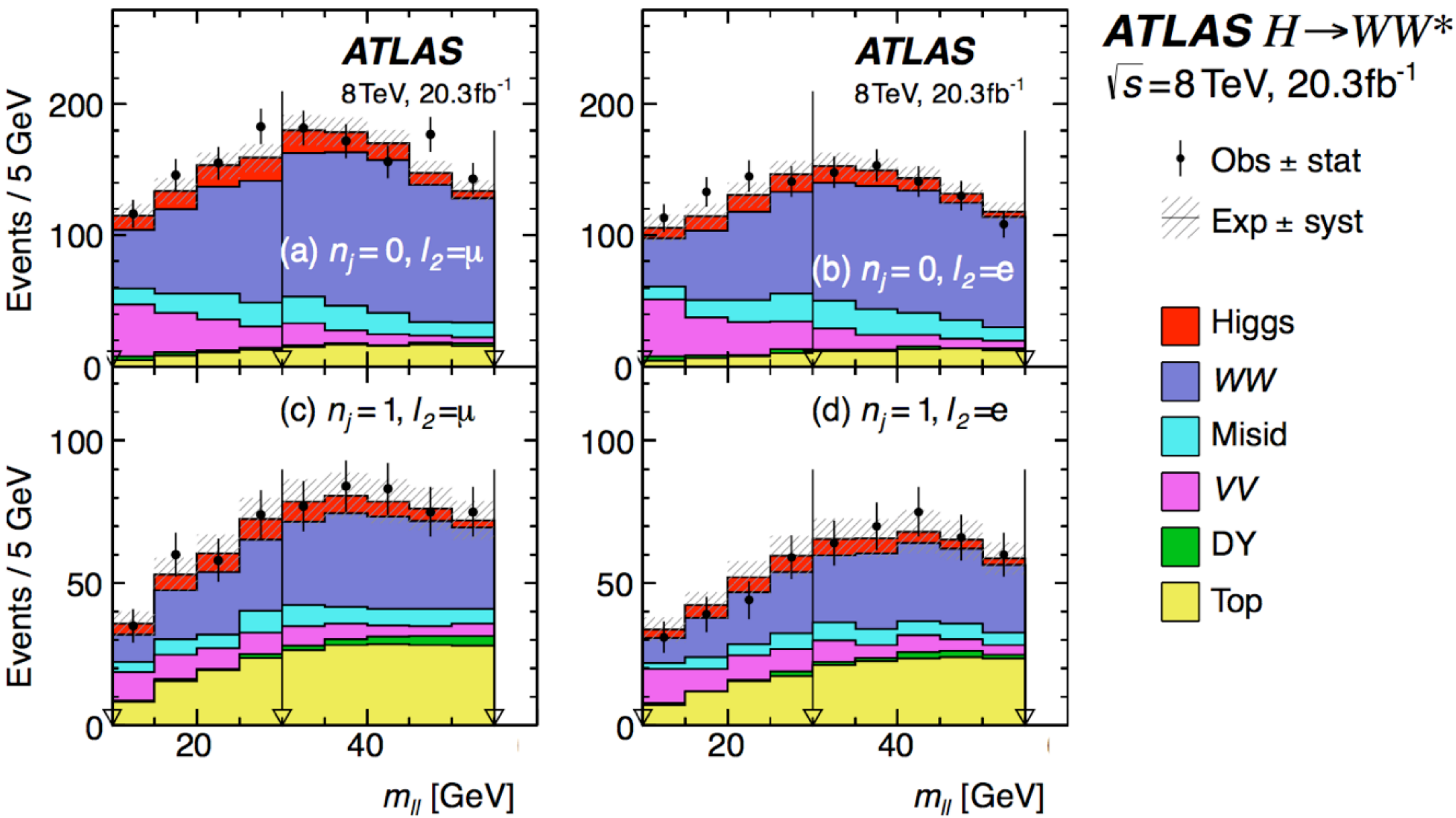
Once we are convinced that the Higgs resonance is actually present at a mass of 125 GeV, we can look for its signatures in other decay modes. These have larger rates, but they produce events that are not obviously distinguishable from other Standard Model reactions.

An example is $pp \to h \to W^+W^- \to \ell^+\ell^-\nu\overline{\nu}$. This is not obviously distinguishable from

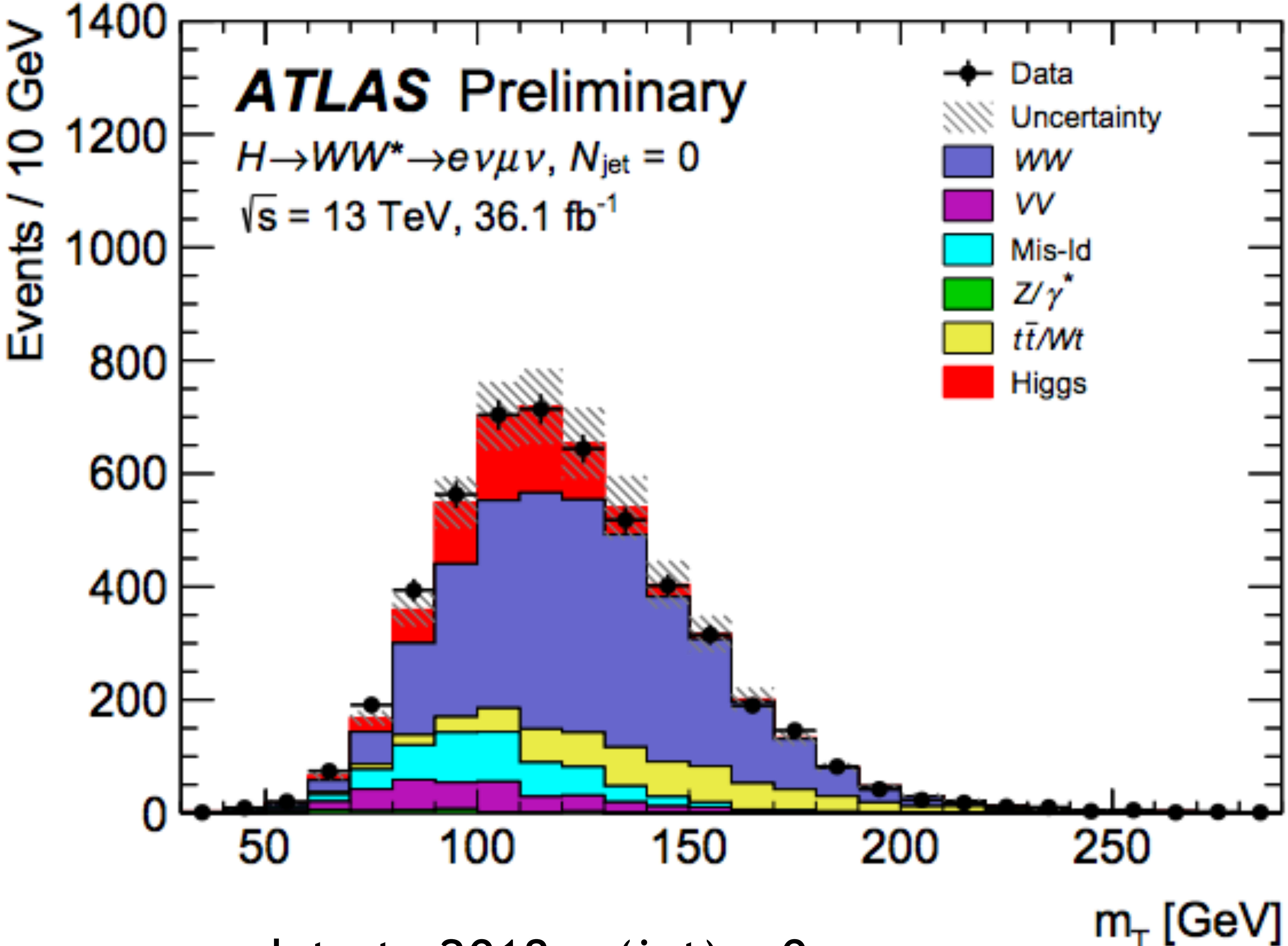
$$pp \to W^+W^- \to \ell^+\ell^-\nu\overline{\nu}$$

The signal to background can be enhanced to going to a region where $m(\ell^+\ell^-)$ and the angle between the two leptons are relatively small. It is also necessary to apply a jet veto ($n_j = 0, 1$) to avoid background from

$$pp \to t\overline{t} \to b\overline{b}\ell^+\ell^-\nu\overline{\nu}$$



ATLAS
8 TeV, 20.3fb-1
(a) $n_j = 0$, $l_2 = \mu$
(b) $n_j = 0$, $l_2 = e$
(c) $n_j = 1$, $l_2 = \mu$
(d) $n_j = 1$, $l_2 = e$
Events / 5 GeV
m_{ll} [GeV]
ATLAS $H \rightarrow WW^*$
$\sqrt{s} = 8$ TeV, 20.3fb^{-1}
Obs ± stat
Exp ± syst
Higgs
WW
Misid
VV
DY
Top

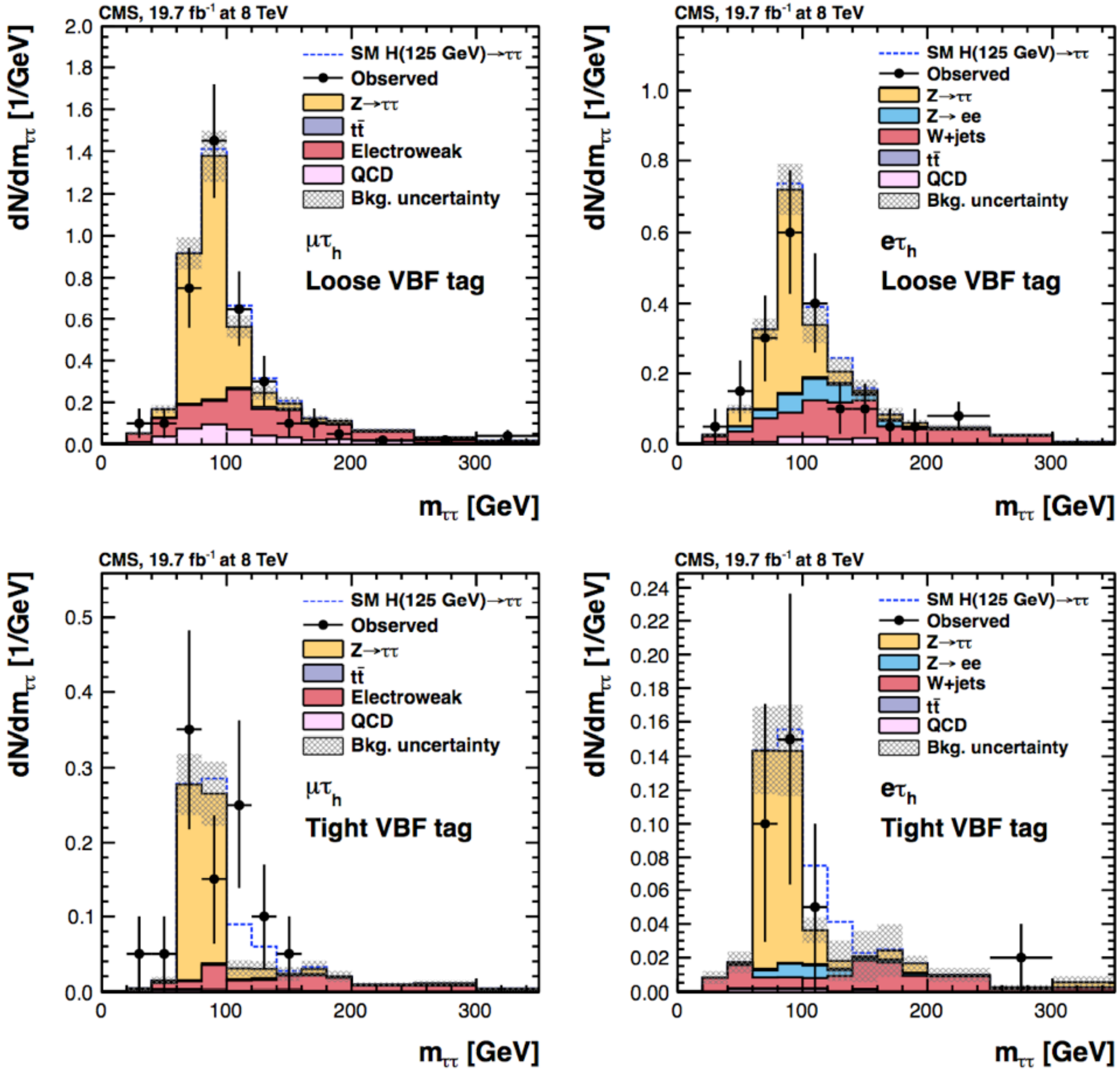


update to 2018, n(jet) = 0

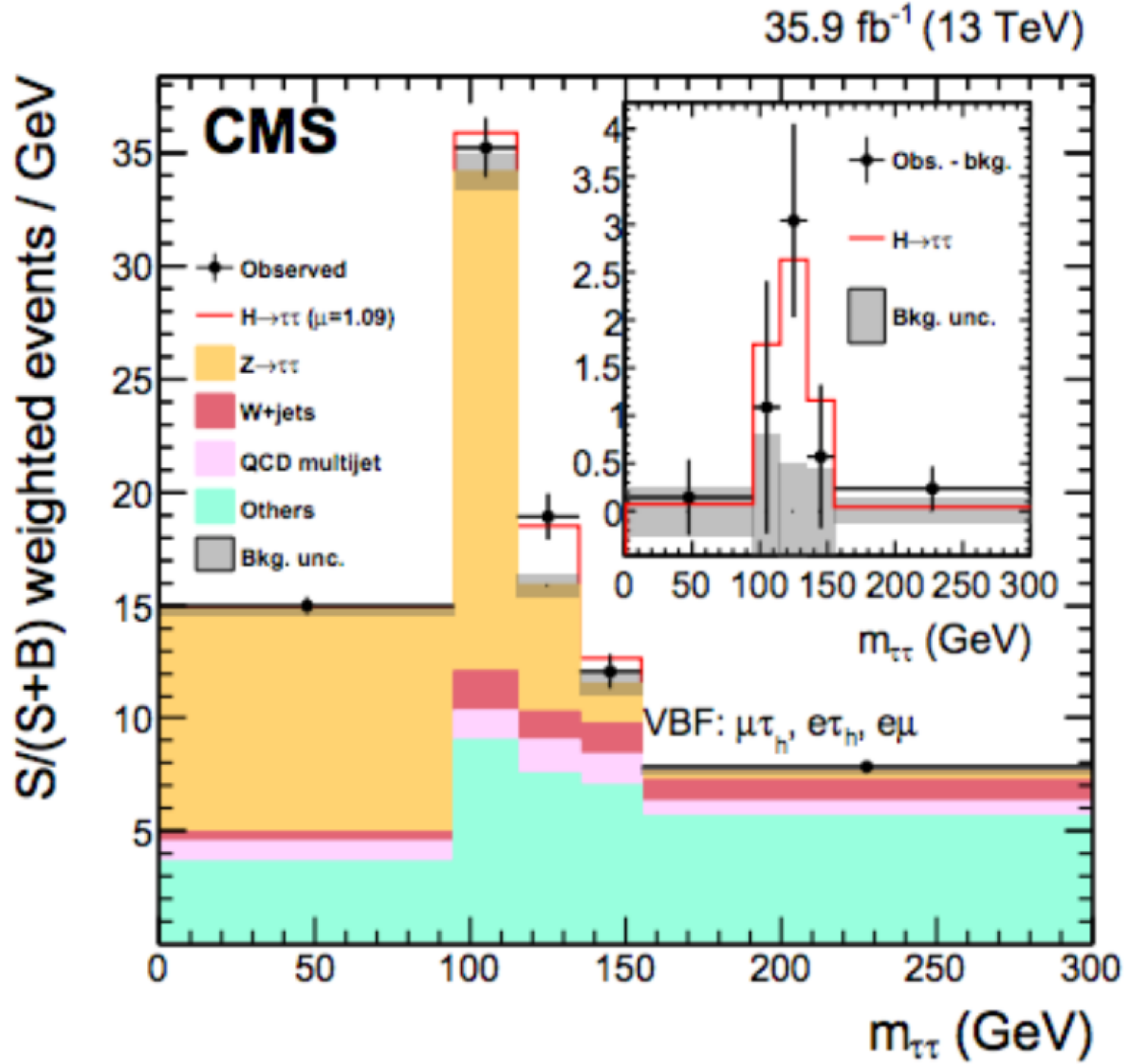
For $pp \to h \to \tau^+\tau^-$, important backgrounds are

$$pp \to Z \to \tau^+\tau^- \qquad\qquad pp \to W^+W^-$$

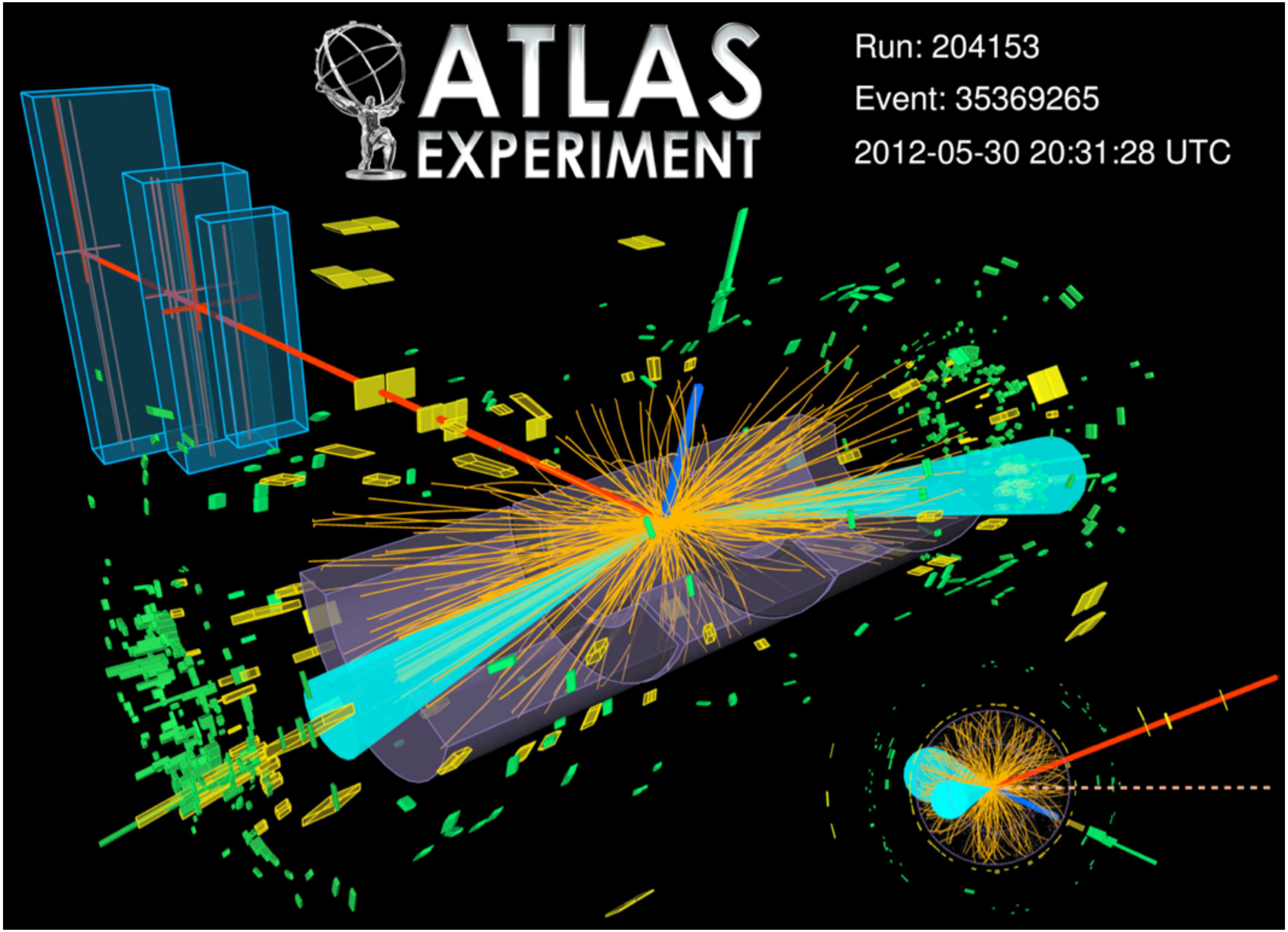
and QCD reactions where jets fake the τ signature. The strongest analyses use the vector boson fusion signature, with forward jets, to minimize the QCD background.



CMS, 19.7 fb-1 at 8 TeV
dN/dmττ [1/GeV]
SM H(125 GeV)→ττ
Observed
Z→ττ
tt̄
Electroweak
QCD
Bkg. uncertainty
μτh
Loose VBF tag
mττ [GeV]
CMS, 19.7 fb-1 at 8 TeV
SM H(125 GeV)→ττ
Observed
Z→ττ
Z→ ee
W+jets
tt̄
QCD
Bkg. uncertainty
eτh
Loose VBF tag
mττ [GeV]
CMS, 19.7 fb-1 at 8 TeV
μτh
Tight VBF tag
mττ [GeV]
CMS, 19.7 fb-1 at 8 TeV
eτh
Tight VBF tag
mττ [GeV]



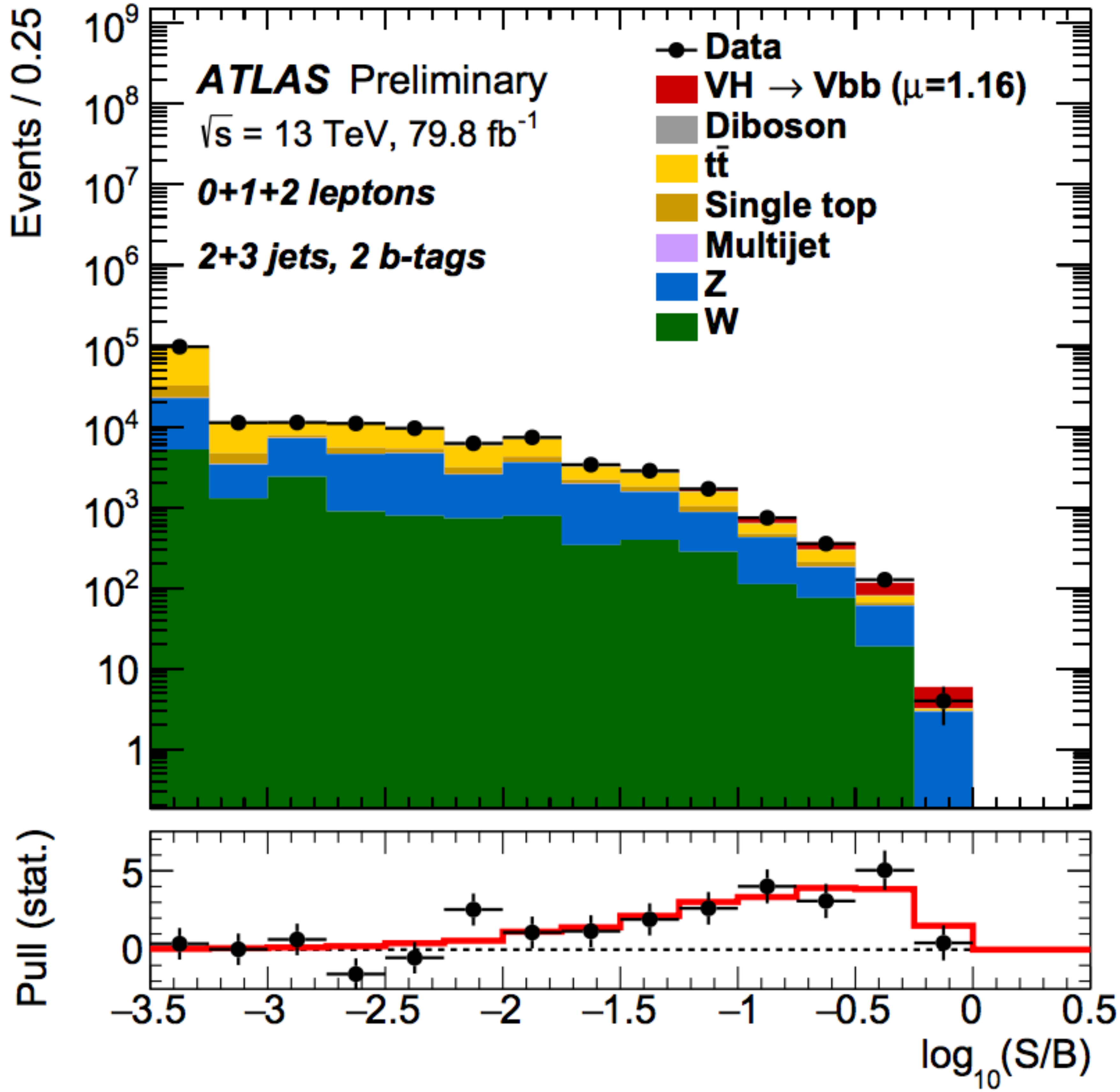
35.9 fb⁻¹ (13 TeV)
CMS
S/(S+B) weighted events / GeV
Observed
H→ττ (μ=1.09)
Z→ττ
W+jets
QCD multijet
Others
Bkg. unc.
Obs. - bkg.
H→ττ
Bkg. unc.
m_ττ (GeV)
VBF: μτ_h, eτ_h, eμ
m_ττ (GeV)


ATLAS
EXPERIMENT
Run: 204153
Event: 35369265
2012-05-30 20:31:28 UTC

The most challenging of the major modes is the largest one, $h \to b\bar{b}$. Observing this mode in gg production is probably hopeless, since $gg \to b\bar{b}$ with 125 GeV mass jets is about a million times larger. Current analyses use associated production with W or Z. However, the reactions

$$
\begin{aligned}
pp &\to Vh \ , \ h \to b\bar{b} \\
pp &\to VZ \ , \ Z \to b\bar{b} \\
pp &\to Vg \ , \ g \to b\bar{b}
\end{aligned}
$$

are difficult to distinguish. It is thought that this can be done using properties of boosted h, Z, g systems including the jet mass and color flow.



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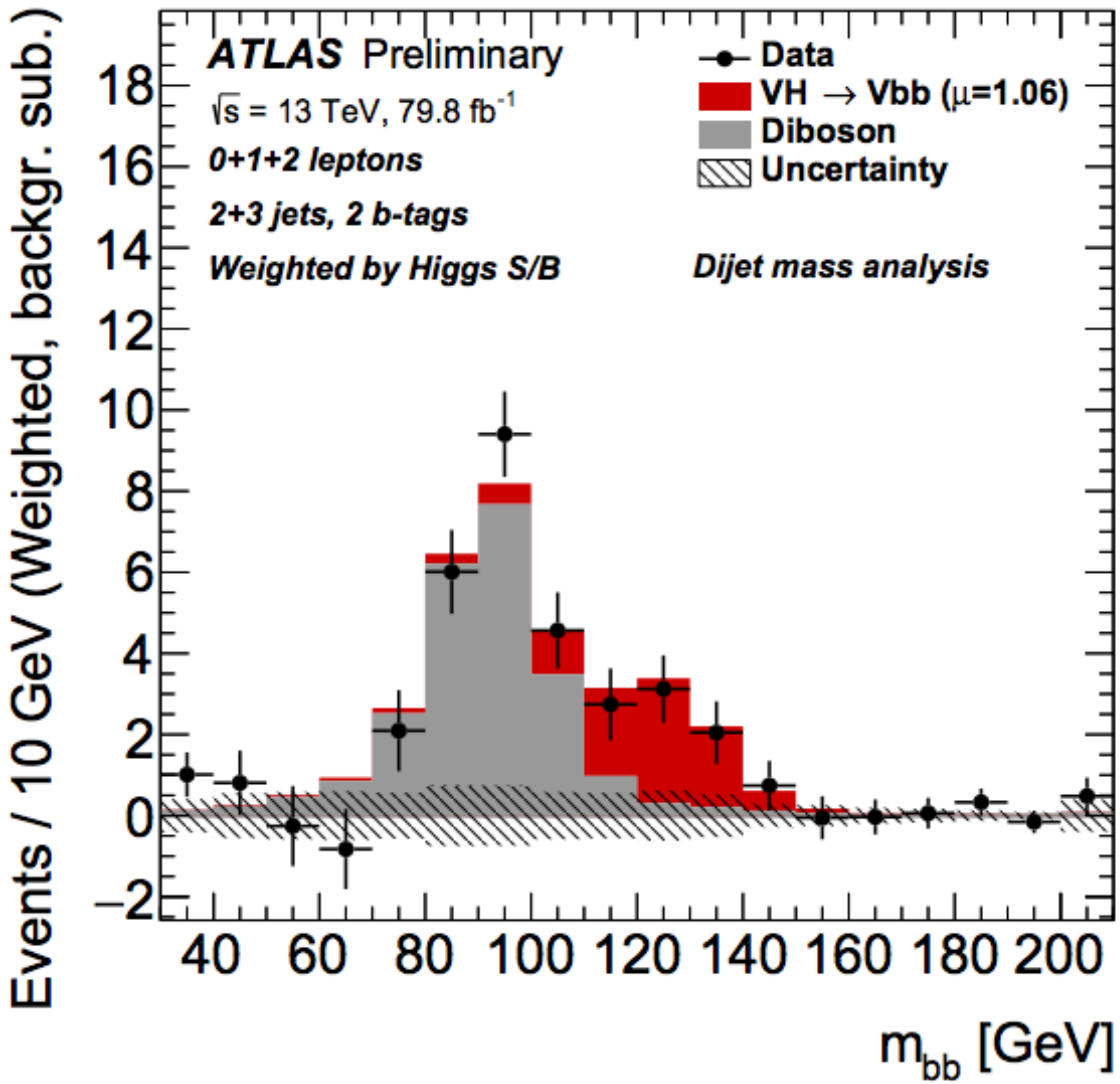
background subtracted:



$$\mu_{VH}^{bb} = 1.16^{+0.27}_{-0.25} = 1.16 \pm 0.16(\text{stat.})^{+0.21}_{-0.19}(\text{syst.})$$

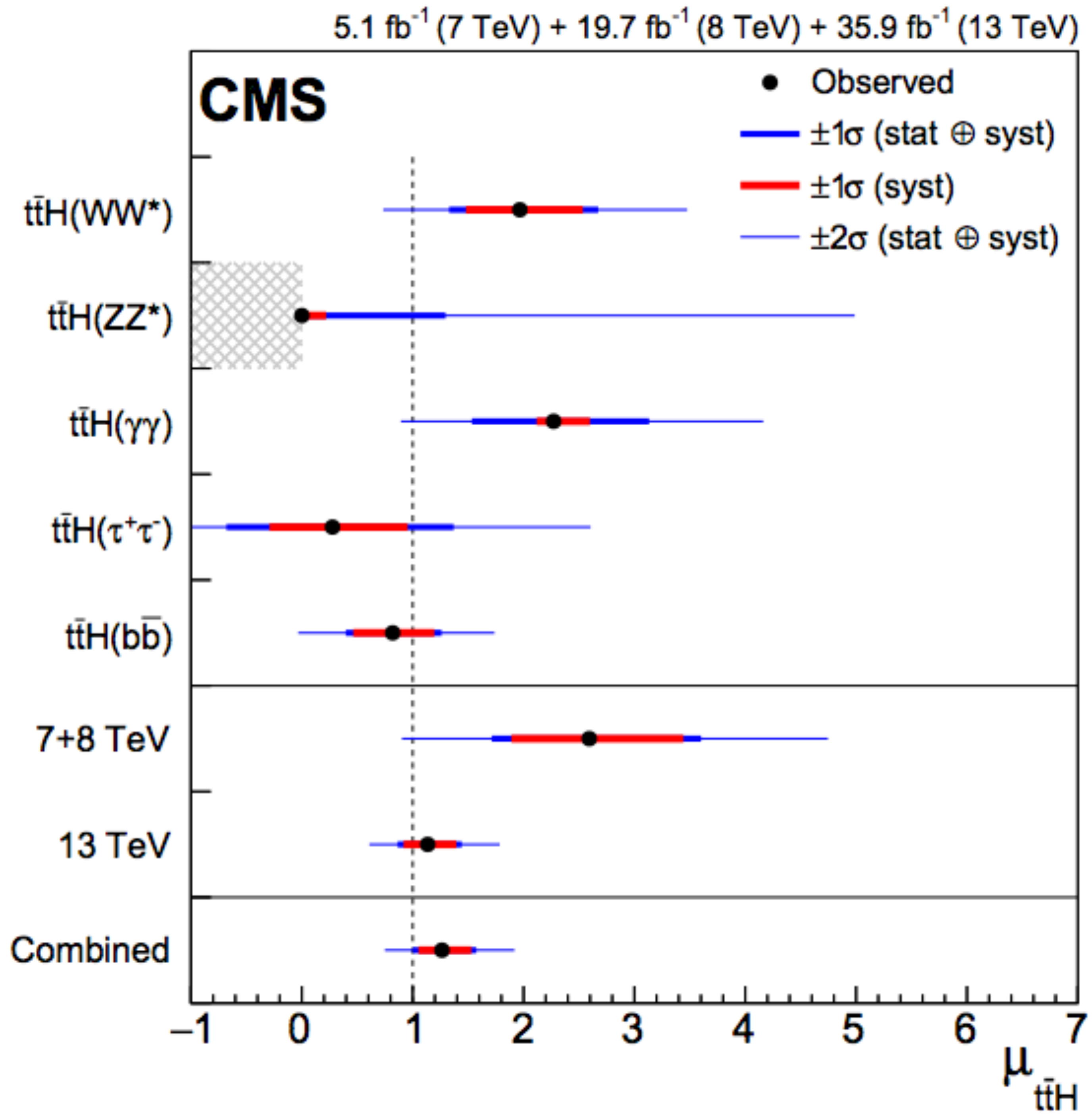
Finally, we can search for the Higgs boson coupling to the top quark through the reaction

$$pp \to t\bar{t}h$$

utilizing as many h decay modes as possible. The decay

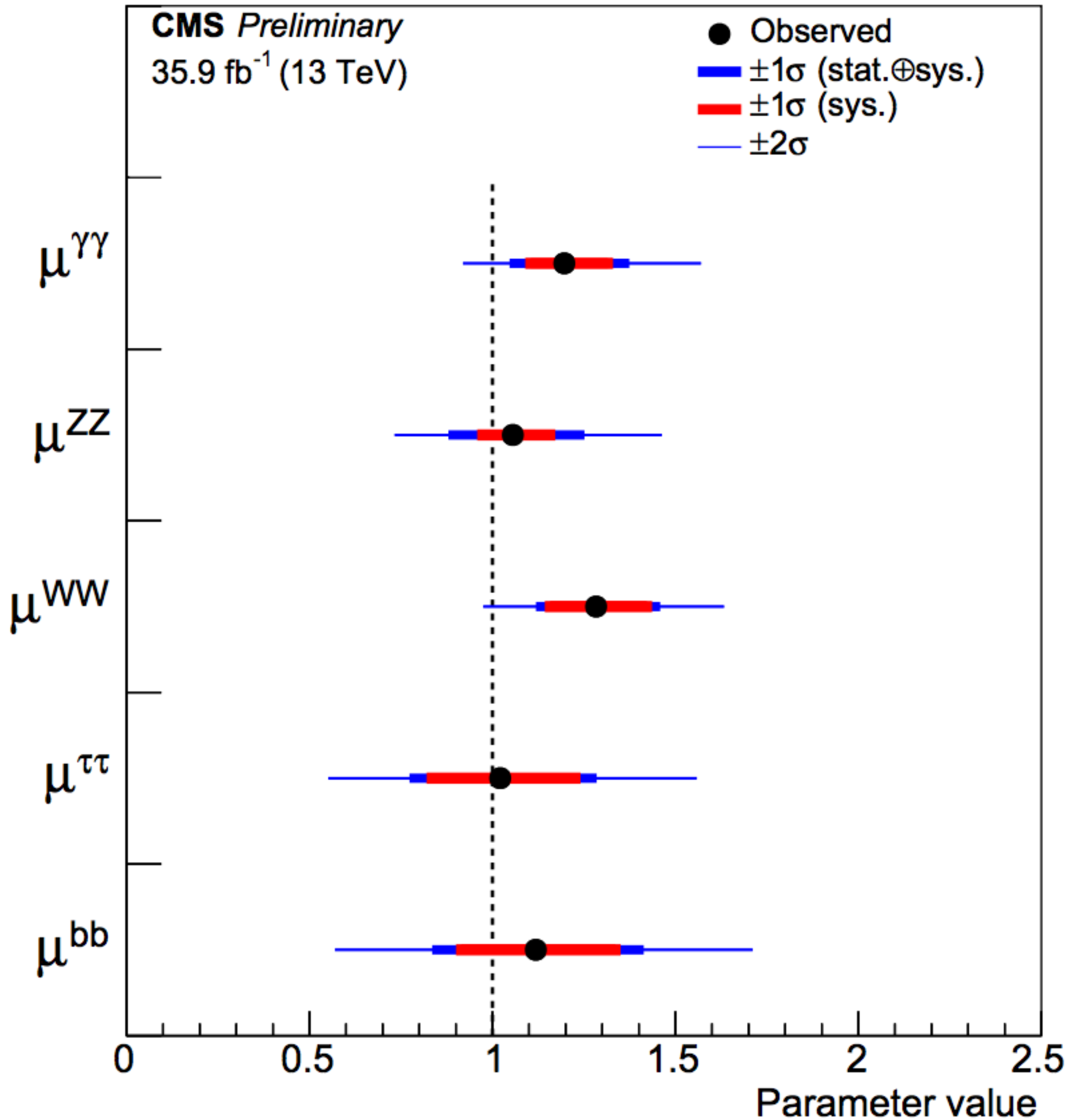
$$h \to b\bar{b}$$

gives the largest cross section, but these are confusing events with 6 or 8 hadronic jets in the signal, subject to combinatoric ambiguity in jet assignments. In leptonic or $\gamma\gamma$ decays, it is clearer how to identify the Higgs boson.



$$\mu = 1.26^{+0.31}_{-0.26}$$

Here is the current CMS summary of μ values:



so, the 125 GeV resonance is the Higgs boson, with all needed couplings within ~30% of the Standard Model predictions.