The Standard Model and the Higgs Boson

1. The SU(2)xU(1) Model of Weak Interactions

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The purpose of this course is to describe the Standard Model of weak interactions and its implications for the properties of the Higgs boson. The outline of the course is:

1. The V-A structure of the weak interactions and the precision electroweak experiments that support the SU(2)xU(1) gauge theory of weak interactions.

2. The Goldstone Boson Equivalence Theorem, and related ideas and applications

3. The properties of the Higgs boson within the Standard Model

4. The description of effect of physics beyond the Standard Model by Effective Field Theory.

Some useful references for this material are:

my CERN school lecture notes:

M. E. Peskin, "Lectures on the Theory of the Weak Interactions", arXiv:1708.09043 .

my forthcoming book on elementary particle physics:

M. E. Peskin, "Concepts of Elementary Particle Physics",
http://www.slac.stanford.edu/~mpeskin/Physics152/theBook.pdf

a very useful introduction to the Standard Model Effective Field Theory:

B. Henning, X. Lu, and H. Murayama, arXiv:1412.1837

You all know that the weak interactions are described by a Yang-Mills theory based on the group SU(2)xU(1).

In Yang-Mills theory, the coupling of any field to the vector bosons is determined by the covariant derivative

$$D_\mu = \partial_\mu - igA_\mu^a t^a$$

The gauge charges t^a depends on the quantum numbers of the field.

For SU(2)xU(1), an essential field is the Higgs field $\varphi(x)$, which obtains a constant value throughout space. This nonzero value gives mass to the weak interaction vector bosons and to the quarks and leptons.

The mass spectrum of vector bosons is especially easy to work out. We assign φ the quantum numbers

$$I = \frac{1}{2} \qquad Y = \frac{1}{2}$$

The action of SU(2)xU(1) is

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \to \exp[-i\alpha^a\sigma^a/2 - i\beta/2] \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Then if φ obtains a nonzero vacuum value, we can write this as

$$\varphi(x) = \exp[-i\alpha^a(x)\sigma^a/2] \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

The covariant derivative on φ is

$$D_\mu\varphi = (\partial_\mu - ig(\sigma^a/2)A^a_\mu - (g'/2)B_\mu)\ \varphi$$

and this forms the kinetic term for φ in the Lagrangian

$$\mathcal{L} = |D_\mu\varphi|^2$$

Replacing φ by its vacuum value, this becomes

$$\frac{1}{8}\begin{pmatrix} 0 & v \end{pmatrix}(g\sigma^a A^a_\mu + g' B_\mu)^2 \begin{pmatrix} 0 \\ v \end{pmatrix}$$

The σ^1 , σ^2 terms give

$$\frac{g^2}{8}[(A^1_\mu)^2 + (A^2_\mu)^2]v^2$$

The remaining terms give

$$\frac{1}{8}(-gA^3_\mu + B_\mu)^2 v^2$$

So we find masses for the vector fields, of the form

$$\mathcal{L} = \frac{1}{2} m^2_{ab} V^a_\mu V^{b\mu}$$

The mass eigenstates are

$$W^\pm = (A^1 \mp iA^2)/\sqrt{2} \qquad m^2_W = g^2 v^2/4$$

$$Z = (gA^3 - g'B)/\sqrt{g^2 + g'^2} \qquad m^2_Z = (g^2 + g'^2)v^2/4$$

$$A = (g'A^3 + gB)/\sqrt{g^2 + g'^2} \qquad m^2_A = 0$$

We introduce the weak mixing angle θ_w , with

$$\cos\theta_w \equiv c_w = g/\sqrt{g^2 + g'^2}$$

$$\sin\theta_w \equiv s_w = g'/\sqrt{g^2 + g'^2}$$

These factors will appear throughout all of the formulae in this course.

An important relation is : $m_W = m_Z \, c_w$

This is a nontrivial consequence of the quantum number assignment for the Higgs field. From the PDG values:

$$80.385 \approx 79.965$$

We will see in the next lecture that, when radiative corrections are included, this relation is satisfied to better than 1 part per mil.

The couplings of quarks and leptons to these vector bosons is also given by the covariant derivative. For a fermion with quantum numbers (I, Y) :

$$D_\mu f = (\partial_\mu - igA^a_\mu \sigma^a/2 - ig' B_\mu Y)f$$

The W couples only to fermions with I = 1/2

$$-i\frac{g}{\sqrt{2}}(W^+_\mu \sigma^+ + W^-_\mu \sigma^-)$$

The diagonal elements give couplings both to Z and A

$$-igA^3_\mu I^3 - ig' B_\mu Y$$

$$= -i\sqrt{g^2 + g'^2}[c_w(c_w Z_\mu + s_w A_\mu)I^3 + s_w(-s_w Z_\mu + c_w A_\mu)]$$

$$= -i\sqrt{g^2 + g'^2}[s_w c_w A_\mu(I^3 + Y) + Z_\mu(c^2_w I^3 - s^2_w Y)]$$

$$= -i\sqrt{g^2 + g'^2}[s_w c_w A_\mu(I^3 + Y) + Z_\mu(I^3 - s^2_w(I^3 + Y))]$$

From these relations, we find the following simple prescriptions:

A couples to $Q = (I^3 + Y)$; the coupling strength is

$$e = \sqrt{g^2 + g'^2}\; s_w c_w = gg'/\sqrt{g^2 + g'^2}$$

This is the photon field, and we can identify e with the electron charge and Q with the electric charge of f.

W couples only to SU(2) doublets, with the universal strength

$$g/\sqrt{2}\;, \qquad g = e/s_w$$

Z couples with strength $g/c_w = e/(c_w s_w)$ to the quantum number

$$Q_Z = I^3 - s_w^2 Q$$

To complete the specification of the Standard Model, we assign the fermions in each generation of quarks and leptons the quantum numbers

$$\nu_{eL} : \quad I^3 = +\frac{1}{2}, \; Y = -\frac{1}{2}, \; Q = 0 \qquad \nu_{eR} : \quad I^3 = 0, \; Y = 0, \; Q = 0$$

$$e^-_L : \quad I^3 = -\frac{1}{2}, \; Y = -\frac{1}{2}, \; Q = -1 \qquad e^-_R : \quad I^3 = 0, \; Y = -1, \; Q = -1$$

$$u_L : \quad I^3 = +\frac{1}{2}, \; Y = \frac{1}{6}, \; Q = \frac{2}{3} \qquad u_R : \quad I^3 = 0, \; Y = \frac{2}{3}, \; Q = \frac{2}{3}$$

$$d_L : \quad I^3 = -\frac{1}{2}, \; Y = \frac{1}{6}, \; Q = -\frac{1}{3} \qquad d_R : \quad I^3 = 0, \; Y = -\frac{1}{3}, \; Q = -\frac{1}{3}$$

This gives the correct electric charge assignments for all species.

The other important feature is that the left-handed fermions are assigned to SU(2) doublets, while the right-handed fermions are assigned to SU(2) singlets.

The fact that the W couples only to left-handed species is a crucial property that shapes the Standard Model, both positively and negatively. It is therefore important to understand that this feature is extremely well supported experimentally. In the next part of this lecture, I will review some surprisingly strong pieces of evidence for this structure.

For these applications, I will go to energies $E \ll m_W$ and approximate

$$\frac{1}{q^2 - m_W^2} \to -\frac{1}{m_W^2}$$

In this limit, the W exchange can be written as the dimension-6 operator

$$\delta\mathcal{L} = \frac{g^2}{2m_W^2}\; J^+_\mu J^{-\mu}$$

where

$$J^+_\mu = \nu_L^\dagger \overline{\sigma}_\mu e_L + u_L^\dagger \overline{\sigma}_\mu d_L + \cdots$$
$$J^-_\mu = e_L^\dagger \overline{\sigma}_\mu \nu_L + d_L^\dagger \overline{\sigma}_\mu u_L + \cdots$$

and the coefficient is conventionally defined as

$$\frac{g^2}{2m_W^2} = \frac{4G_F}{\sqrt{2}}$$

This theory is called the V-A theory, since

$$u_L^\dagger \overline{\sigma}^\mu d_L = \overline{u}\gamma^\mu \frac{1-\gamma^5}{2} d$$

It reflects maximal parity violation for the charge-changing weak interactions.

To discuss the consequences of V-A theory, I should first explain my conventions for fermions. For a Dirac fermion, I set

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}$$

with

$$\sigma^\mu = (1, \vec{\sigma})^\mu \qquad \overline{\sigma}^\mu = (1, -\vec{\sigma})^\mu$$

Then, for example, a vector current takes the form

$$j^\mu = \overline{\psi}\gamma^\mu\psi = \psi_L^\dagger \overline{\sigma}^\mu \psi_L + \psi_R^\dagger \overline{\sigma}^\mu \psi_R$$

and divides neatly into L and R pieces. The L and R fields are linked by the fermion mass term. If we can ignore masses, the L and R fermion numbers are separately conserved.

The labels L,R here is called chirality. For a massless fermion, this is identical to the fermion helicity; for a massive fermion, there is a change of basis.

Some properties of these fermions are

For massive fermions moving in the 3 direction

$$p = (E, 0, 0, p)$$

$$U_L = \begin{pmatrix} \sqrt{E+p}\,\xi_L \\ \sqrt{E-p}\,\xi_L \end{pmatrix} \qquad V_L = \begin{pmatrix} \sqrt{E-p}\,\xi_R \\ -\sqrt{E+p}\,\xi_R \end{pmatrix}$$

$$U_R = \begin{pmatrix} \sqrt{E-p}\,\xi_R \\ \sqrt{E+p}\,\xi_R \end{pmatrix} \qquad V_R = \begin{pmatrix} \sqrt{E+p}\,\xi_L \\ -\sqrt{E-p}\,\xi_L \end{pmatrix}$$

with

$$\xi_L = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \xi_R = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Here U_L , for example, is the L helicity spinor, written in the chirality basis. For massless fermions, we use only the top (L) or the bottom (R) two components, which I call u, v.

The matrix element $\langle 0| \, j^\mu \left| e_L^- e_R^+ \right\rangle$ is given by

$$v_R^\dagger \overline{\sigma}^\mu u_L = \sqrt{2E}(-1 \; 0) \; (1, -\sigma^1, -\sigma^2, -\sigma^3)^\mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sqrt{2E}$$
$$= 2E \; (0, 1, -i, 0)^\mu \quad = 2E \; \sqrt{2}\epsilon_-^\mu$$

the polarization vector for the spin 1 virtual photon.

So for a current-current annihilation process such as

$$e_L^- e_R^+ \to \mu_L^- \mu_R^+$$

we find $(u_L^\dagger \overline{\sigma}_\mu v_R)(v_R^\dagger \overline{\sigma}^\mu u_L) \quad = (2E)^2 \; 2 \; \epsilon_-' \cdot \epsilon_-$

$$= s(1 + \cos\theta) = -2u$$

Another way to write this is

$$|(u_L^\dagger \overline{\sigma}_\mu v_R)(v_R^\dagger \overline{\sigma}^\mu u_L)|^2 = 4(2p_{e-} \cdot p_{\mu+})(2p_{e+} \cdot p_{\mu-})$$

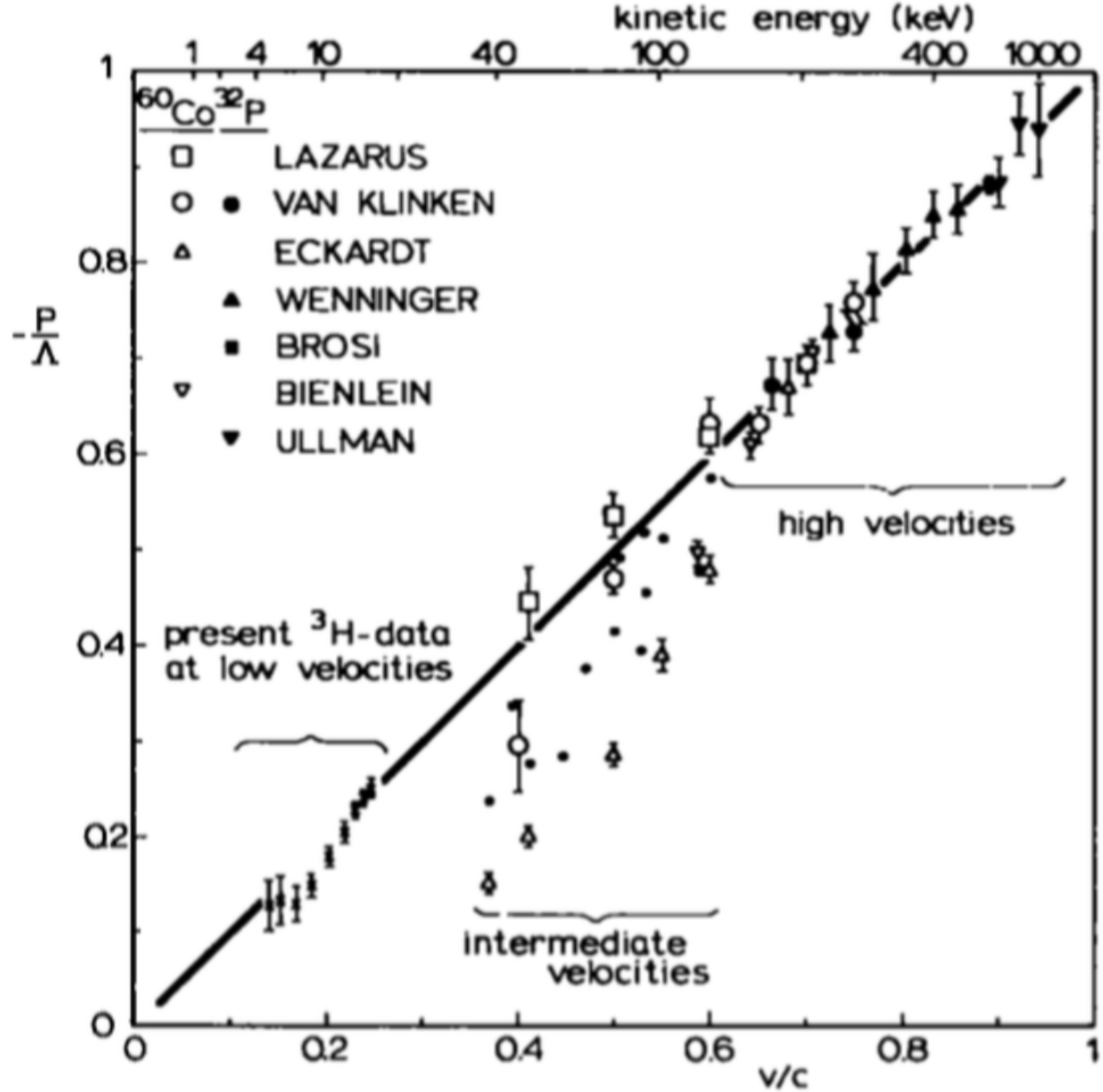
It is a nice exercise to check this answer using the usual trace theorems.

Now we can look into the consequences of the V-A theory.

1. The V-A theory implies that electrons emitted in ß decay are left-handed. More precisely, for an electron that is not completely relativistic,

$$Pol(e^-) = \frac{(\sqrt{E+p})^2 - (\sqrt{E-p})}{(\sqrt{E+p})^2 + (\sqrt{E-p})} = \frac{p}{E} = v$$

By looking at a variety of ß transitions, we can test the dependence on v.



Koks and van Klinken

2. The V-A structure of the weak coupling leads to a matrix element for muon decay

$$|\mathcal{M}(\mu^- \to \nu_\mu e^- \overline{\nu}_e)|^2 \sim (2p_\mu \cdot p_{\overline{\nu}})(2p_e \cdot p_\nu)$$

The neutrinos emitted in muon decay are not visible, but still this expression leads to a characteristic shape.

Recall formulae for 3-body phase space:

$$p_\mu = (m_\mu, \vec{0}) = p_e + p_\nu + p_{\overline{n}u}$$

$$x_i = \frac{2p_i \cdot p_\mu}{m_\mu^2} \qquad x_e + x_\nu + x_{\overline{\nu}} = 2$$

$$2p_e \cdot p_\nu = (p_e + p_\nu)^2 = (p_\mu - p_{\overline{\nu}})^2 = m_\mu^2(1 - x_{\overline{\nu}})$$

and (Dalitz!)

$$\int \Pi_3 = \frac{m_\mu^2}{128\pi^2} \int dx_e dx_{\overline{\nu}}$$

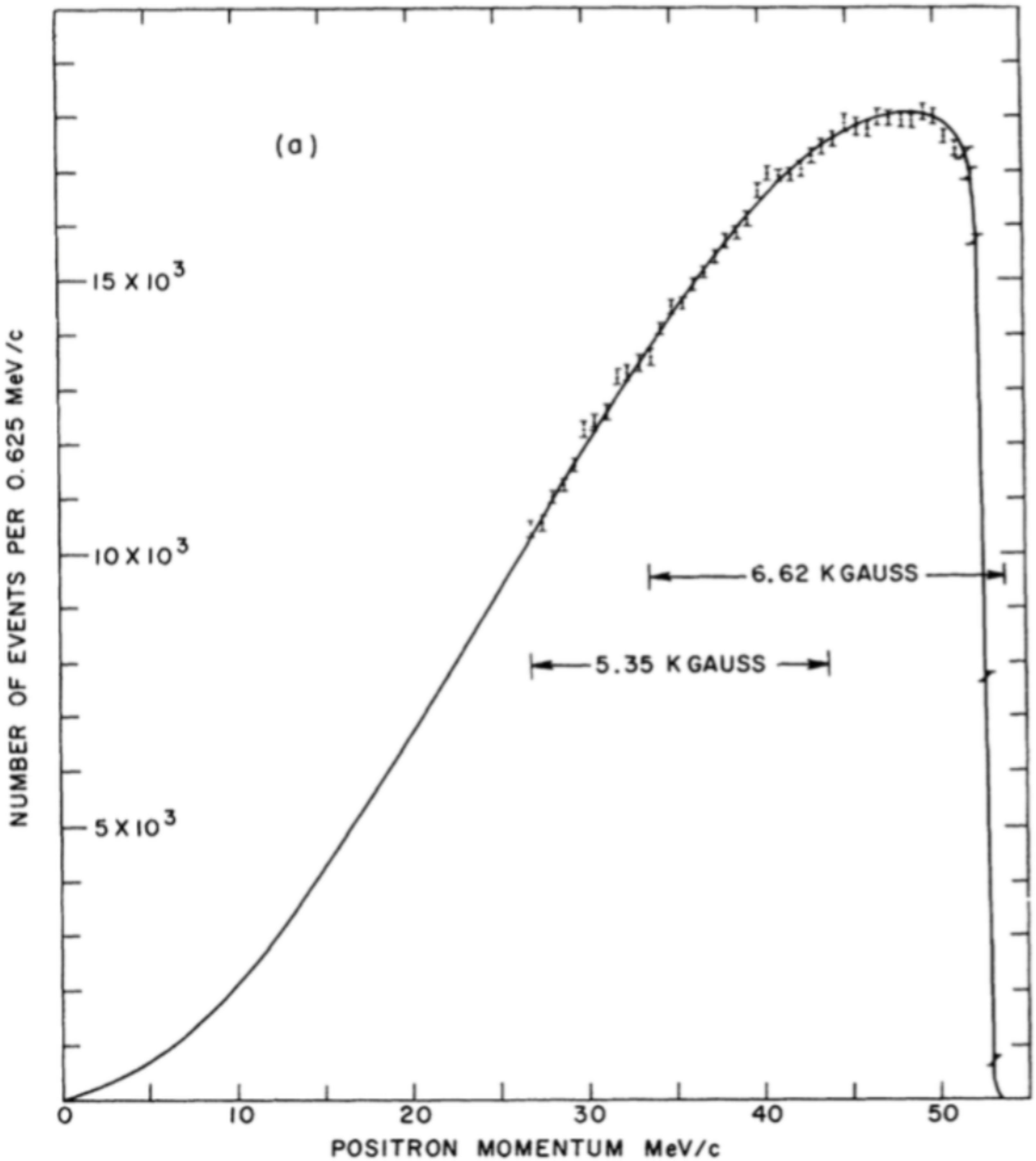
Then the muon decay rate is proportional to

$$\int_0^1 dx_e \int_{1-x_e}^1 dx_{\overline{\nu}}\; x_{\overline{\nu}}(1 - x_{\overline{\nu}})$$

that is,

$$\frac{d\Gamma}{dx_e} \sim \int_{1-x_e}^1 dx_{\overline{\nu}}\; x_{\overline{\nu}}(1 - x_{\overline{\nu}}) = (\frac{x_e^2}{2} - \frac{x_e^3}{3})$$

This shape, with a double zero at x = 0 and zero slope at the endpoint, is seen in the data.



(a)
NUMBER OF EVENTS PER 0.625 MeV/c
15 X 10^3
10 X 10^3
5 X 10^3
6.62 K GAUSS
5.35 K GAUSS
0
10
20
30
40
50
POSITRON MOMENTUM MeV/c

Bardon et al.

3. Charged pion decay is mediated by the V-A operator

$$\frac{4G_F}{\sqrt{2}}\ (d_L^\dagger \overline{\sigma}^\mu u_L)[\nu_{eL}^\dagger \overline{\sigma}_\mu e_L + \nu_{\mu L}^\dagger \overline{\sigma}_\mu \mu_L]$$

At first sight, it might seem that the pion must decay equally often to e and μ. This would contradict experiment, which says that almost all decays are to μ. But, what is the real prediction of V-A ?

The pion matrix element is

$$\langle 0|\,(d_L^\dagger \overline{\sigma}^\mu u_L)\,\big|\pi^+(p)\big\rangle = -i\frac{1}{2}F_\pi p^\mu$$

where F_π is the pion decay constant, equal to 135 MeV. Then the complete matrix element involves

$$p_\mu\ u_{\nu L}^\dagger(p_\nu)\overline{\sigma}^\mu v(p_{\ell+})$$

The neutrino must be left-handed, by V-A. But, the pion is spin 0, so the lepton must also be left-handed. The neutrino and lepton spinors are

$$U_L = \begin{pmatrix} \sqrt{2E_\nu}\ \xi_L \\ 0 \end{pmatrix} \qquad V_L = \begin{pmatrix} \sqrt{E_\ell - p_\ell}\xi_R \\ \times \end{pmatrix}$$

Then the matrix element is proportional to

$$|\mathcal{M}|^2 \sim (E_\ell - p_\ell)E_\nu = \frac{m_\ell^2}{m_\pi^2}(\frac{m_\pi^2 - m_\ell^2}{2m_\pi^2})$$

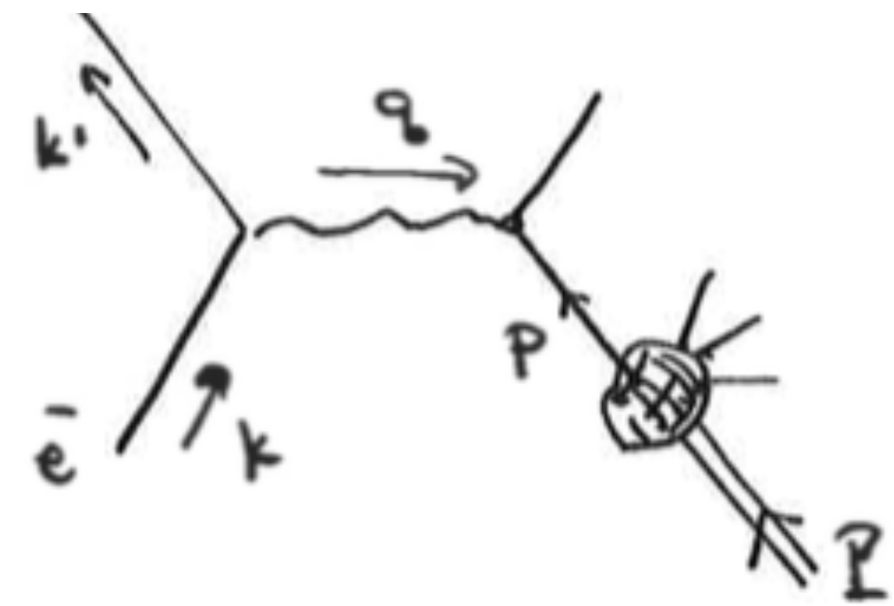
There is another factor of E_ν from phase space. Then V-A predicts

$$\frac{BR(\pi^- \to e^- \overline{\nu})}{BR(\pi^- \to \mu^- \overline{\nu})} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.28 \times 10^{-4}$$

comppared to experiment: 1.23×10^{-4}

4. The helicity structure of the V-A interaction between leptons and quarks is also seen in neutrino deep inelastic scattering. Electron deep inelastic scattering has the kinematics:

in leading order in QCD:



In neutrino deep inelastic scattering, we create this kinematic situation by producing neutrinos from pion decay, using an absorber (iron from a battleship) to remove muons, and then impinging the beam on a large target.

The kinematic variables of deep inelastic scattering are

$$s = (k+p)^2 \qquad\qquad Q^2 = -q^2$$

$$y = \frac{q^0}{k^0} = \frac{2P\cdot q}{2P\cdot k} \qquad\qquad x = \frac{Q^2}{2P\cdot q}$$

so, $Q^2 = xys$. The quark is a parton with momentum fraction ξ , $0 < \xi < 1$. Then in the lepton-parton reaction

$$\hat{s} = 2p\cdot k = 2\xi P\cdot k$$

$$\hat{t} = q^2 = -Q^2$$

$$\hat{u} = -2p\cdot k' = -2\xi P\cdot(k-q) = -\hat{s}(1-y)$$

The final quark is on shell, so

$$0 = (p+q)^2 = 2p\cdot q - Q^2 = 2\xi P\cdot q - Q^2$$

and ξ is equal to the observable x !

What concerns us here is the distribution in y. For the reaction

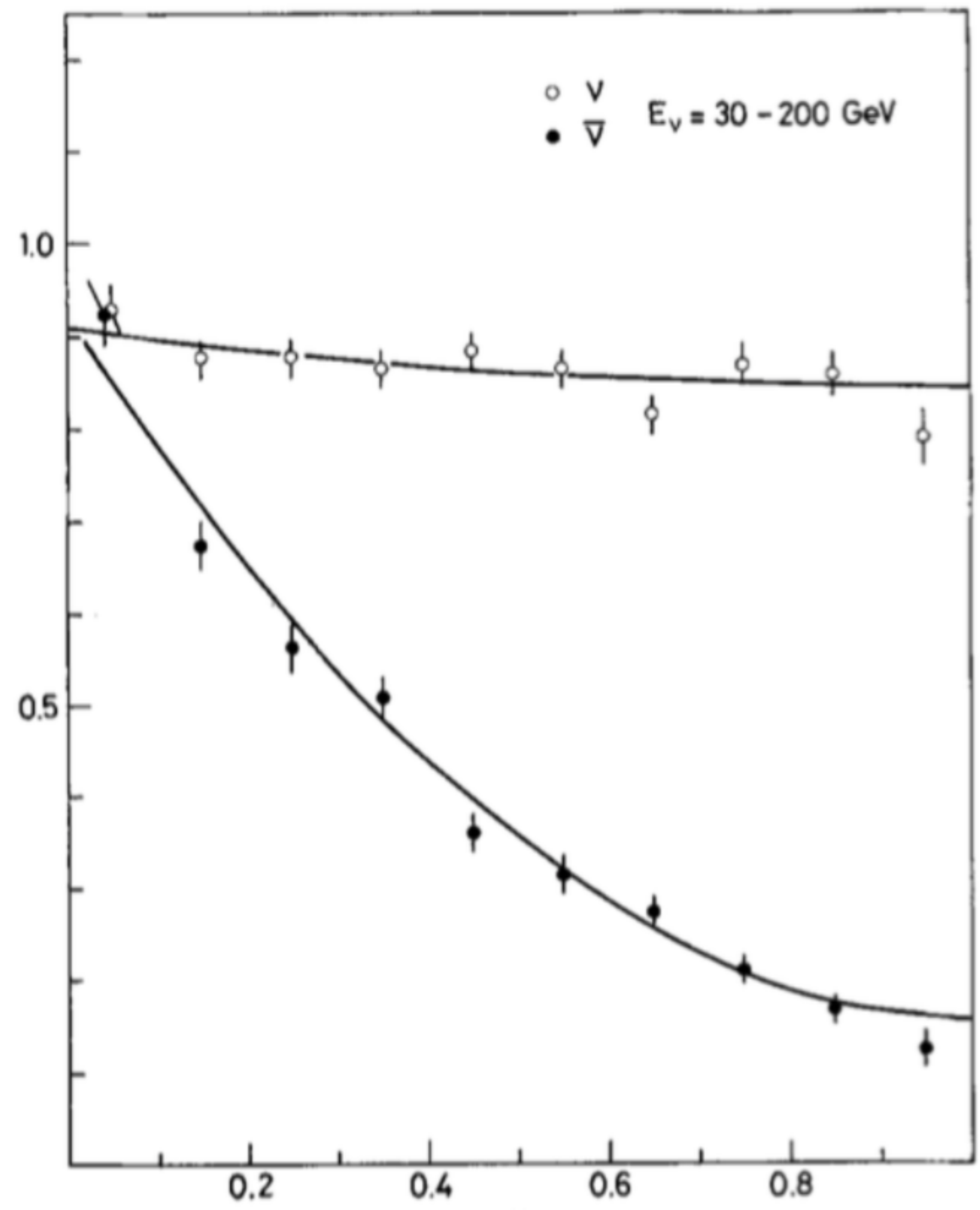
$$\nu + d \to \mu^- + u$$

the basic current-current amplitudes would be

$$|(u_L^\dagger(p_\mu)\overline{\sigma}^\mu u_L(\nu))(u_L^\dagger(u)\overline{\sigma}_\mu u_L(d))|^2$$
$$= 4(2p_\mu \cdot p_u)(2p_\nu \cdot p_d) = 4s^2$$

$$|(u_L^\dagger(p_\mu)\overline{\sigma}^\mu u_L(\nu))(u_R^\dagger(u)\overline{\sigma}_\mu u_R(d))|^2$$
$$= 4(2p_\mu \cdot p_d)(2p_\nu \cdot p_u) = 4u^2 = 4s^2(1-y)^2$$

Neutrinos from π^+decay are L. V-A says that they have no charge-changing weak coupling to R quarks. Then the $(1-y)^2$ term should be absent. Conversely, antineutrinos are R, so the deep inelastic cross section should be proportional to $(1-y)^2$.



CDHS
experiment

In the more modern era, we test these predictions in collider physics. For example, the Standard Model predicts that

$$\frac{d\sigma}{d\cos\theta_*}(d\overline{u} \to W^- \to \mu^-\overline{\nu}) \sim u^2 \sim (1+\cos\theta_*)^2$$

$$\frac{d\sigma}{d\cos\theta_*}(u\overline{d} \to W^+ \to \mu^+\nu) \sim t^2 \sim (1-\cos\theta_*)^2$$

These angular distributions are well verified at the LHC.

The neutral current amplitudes are more complex, because the photon and Z couple to both L and R fermions. In e^+e^- annihilation (for example, at LEP), the angular distributions are

$$\frac{d\sigma}{d\cos\theta}(e^-_L e^+_R \to f_L \overline{f}_R) = \frac{\pi\alpha^2}{2s} F_{LL}(s)(1+\cos\theta)^2$$

$$\frac{d\sigma}{d\cos\theta}(e^-_R e^+_L \to f_L \overline{f}_R) = \frac{\pi\alpha^2}{2s} F_{RL}(s)(1-\cos\theta)^2$$

$$\frac{d\sigma}{d\cos\theta}(e^-_L e^+_R \to f_R \overline{f}_L) = \frac{\pi\alpha^2}{2s} F_{LR}(s)(1-\cos\theta)^2$$

$$\frac{d\sigma}{d\cos\theta}(e^-_R e^+_L \to f_R \overline{f}_L) = \frac{\pi\alpha^2}{2s} F_{RR}(s)(1+\cos\theta)^2$$

where

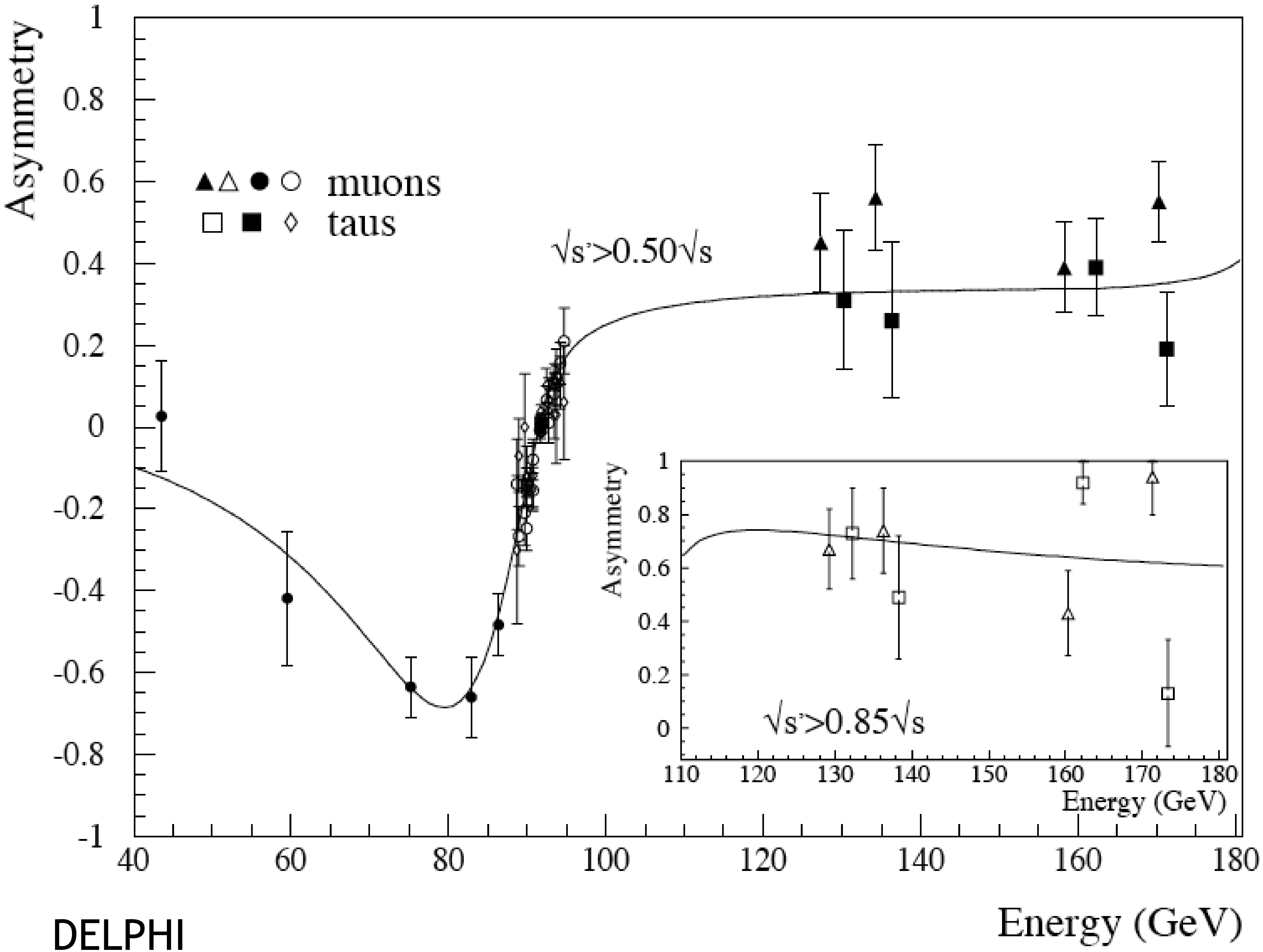
$$F_{LL} = \left| Q_f \;+\; \frac{(1/2 - s_w^2)(I_f^3 - s_w^2 Q_f)}{s_w^2 c_w^2} \frac{s}{s - m_Z^2} \right|^2$$

$$F_{RL} = \left| Q_f \;+\; \frac{(-s_w^2)(I_f^3 - s_w^2 Q_f)}{s_w^2 c_w^2} \frac{s}{s - m_Z^2} \right|^2$$

$$F_{LR} = \left| Q_f \;+\; \frac{(1/2 - s_w^2)(-s_w^2 Q_f)}{s_w^2 c_w^2} \frac{s}{s - m_Z^2} \right|^2$$

$$F_{RR} = \left| Q_f \;+\; \frac{(-s_w^2)(-s_w^2 Q_f)}{s_w^2 c_w^2} \frac{s}{s - m_Z^2} \right|^2$$

Note, for $s > m_Z^2$, constructive interference for LL and RR, destructive interference for RL and LR. Then, with unpolarized beams (as at LEP), we expect a positive forward-backward asymmetry.


Asymmetry
muons
taus
√s'>0.50√s
Energy (GeV)
Asymmetry
√s'>0.85√s
Energy (GeV)

DELPHI

It is interesting to explore the high energy limits of the expressions $F_{IJ}(s)$. Begin with $F_{RL}(s)$. In the limit $s \gg m_Z^2$, this becomes

$$F_{RL} \to \left| \frac{s_w^2 c_w^2 (I_f^3 + Y_f) - s_w^2 I_f^3 + s_w^4 (I_f^3 + Y_f)}{s_w^2 c_w^2} \right|^2$$

$$= \left| \frac{s_w^2 Y_f}{s_w^2 c_w^2} \right|^2 = \left| \frac{(-1) Y_f}{c_w^2} \right|^2 = \left| \frac{g'^2}{e^2} Y_{eR} Y_f \right|^2$$

This is exactly the amplitude for s-channel B boson exchange, in the situation where the original SU(2)xU(1) symmetry of the model is not broken.

The simplicity of this expression tells us that it is useful to analyze the high-energy limit of the weak interactions from the viewpoint that broken symmetry is restored at high energy.

Here is the same analysis for $F_{LL}(s)$:

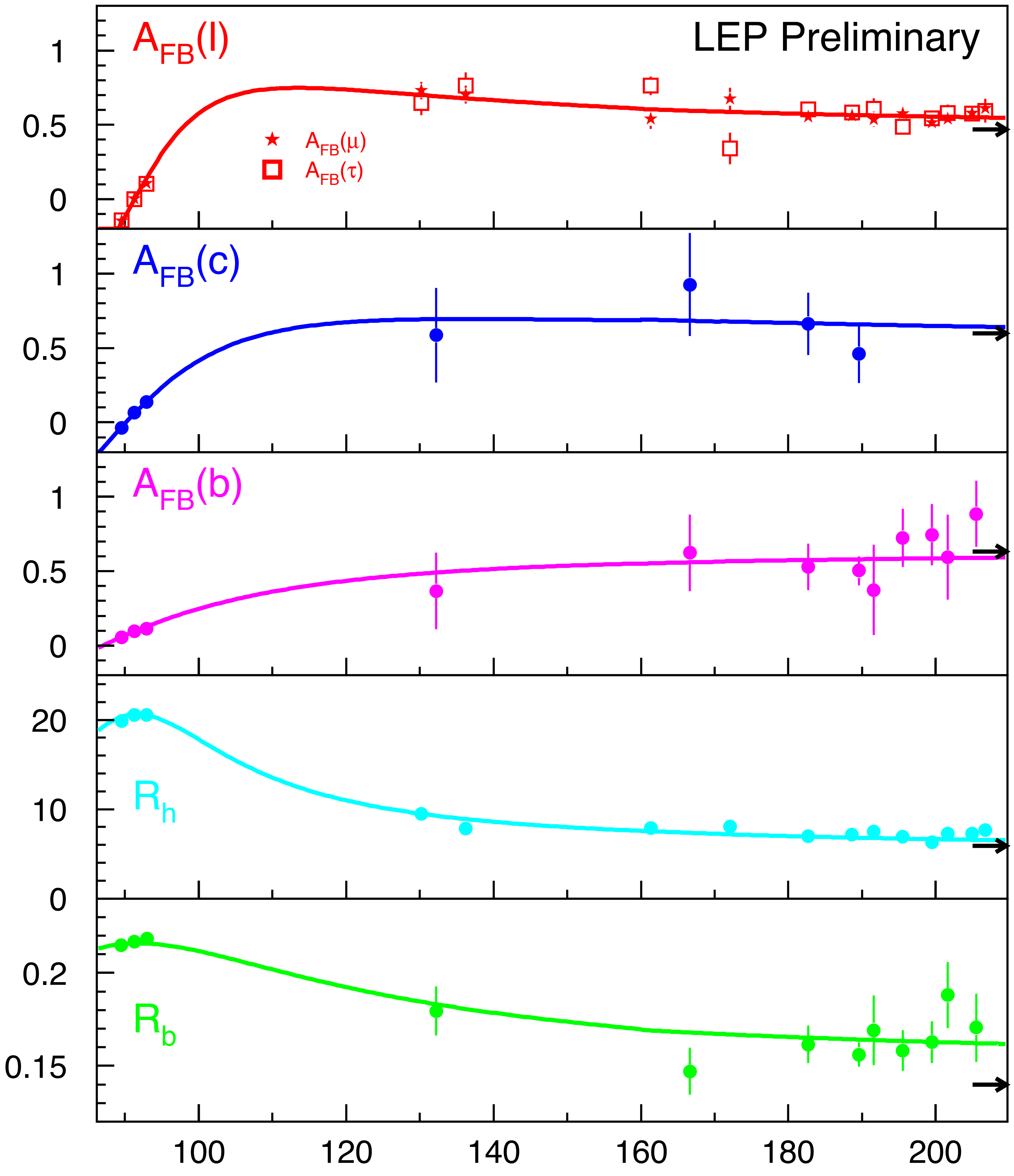
$$F_{LL} \to \left| \frac{s_w^2 c_w^2 (I_f^3 + Y_f) + (1/2 - s_w^2)(I_f^3 - s_w^2(I_f^3 + Y_f))}{s_w^2 c_w^2} \right|^2$$

$$= \left| \frac{(1/2) c_w^2 I_f^3 + (1/2) s_w^2 Y_f}{s_w^2 c_w^2} \right|^2$$

$$= \left| \frac{g^2}{e^2} I_{eL}^3 I_f^3 + \frac{g'^2}{e^2} Y_{eL} Y_f \right|^2$$

so the result is a coherent sum of A^3 and B exchanges as expected in the theory with unbroken symmetry.

Here is the approach to the limit of the symmetric theory as measured at LEP:



data compilation
by Hildreth

The Z boson appears as a resonance in e^+e^- annihilation. In the 1990's, the accelerators LEP and SLC tuned their energy to the Z mass to produce large numbers of Z bosons at rest in the lab, in an appropriate setting for precision measurements.

LEP also operated above 200 GeV, to study the electroweak pair production of W and Z bosons. I will discuss that program in the next lecture.

I will now review the precision weak interaction experiments at the Z, which continue to provide important constraints on the Standard Model and its generalizations.

The e^+e^- cross section in the vicinity of the Z resonance.

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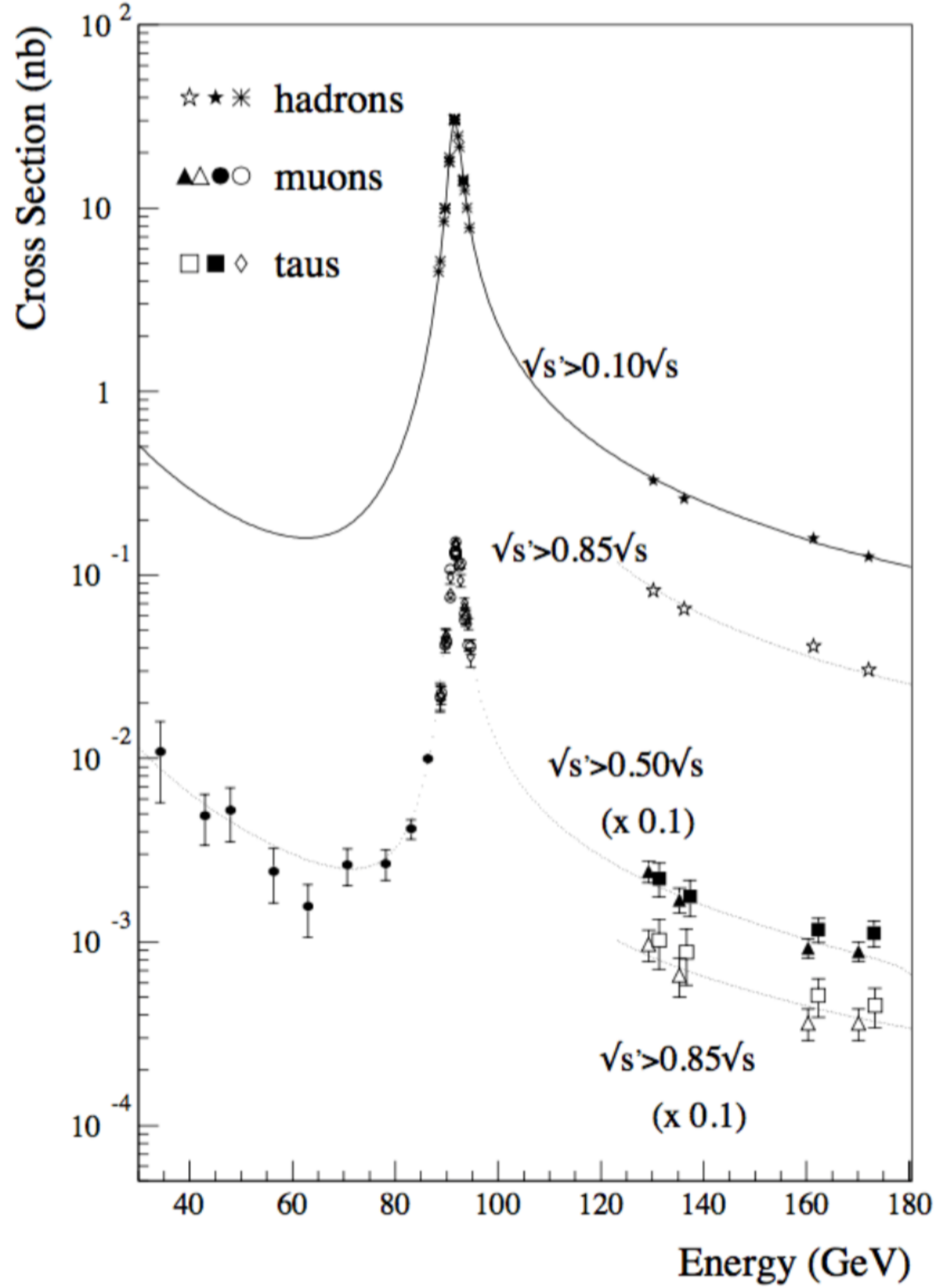


To begin, we should work out the Z width and branching fractions at leading order.

The leading order matrix element for Z decay to $f_L \overline{f}_R$ is

$$\mathcal{M}(Z \to f_L \overline{f}_R) = i \frac{g}{c_w} \, Q_{Zf} \; u_L^\dagger \overline{\sigma}^\mu v_R \; \epsilon_{Z\mu}$$

with

$$Q_Z = I^3 - s_w^2 Q$$

Recall from the previous lecture that

$$u_L^\dagger \overline{\sigma}^\mu v_R = (2E) \; \sqrt{2} \; (\epsilon_-^\mu)^*$$

We can integrate over the fermion direction, but it is simpler, and equivalent, to average over the direction of the Z polarization. Then

$$\langle |\mathcal{M}|^2 \rangle = \frac{2}{3} \frac{g^2}{c_w^2} Q_Z^2 m_Z^2$$

Then

$$\Gamma(Z \to f_L \overline{f}_R) = \frac{1}{2m_Z} \frac{1}{8\pi} \left\langle |M|^2 \right\rangle$$

So, finally,

$$\Gamma(Z \to f_L \overline{f}_R) = \frac{\alpha_w m_Z}{6 c_w^2} \; Q_Z^2 \; N_f$$

where

$$\alpha_w = \frac{g^2}{4\pi} \qquad N_f = \begin{cases} 1 & \text{lepton} \\ 3(1 + \alpha_s/\pi + \cdots) & \text{quark} \end{cases}$$

The widths to right-handed species $f_R \overline{f}_L$ obey the same formulae. Now we only need to evaluate these formulae and sum over all Standard Model species that can appear in Z decays.

It is worth pausing to ask what values of coupling constants we should use to evaluate this formula.

Begin with α . You all know that $\alpha = 1/137$. However, α is a running coupling constant that takes larger values as the length scale on which it is considered decreases. At $Q = 91.\ \mathrm{GeV}$, $\alpha(Q) = 1/128$. Later in the lecture, I will defend a value of s_w^2

$$s_w^2 = 0.23$$

For this value, we find

$$\alpha_w = \frac{1}{29.6} \qquad \alpha' = \frac{1}{98.}$$

It is interesting to compare these to other fundamental Standard Model couplings at the same scale:

$$\alpha_s = \frac{1}{8.5} \qquad \alpha_t = \frac{y_t^2}{4\pi} = \frac{1}{12.7}$$

We combine with these values the values of the Q_Z . It is useful to tabulate these for one Standard Model generation:

species	Q_{ZL}	Q_{ZR}	S_f	A_f
ν	$+\frac{1}{2}$	$-$	0.250	1.00
e	$-\frac{1}{2} + s_w^2$	$+s_w^2$	0.126	0.15
u	$+\frac{1}{2} - \frac{2}{3}s_w^2$	$-\frac{2}{3}s_w^2$	0.144	0.67
d	$-\frac{1}{2} + \frac{1}{3}s_w^2$	$+\frac{1}{3}s_w^2$	0.185	0.94

In this table, the quantities evaluated numerically are

$$S_f = Q_{ZL}^2 + Q_{ZR}^2 \qquad A_f = \frac{Q_{ZL}^2 - Q_{ZR}^2}{Q_{ZL}^2 + Q_{ZR}^2}$$

The first of these gives the total decay rate for the species f . The second gives the polarization asymmetry, the preponderance of f_L over f_R in Z decays.

It is possible to measure both the rates and the asymmetries in Z resonance experiments.

The S_f are tested by the Z total width and branching ratios. At the level of our leading-order theory, the width is

$$\Gamma_Z = \frac{\alpha_w m_Z}{6c_w^2}\Big[\underset{\nu}{3\cdot 0.25} + \underset{e}{3\cdot 0.126} + \underset{u}{2\cdot(3.1)\cdot 0.144} + \underset{d}{3\cdot(3.1)\cdot 0.185}\Big]$$

The separate terms in this formula give the branching ratios

$$BR(\nu_e\overline{\nu}_e) = 6.7\% \qquad BR(e^+e^-) = 3.3\%$$

$$BR(u\overline{u}) = 11.9\% \qquad BR(d\overline{d}) = 15.3\%$$

The numerical value of the total is $\Gamma_Z = 2.49\ \mathrm{GeV}$

This can be compared to the value obtained from the Z resonance lineshape

$$\Gamma_Z = 2.4952 \pm 0.0023\ \mathrm{GeV}$$

The precision of the Z resonance measurements is quite remarkable, reaching parts per mil for many variables. To discuss the rapport between theory and experiment at this level, we need to include electroweak radiative corrections, which typically are of order 1%.

As I continue to discuss the experimental results, I will make reference to radiative corrections that are particularly important.

To give a complete accounting of radiative corrections, I should give a precise account of the renormalization conventions used. Please let me postpone that discussion to later in the lecture (where, in any event, I will still not treat it completely).

To begin the review of experiments, I should discuss the measurement of the Z mass and width in more detail.

Ideally, the Z is a Breit-Wigner resonance,

$$\sigma \sim \left| \frac{1}{s - m_Z^2 + i m_Z \Gamma_Z} \right|^2$$

however, the line shape is distorted by initial state radiation. The magnitude of collinear photon radiation is given by the parameter

$$\beta = \frac{2\alpha}{\pi}(\log \frac{s}{m_e^2} - 1) = 0.108 \text{ at the Z}$$

In addition, since the Z is narrow, the effect is magnified, since relatively soft radiation can push the CM energy off resonance. The size of the correction on the Z peak can be roughly estimated as $-\beta \cdot \log \frac{m_Z}{\Gamma_Z} = -40\%$

To make a proper accounting, we need to resum collinear photon radiation just as we resum collinear gluon radiation in parton distributions.

Fadin and Kuraev computed the parton distribution of an electron in the electron and computed this in QED

$$f_e(z,s) = \frac{\beta}{2}(1-z)^{\beta/2-1}(1+\frac{3}{8}\beta) - \frac{1}{4}\beta(1+z) + \cdots$$

This function, for each electron, would be convolved with the Breit-Wigner. The theory was extended to include 2 orders of subleading logs and finite corrections of order α^2.

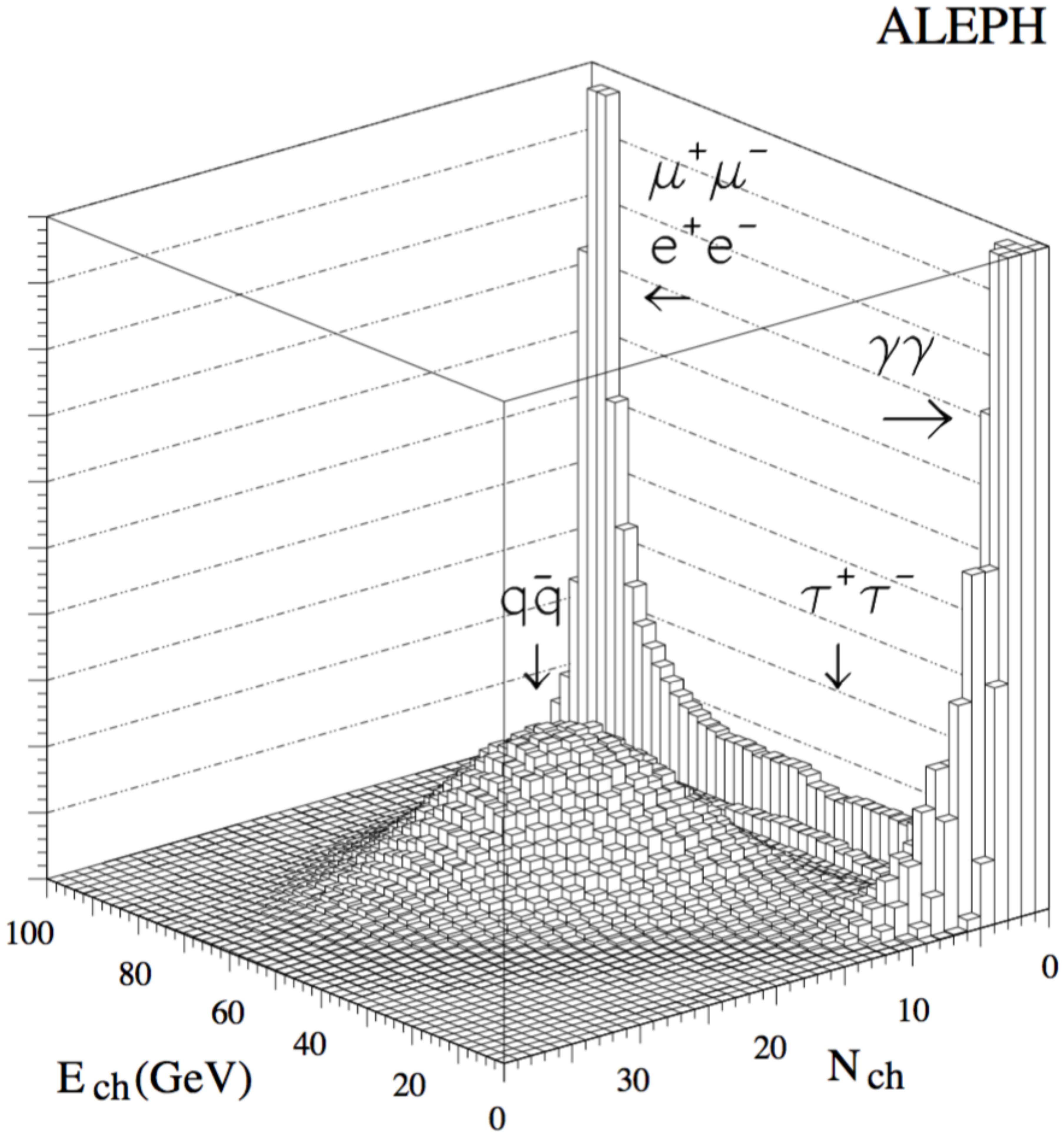
The experimental aspects of the measurement were also very challenging. The energy of the LEP ring was calibrated using resonant depolarization of a single beam and then corrected for 2-beam effects.

However, this calibration was found to depend on the season and the time of day. Some contributing effects were the changes in the size of the LEP/LHC tunnel due to the annual change in the water level of Lake Geneva and current surges in the magnets due to the passage of the TGV.

To measure the branching ratios, we need only collect Z events and sort them into categories.

The major backgrounds are from Bhabha and 2-gamma events; these do not resemble Z events (unlike the situation at LHC !).   Nonresonant annihilations are at the level of parts per mil (except for tau - few %).

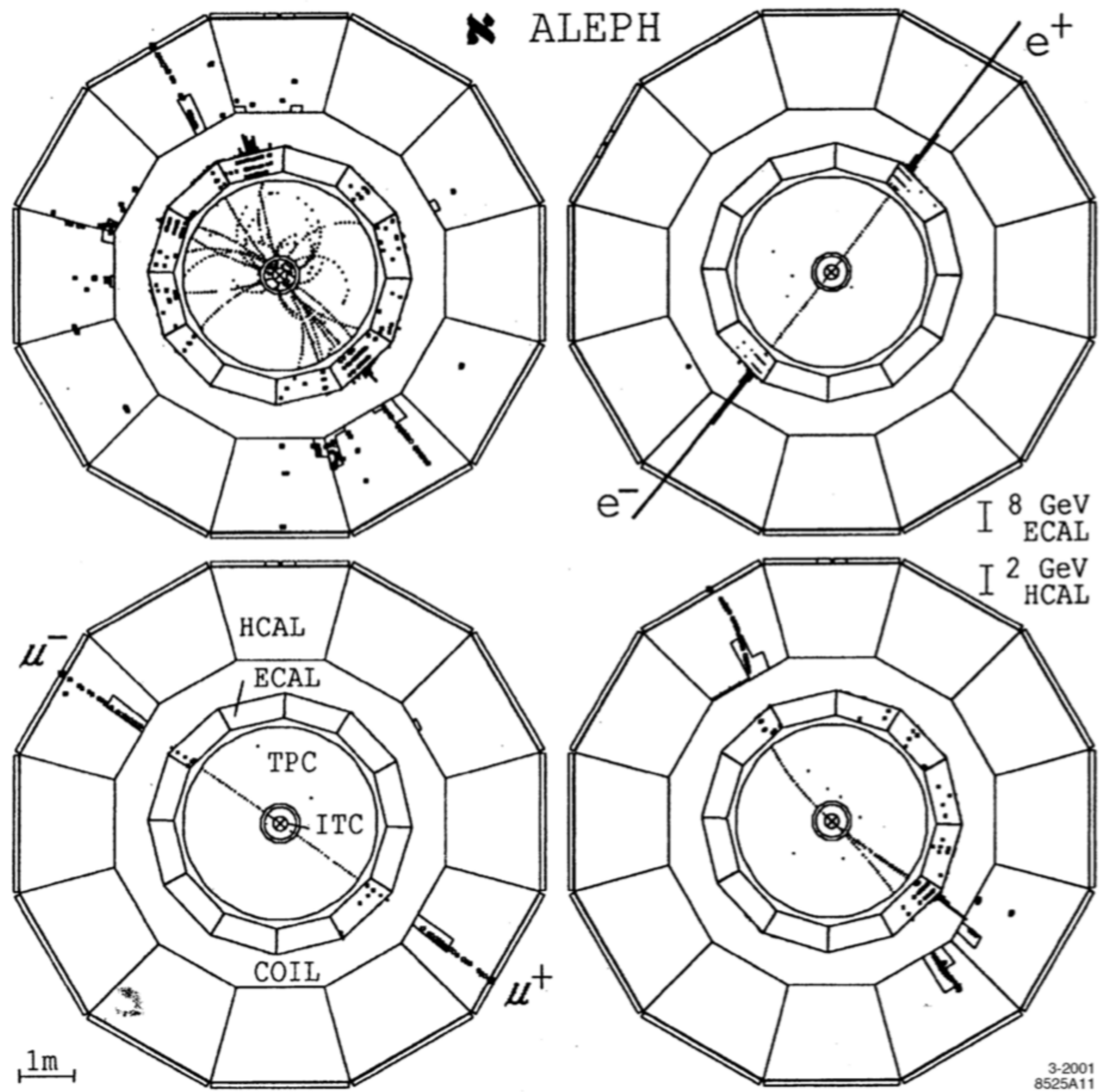
The various leptonic and hadronic decay modes have different, characteristic, forms.

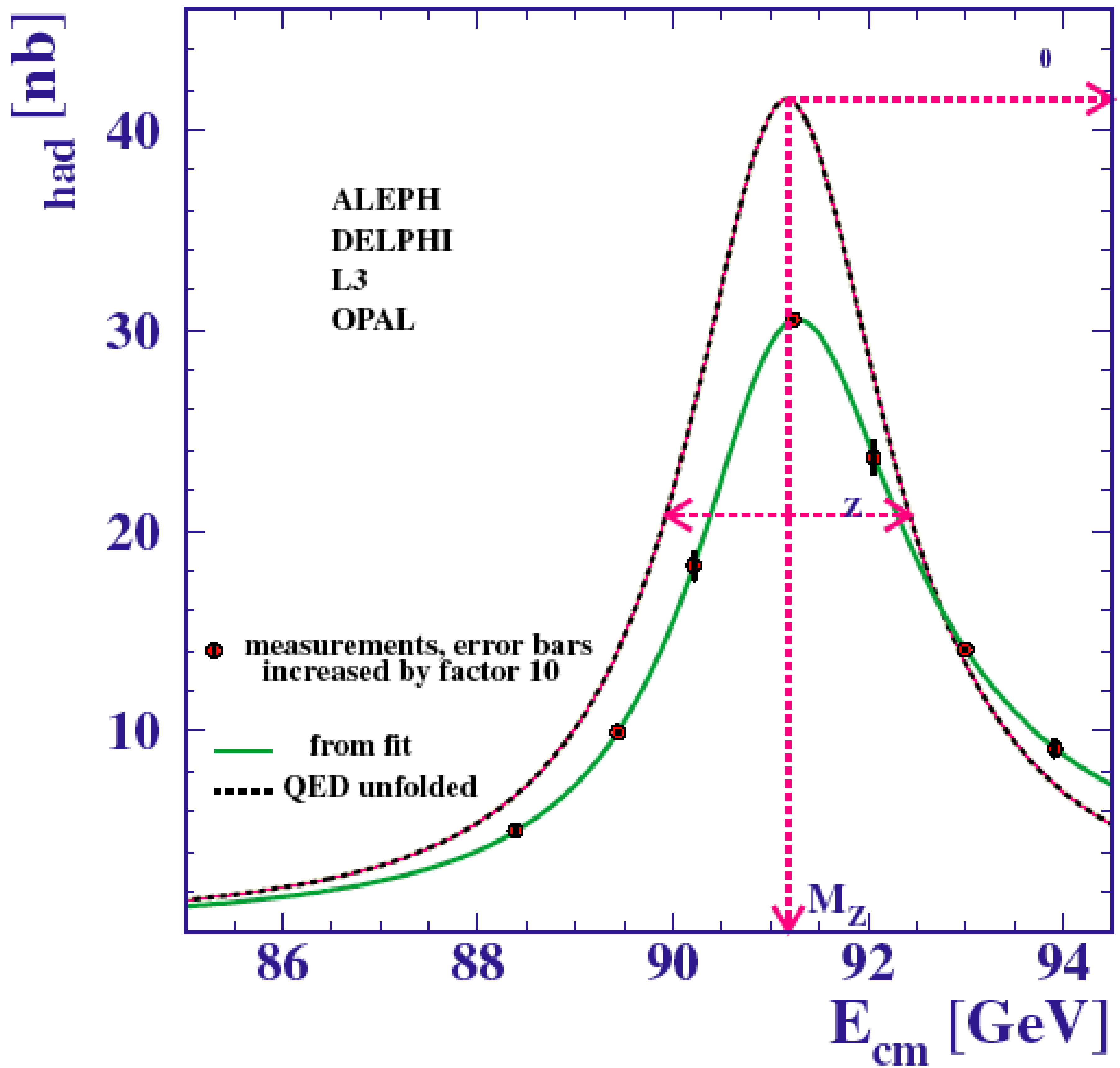


ALEPH
$\mu^+\mu^-$
e^+e^-
$\gamma\gamma$
$q\bar{q}$
$\tau^+\tau^-$
100
80
60
40
20
0
E_{ch}(GeV)
0
10
20
30
N_{ch}

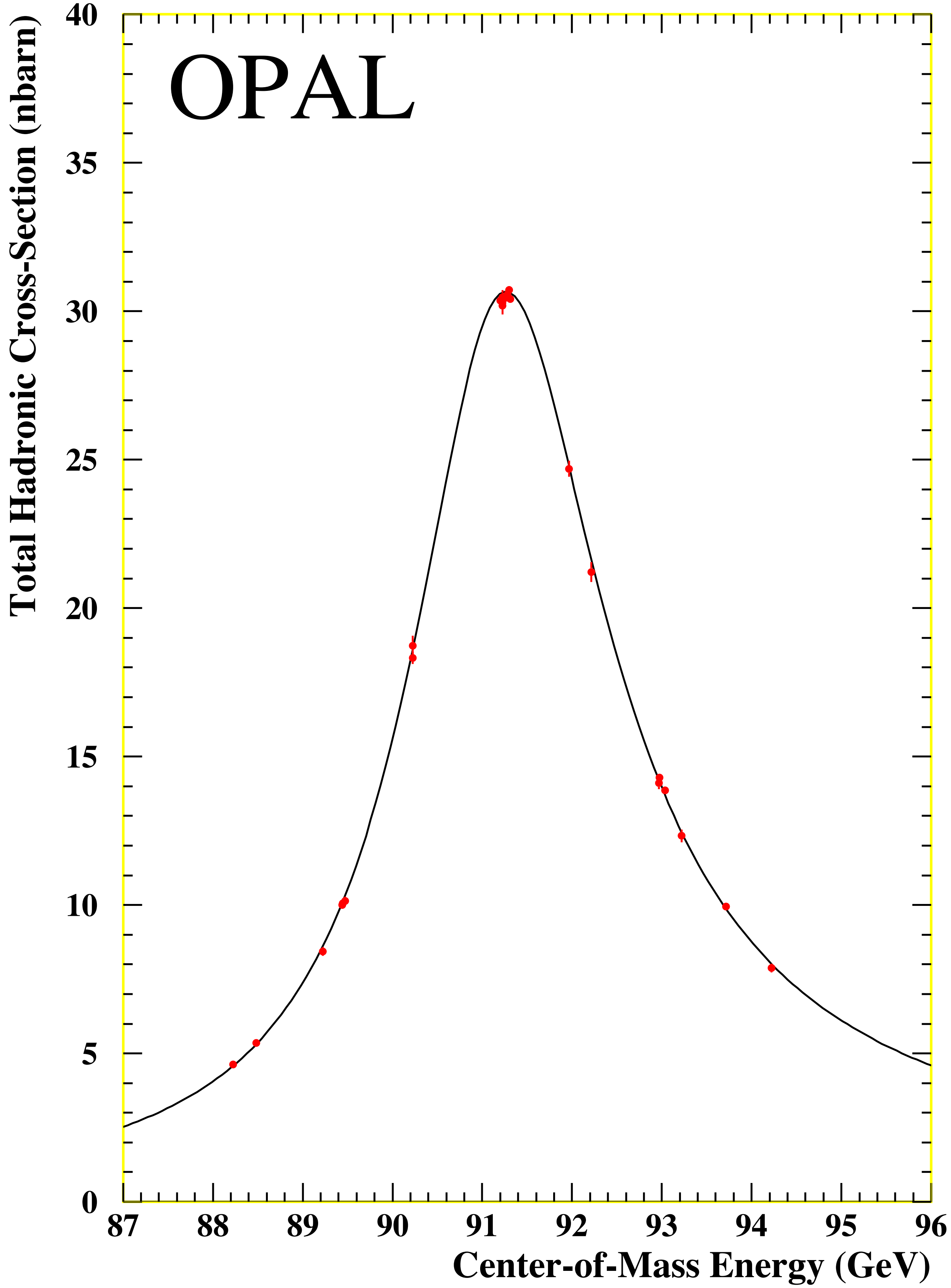
	ALEPH	DELPHI	L3	OPAL
$q\bar{q}$ final state				
acceptance	$s'/s > 0.01$	$s'/s > 0.01$	$s'/s > 0.01$	$s'/s > 0.01$
efficiency [%]	99.1	94.8	99.3	99.5
background [%]	0.7	0.5	0.3	0.3
e^+e^- final state				
acceptance	$-0.9 < \cos\theta < 0.7$ $s' > 4m_\tau^2$	$\lvert\cos\theta\rvert < 0.72$ $\eta < 10°$	$\lvert\cos\theta\rvert < 0.72$ $\eta < 25°$	$\lvert\cos\theta\rvert < 0.7$ $\eta < 10°$
efficiency [%]	97.4	97.0	98.0	99.0
background [%]	1.0	1.1	1.1	0.3
$\mu^+\mu^-$ final state				
acceptance	$\lvert\cos\theta\rvert < 0.9$ $s' > 4m_\tau^2$	$\lvert\cos\theta\rvert < 0.94$ $\eta < 20°$	$\lvert\cos\theta\rvert < 0.8$ $\eta < 90°$	$\lvert\cos\theta\rvert < 0.95$ $m_{f\bar{f}}^2/s > 0.01$
efficiency [%]	98.2	95.0	92.8	97.9
background [%]	0.2	1.2	1.5	1.0
$\tau^+\tau^-$ final state				
acceptance	$\lvert\cos\theta\rvert < 0.9$ $s' > 4m_\tau^2$	$0.035 < \lvert\cos\theta\rvert < 0.94$ $s' > 4m_\tau^2$	$\lvert\cos\theta\rvert < 0.92$ $\eta < 10°$	$\lvert\cos\theta\rvert < 0.9$ $m_{f\bar{f}}^2/s > 0.01$
efficiency [%]	92.1	72.0	70.9	86.2
background [%]	1.7	3.1	2.3	2.7

final LEPEWWG Phys. Rept. 2005


ALEPH
e+
e−
8 GeV
ECAL
2 GeV
HCAL
HCAL
ECAL
TPC
ITC
COIL
μ−
μ+
1m
3-2001
8525A11



composite of the four LEP experiments, showing the effect of ISR



OPAL
Total Hadronic Cross-Section (nbarn)
Center-of-Mass Energy (GeV)
0
5
10
15
20
25
30
35
40
87
88
89
90
91
92
93
94
95
96

Two particular branching ratios merit special attention.

First, the Z decays invisibly, to neutrinos, 20% of the time. This decay affects the cross section

$$\sigma(e^+e^- \to Z \to \text{ hadrons})$$

by decreasing the Z peak height and increasing the width. Measurement of these parameters and comparison to Standard Model predictions gives

$$n_\nu = 2.9840 \pm 0.0082$$

Second, the Z branching ratio to b quarks is of special interest, particularly because the b belongs to the same SU(2)xU(1) multiplet as the t_L .

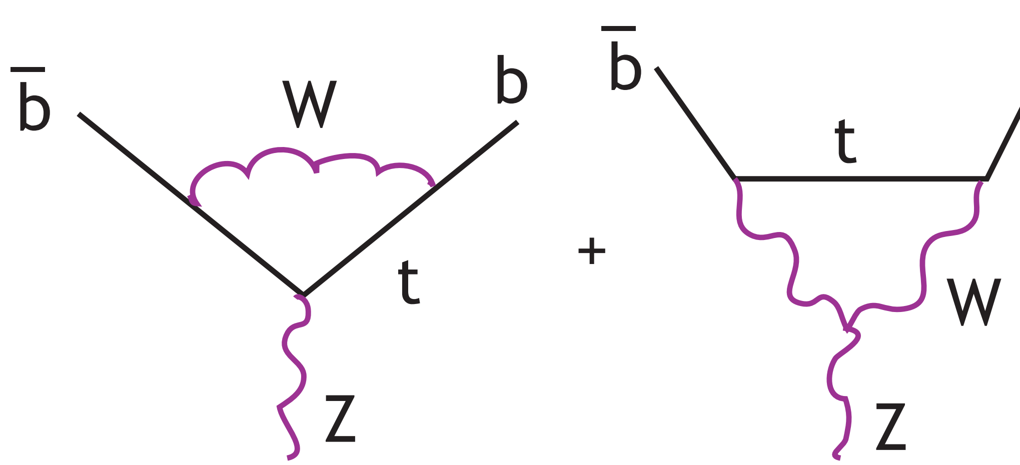
An observable that specifically tracks this effect is

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{ hadrons})}$$

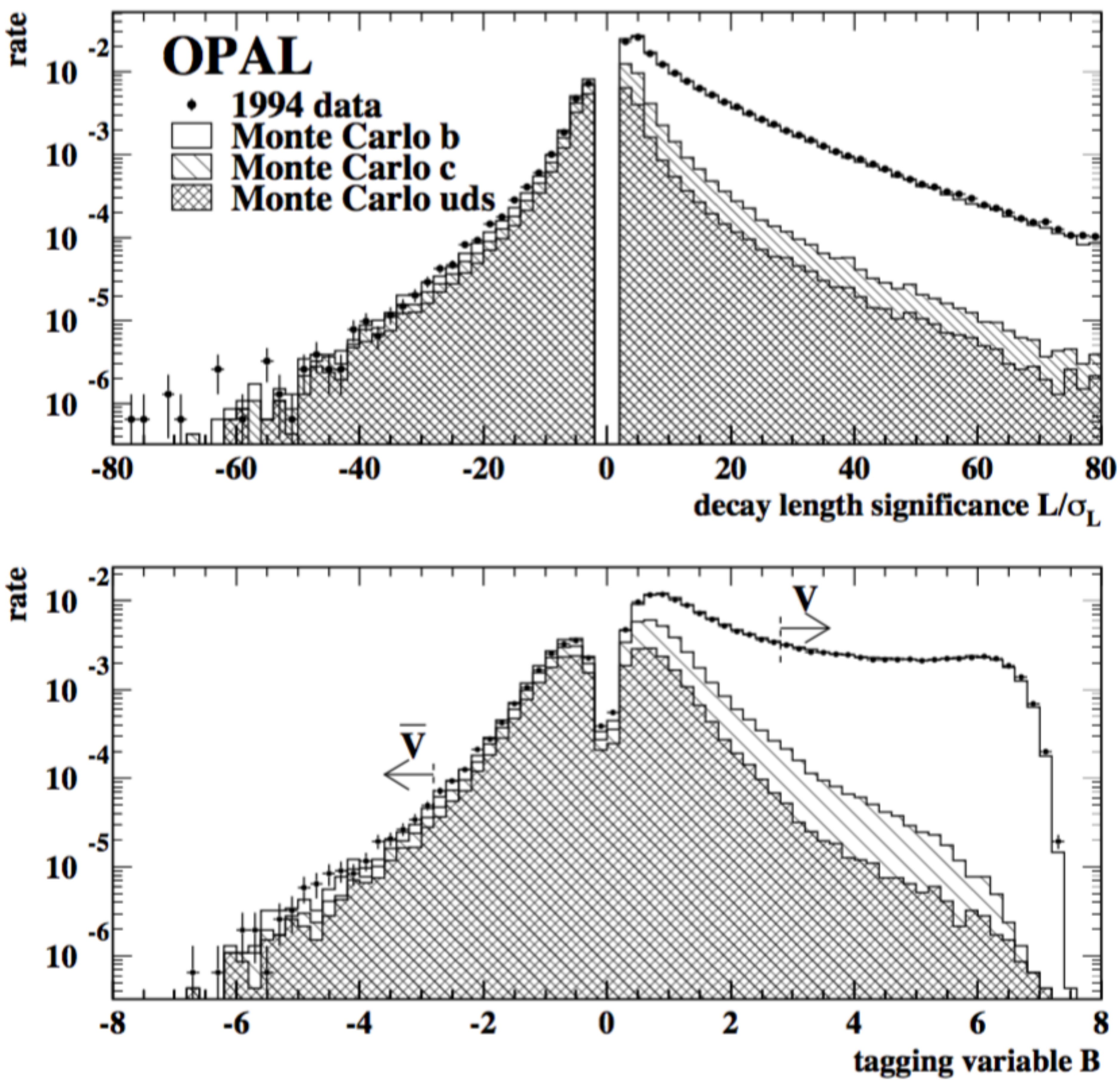
In the leading-order model, this quantity has the value

$$R_b = 0.22$$

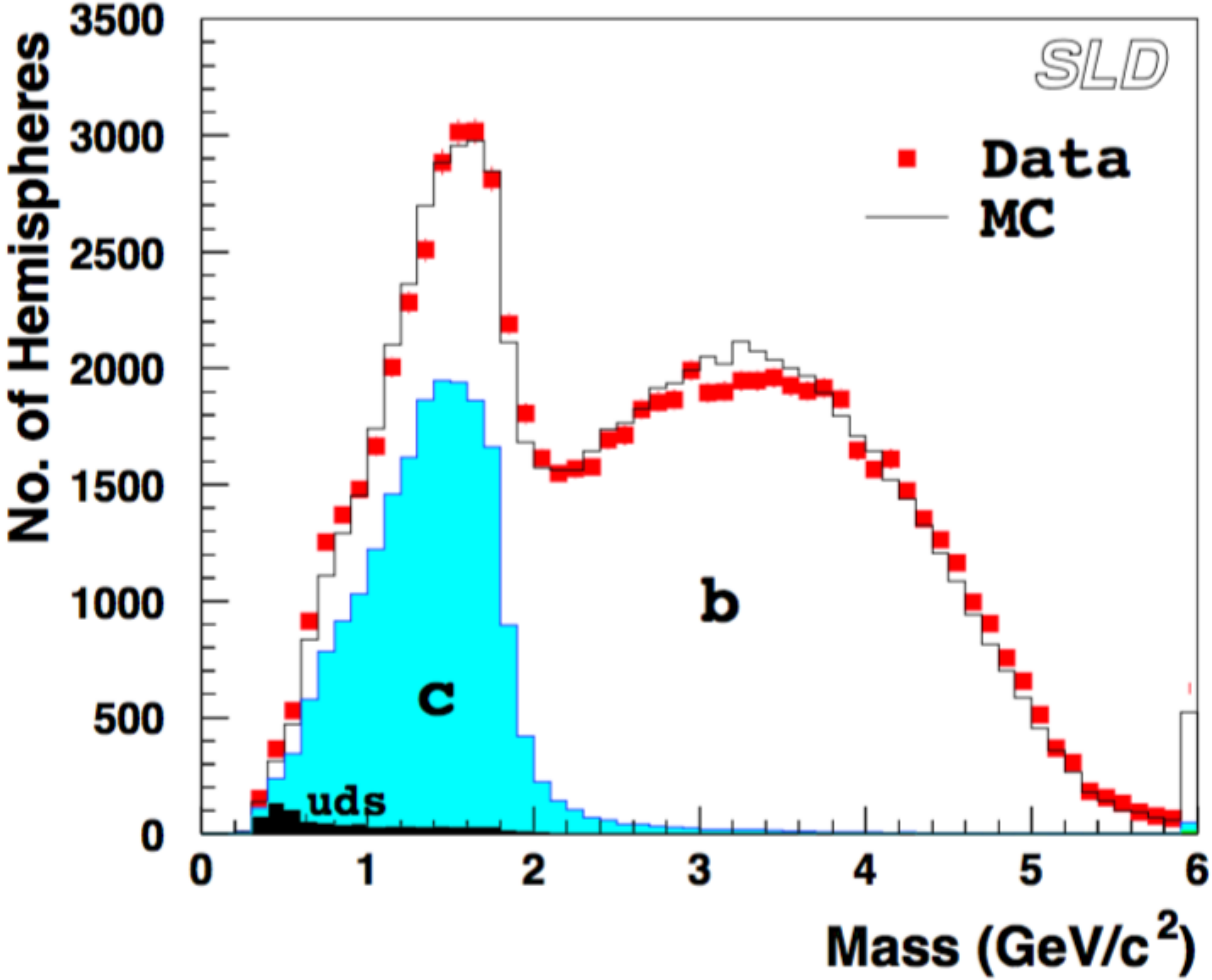
However there is a large radiative correction from diagrams involving the top quark



$$Q_{ZbL} = -(\frac{1}{2} - \frac{1}{3}s_w^2 - \frac{\alpha}{16\pi s_w^2}\frac{m_t^2}{m_W^2})$$



OPAL
1994 data
Monte Carlo b
Monte Carlo c
Monte Carlo uds
rate
decay length significance L/σ_L
rate
V
$\overline{V}$
tagging variable B


SLD
Data
MC
No. of Hemispheres
3500
3000
2500
2000
1500
1000
500
0
0
1
2
3
4
5
6
Mass (GeV/c^2)
b
c
uds

b-tag working points used in these studies.

	ALEPH	DELPHI	L3	OPAL	SLD
b Purity [%]	97.8	98.6	84.3	96.7	98.3
b Efficiency [%]	22.7	29.6	23.7	25.5	61.8

The performance of SLD was much better due to its pixel vertex detector at 2 cm; however, the SLD statistics was 10 times smaller.

Final LEP/SLC results:

$$R_b = 0.21629 \pm 0.00066$$ (-2% from LO)

$$R_c = 0.1721 \pm 0.0030$$

Now turn to the Z asymmetries.   These take very different values for l, c, b – all predicted by a common value of s_w^2 .

There are three very different methods to measure the lepton asymmetries:

from forward-backward asymmetries, esp. to quarks

from direct measurement using beam polarization

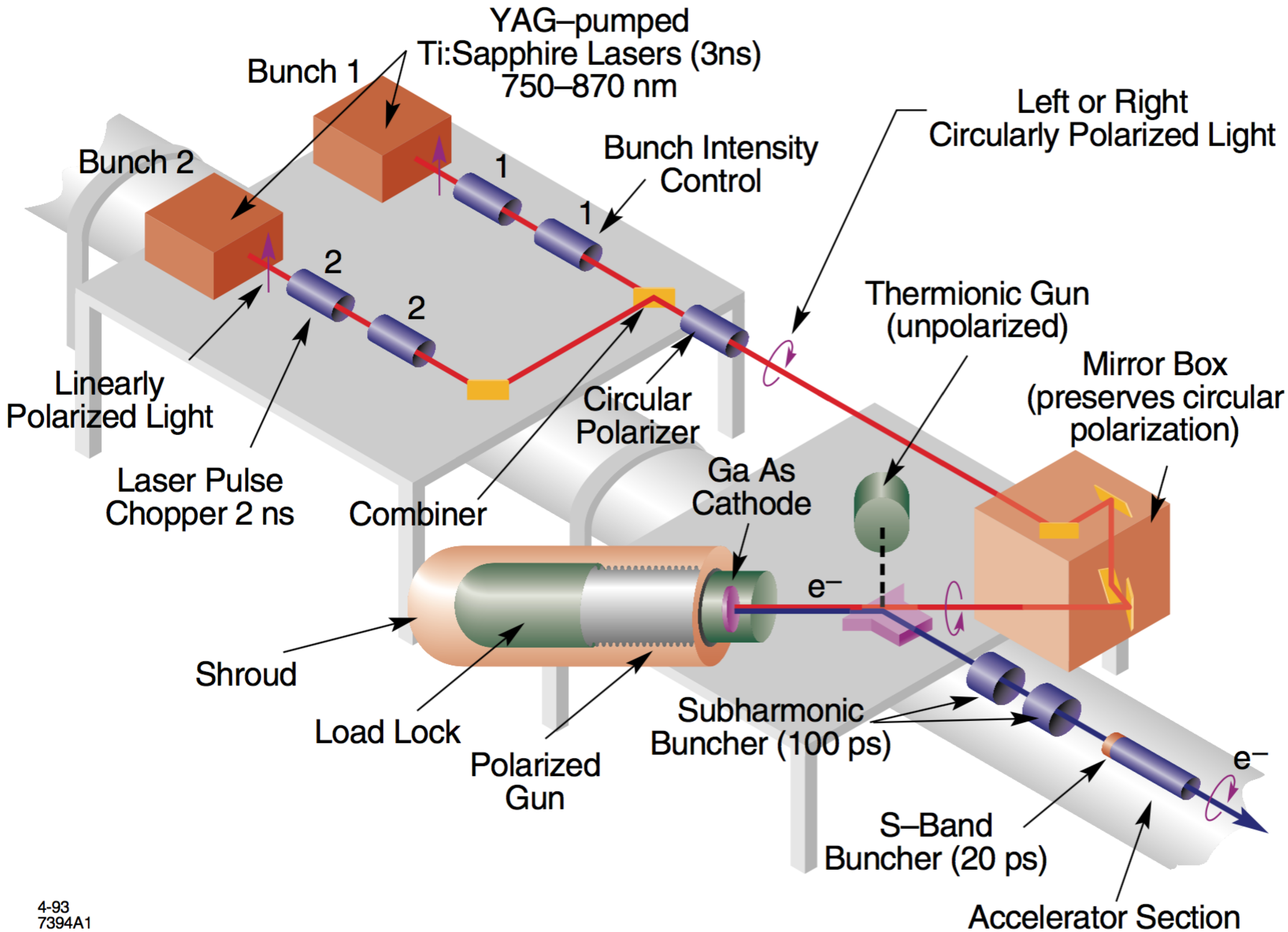
from tau lepton polarimetry

For unpolarized beams, the angular distribution for $e^+e^- \to f\overline{f}$ is :

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to f\overline{f}) \sim \left(\frac{1+A_e}{2}\right)\left(\frac{1+A_f}{2}\right)(1+\cos\theta)^2 + \left(\frac{1-A_e}{2}\right)\left(\frac{1+A_f}{2}\right)(1-\cos\theta)^2 + \left(\frac{1+A_e}{2}\right)\left(\frac{1-A_f}{2}\right)(1-\cos\theta)^2 + \left(\frac{1-A_e}{2}\right)\left(\frac{1-A_f}{2}\right)(1+\cos\theta)^2$$

This leads to

$$A_{FB} = \frac{3}{4}\ A_e\ A_f$$

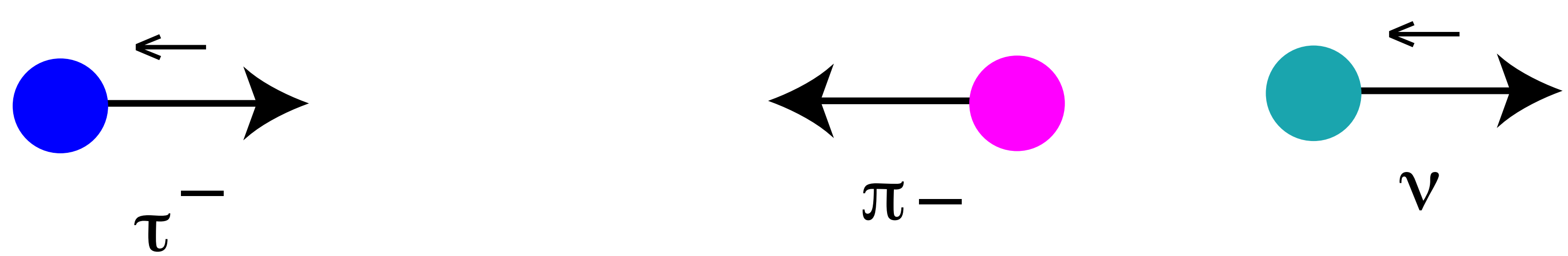


4 km to the right, measure a cross section asymmetry.

$$A_\ell = 0.1513 \pm 0.0021$$

Since τ leptons decay through V-A weak interactions, their decays are sensitive to the τ polarization.

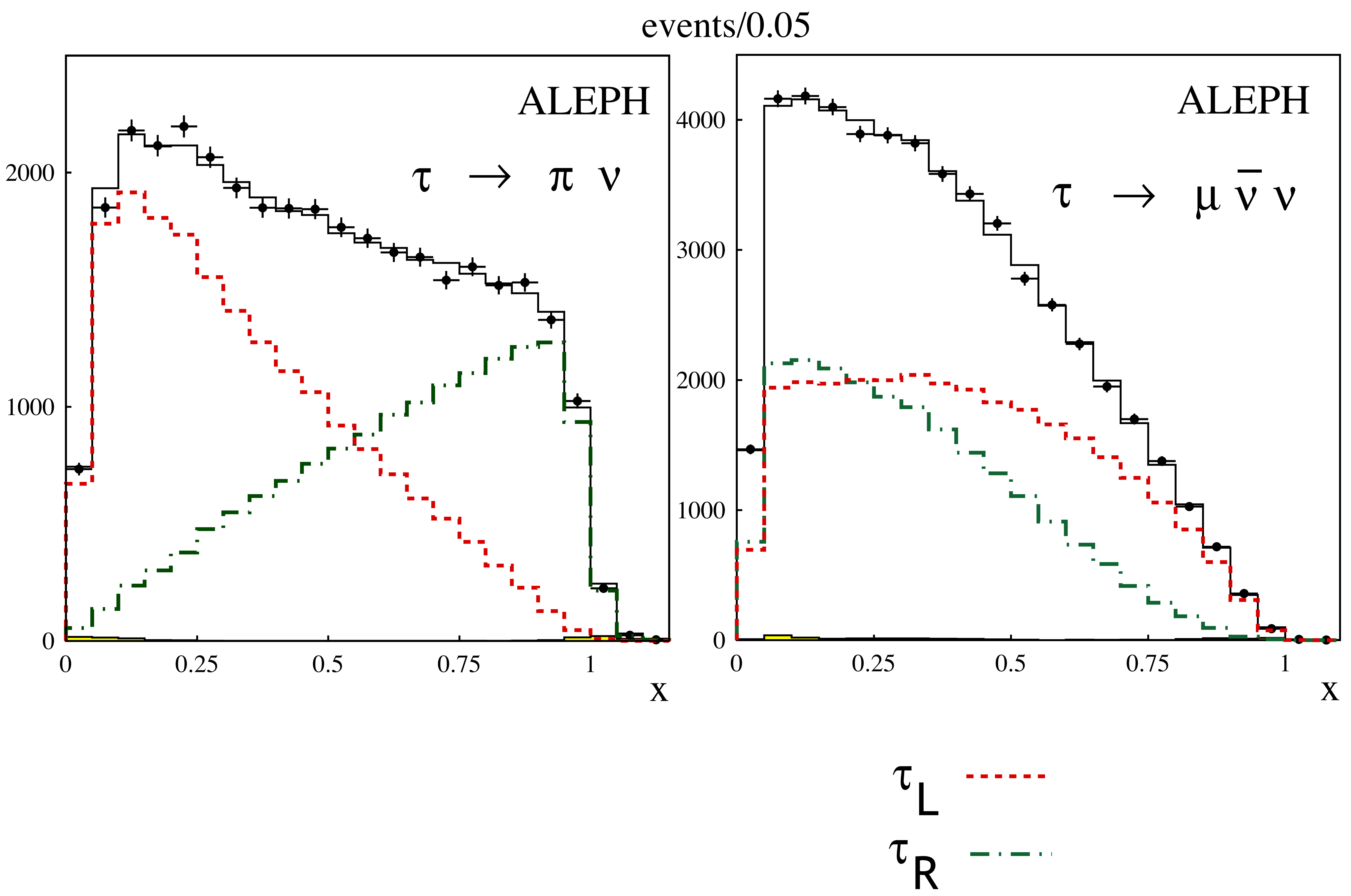
The easiest case to understand is $\tau^- \to \nu_\tau \pi^-$. A τ at rest with $S^3 = -\frac{1}{2}$ decays to a forward ν_τ and a backward π^-.



A highly boosted τ has then has

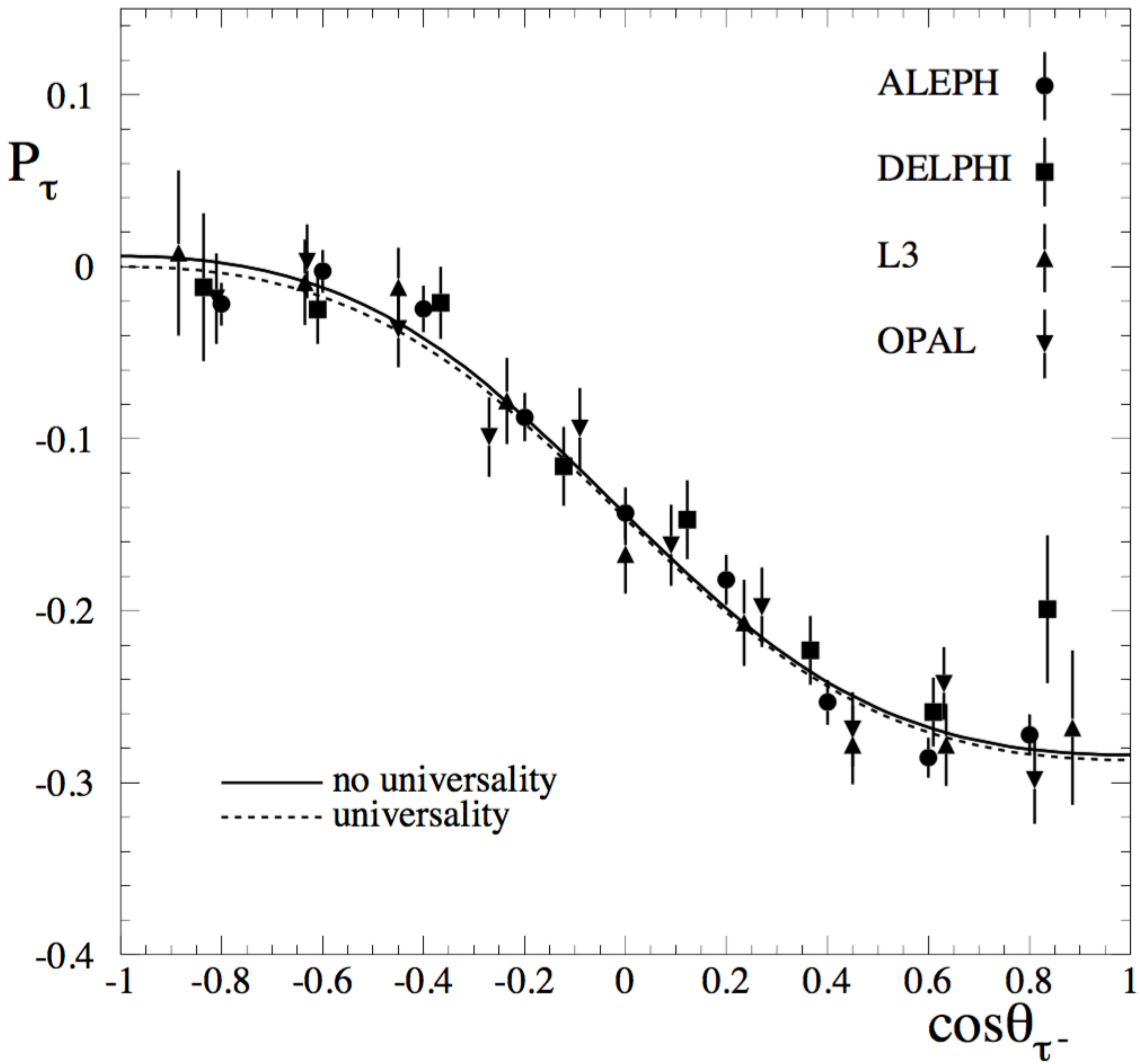
$$\tau_L \ : \quad \frac{d\Gamma}{dx} \sim (1-x) \qquad \tau_R \ : \quad \frac{d\Gamma}{dx} \sim x$$

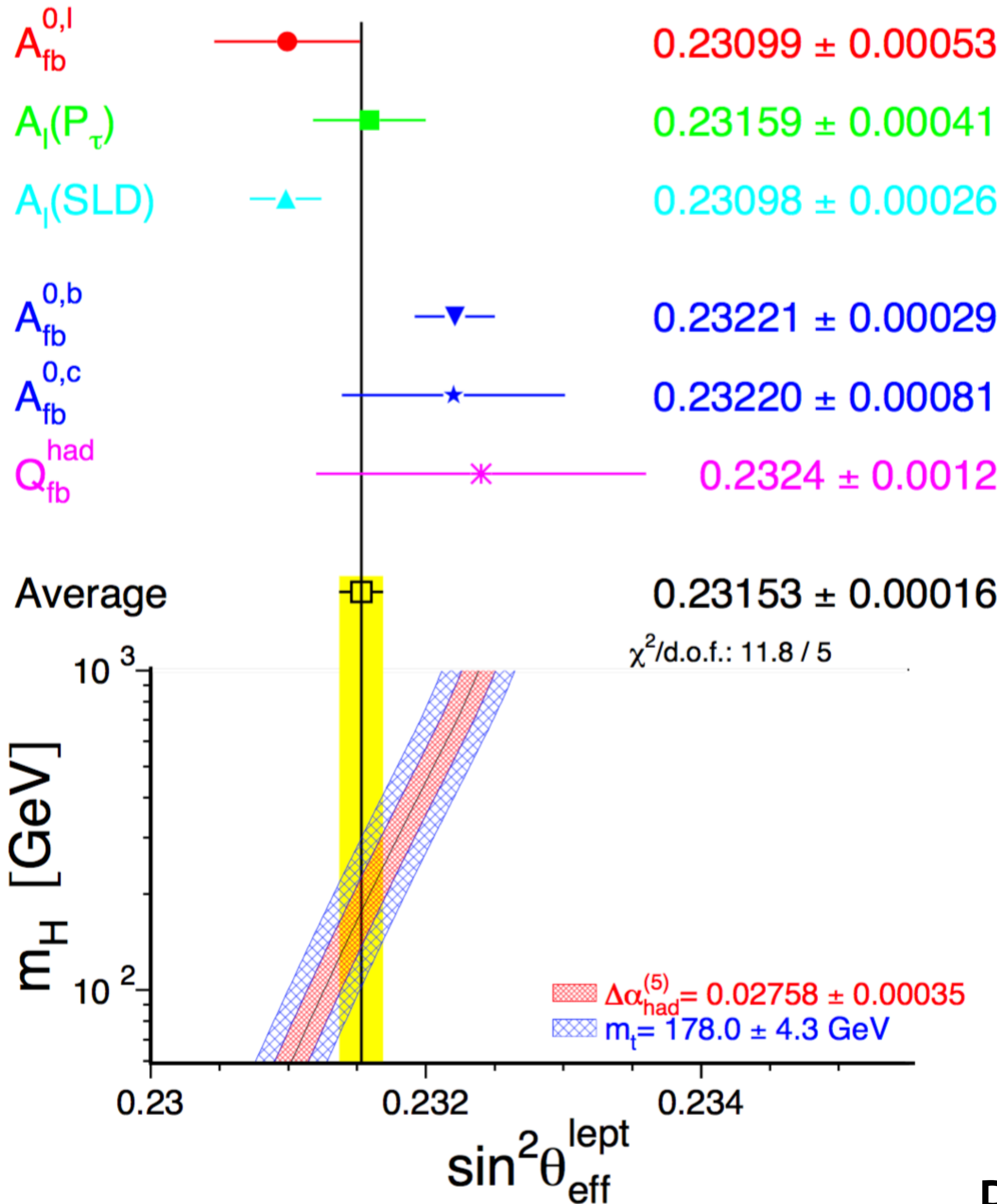
where $x = E_\pi / E_\tau$. Similar asymmetries appear in the other prominent τ decay modes.



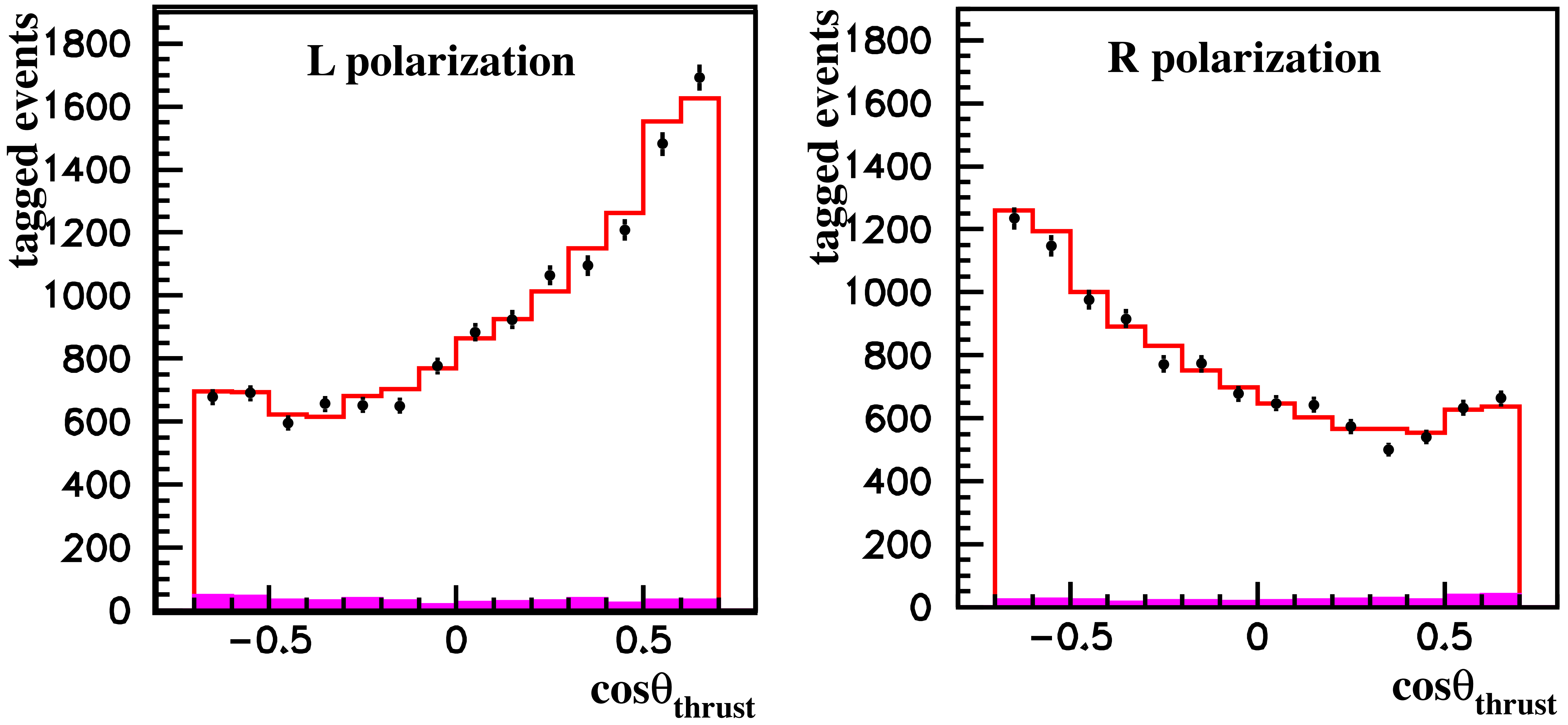
events/0.05
ALEPH
τ → π ν
ALEPH
τ → μ ν̄ ν
x
x
τL
τR

There is also a correlation between τ polarization and $\cos\theta$ that can be used to improve the measurement.

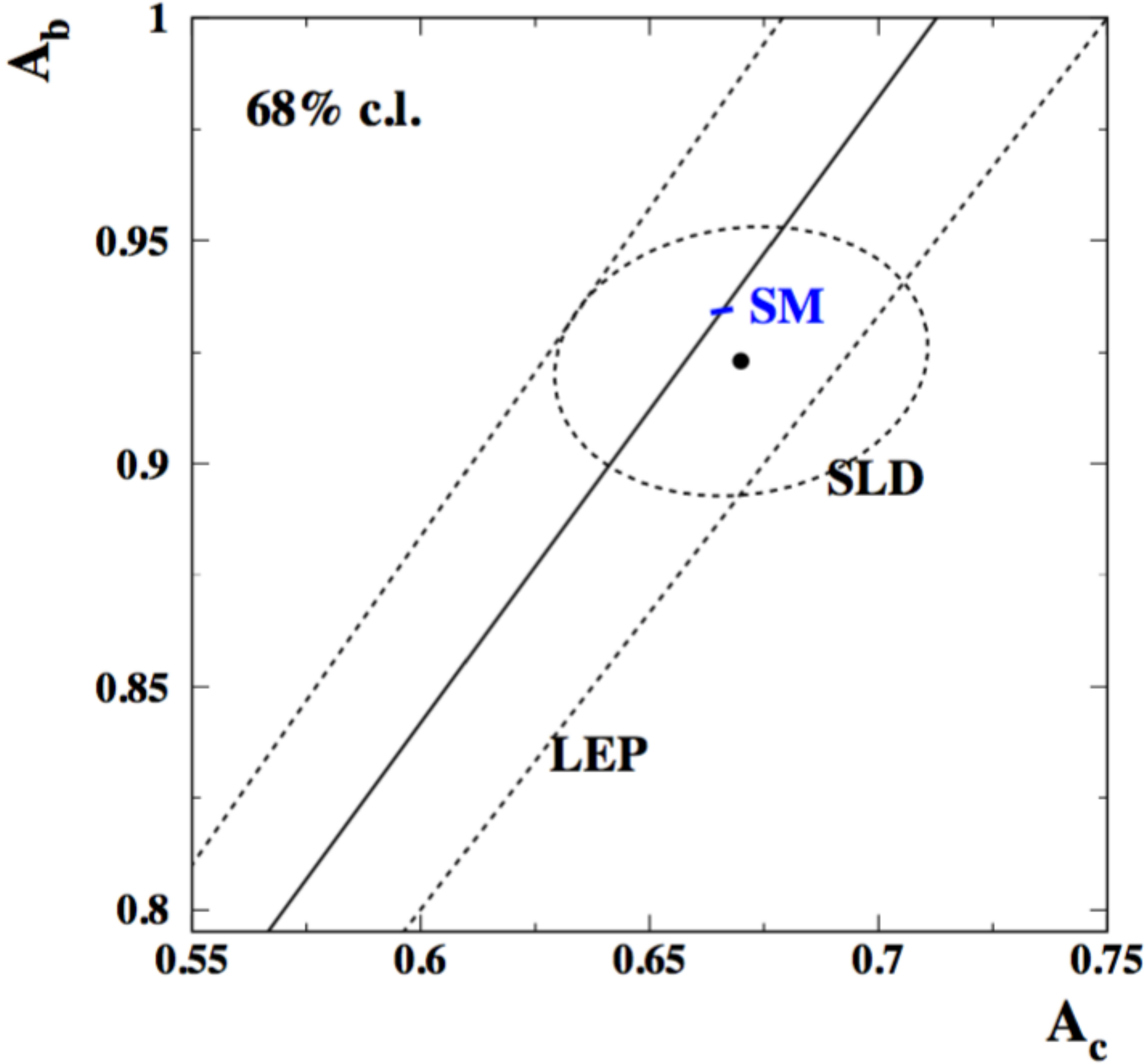




final LEPEWWG
Phys. Rept. 2006


L polarization
R polarization
tagged events
cosθ_{thrust}
1800
1600
1400
1200
1000
800
600
400
200
0
−0.5
0
0.5

SLD

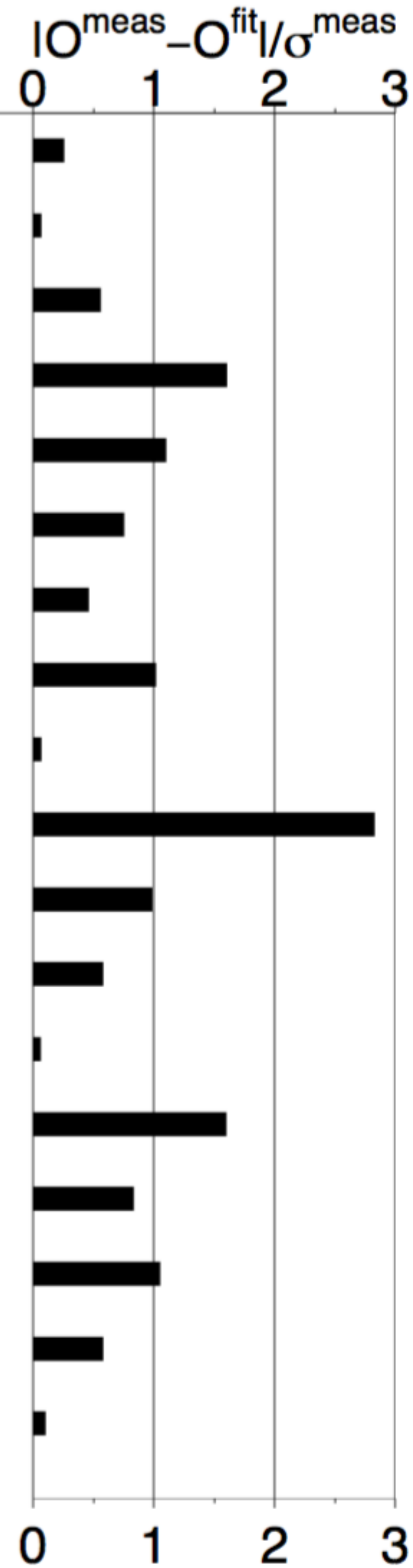


A_b
68% c.l.
SM
SLD
LEP
1
0.95
0.9
0.85
0.8
0.55
0.6
0.65
0.7
0.75
A_c

Here is a summary of the LEP and SLC precision measurements, compiled in the LEP EWWG summary report: Phys.Rept. 427, 257 (2006).

Measurements are shown in terms of the pull (in σ) with respect to the best-fit Standard Model parameters.

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ_{had}^0 [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21562
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.389
Γ_W [GeV]	2.133 ± 0.069	2.093
m_t [GeV]	178.0 ± 4.3	178.5



final LEPEWWG
Phys. Rept. 2006

Now we must discuss the renormalization prescription for the computation of 1-loop radiative corrections.

The Standard Model has a large number of parameters. However, for the specific processes that I have discussed in this lecture, the tree-level predictions depend only on 3 parameters

$$g \ , \ g' \ , \ v$$

The 1-loop corrections will include divergent corrections, included quadratically divergent corrections from v^2 . However, when the corrections to these three parameters are fixed, all 1-loop corrections are made finite. Each specific reaction will obtain a finite correction, which is a prediction of the Standard Model.

Different schemes are used to fix the three underlying divergent amplitudes. Each gives different expressions for the cross sections. These expressions become identical when observables are related to other observables. Three common schemes are

Marciano-Sirlin: fix $\alpha(m_Z), m_Z, m_W$ to their experimental values

on-shell Z: fix $\alpha(m_Z), G_F, m_Z$ to their experimental values

$\overline{MS}$ subtraction

In most analyses today, the 3 unknown constants in each scheme are varied to give the best global fit to the corpus of precision data.

There are many possible definitions of θ_w .

Marciano-Sirlin scheme: define θ_w by $c_w = m_W / m_Z$

this leads to: $s_w^2 = 0.22290 \pm 0.00008$

on-shell Z scheme: define θ_w by

$$\sin^2 2\theta_w = (2c_w s_w)^2 = \frac{4\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2}$$

this leads to

$$s_w^2 = 0.231079 \pm 0.000036$$

Both definitions lead to the same expressions relating observables to observables, but only when finite 1-loop corrections are included.

One particular class of radiative corrections is very simple to analyze.   This is the case in which new particles have no direct coupling to light fermions but appear in Z processes only through vector boson vacuum polarization amplitudes.

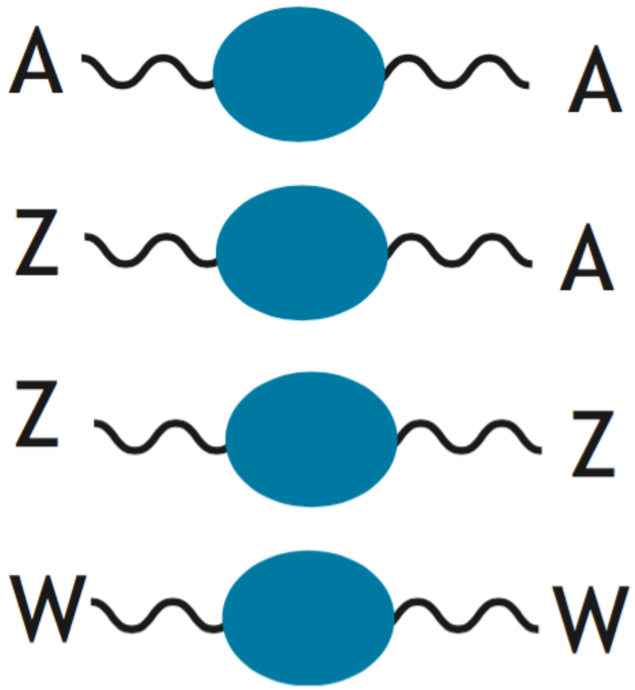
These are called oblique radiative corrections .

They are most simply discussed as a power series in

$$m_Z^2/M^2$$

where M is the mass of a new particle from beyond the Standard Model.

Define the vacuum polarization amplitudes



$$
\begin{aligned}
&= ie^2\Pi_{QQ}g^{\mu\nu} \\
&= i\frac{e^2}{s_w c_w}(\Pi_{3Q} - s_w^2\Pi_{QQ})g^{\mu\nu} \\
&= i\frac{e^2}{s_w^2 c_w^2}(\Pi_{33} - 2s_w^2\Pi_{3Q} + s_w^2\Pi_{QQ})g^{\mu\nu} \\
&= i\frac{e^2}{s_w^2}\Pi_{11}g^{\mu\nu}
\end{aligned}
$$

Each amplitude has a Taylor expansion in q^2/M^2 :

$$\Pi_{QQ}(q^2) = Aq^2 + \cdots$$

$$\Pi_{3Q}(q^2) = Bq^2 + \cdots$$

$$\Pi_{33}(q^2) = C + Dq^2 + \cdots$$

$$\Pi_{11}(q^2) = E + Fq^2 + \cdots$$

Of the 6 constants on the previous slide, 3 contribute to the renormalizations of g , g' , v . This leaves 3 combinations that are finite at 1 loop. These are

$$S = \frac{16\pi}{m_Z^2}[\Pi_{33}(m_Z^2) - \Pi_{33}(0) - \Pi_{3Q}(m_Z^2)]$$

$$T = \frac{4\pi}{s_w^2 m_W^2}[\Pi_{11}(0) - \Pi_{33}(0)]$$

$$U = \frac{16\pi}{m_Z^2}[\Pi_{11}(m_Z^2) - \Pi_{11}(0) - \Pi_{33}(m_Z^2) + \Pi_{33}(0)]$$

Roughly, T parametrizes the correction to $m_W/m_Z c_w$, S parametrizes the q^2/M^2 correction, and U, with both suppressions, is very small in most BSM models.

The leading oblique corrections to electroweak observables can then be expressed as, for example,

$$\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha c^2}{c^2 - s^2}\left(-\frac{1}{2}S + c^2 T\right)$$

$$s_*^2 - s_0^2 = \frac{\alpha}{c^2 - s^2}\left(\frac{1}{4}S - s^2 c^2 T\right)$$

This allows experiment to place constraints that can then be applied to a large class of models.

Some guidance about the expected sizes of S and T is given by the result for one new electroweak doublet:

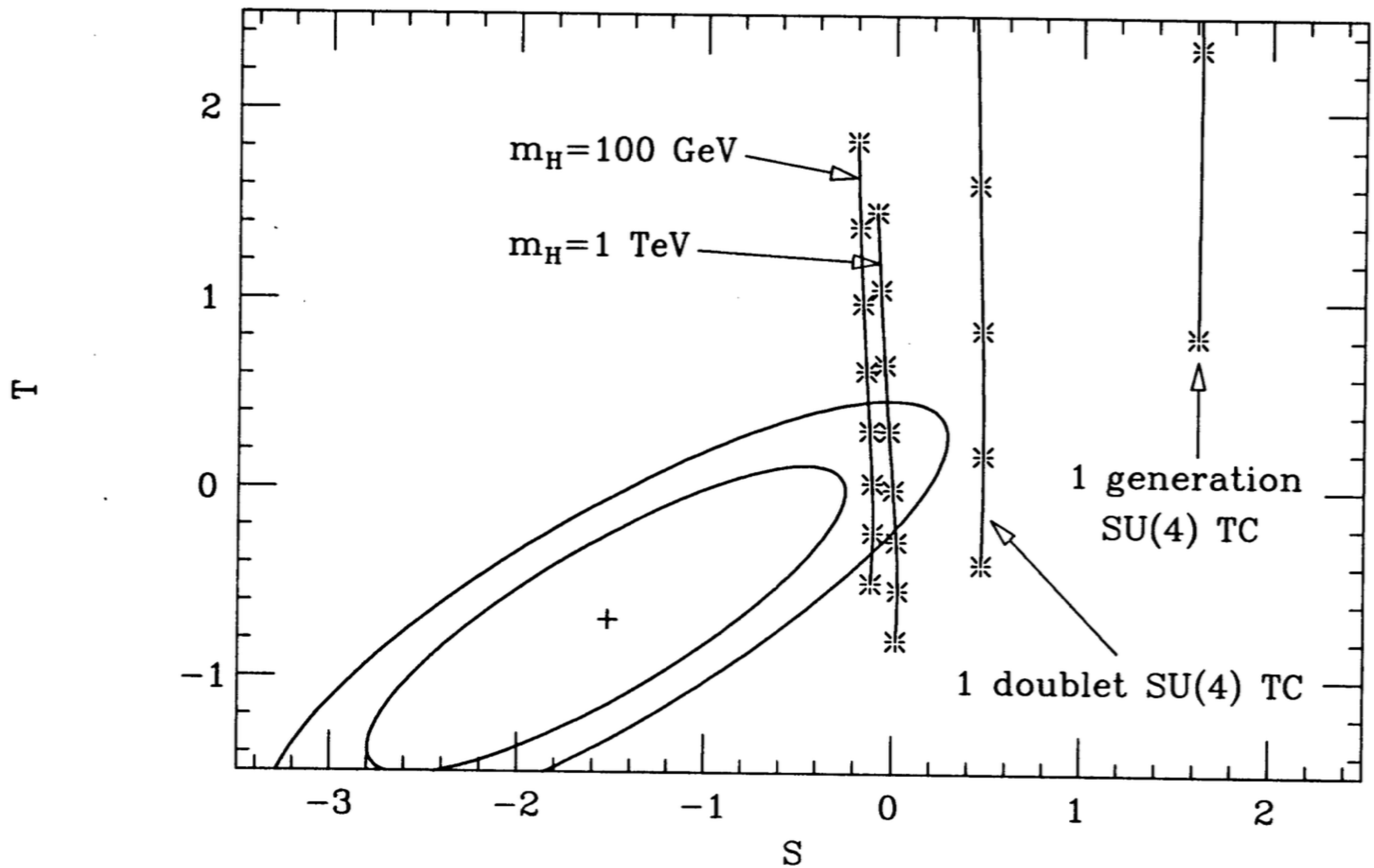
$$S = \frac{1}{6\pi} \qquad\qquad T = \frac{|m_U^2 - m_D^2|}{m_Z^2}$$

The effects of the SM top quark and Higgs boson can also be expressed (approximately) in the S,T framework

top: $$S = \frac{1}{6\pi}\log\frac{m_t^2}{m_Z^2} \qquad T = \frac{3}{16\pi s^2 c^2}\frac{m_t^2}{m_Z^2}$$

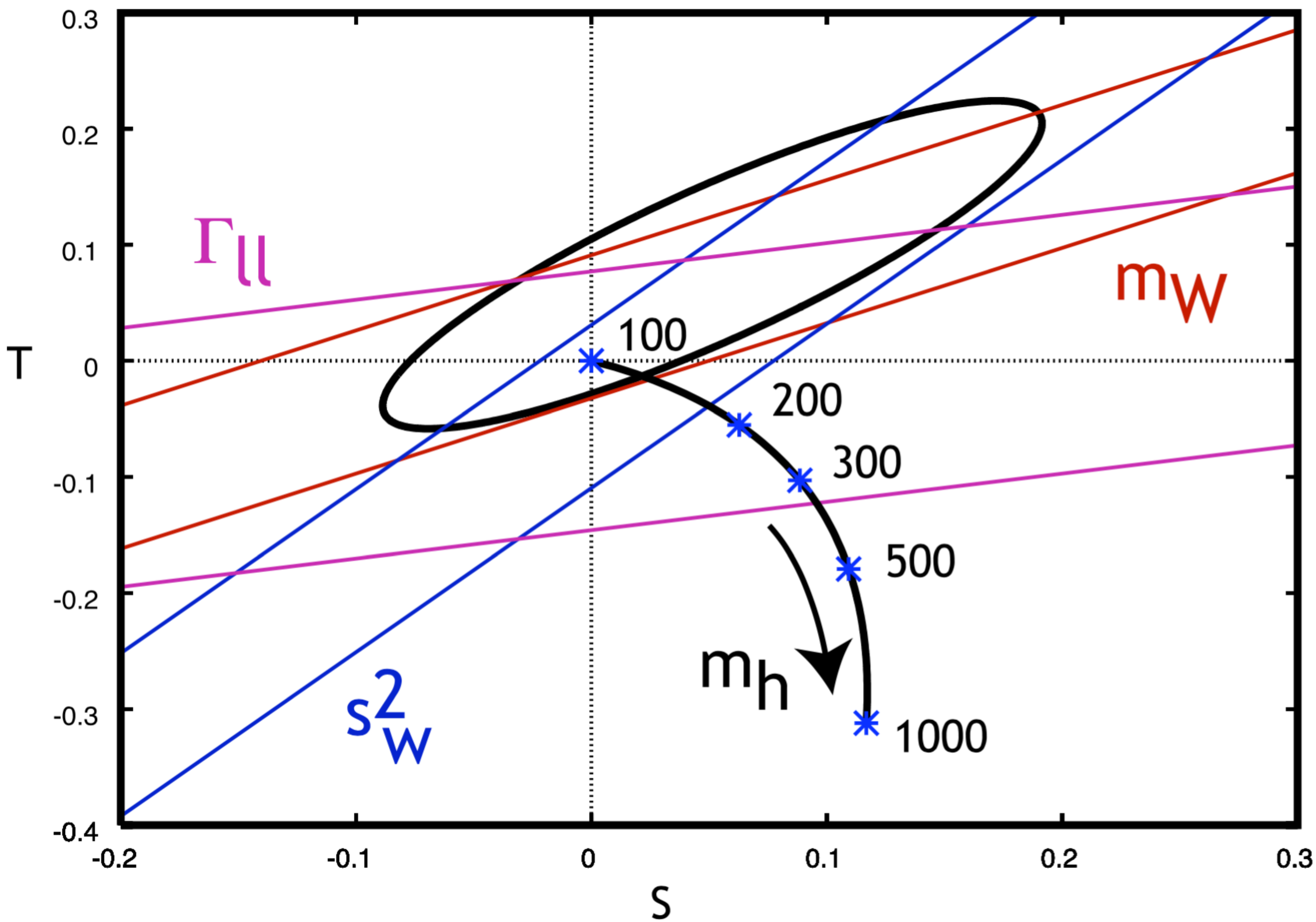
Higgs: $$S = \frac{1}{12\pi}\log\frac{m_h^2}{m_Z^2} \qquad T = -\frac{3}{16\pi c^2}\log\frac{m_h^2}{m_Z^2}$$

S,T fit c. 1991



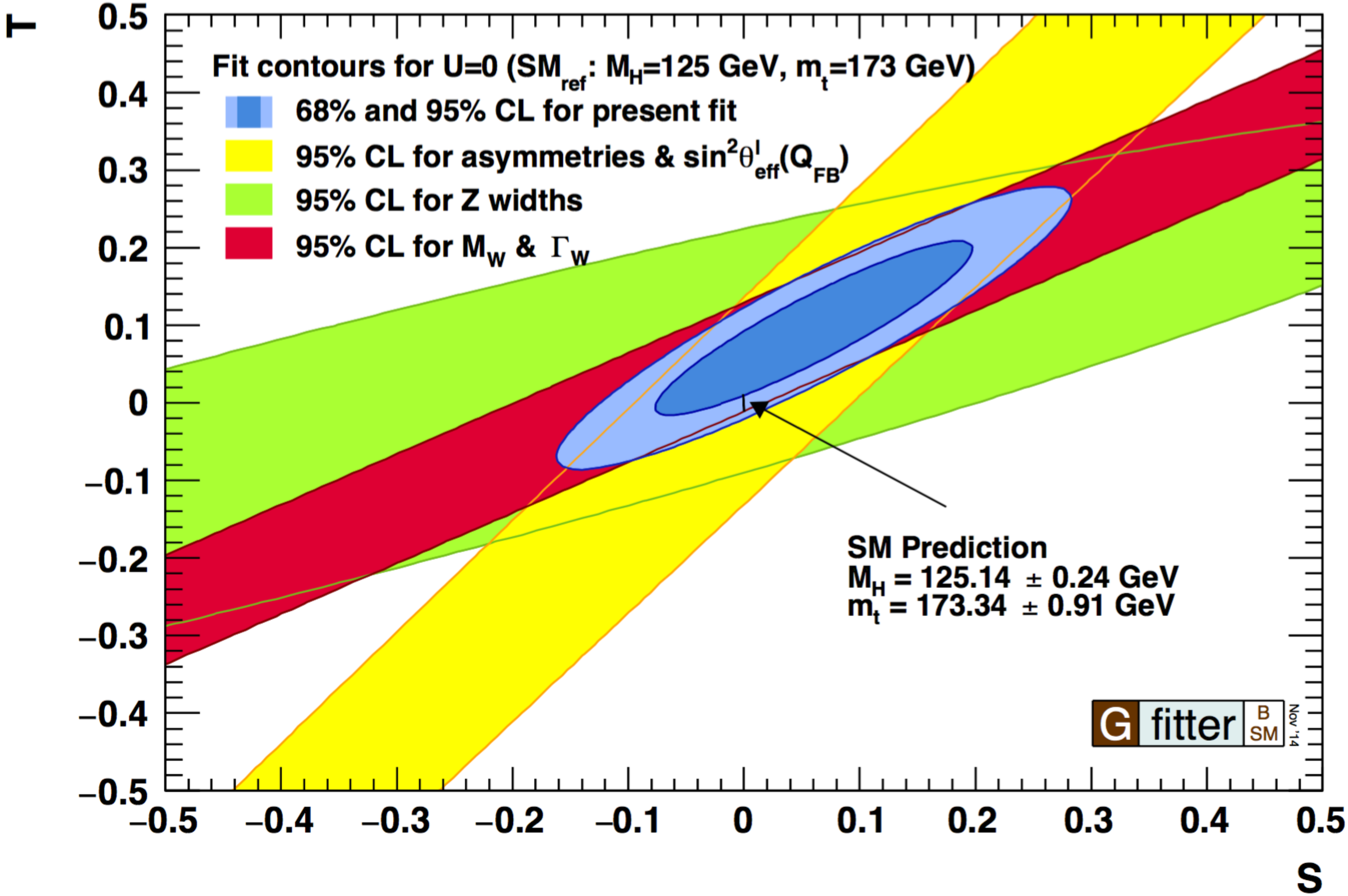
2
1
0
−1
−3
−2
−1
0
1
2
S
T
m_H=100 GeV
m_H=1 TeV
1 generation
SU(4) TC
1 doublet SU(4) TC

S,T fit c. 2008



LEP EWWG: within the MSM $m_h < 144\ (182)$ GeV (95% CL)

S,T fit c. 2014



Fit contours for U=0 (SM$_{ref}$: M_H=125 GeV, m_t=173 GeV)
68% and 95% CL for present fit
95% CL for asymmetries & $\sin^2\theta^l_{eff}(Q_{FB})$
95% CL for Z widths
95% CL for M_W & Γ_W
SM Prediction
M_H = 125.14 ± 0.24 GeV
m_t = 173.34 ± 0.91 GeV
G fitter B SM
Nov '14
T
S
0.5
0.4
0.3
0.2
0.1
0
–0.1
–0.2
–0.3
–0.4
–0.5