Four-Dimensional F-Theory Compactifications with Discrete Gauge Symmetries via Higgsing

joint work with E. Palti, O. Till and T. Weigand:
arXiv:1408.6831

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General motivation for F-theory GUT models

- $SU(5)$: **10 10 5** (top) Yukawa difficult to get in IIB (only via instantons);
- $SO(10)$: quarks and leptons sit in **16** of $SO(10)$; cannot be realized with pert. open strings;
- E_6 gauge groups are excluded in type IIB;
- Takes back-reaction of D-branes into account;
- Allows for 'local models'—do not take any gravitational interaction into account;
  $M_{\text{pl, }4}/\mu \to \infty$ and $g^2_{\text{YM}}(\mu) = \text{const}$
- ...

Possibility of **exceptional groups** and **locality** are the crucial reasons to consider F-theory!

Motivation for additional symmetries

- Have to forbid dim. 4 proton decay operators;
  - R-parity violating terms $u_R^c d_R^c d_R^c$, LLe_R^c, QLd_R^c ($\leftrightarrow$ $\mathbf{10}\,\bar{\mathbf{5}}_m \bar{\mathbf{5}}_m$) must be absent due to experimental bound;
  - Necessary condition: $\bar{\mathbf{5}}_m$, $\bar{\mathbf{5}}_H$ on different curves otherwise: $\mathbf{10}\,\bar{\mathbf{5}}_m \bar{\mathbf{5}}_H$ implies $\mathbf{10}\,\bar{\mathbf{5}}_m \bar{\mathbf{5}}_m$
  - Note: Not sufficient; To forbid effective dim. 5 operators (from triplet exchange) need H_u and H_d on two different curves;
- In general, want selection rules to exclude unwanted stuff;

Will omit references here; Can be found in arXiv:1408.6831;

Basics of F-theory: From IIB to F-theory

- Type IIB string theory with varying axion-dilaton:

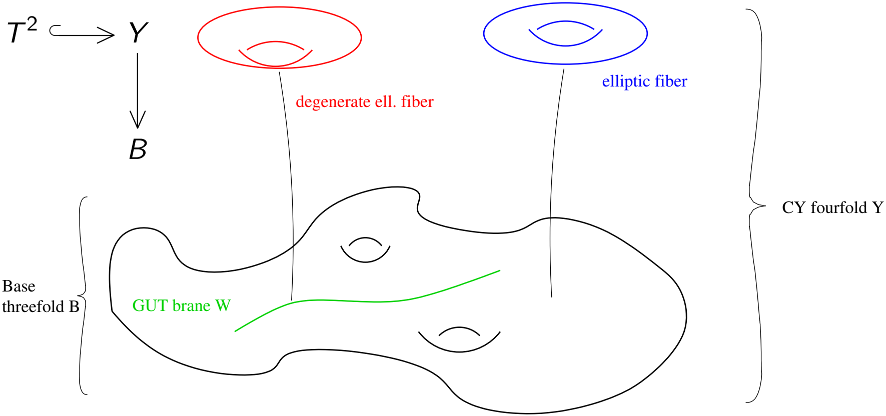
$$\tau := C_0 + i\, e^{-\phi} \qquad\qquad g_s = e^{\phi}$$

- Add conjectured exact SL(2,$\mathbb{Z}$) S-duality of IIB

$$\tau \to \frac{a\tau + b}{c\tau + d}, \begin{pmatrix} H \\ F \end{pmatrix} \to \begin{pmatrix} d & c \\ b & a \end{pmatrix} \begin{pmatrix} H \\ F \end{pmatrix}, \begin{matrix} \tilde{F}_5 & \to & F_5 \\ g_{MN} & \to & g_{MN} \end{matrix} ,$$

- Led to idea of 12d theory—**F-theory**—where information of τ encoded into elliptic curve over every point of $M_4 \times B$;
- F-theory basically 'book-keeping' device to describe vacua of IIB;
- From duality via M-theory and assumptions on non-compact 4d space, we find that compact space Y_4 has to be elliptically fibred CY_4;

Picture of elliptically fibred CY_4



by J.K.

$U(1)$-fibration vs. $\mathbb{Z}_2$-fibration I

- Fibration with $U(1)$ given by non-generic quartic in $\mathbb{P}_{1\,1\,2}$:

  $$\mathrm{w}^2 + b_0 \mathrm{w}\mathrm{u}^2 + b_1 \mathrm{u}\mathrm{v}\mathrm{w} + b_2 \mathrm{v}^2\mathrm{w} + c_0 \mathrm{u}^4 + c_1 \mathrm{u}^3\mathrm{v} + c_2 \mathrm{u}^2\mathrm{v}^2 + c_3 \mathrm{u}\mathrm{v}^3 = 0$$

  with $[\mathrm{u} : \mathrm{v} : \mathrm{w}] \in \mathbb{P}_{1\,1\,2}$ and b_i's and c_i's base sections;
- Curve has two rational points:

  $$\mathrm{Sec}_0 : \ [0 : 1 : 0]\,, \qquad \mathrm{Sec}_1 : \ [0 : 1 : -b_2];$$

  Hence, describes torus fibrations with two sections; Such a fibration gives a $U(1)$;
- If we add $c_4\, v^4$, above rational points become

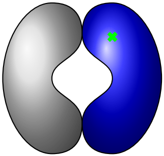
  $$[0 : 1 : -\tfrac{1}{2}(b_2 - \sqrt{b_2^2 - 4\, c_4})]\,, \qquad 0 : 1 : -\tfrac{1}{2}(b_2 + \sqrt{b_2^2 - 4\, c_4})];$$

  Not rational any more, hence, no sections but bi-section; Obtain $\mathbb{Z}_2$-fibration instead of $U(1)$-model;

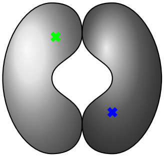
$U(1)$-fibration vs. $\mathbb{Z}_2$-fibration II

- What are the geometric changes?
- $U(1)$-fibration: two codim-two loci, C_1 and C_2, over which fibre degenerates/factorises in two distinct ways:

fibre over C_1

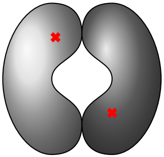


fibre over C_2



- $\mathbb{Z}_2$-model: only one distinct degeneration/factorisation (over C)

fibre over C



$U(1)$-fibration vs. $\mathbb{Z}_2$-fibration III

- What is field theoretically going on?
- All factorised fibre $\mathbb{P}^1$'s can be wrapped by M2-branes; Obtain additional DOF, which at codim-two interpreted as matter states;
- Intersection pattern of this M2 branes with sections/fibral divisors gives $U(1)$-charge/weights of these states;
- Hence, for $U(1)$-model two states:
  - over C_1: $\mathbf{1}_2$;
  - over C_2: $\mathbf{1}_1$;
- Going from $U(1)$-fibration to $\mathbb{Z}_2$-fibration, lose $\mathbf{1}_2$-state;
- Interpreted as Higgsing of $U(1)$ where $\langle \mathbf{1}_2 \rangle \neq 0$; Therefore $U(1) \to \mathbb{Z}_2$ because Higgsing with charge two state;

$U(1)$-fibration vs. $\mathbb{Z}_2$-fibration IV

- That there is such a $\mathbb{Z}_2$ can be seen from circle compactification; Discrete Wilson lines around S^1 give two distinct vacua;
- Can identify these two vacua in geometry;
- Use that F-theory on CY plus S^1 is equivalent to M-theory on same CY;
- If we use this dictionary and shrink blue $\mathbb{P}^1$ over C_1 to obtain massless Higgs mode, find that both $\mathbb{P}^1$'s over C_2 remain massive/finite size;
- If we shrink gray $\mathbb{P}^1$ over C_1 to obtain massless Higgs mode, find that one $\mathbb{P}^1$ over C_2 must shrink too.
- Actually this are two different elements in the Tate-Shafarevich group (i.e. fibrations with same Jacobian fibration) of this fibration; Hence these are the two vacua;
- In S^1 compactification picture, Higgs modes are identified with different KK-modes (massless due to gauge field in S^1);
- In general: if $\mathbf{1}_n$ Higgs, n different KK-modes for Higgs ; Can be checked for $n = 3$ in F_3-fibration where $\mathbf{1}_3$ appears;

$\mathbb{Z}_2$-selection rules in the presents of an $SU(5)$ I

- To obtain further understanding of $\mathbb{Z}_2$ add $SU(5)$;
- For toric torus hypersurfaces fibrations (such as $\mathbb{P}_{1,1,2}[4]$) there exists canonical procedure—so-called tops constructions;
- For $\mathbb{P}_{1,1,2}[4]$-fibration this leads to three different embeddings of $SU(5)$, one of which gives following matter spectrum:

$$\mathbf{10}\,, \qquad \mathbf{5}_A\,, \qquad \mathbf{5}_B\,, \qquad \mathbf{1}\,,$$

with the Yukawa couplings:

$$\mathbf{10}\,\bar{\mathbf{5}}_A\,\bar{\mathbf{5}}_A\,, \quad \mathbf{10}\,\bar{\mathbf{5}}_B\,\bar{\mathbf{5}}_B\,, \quad \mathbf{10}\,\mathbf{10}\,\mathbf{5}_B\,, \quad \bar{\mathbf{5}}_A\mathbf{5}_B\,\mathbf{1}\,;$$

- There are no $\mathbf{10}\,\mathbf{10}\,\mathbf{5}_A$, $\bar{\mathbf{5}}_A\mathbf{5}_A\,\mathbf{1}$, $\bar{\mathbf{5}}_B\mathbf{5}_B\,\mathbf{1}$ couplings which should be there if just an $SU(5)$-symmetry;
- These couplings naturally absent if assign $\mathbb{Z}_2$-charges:

$+$ or 0 $\mathbf{10}$, $\mathbf{5}_B$
$-$ or 1 $\mathbf{5}_A$, $\mathbf{1}$

$\mathbb{Z}_2$-selection rules in the presents of an $SU(5)$ II

- To understand that look again at Higgsing picture;
- If we start from $U(1)$-model with same $SU(5)$-top, obtain matter spectrum:

$$\mathbf{10}_{-2}\,, \qquad \mathbf{5}_{-6}\,, \qquad \mathbf{5}_{-1}\,, \qquad \mathbf{5}_{4}\,, \qquad \mathbf{1}_{5}\,, \qquad \mathbf{1}_{10}\,,$$

  where subscripts are $U(1)$-charges; Furthermore, have all allowed Yukawas;
- Switching on VEV for $\mathbf{1}_{10}$ gives one massive and one mass-less combination of $\mathbf{5}_{-6}$ and $\mathbf{5}_{4}$ via $\bar{\mathbf{5}}_{-4}\mathbf{5}_{-6}\mathbf{1}_{10}$-coupling;

$\Rightarrow$ Get same amount of massless states as in $\mathbb{Z}_2$-case whereupon the $\mathbb{Z}_2$-charge is $U(1)$-charge mod 2 (up to subtleties involving the center of $SU(5)$);

G_4-flux in the recombination picture I

- Above described Higgsing is also known as brane recombination;
- Going from $U(1)$-fibration to $\mathbb{Z}_2$-fibration changes Euler number by

$$\chi(CY_{U(1)}) - \chi(CY_{\mathbb{Z}_2}) = 3\chi(C_1)$$

  where $\chi(C_1)$ is Euler characteristic of C_1 locus;
- Under brane recombination charges must be conserved, e.g.

$$n_{D3} = \frac{1}{24}\chi(CY) - \frac{1}{2}\int_{CY} G_4 \wedge G_4$$

- Hence, if start w/o, flux must have flux after recombination;

G_4-flux in the recombination picture II

- Candidate for such flux:

$$G_4 = \sigma_1 - \tfrac{1}{2}[u] \wedge [\rho]$$

with $\sigma_1 = \{u = 0\} \cap \{w = -b_2 v^2\} \cap \{\rho = 0\}$ and $[\rho] = \frac{1}{2}[c_4]$ where $c_4 = \tau\, \rho$ such that flux well quantised and F-flat;

- Generalisation to case where start w/ flux F:

$$G_4([\rho]) = \sigma_1 - \tfrac{1}{2}[u] \wedge [\rho]$$

with $[\rho] = 2F + \frac{1}{2}[c_4]$;

- Fits perfectly with states needed for recombination; Because need both chiralities for states along C_1 for recombination:

$$-\tfrac{1}{2} c_1(\mathcal{K}_{C_1}) \leq 2\,F \leq \tfrac{1}{2} c_1(\mathcal{K}_{C_1})\,;$$

Summary & Outlook

- Showed you realisation of discrete gauge symmetries in F-theory via Higgsing in compactifications to 4d;
  - Details of identification of Higgs modes;
  - Appearance of discrete selection rules on Yukawas;
  - G_4-flux for $\mathbb{Z}_2$-model;
- Extend flux computation by adding $SU(5)$;
- Work out details for $\mathbb{Z}_3$-case;

Thank you for your attention!