Non-global logarithms in jet and isolation cone cross sections

arXiv: 1803.07045 (MB, Thomas Becher, Ding Yu Shao)

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AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

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Overview

Non-Global Logrithms

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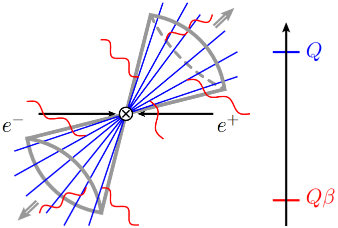
Logarithms appearing in the calculation of non-global obervables are called non-global logarithms.



terms $\propto \alpha_s^n \ln^n \left(\frac{E_{out}}{E_{in}}\right) = \alpha_s^n \ln^n (\beta)$

Classical perturbation theory **breaks down** **for small** β

"Order-By-Order" versus "Logarithm-by-Logarithm"

$a_i \propto \mathcal{O}(1)$

LO: $\sigma = \quad a_0 + \mathcal{O}(\alpha_s)$

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\text{N}^4\text{LO:} \quad \sigma &= a_0 + a_1\alpha_s + a_2\alpha_s^2 + a_3\alpha_s^3 + a_4\alpha_s^4 + \mathcal{O}(\alpha_s^5) \\
\text{N}^5\text{LO:} \quad \sigma &= a_0 + a_1\alpha_s + a_2\alpha_s^2 + a_3\alpha_s^3 + a_4\alpha_s^4 + a_5\alpha_s^5 + \mathcal{O}(\alpha_s^6)
\end{aligned}$$

....

"Order-By-Order" versus "Logarithm-by-Logarithm"

$$L = \ln(\beta)$$
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$$\text{N}^3\text{LO:} \quad \sigma = a_0 + a_{10}\alpha_s + a_{11}\alpha_s L + a_{20}\alpha_s^2 + a_{21}\alpha_s^2 L + a_{22}\alpha_s^2 L^2 + a_{30}\alpha_s^3 + a_{31}\alpha_s^3 L + a_{32}\alpha_s^3 L^2 + a_{33}\alpha_s^3 L^3 + \mathcal{O}(\alpha_s^4 L^4)$$

....

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LL: $\sigma = \quad a + \sum b_m \alpha_s^m L^m + \mathcal{O}\left(\alpha_s^n L^{n-1}\right)$

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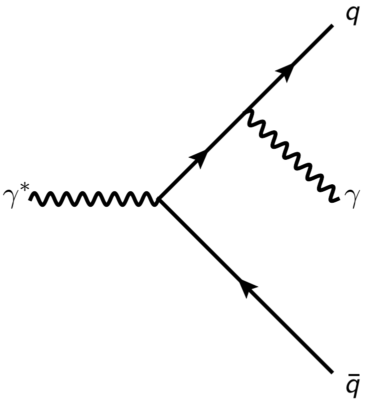
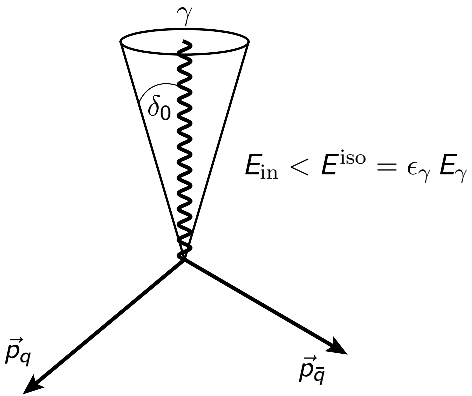
Leading-Logarithm technique fixes perturbative approach

Photon Production



Isolated Photon

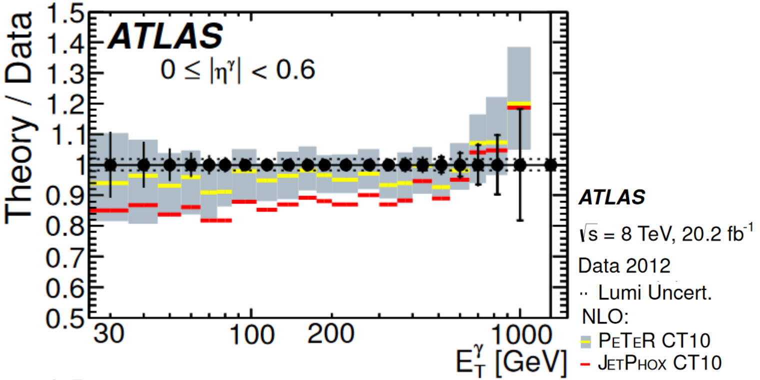


Why should we care?

No experimental measurements possible of direct photon production processes without such cuts!

- $E^{\text{iso}}_{ATLAS} = 4.8\,\text{GeV} + 4.2 \times 10^{-3} \times E_T^{\gamma}$
- $\delta_0^{ATLAS} \approx 0.42 \approx 0.13\,\pi \approx 25^\circ$

ATLAS Results



arXiv: 1605.03495

Factorization formula

Factorization formula for e^+e^--Annihilation

$$\frac{\mathrm{d}\sigma(\epsilon_\gamma,\delta_0)}{\mathrm{d}E_\gamma} = \sum_{m=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_{\gamma+m}(\{\underline{n}\}, E_\gamma, Q, \delta_0) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, \epsilon_\gamma E_\gamma, \delta_0)\rangle$$

$\boldsymbol{\mathcal{H}}_{\gamma+m}$	"Hard function": Squared amplitude of hard process (consisting of m partons and the isolated photon)
$\{\underline{n}\}$	Directions of the m hard partons ($\{\underline{n}\} = \{n_1, \ldots, n_m\}$)
$\boldsymbol{\mathcal{S}}_m$	"Soft function": soft radiation (inside cone)
$\otimes$	Integration over $\{\underline{n}\}$
$\langle \ldots \rangle$	Sum over Colors

Renormalization group equation

$\boldsymbol{\mathcal{H}}_{\gamma+m}$ fulfill renormalization group equations

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_{\gamma+m}(\{\underline{n}\},Q,\mu) = -\sum_{l=2}^{m}\boldsymbol{\mathcal{H}}_{l}(\{\underline{n}\},Q,\mu)\,\boldsymbol{\Gamma}^{H}_{lm}(\{\underline{n}\},Q,\mu)$$

- Compute $\boldsymbol{\mathcal{H}}_{\gamma+m}$ at characteristic high scale (such as $\mu_h = E_\gamma$)
- Evolve $\boldsymbol{\mathcal{H}}_{\gamma+m}$ to the scale of $\boldsymbol{\mathcal{S}}_m$ using RGE
- Compute $\boldsymbol{\mathcal{S}}_m$ at characteristic low scale (such as $\mu_s = \epsilon_\gamma E_\gamma$)

Ingredients for Leading Logarithmic accuracy (large N_c)

$\boldsymbol{\mathcal{H}}_{\gamma+m}(\mu_h \approx E_\gamma)$ at LL

$$\boldsymbol{\mathcal{H}}_{\gamma+2}(\mu_h \approx E_\gamma) = \sigma_0 + \mathcal{O}(\alpha_s)$$
$$\boldsymbol{\mathcal{H}}_{\gamma+m}(\mu_h \approx E_\gamma) = \mathcal{O}(\alpha_s)\ \forall\, m \neq 2$$

Anomalous Dimension at LL

$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$\boldsymbol{\mathcal{S}}_m(\mu_s \approx \epsilon_\gamma E_\gamma)$ at LL

$$\boldsymbol{\mathcal{S}}_m(\mu_s \approx \epsilon_\gamma E_\gamma) = 1 + \mathcal{O}(\alpha_s)$$

What does the anomalous Dimension represent?

Formal:

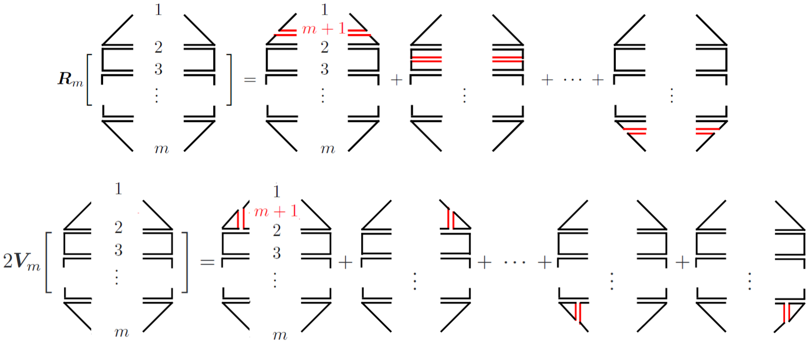
$$\boldsymbol{R}_m = 4N_c \sum_i W_{i,i+1}^{m+1} \Theta_{OUT}(n_{m+1})$$

$$\boldsymbol{V}_m = -4N_c \sum_i \int \frac{d\Omega(n_{m+1})}{4\pi} W_{i,i+1}^{m+1}$$

$$W_{i,j}^{k} = \frac{n_i \cdot n_j}{(n_i \cdot n_k)(n_k \cdot n_j)}$$

What does the anomalous Dimension represent?

Picture:



Resummation by RGE at Leading Logarithmic accuracy

Renormalization group evolution

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_{\gamma+m}(\{\underline{n}\},Q,\mu) = -\sum_{l=2}^{m}\boldsymbol{\mathcal{H}}_l(\{\underline{n}\},Q,\mu)\,\boldsymbol{\Gamma}^H_{lm}(\{\underline{n}\},Q,\mu)$$

Renormalization group evolution at leading log

$$\frac{d}{dt}\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t)\boldsymbol{V}_m + \boldsymbol{\mathcal{H}}_{m-1}(t)\boldsymbol{R}_{m-1}$$

$$t = \frac{1}{2\beta_0}\ln\frac{\alpha(\mu_s)}{\alpha(\mu_h)} = \frac{\alpha_s}{4\pi}\ln\frac{\mu_h}{\mu_s} + \mathcal{O}(\alpha_s^2)$$

Solution

$$\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t_0)\,e^{(t-t_0)\boldsymbol{V}_m} + \int_{t_0}^{t} dt'\,\boldsymbol{\mathcal{H}}_{m-1}(t')\,\boldsymbol{R}_{m-1}\,e^{-(t-t')\boldsymbol{V}_m}$$

Resummation by RGE at Leading Logarithmic accuracy

Solution

$$\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t_0)\, e^{(t-t_0)\boldsymbol{V}_m} + \int_{t_0}^{t} dt'\, \boldsymbol{\mathcal{H}}_{m-1}(t')\, \boldsymbol{R}_{m-1}\, e^{-(t-t')\boldsymbol{V}_m}$$

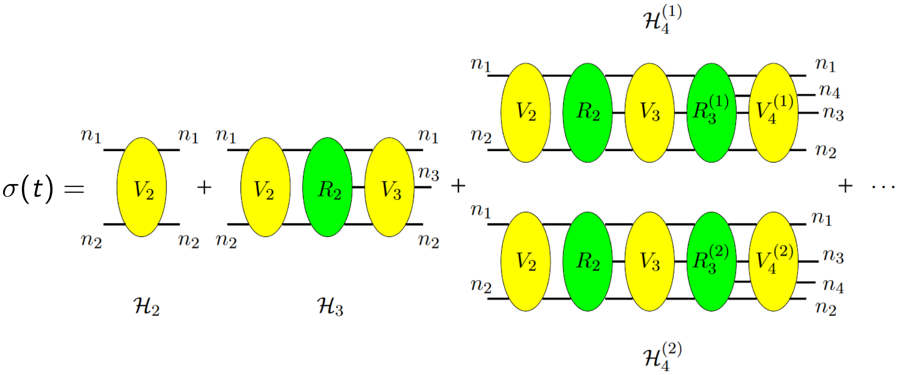
Iterative procedure

$$\begin{aligned}
\boldsymbol{\mathcal{H}}_2(t) &= \boldsymbol{\mathcal{H}}_2(0)\, e^{t\boldsymbol{V}_2} \\
\boldsymbol{\mathcal{H}}_3(t) &= \int_0^t dt'\, \boldsymbol{\mathcal{H}}_2(t')\, \boldsymbol{R}_2\, e^{(t-t')\boldsymbol{V}_3} \\
\boldsymbol{\mathcal{H}}_4(t) &= \int_0^t dt'\, \boldsymbol{\mathcal{H}}_3(t')\, \boldsymbol{R}_3\, e^{(t-t')\boldsymbol{V}_4} \\
\boldsymbol{\mathcal{H}}_5(t) &= \dots
\end{aligned}$$

Cross Section

$$\sigma(t) = \boldsymbol{\mathcal{H}}_2(t) + \int \frac{d\Omega_1}{4\pi} \boldsymbol{\mathcal{H}}_3(t) + \int \frac{d\Omega_1}{4\pi}\frac{d\Omega_2}{4\pi} \boldsymbol{\mathcal{H}}_4(t) + \dots$$

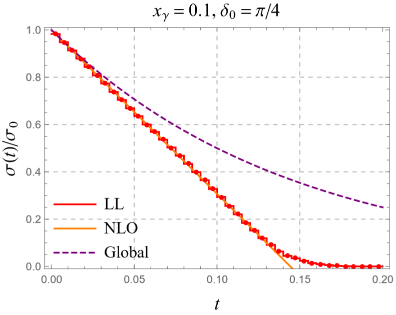
Resummation by RGE at Leading Logarithmic accuracy



Monte Carlo!

- generate a lot of $\mathcal{H}_{[\gamma+]2}(t=0)$ (using event generator [Madgraph])
- for every event
  - calculate $\boldsymbol{V}_2$
  - generate $\Delta t'$
  - generate $\boldsymbol{R}_2$ and calculate $\boldsymbol{V}_3$
  - calculate $\mathcal{H}_3(\Delta t')$
  - repeat for $\mathcal{H}_4(\Delta t' + \Delta t'')$ ...
  - restart the "showering", as soon as we emit into the cone

$e^+ e^- \to \gamma + X$


$x_\gamma = 0.1, \delta_0 = \pi/4$
$\sigma(t)/\sigma_0$
t
LL
NLO
Global
$x_\gamma = 0.9, \delta_0 = \pi/4$
$\sigma(t)/\sigma_0$
t
LL
NLO
Global

Summary

- we resum non-global logarithms at leading logarithmic accuracy using a parton shower;
- flexible implementation by using Madgraph, Code can be easily adapted to different non-global observables.

- small-angle observables not (yet) accounted for;
- going to NLL: higher-order corrections to matching coefficients and anomalous dimension;
- $N_c = 3$ difficult to achieve.

Thank you!