The Standard Model and the Higgs Boson

2. the Goldstone Boson Equivalence Theorem

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MITP Summer School
July 2018

In this lecture, I will describe the properties of the weak interactions at energies much greater than $m_W, \ m_Z$.

Some important conceptual issues arise here, especially when we search for new physics beyond the Standard Model. In particular, how do we parametrize possible deviations of the W, Z properties from the Standard Model predictions ? There are dangers if you do this in the wrong way.

I would like to recommend a skeleton key for thinking about these issues, called the Goldstone Boson Equivalence Theorem.

Let's begin with the following question, which was one of the most difficult aspects to understand about spontaneously broken gauge theories:

In the rest frame, a massive vector boson has 3 polarization states

$$\epsilon^\mu_\pm = (1/\sqrt{2})(0, 1, \pm i, 0)^\mu$$
$$\epsilon^\mu_0 = (0, 0, 0, 1)^\mu$$

representing the 3 possible states of a spin 1 particle with $J^3 = \pm 1, 0$.

Now boost along the 3 axis to high energy. The boosts of the polarization vectors are

$$\epsilon^\mu_\pm = (1/\sqrt{2})(0, 1, \pm i, 0)^\mu$$
$$\epsilon^\mu_0 = (\frac{p}{m}, 0, 0, \frac{E}{m})^\mu$$

Note that, as E becomes large, the components of ϵ_0^μ grow without bound; in fact,

$$\epsilon_0^\mu \rightarrow \frac{p^\mu}{m}$$

Another way to express this is that the polarization sum is

$$\sum \epsilon_i^\mu \epsilon_j^\nu = -\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m^2}\right)$$

and the second term on the right has unbounded matrix elements.

This potentially leads to very large contributions to vector boson amplitudes, even threatening violation of unitarity.

For example, the amplitude for production of a scalar in e^+e^- annihilation is

$$\mathcal{M}(e^+e^- \to \phi^+\phi^-) = i\frac{e^2}{s}(2E)\sqrt{2}\epsilon_- \cdot (k_+ - k_-)$$

In $e^+e^- \to W^+W^-$, we might expect

$$\mathcal{M} \sim i\frac{e^2}{s}(2E)\sqrt{2}\epsilon_- \cdot (k_+ - k_-)\ \epsilon^*(W^-) \cdot \epsilon^*(W^+)$$

But, for longitudinally polarized W bosons, this extra factor becomes

$$\frac{k_- \cdot k_+}{m_W^2} = \frac{s - 2m_W^2}{2m_W^2}$$

and this really does violate unitarity at high energy.

So, the question is: When are these enhancements from the form of ϵ_0^μ real — always, sometimes, or never ?

The answer is given by the Goldstone Boson Equivalence Theorem of Cornwall and Tiktopoulos and Vayonakis:

In the Higgs mechanism, a massive W boson acquired its longitudinal component by absorbing a Goldstone boson from the Higgs sector. When the W is at rest, it is not so clear which polarization state comes from the original vector boson and which comes from the Higgs boson. However, for a highly boosted W, there is a clear distinction between the transverse and longitudinal polarization states. Then,

$$\mathcal{M}(X \to Y + W_0^+(p)) = \mathcal{M}(X \to Y + \pi^+(p)) \; (1 + \mathcal{O}(m_W/E_W))$$

The proof is too complicated to give in this lecture; an excellent reference is Chanowitz and Gaillard, Nucl. Phys. B261, 379 (1985). The important point is the proof makes essential use of gauge invariance.

I will discuss three examples that probe different aspects of the application of this theorem.

The first is the theory of W polarization in top quark decay.

The matrix element is

$$\mathcal{M}(t \to bW^+) = i\frac{g}{\sqrt{2}} u_L^\dagger(b)\overline{\sigma}^\mu u_L(t)\ \epsilon_\mu^*(W)$$

It is a good approximation to ignore the b quark mass. I will use coordinates in which the t is at rest with spin and the W moves in the 3 direction. Then

$$u_L(b) = \sqrt{2E_b}\begin{pmatrix} -1 \\ 0 \end{pmatrix} \qquad u_L(t) = \sqrt{m_t}\ \xi$$

For a W^{+}_{-}, $\quad \overline{\sigma}^{\mu}\,\epsilon^{*}_{-\mu} = \frac{1}{\sqrt{2}}(\sigma^1 + i\sigma^2) = \sqrt{2}\sigma^{+}$

and the amplitude is : $\mathcal{M} = -ig\sqrt{2m_t E_b}\;\xi_2$

For a W^{+}_{+}, $\quad \overline{\sigma}^{\mu}\,\epsilon^{*}_{+\mu} = \frac{1}{\sqrt{2}}(\sigma^1 - i\sigma^2) = \sqrt{2}\sigma^{-}$

and the amplitude is : $\mathcal{M} = 0$

For a W^{+}_{0}, $\quad \overline{\sigma}^{\mu}\epsilon^{*}_{0\mu} = -(\frac{p + E\sigma^3}{m_W})$

and the amplitude is :

$$\mathcal{M} = ig\sqrt{2m_t E_b}\;(\frac{m_t}{m_W})\;\xi_1$$

Averaging over the t spin direction and integrating over phase space, we find

$$\Gamma(t \to bW_-^+) = \frac{1}{2m_t}\frac{1}{16\pi}\frac{2p}{m_t} \cdot g^2(2pm_t)$$

Using the kinematic relation $2pm_t = m_t^2 - m_W^2$
we then find

$$\Gamma(t \to bW_-^+) = \frac{\alpha_w}{8}\ m_t\ (1 - \frac{m_W^2}{m_t^2})^2$$

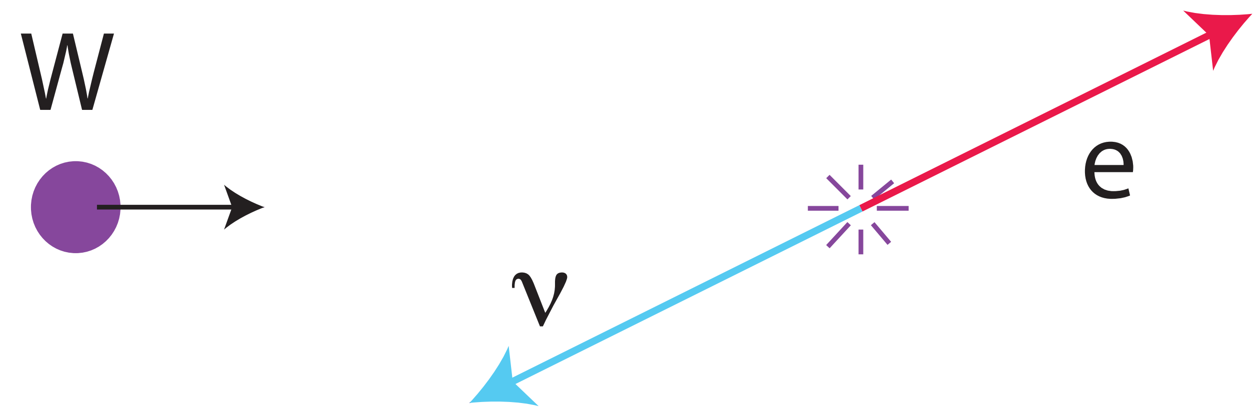
and similarly $\Gamma(t \to bW_+^+) = 0$

$$\Gamma(t \to bW_0^+) = \frac{\alpha_w}{8}\ m_t\ (1 - \frac{m_W^2}{m_t^2})^2 \cdot \frac{m_t^2}{2m_W^2}$$

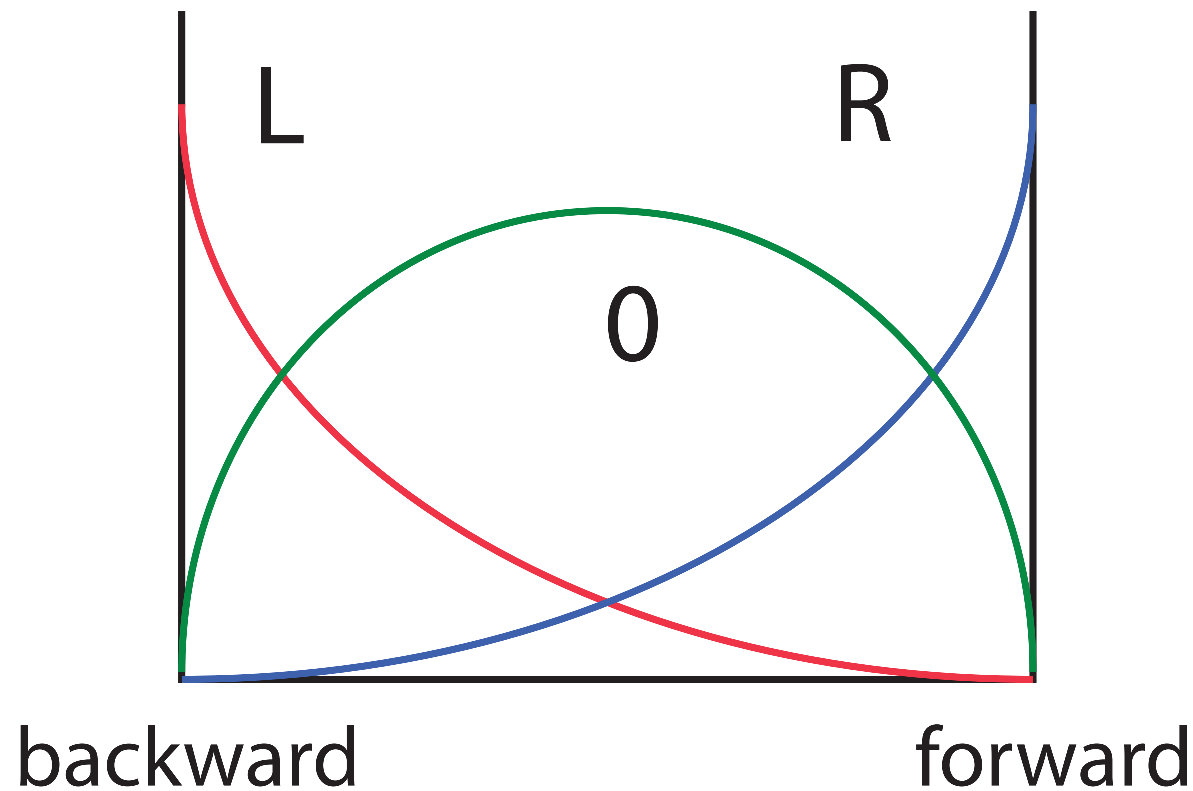
so the enhancement of the longitundinal polarization state is really predicted.

$$\frac{\Gamma(t \to bW_0^+)}{\Gamma(t \to bW^+)} = \frac{m_t^2/2m_W^2}{1 + m_t^2/2m_W^2} \approx 70\%$$

What does experiment say ? We can test this by reconstructing $pp \to t\bar{t} \to \ell\nu 4j$ and measuring the angular distribution of the W decay products



$$\frac{d\Gamma}{d\cos\theta} \sim \begin{cases} (1+\cos\theta)^2 & + \\ \sin^2\theta/2 & 0 \\ (1-\cos\theta)^2 & - \end{cases}$$



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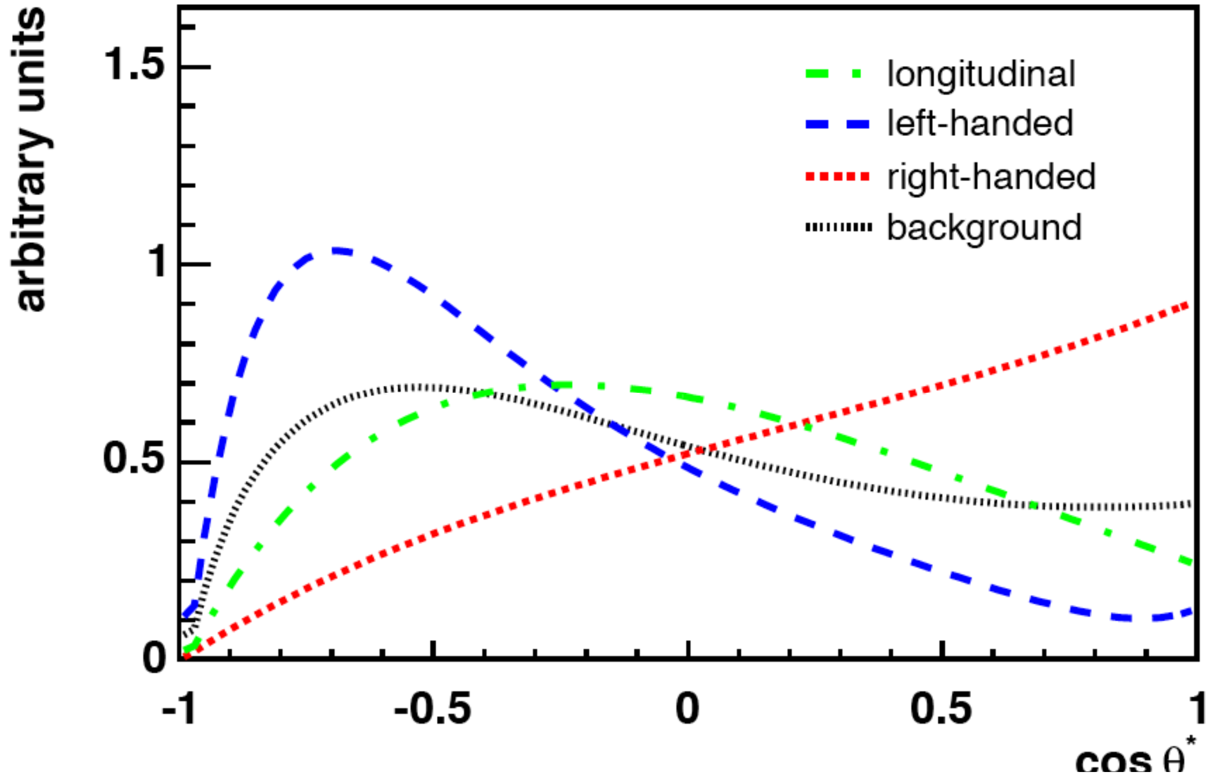
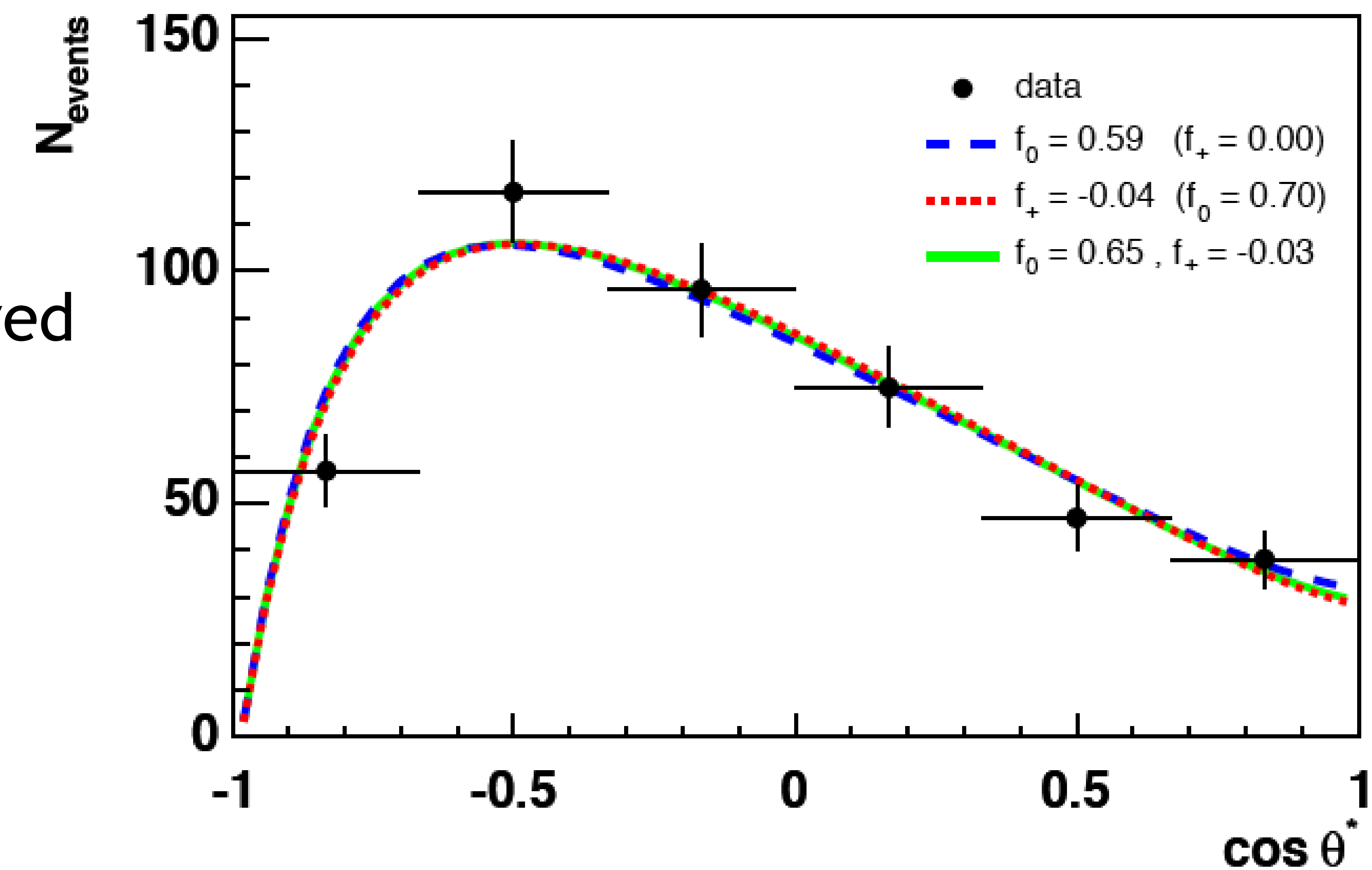
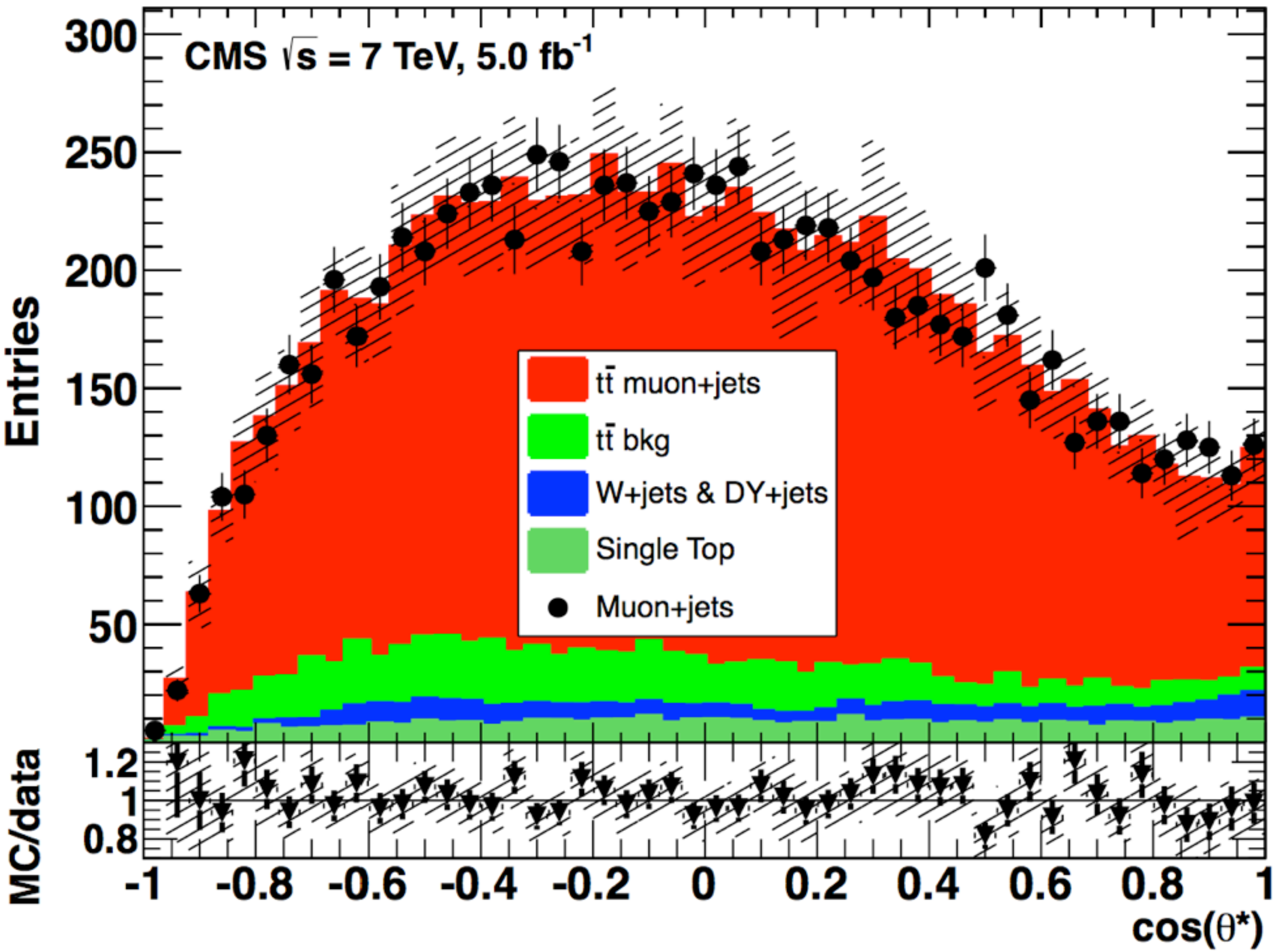


CDF experiment

observed





CMS $\sqrt{s}$ = 7 TeV, 5.0 fb^{-1}
Entries
$t\bar{t}$ muon+jets
$t\bar{t}$ bkg
W+jets & DY+jets
Single Top
Muon+jets
MC/data
cos(θ*)
300
250
200
150
100
50
1.2
1
0.8
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

What does the GBET have to say ?

According to the GBET, we should have

$$\mathcal{M}(t \to bW_0^+) \to \mathcal{M}(t \to b\pi^+)$$

The amplitude for emission of a Higgs boson should be proportional to the top quark Yukawa coupling, given by

$$m_t = \frac{y_t v}{\sqrt{2}}$$

So, the W_0^+ amplitude should be larger by the factor

$$\frac{y_t^2}{g^2} = \frac{2m_t^2/v^2}{4m_W^2/v^2} = \frac{m_t^2}{2m_W^2}$$

which is exactly what we found.

Turn next to the process

$$e^+e^- \to W_0^- W_0^+$$

I argued earlier that the apparently enhancement in this process is probably spurious, since it violates unitarity.

The GBET says:

$$\mathcal{M}(e^+e^- \to W_0^- W_0^+) \to \mathcal{M}(e^+e^- \to \pi^+\pi^-)$$

This implies (using also the high energy limit of SU(2) x U(1) and the Higgs quantum nos $(I, Y) = (\frac{1}{2}, \frac{1}{2})$

for $e_R^- e_L^+$:

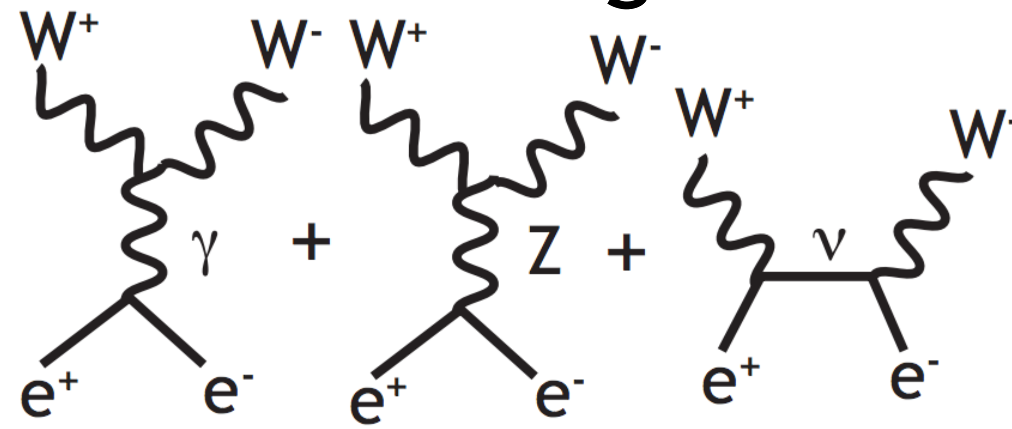
$$\mathcal{M} = -i(2E)\sqrt{2}\epsilon_+ \cdot (k_- - k_+) \cdot \frac{e^2}{2c_w^2}\frac{1}{s}$$

for $e_L^- e_R^+$:

$$\mathcal{M} = -i(2E)\sqrt{2}\epsilon_- \cdot (k_- - k_+) \cdot \left(\frac{e^2}{4c_w^2}\frac{1}{s} + \frac{e^2}{4s_w^2}\frac{1}{s}\right)$$

so some cancellation of the effect discussed above must occur.

In fact, at leading order in the Standard Model, the amplitude is the sum of 3 diagrams.



Work this out carefully for $e^-_R e^+_L$, where the ν diagram is absent.

$$i\mathcal{M} = (-ie)(ie)2E\sqrt{2}\epsilon_{+\mu}\left[\frac{-i}{s} + \frac{-s_w^2}{s_w c_w}\frac{c_w}{s_w}\frac{-i}{s-m_Z^2}\right]$$

$$\cdot\left[\epsilon^*_-\epsilon^*_+(k_- - k_+)^\mu + \epsilon^{*\mu}_-(-q-k_-)\cdot\epsilon^*_+ + \epsilon^{*\mu}_+(q+k_+)\cdot\epsilon^*_-\right]$$

where $q = k_- + k_+$ and, in the 2nd line, ϵ^*_- and ϵ^*_+ are the W polarizations. Send

$$\epsilon^*_- \to \frac{k_-}{m_W} \qquad \epsilon^*_+ \to \frac{k_+}{m_W}$$

Then the second term in brackets becomes

$$\frac{1}{m_W^2}\left[2k_-\cdot k_+(k_- - k_+)^\mu + k_-^\mu(-2k_-\cdot k_+) + k_+^\mu(2k_+\cdot k_-)\right]$$
$$= -\frac{k_+\cdot k_-}{m_W^2}(k_- - k_+)^\mu = -\frac{s-2m_W^2}{2m_W^2}(k_- - k_+)^\mu$$

On the other hand, the first term in brackets becomes

$$\left[\frac{-i}{s} - \frac{-i}{s-m_Z^2}\right] = \frac{im_Z^2}{s(s-m_Z^2)}$$

Assembling the pieces and using $m_Z^2/m_W^2 = 1/c_w^2$, we find

$$i\mathcal{M} = i(e^2)2E\sqrt{2}\epsilon_{+\mu}(k_- - k_+)^\mu(\frac{1}{s(s-m_Z^2)})(-\frac{s-2m_W^2}{2c_w^2})$$

which gives the predicted expression at high energy.

For $e^-_L e^+_R$, we must work a little harder. The first two diagrams contribute

$$i\mathcal{M} = (-ie)(ie)2E\sqrt{2}\epsilon_{-\mu}\left[\frac{-i}{s} + \frac{(1/2 - s_w^2)}{s_w c_w}\frac{c_w}{s_w}\frac{-i}{s - m_Z^2}\right]$$

$$\cdot\left[\epsilon^*_-\epsilon^*_+(k_- - k_+)^\mu + \epsilon^{*\mu}_-(-q - k_-)\cdot\epsilon^*_+ + \epsilon^{*\mu}_+(q + k_+)\cdot\epsilon^*_-\right]$$

After the reductions described above, there is a term that does not cancel its high energy behavior

$$i\mathcal{M} = (-ie^2)2E\sqrt{2}\epsilon_{-\mu}\left[\frac{1}{2s_w^2}\frac{1}{s}\right](-\frac{s}{2m_W^2}(k_- - k_+)^\mu)$$

$$= (\frac{ie^2}{4s_w^2})2E\sqrt{2}\epsilon_{-\mu}\frac{1}{m_W^2}(k_- - k_+)^\mu$$

However, now we must add the ν diagram, which contributes

$$i\mathcal{M} = (i\frac{g}{\sqrt{2}})^2 v_R^\dagger \overline{\sigma}\cdot\epsilon_+^* \frac{i\sigma\cdot(p-k_-)}{(p-k_-)^2}\overline{\sigma}\cdot\epsilon_-^* u_L(p)$$

Substitute $\epsilon_-^* \to k_-/m_W$ and simplify

$$\frac{\sigma\cdot(p-k_-)}{(p-k_-)^2}\overline{\sigma}\cdot\frac{k_-}{m_W}u(p) = \frac{\sigma\cdot(p-k_-)}{(p-k_-)^2}\overline{\sigma}\cdot\frac{(k_- - p)}{m_W}u(p) = -\frac{1}{m_W}u(p)$$

Also $\epsilon_+^* \to k_+/m_W$ and

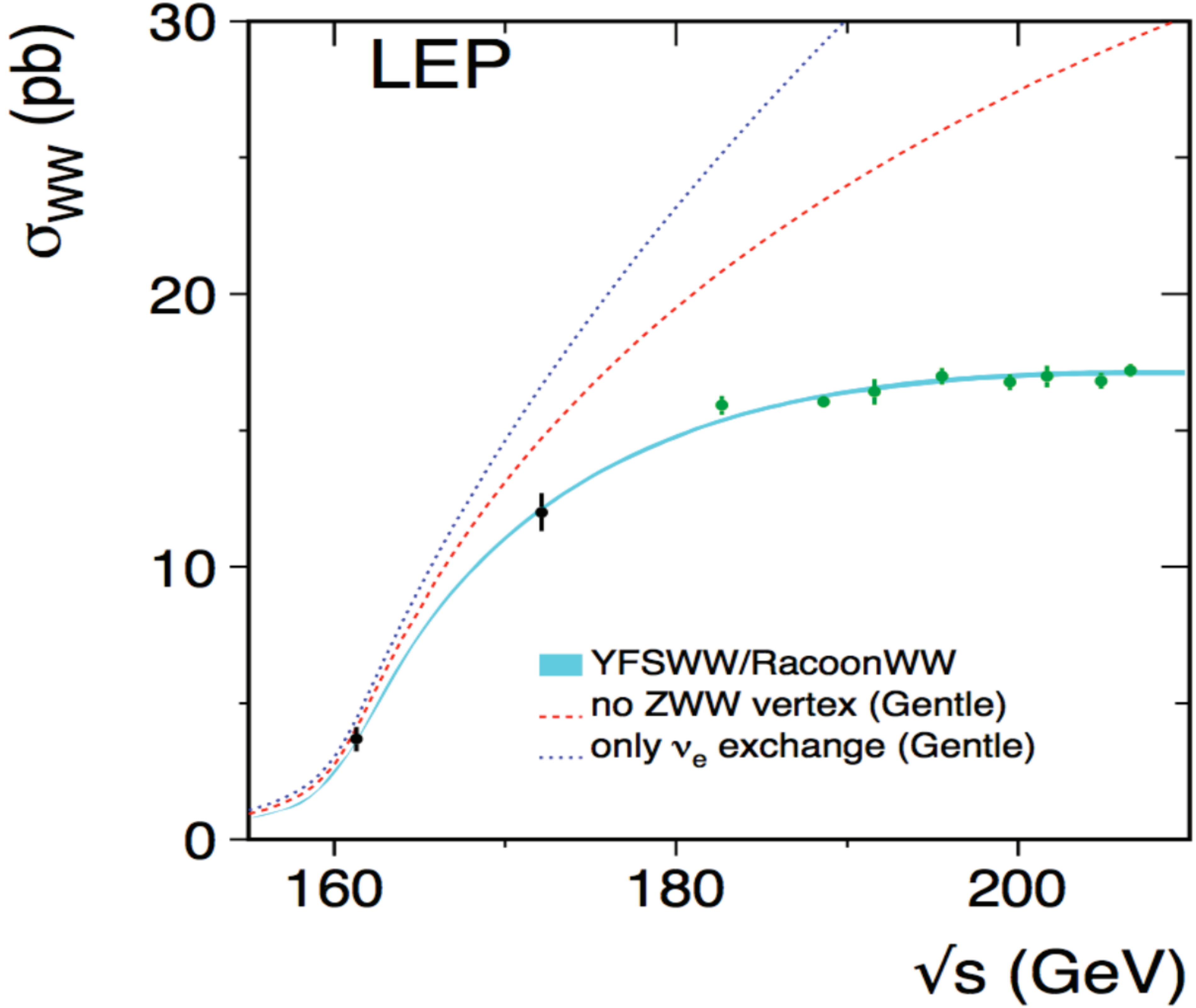
$$k_+ = (k_+ - k_- + (p+\overline{p}))/2$$

so, finally, we find

$$i\mathcal{M} = -i\frac{e^2}{2s_w^2}(2E)\sqrt{2}\epsilon_{-\mu}\frac{1}{2}(k_- - k_+)^\mu\frac{1}{m_W^2}$$

which indeed cancels the term on the previous page. In the end, these cancellations gives the GBET prediction.

The cross section for $e^+e^- \to W^+W^-$ was measured by the LEP experiments, with this result:



Before these LEP measurements were made, theorists tried to predict how deviations from the Standard Model might show up. One idea was to extend the 3-vector boson vertex by adding terms:

$$\delta\mathcal{L} = e[ig_{1\gamma}A_\mu(W_\mu^- W^{+\mu\nu} - W_\mu^+ W^{-\mu\nu}) + i\kappa_\gamma W_\mu^- W_\nu^+ A_{\mu\nu}$$
$$+ i\frac{\lambda_\gamma}{m_W^2} W_{\lambda\mu}^- W^{+\mu\nu} A_\nu{}^\lambda]$$

and similarly for Z. Here $V_{\mu\nu} = (\partial_\mu V_\nu - \partial_\nu V_\mu)$. Setting

$$g_{1\gamma} = \kappa_\gamma = 1$$

we have the Standard Model Lagrangian; any deviations are "extra". If CP conservation is relaxed, more terms can be added.

It was quickly realized that the extra terms in the Lagrangian imply extra terms in the amplitudes enhanced by the factor s/m_W^2 . This would seem to imply high sensitivity to the new terms.

We now understand the origin of these terms. Modifying the Yang-Mills vertices breaks gauge invariance. Then the GBET does not apply, and the delicate cancellations that is requires do not happen.

Today, we have strong evidence for SU(2)xU(1). So, is are tests for these terms useless, or do they give some measure of the presence of new physics ?

I will address this question in my lecture #4.

For the next topic in this lecture, I will describe an analysis in which the GBET might be expected to apply, but actually it does not. This is in collinear W radiation from a quark line.

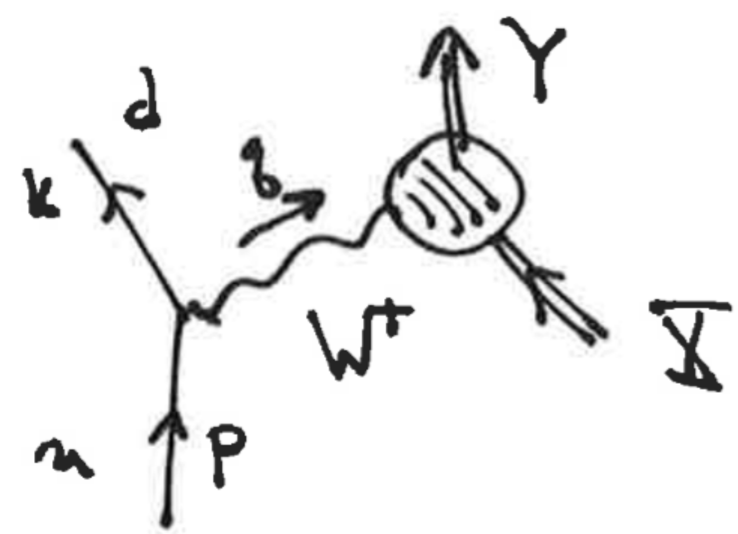
In QCD, quarks easily radiate photons and gluons in collinear directions, giving rise to initial- and final-state radiation described by the Altarelli-Parisi equations. This is the mechanism by which quarks become jets at the LHC.

At high energy, W bosons can also be radiated. An interesting question is: can the W_0^+ be radiated ? By the GBET, this is a Higgs boson state that does not couple to light quarks.

Consider the almost-collinear radiation

$$u(p) \to d(k) + W^+(q)$$

This can yield a W parton distribution in the proton, allowing W-induced reactions at the LHC.



An important one is WW fusion: $W^+W^- \to h$

To produce the Higgs boson, we would like to have W partons in states of longitudinal polarization.

Write the momentum vectors of u, d, W for $p_T << E$, with u, d on shell and W off shell

$$p = (E, 0, 0, E)$$

$$k = ((1-z)E, -p_T, 0, (1-z)E - \frac{p_T^2}{2(1-z)E})$$

$$q = (zE, p_T, 0, zE + \frac{p_T^2}{2(1-z)E})$$

The denominator of the W propagator is

$$q^2 - m_W^2 = -p_T^2 - \frac{z}{(1-z)} p_T^2 - m_W^2 = -(\frac{p_T^2}{(1-z)} + m_W^2)$$

Now we must compute the matrix elements for W emission

$$i\mathcal{M} = ig\; u_L^\dagger(k)(\overline{\sigma}\cdot\epsilon_W^*)u_L(p)$$

to first order in $(p_T, m_W)/E$. Use the explicit spinors

$$u_L(k) = \sqrt{2(1-z)E}\begin{pmatrix} p_T/2(1-z) \\ 1 \end{pmatrix} \qquad u_L(p) = \sqrt{2E}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The W polarization vectors are

$$\epsilon_\pm^{*\mu} = (0, 1, \mp i, -p_T/zE)^\mu/\sqrt{2}$$
$$\epsilon_0^{*\mu} = (q, p_T, 0, zE)^\mu/m_W$$

with

$$q = [(zE)^2 - m_W^2]^{1/2} = zE - \frac{m_W^2}{2zE}$$

Then

$$\overline{\sigma}\cdot\epsilon_+^* = \frac{1}{\sqrt{2}}\begin{pmatrix} -p_T/zE & 0 \\ 2 & p_T/zE \end{pmatrix}$$

$$\overline{\sigma}\cdot\epsilon_-^* = \frac{1}{\sqrt{2}}\begin{pmatrix} -p_T/zE & 2 \\ 0 & p_T/zE \end{pmatrix}$$

$$\overline{\sigma}\cdot\epsilon_0^* = \frac{1}{m_W}\begin{pmatrix} q+zE & p_T \\ p_T & q-zE \end{pmatrix}$$

With these ingredients, it is straightforward to work out the matrix elements

$$i\mathcal{M} = ig \begin{cases} \frac{\sqrt{1-z}\ p_T}{z(1-z)} & - \\ \frac{\sqrt{1-z}\ p_T}{z} & + \\ -\frac{1}{\sqrt{2}}\frac{\sqrt{1-z}\ m_W}{z} & 0 \end{cases}$$

The first two lines here are exactly what one finds in the derivation of the Altarelli-Parisi equations, with the substitution $g_s t^a \to g/\sqrt{2}$. The last line is new for a massive vector boson.

Now let's embed these results into the formula for the cross section. We begin from

$$\sigma(uX \to dY) = \frac{1}{2s}\int \frac{d^3k}{(2\pi)^3 2k}\int d\Pi_Y \ (2\pi)^4\delta^{(4)}(p+p_X-k-p_Y)$$
$$\left|\mathcal{M}(u\to W^+d)\frac{1}{q^2-m_W^2}\mathcal{M}(W^+X\to Y)\right|^2$$

Using the collinear kinematics,

$$\frac{1}{2s}\int\frac{d^3k}{(2\pi)^3 2k} = \frac{1}{2(\hat{s}/z)}\int\frac{dzEd^2p_T}{16\pi^3 E(1-z)} = \frac{1}{2\hat{s}}\int\frac{dzdp_T^2\pi}{16\pi^3}\frac{z}{(1-z)}$$

Then

$$\sigma(uX\to dY) = \int dz\int\frac{dp_T^2}{(4\pi)^2}\frac{z}{(1-z)}|\mathcal{M}(u\to W^+d)|^2\frac{1}{(p_T^2/(1-z)+m_W^2)^2}$$
$$\cdot\frac{1}{2\hat{s}}\int d\Pi_Y(2\pi)^4\delta^{(4)}(q+p_X-p_Y)\ |\mathcal{M}(W^+X\to Y)|^2$$

The last line is $\sigma(W^+(q)X\to Y)$.

So we have an expression for the cross section in the form

$$\sigma(uX \to dY) = \int dz\ f_{W\leftarrow u}(z)\ \sigma(W^+(q)X \to Y)$$

where

$$f_{W\leftarrow u}(z) = \int \frac{dp_T^2}{(4\pi)^2} \frac{z}{(1-z)} \frac{(1-z)^2}{(p_T^2 + (1-z)m_W^2)^2} |\mathcal{M}(u \to W^+ d)|^2$$

We can evaluate this using the formulae for the matrix element given on a previous slide.

The result is:

$$f_{W-}(z) = \frac{\alpha_w}{4\pi} \int \frac{dp_T^2 p_T^2}{[p_T^2 + (1-z)m_W^2]^2} \; \frac{1}{z}$$

$$f_{W+}(z) = \frac{\alpha_w}{4\pi} \int \frac{dp_T^2 p_T^2}{[p_T^2 + (1-z)m_W^2]^2} \; \frac{(1-z)^2}{z}$$

$$f_{W0}(z) = \frac{\alpha_w}{8\pi} \int \frac{dp_T^2 m_W^2}{[p_T^2 + (1-z)m_W^2]^2} \; \frac{(1-z)^2}{z}$$

For the transverse W polarizations, we find a result very similar to the Altarelli-Parisi splitting functions.

Integrating over p_T ; we find

$$f_{WT}(z) = \frac{\alpha_w}{4\pi} \log \frac{Q^2}{m_W^2} \frac{1 + (1-z)^2}{z}$$

For a longitudinal W, the p_T integral is not divergent. The emission is restricted to an interval of p_T where the W can be thought to be approximately at rest in a collinearly moving frame. Here, despite the GBET, we get a nonzero answer

$$f_{W0}(z) = \frac{\alpha_w}{8\pi}\frac{1-z}{z}$$

with the W having characteristic $p_T \sim m_W$. This formula (due to Sally Dawson) is the basis for the analysis of W fusion processes at the LHC.

Finally, I would like to point out that m_t^2/m_W^2 enhancements often occur in perturbation theory.

As an example, let's look back at the top quark contributions to the S and T parameters of precision electroweak.

To do this, we compute the vacuum polarization diagrams with chiral currents.

Recall that the QED vacuum polarization for a charge 1 massive fermion (the electron) is

$$\Pi^{\mu\nu}_{QQ}(q^2) = -\frac{1}{(4\pi)^2}\int_0^1 dx\ \log\left[\frac{\Lambda^2}{m^2 - x(1-x)q^2}\right] \cdot 8x(1-x) \cdot (q^2 g^{\mu\nu} - q^\mu q^\nu)$$

This expression is explicitly transverse (current-conserving). But, for massive particles, chiral currents are not conserved, so, in the vacuum polarizations of chiral currents, more terms can appear.

Let's now compute the electroweak vacuum polarizations of the (t,b) multiplet. I include the color factor of 3. I quote only the terms proportional to $g^{\mu\nu}$.

$$\Pi_{11}(q^2) = -\frac{3}{(4\pi)^2}\int_0^1 dx\ \log\left[\frac{\Lambda^2}{xm_t^2 - x(1-x)q^2}\right]$$

$$\cdot(\frac{2}{4})(4x(1-x)q^2 - 2xm_t^2)$$

$$\Pi_{33}(q^2) = -\frac{3}{(4\pi)^2}\int_0^1 dx\ \log\left[\frac{\Lambda^2}{m_t^2 - x(1-x)q^2}\right]$$

$$\cdot(\frac{1}{4})(4x(1-x)q^2 - 2m_t^2) \qquad + (t \to b)$$

$$\Pi_{3Q}(q^2) = -\frac{3}{(4\pi)^2}\int_0^1 dx\ \log\left[\frac{\Lambda^2}{m_t^2 - x(1-x)q^2}\right]$$

$$\cdot(\frac{1}{2}Q_t)(4x(1-x)q^2) \qquad + (t \to b)$$

Compute the T parameter:

$$T = \frac{4\pi}{s_w^2 m_W^2}(\Pi_{11}(0) - \Pi_{33}(0))$$

$$= \frac{-3}{4\pi s_w^2 m_W^2}\int_0^1 dx\, \left(\log[\frac{\Lambda^2}{x m_t^2}](-x m_t^2) - \log[\frac{\Lambda^2}{m_t^2}](-\frac{1}{2}m_t^2)\right)$$

$$= \frac{-3}{4\pi s_w^2 m_W^2}(-\frac{1}{4}m_t^2)$$

$$= \frac{3}{16\pi s_w^2 m_W^2}$$

This comes entirely from the additional non-transverse terms in the numerator.

Compute the S parameter:

$$S = 16\pi[\Pi'_{33}(0) - \Pi'_{3Q}(0)]$$

$$= \frac{d}{dq^2}\left\{ \frac{-3}{\pi} \int_0^1 dx \ \log\left[\frac{\Lambda^2}{m_t^2 - x(1-x)q^2} \right] \right.$$

$$\left. \cdot [\frac{1}{4}(4x(1-x)q^2 - 2m_t^2) - \frac{1}{2} Q_t (4x(1-x)q^2)] \right\} + \cdots$$

$$= \frac{-3}{\pi} \int_0^1 dx \ \left[\log[\frac{\Lambda^2}{m_t^2}] x(1-x)(1-2Q_t) + \frac{x(1-x)}{4m_t^2}(-2m_t^2) \right] + \cdots$$

$$= \frac{-3}{\pi} \left[-\log m_t^2 \cdot \frac{1}{6}(-\frac{1}{3}) + \frac{1}{6} \cdot (-\frac{1}{2}) \right]$$

$$\approx -\frac{1}{6\pi} \log \frac{m_t^2}{m_Z^2}$$

If we had a very heavy 4th generation quark doublet, the log terms would cancel but the last term in the previous calculation, again from non-transverse terms, would survive.   In this case we would find

$$S = \frac{3}{6\pi}$$

a nonzero constant, even when the 4th generation quark masses go to infinity.

The moral of the story is that we must watch out for fermions chirally coupled to the Standard Model that are taken to be very heavy.

Without the top quark, the Standard Model is incomplete. Then we can expect to find factors $\log m_t^2$ in loop diagrams.

For any fermion whose mass is generated by the Higgs field but nevertheless becomes large, we can expect to find terms $y_f^2/g^2 = m_f^2/2m_W^2$ in perturbation theory. Sometimes we will even find

$$y_f^2/m_f^2 \to \text{const.} \neq 0$$

so that the heavy fermions do not decouple but rather leave ghostly remnants.