# Superfluid Dark Matter

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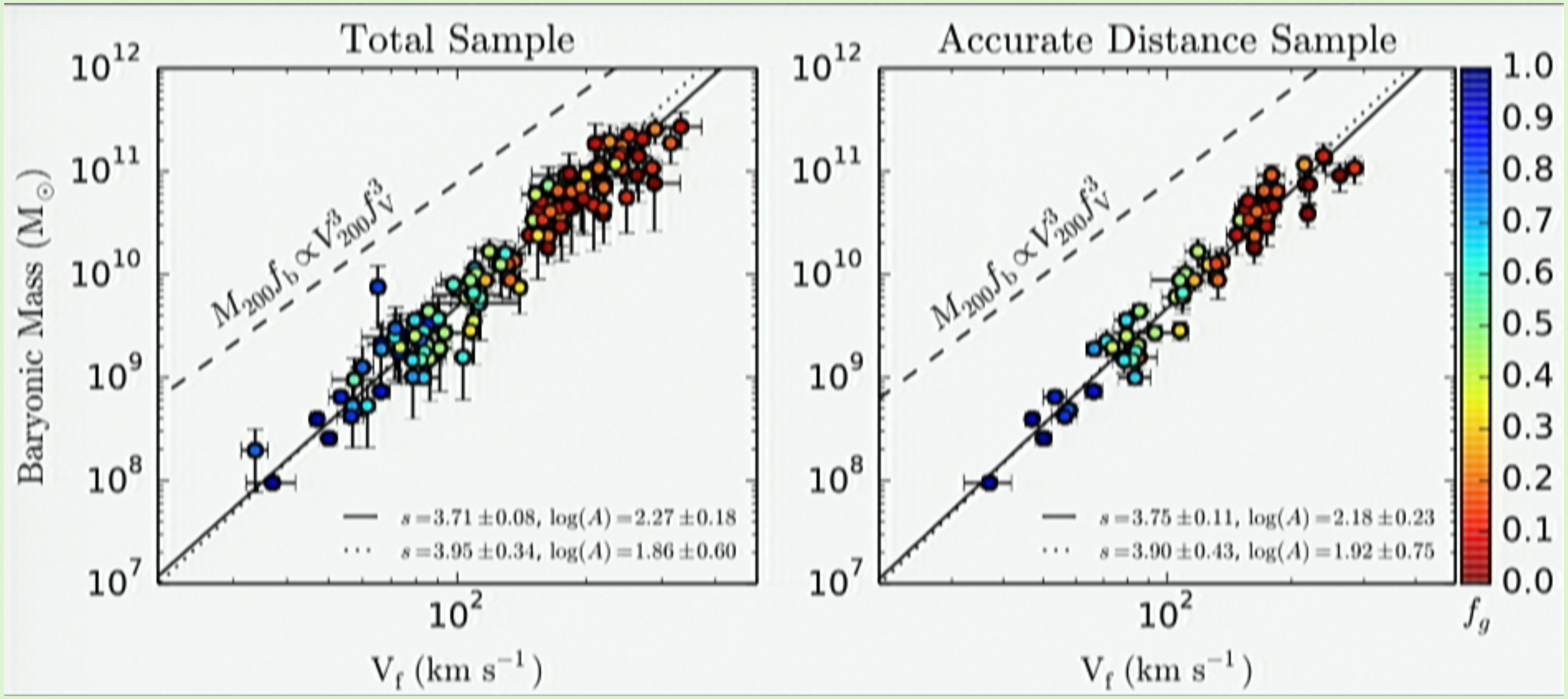
(To appear (very soon!))

# Large Scale Success of CDM

- CMB
- Matter power spectra
- Expansion history of the universe

# Baryonic Tully Fischer Relation

McGaugh (2015)




CDM predicts $v_f^3 \sim M_b$

Observed $v_f^4 = a_0 G_N M_b$ with little scatter

# MOND (a brief introduction)

Milgrom (1983)

$$F = m\,f\left(\frac{a}{a_0}\right)a$$

$$F = ma$$ For $a \gg a_0$ $\longleftrightarrow$

$$F = m\frac{a^2}{a_0}$$ For $a \ll a_0$ $\longleftrightarrow$

Smooth mapping between the two extremes

$$a_0 \approx \frac{1}{6}H_0 = 1.2\times 10^{-10}\,\mathrm{m/s^2}$$

# Three Approaches

## CDM with Feedback

- Stellar evolution
- Black Holes
- AGN feedback
- Supernovae

## Modified gravity

- Bekenstein and Milgrom (1984)
- Zlosnik et al. (2007)
- Hossenfelder (2017)
- Verlinde (2017)

## Hybrid approach

- Blanchet (2007)
- Zhao (2008)
- Ho et al. (2008)
- This work!

# Motivation For Superfluid DM from MOND

$$\mathcal{L}_{\text{MOND}} = -\frac{2M_{\text{Pl}}^2}{3a_0}((\partial\phi)^2)^{3/2} + \frac{\phi}{M_{\text{Pl}}}\rho_b$$

Bekenstein and Milgrom (1983)

$$\mathcal{L}_s = P(X); \quad X = \mu + \dot{\phi} - \frac{(\nabla\phi)^2}{2m}$$

Son and Wingate (2009)

$$\mathcal{L} = \frac{2\Lambda(2m)^{3/2}}{3} X\sqrt{|X|} - \frac{\Lambda}{M_{\text{Pl}}}\phi\,\rho_b$$

Berezhiani and Khoury (2015)

# Condensate Properties

$$P_{\mathrm{cond}} = \frac{2\Lambda}{3}(2m\mu)^{3/2}$$

$$n_{\mathrm{cond}} = \frac{\partial P_{\mathrm{cond}}}{\partial \mu} \quad \Rightarrow \quad P_{\mathrm{cond}} = \frac{\rho_{\mathrm{cond}}^{3}}{12\Lambda^{2} m^{6}}$$

$$\Downarrow$$

Predomiant 3-body interactions

Different from other BEC DM approaches with $P_{\mathrm{cond}} \propto \rho_{\mathrm{cond}}^{2}$

Sin (1994), Goodman (2000), Peebles (2000), Boehmer and Harko (2007)

# Bose Gas with 3-Body Contact Interactions

$$H - \mu N = \int \mathrm{d}^3x \left[ \psi^\dagger(x) \left( -\frac{\hbar^2 \nabla^2}{2m} \right) \psi(x) - \mu \psi^\dagger(x) \psi(x) + \frac{1}{3} h (\psi^\dagger(x))^3 (\psi(x))^3 \right]$$

First study in detail a Bose gas with 2 body contact interactions

$$H - \mu N = \int \mathrm{d}^3x \left[ \psi^\dagger(x) \left( -\frac{\hbar^2 \nabla^2}{2m} \right) \psi(x) - \mu \psi^\dagger(x) \psi(x) + \frac{1}{2} g (\psi^\dagger(x))^2 (\psi(x))^2 \right]$$

# Bose Einstein Condensation at T = 0

$$H-\mu N = \int d^3x\left[\psi^\dagger(x)\left(-\frac{\hbar^2\nabla^2}{2m}\right)\psi(x)-\mu\psi^\dagger(x)\psi(x)+\frac{1}{2}g(\psi^\dagger(x))^2(\psi(x))^2\right]$$

$$\psi(x)=\Psi(x)+\psi_1(x)$$

At T=0, $\psi_1(x)=0 \quad \Longrightarrow \quad |\Psi(x)|^2=\frac{\mu}{g}$

# Bose Einstein Condensation at Finite T

$$H' = H - \mu N = \int \mathrm{d}^3x \left[ \psi^\dagger(x) \left( -\frac{\hbar^2 \nabla^2}{2m} \right) \psi(x) - \mu \psi^\dagger(x)\psi(x) + \frac{1}{2} g (\psi^\dagger(x))^2 (\psi(x))^2 \right]$$

- Evaluate the Hamiltonian order by order in the normal field.
- Linear term vanishes (similar to the one loop calculation in QFT).

$$\int \mathrm{d}^3x \left( \psi_1^\dagger(x)\Psi(x) + \psi_1(x)\Psi^*(x) \right) = 0\,.$$

- Fourier transform the field to Ladder operators: $\psi_1(x) \to a_k, a_k^\dagger$.
- Diagonalize the resulting Hamiltonian.
- Find the energy spectrum in terms of momenta.

$$H'^{(2)} = \sum_{\mathbf{K}} (\omega_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}}) + \frac{1}{2}\Delta (a_{\mathbf{k}}^\dagger a_{-\mathbf{k}}^\dagger + a_{-\mathbf{k}} a_{\mathbf{k}})$$

$$\omega_{\mathbf{k}} = \frac{k^2}{2m} + 2n_0 g - \mu, \quad \Delta = n_0 g \ , \quad n_0 = |\Psi(x)|^2$$

# Finite T analysis continues

$$\Rightarrow \qquad H'^{(2)} = \sum_{\mathbf{k}\neq 0}\left[\epsilon_{\mathbf{k}} b^{\dagger}_{\mathbf{k}} b_{\mathbf{k}} + \frac{1}{2}\left(\epsilon_{\mathbf{k}} - \omega_{\mathbf{k}}\right)\right]$$

$$\epsilon_{\mathbf{k}} \equiv \sqrt{\omega_{\mathbf{k}}^2 - \Delta^2} = \sqrt{\left(\frac{k^2}{2m} + n_0 g - \mu\right)\left(\frac{k^2}{2m} + 3n_0 g - \mu\right)}$$

And finally solve the equation of motion: $\left\langle \frac{\partial H'}{\partial N_0} \right\rangle = 0$

$$|\Psi|^2 = \frac{\mu}{g} - \frac{4}{3\pi^2}(mg)^{3/2}|\Psi|^3 - \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{\frac{k^2}{m} - 2\mu + 3g|\Psi|^2}{\sqrt{\left(\frac{k^2}{2m} - \mu + g|\Psi|^2\right)\left(\frac{k^2}{2m} - \mu + 3g|\Psi|^2\right)}} \frac{1}{\left(e^{\beta\sqrt{\left(\frac{k^2}{2m} - \mu + g|\Psi|^2\right)\left(\frac{k^2}{2m} - \mu + 3g|\Psi|^2\right)}} - 1\right)}$$

$\boxed{\text{Pathological}}$

# Hohenberg-Martin Dilemma

(1965)

Solution can either be made to obey the equation of motion or be gapless

$$\left\langle \frac{\partial H'}{\partial N_0} \right\rangle = 0$$

$$\lim_{\mathbf{k}\to 0} \epsilon_{\mathbf{k}} = 0$$

Analogous to the problem in QFT for one loop correction to the effective potential of a scalar field

Fujimoto et al. (1983)

Weinberg and Wu (1987)

Hawking and Moss (1983)

Cahill (1995)

# Resolution!

Yukalov (2006)

Separate chemical potentials for the two phases, since the particles in each phase are conserved for a given T, V, N.

Equation of Motion

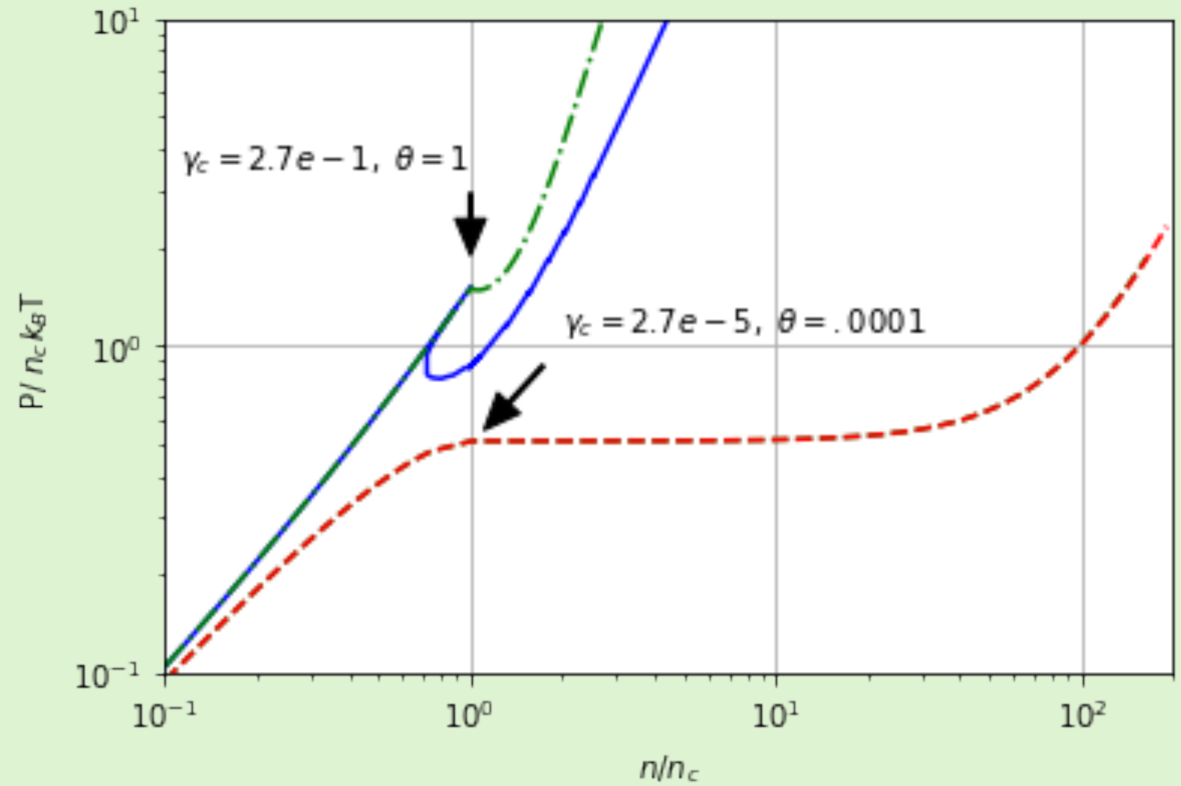
$$\mu_0 = (2n - n_0 + \sigma)g$$

$$\sigma = \frac{1}{V}\sum_{\mathbf{k}} \langle a_{\mathbf{k}} a_{-\mathbf{k}} \rangle$$

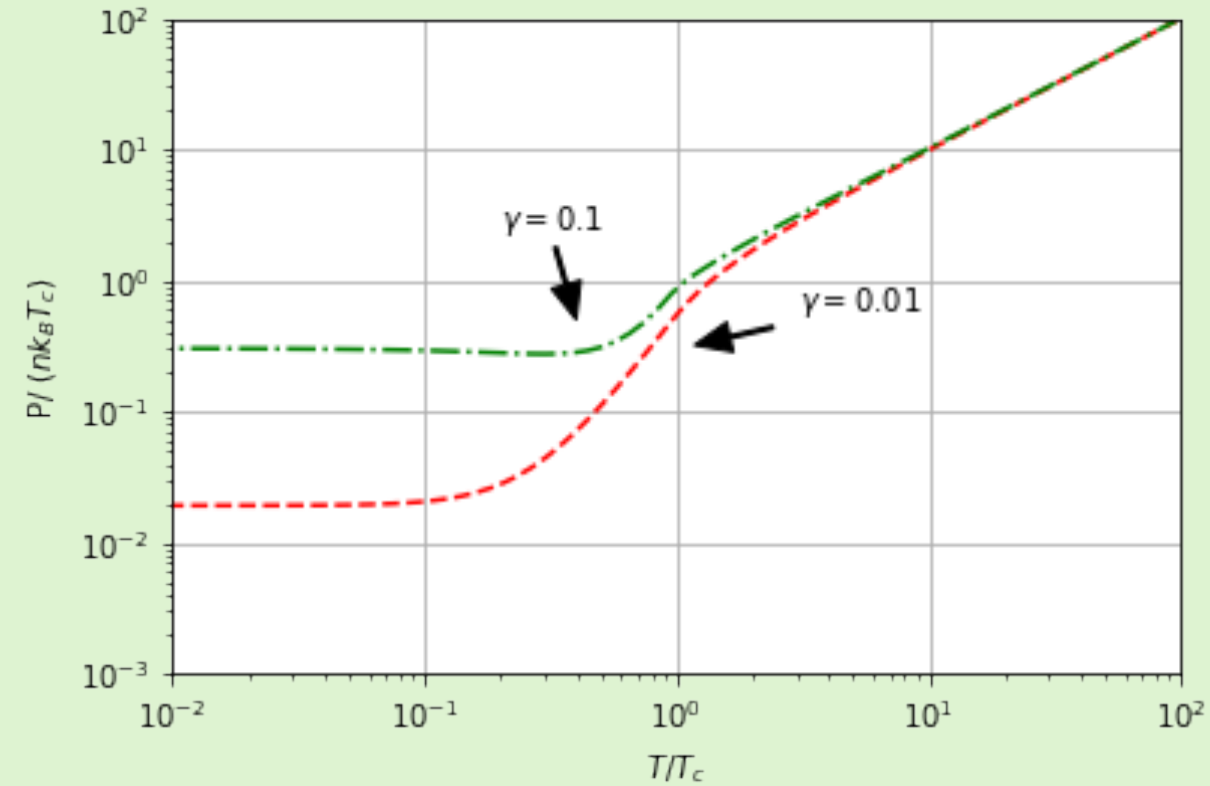
$$\mu_1 = (2n - n_0 - \sigma)g$$

Gapless Spectra

# Well Behaved Equation of State




(a) Pressure versus density

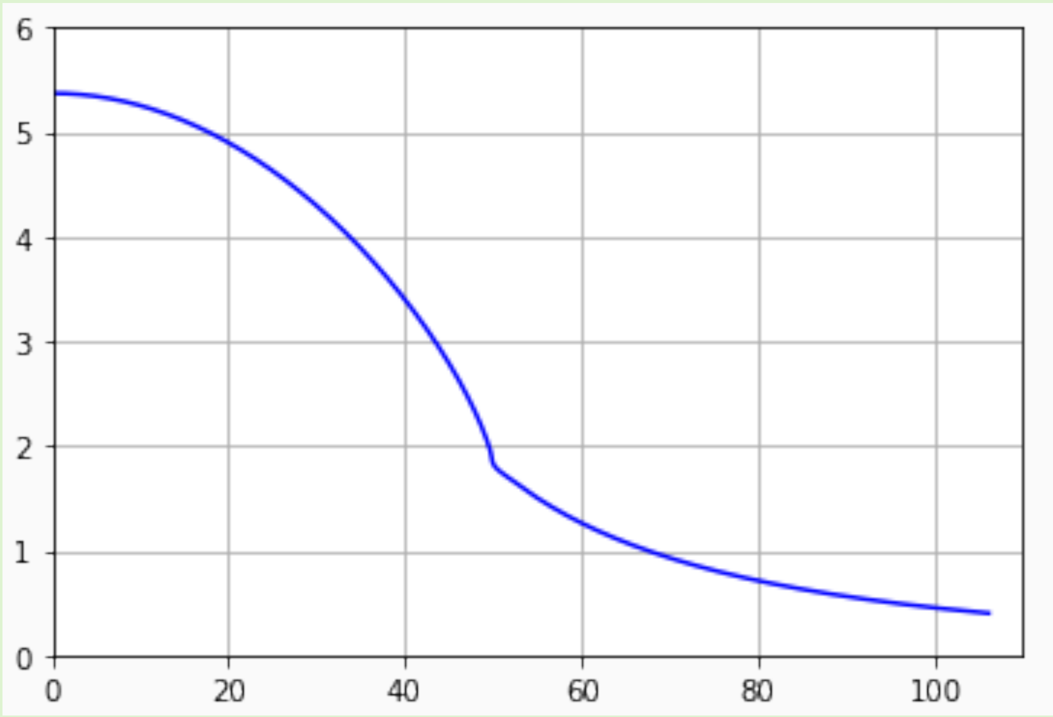



(b) Pressure versus temperature

Comparison with Slepian and Goodman (2011)

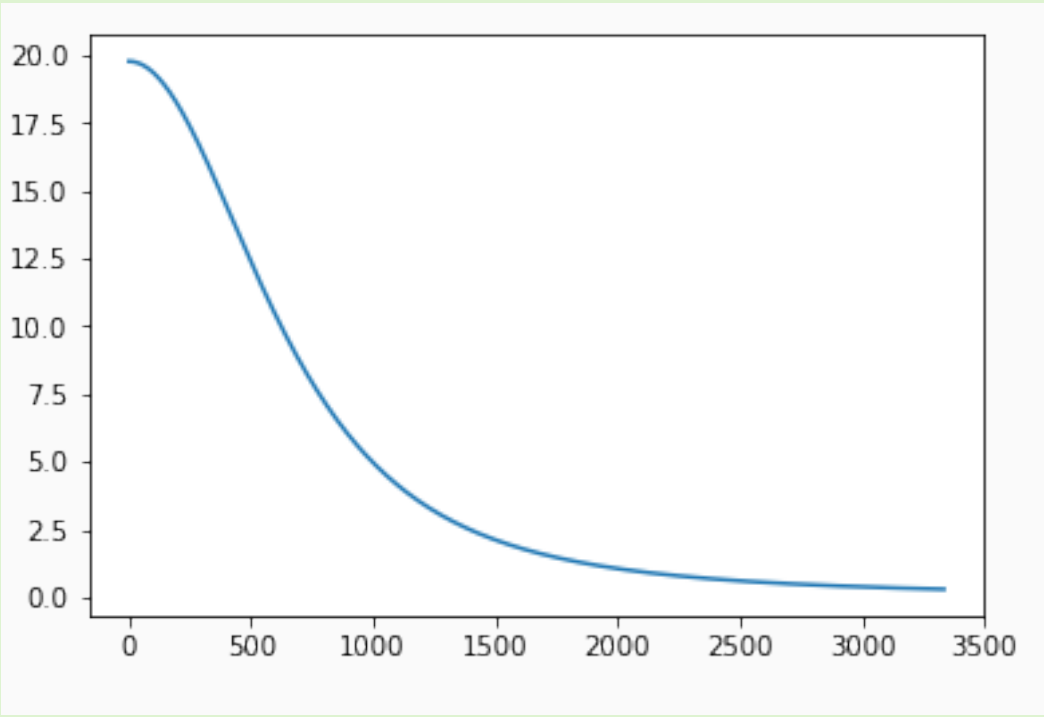
# Density profiles

$$\frac{\rho}{\rho_{\text{virial}}}$$




Radius in kpc

$$\frac{\rho}{\rho_{\text{virial}}}$$




Radius in kpc

- Cored density profiles.
- In MW sized galaxies, the superfluid core $\Longrightarrow$ MOND
- For Galaxy clusters and larger scales, DM described by CDM