Attractors and the renormalization group

A tale of two flows

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The String Theory Universe
Mainz 25/09/2014

Collaborators

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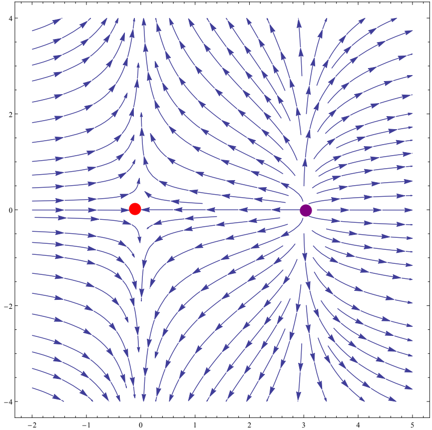
hep-th 1407.0469, 1410...

Book the first

The renormalization group flow

Renormalization group flow

Space of coupling constants



UV conformal field theory ●
IR conformal field theory ●

RG → flow of couplings

$$\frac{d}{d\mu} = -\beta_i(g)\frac{\partial}{\partial g_i}$$

Fixed points

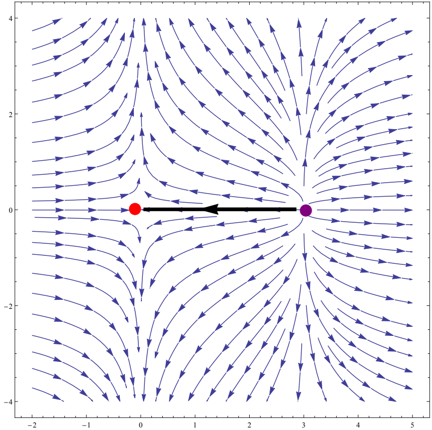
$$\beta_i(g) = 0 \longleftrightarrow \text{CFT}$$

Triggering flow

$$\mathcal{L} = \mathcal{L}_{CFT} + M^{d-\Delta}\mathcal{O}_\Delta$$

Renormalization group flow

Space of coupling constants



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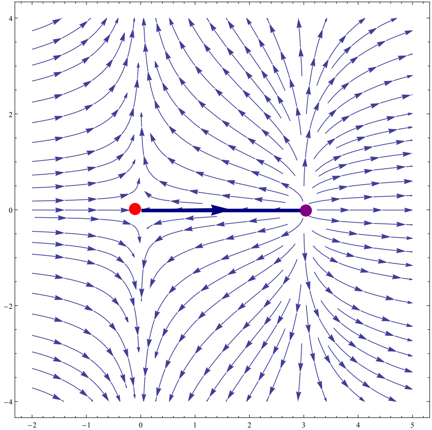
Renormalization group flow (Is it reversible?)

Space of coupling constants



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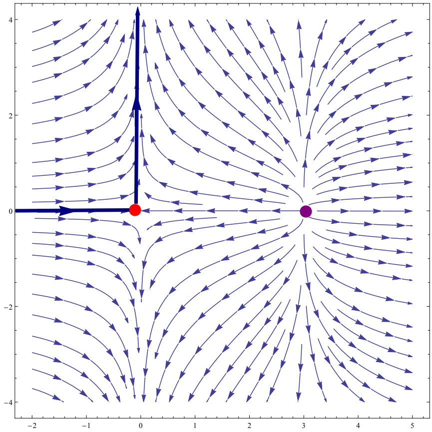
Renormalization group flow (Can there be cycles?)

Space of coupling constants



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Zamolodchikov's c-theorem

$$\text{Anomaly for 2d CFT} \longrightarrow \langle T^{\mu}_{\ \mu} \rangle = c\, R$$

For a unitary 2d CFT exists $\mathcal{C}(\mu)$ such that

- Monotonically decreasing along the flow

$$\frac{d}{d\mu}\mathcal{C}(\mu) \leq 0$$

- Constant on fixed points

$$\beta_i = 0 \;\; \Leftrightarrow \;\; \frac{d}{d\mu}\mathcal{C}(\mu) = 0$$

- Matches the the central charge

$$\mathcal{C}(\mu)_{f.p.} = c_{CFT} \;\; \Rightarrow \;\; \boxed{c_{UV} > c_{IR}}$$

Zamolodchikov 1986

Zamolodchikov's c-theorem (RG is not reversible)

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Zamolodchikov's c-theorem (RG has no cycles)

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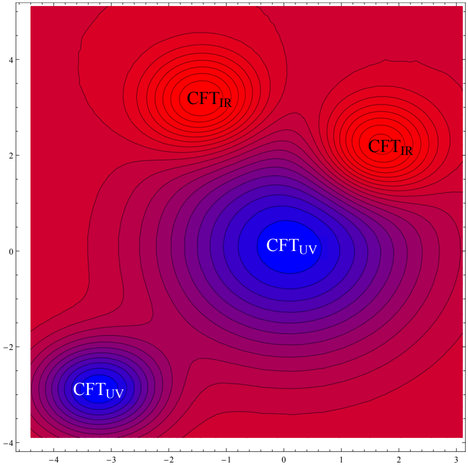
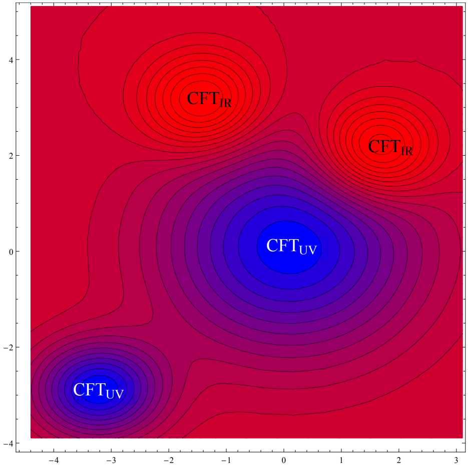
Zamolodchikov's c-theorem

$$\boxed{RG - landscape}$$



Zamolodchikov's c-theorem (What about $d > 2$?)

$$\boxed{RG - landscape}$$



Cardy's proposal

$$\text{Anomaly 4d CFT} \longrightarrow \langle T^{\mu}_{\ \mu} \rangle = aE_4 - bW^2$$

- Euler density $\quad E_4 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$
- Weyl squared $\quad W^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$

Conjecture, the quantity:

$$a \sim \int_{S^4} \langle T^{\mu}_{\ \mu} \rangle$$

Decreases under RG $\implies$ $a_{IR} < a_{UV}$

Cardy 1988

Odd spacetime dimensions the situation is more subtle $\langle T^{\mu}_{\ \mu} \rangle = 0$

The a-theorem

Core idea: RG $\longrightarrow$ SSB of conformal symmetry

Weyl transformations

$$g_{\mu\nu} \longrightarrow e^{2\sigma} g_{\mu\nu} \qquad \tau \to \tau + \sigma$$

Dilaton τ is Nambu-Goldstone

- Write EFT with Diff $\times$ Weyl-invariance
- Add terms to reproduce anomalies (Wess-Zumino problem)
- Take $g_{\mu\nu} \to \eta_{\mu\nu}$ to obtain $S_{eff}[\tau]$

The dilaton couples to matter via

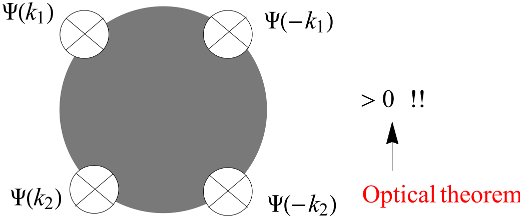
$$S_{coup} \sim \int d^d x \, \tau(x) T^{\mu}_{\ \mu} + \dots$$

The a-theorem

In four dimensions we find

$$S_{eff}[\tau] = 2(a_{UV} - a_{IR}) \int dx^4 \left((\partial\tau)^2 \Box\tau - (\partial\tau)^4 \right) \ldots \quad \Psi = 1 - e^{-\tau}$$

$$\Delta \mathrm{a} = \frac{1}{4\pi} \int_{s>0} \frac{\mathrm{Im}(A(s))}{s^3} =$$



This proves Cardy's conjecture

Schwimmer, Komargodski 2011

Book the second

Attractor flows

Gauged supergravity in a nutshell

$\mathcal{N} = 2$ gSUGRA in 4d and 5d

$$\mathcal{L} = \frac{1}{2} R - G_{i\bar{\jmath}} \partial_\mu \phi^i \partial^\mu \bar{\phi}^j + \frac{1}{4} \operatorname{Im} \mathcal{N}_{IJ} F^I_{\mu\nu} F^{J\,\mu\nu} - g^2 \, V_F(\phi, \bar{\phi})$$

- The field content is:
  - Dynamical metric $g_{\mu\nu}$ two derivative EH
  - Complex scalar fields (Moduli) $\phi^i \;\; i = 1, \ldots, n_v$
  - Abelian gauge fields $A^I_\mu \;\; I = 1, \ldots, n_v + 1$
- Structure determined by holomorphic prepotential:
  - $G_{i\bar{\jmath}}$ is a special Kähler metric.
  - $\mathcal{N}_{IJ}$ scalar dependent gauge couplings.
  - $V_F(\phi, \bar{\phi})$ flux potential.

Black objects in gauged supergravity

Consider metric ansatz (4d and 5d)

$$ds^2 = -a(r)^2 dt^2 + a(r)^{-2} dr^2 + b(r)^2 \sum_{i=1}^{d-2} dx_i^2 + w(r)^2 dz^2$$

Cases:

- Five dimensions $w(r) = a(r)$ or $w(r) = b(r)$
- Four dimensions $w(r) = b(r)$

Driven by first order equations

$$\frac{\partial}{\partial r} \Psi^a(r) = \mathcal{W}^a \left(\Psi, p^I, q_I, h^I, h_I \right) \qquad \Psi^a = (\phi^i(r), a(r), b(r), w(r))$$

Electric/magnetic charge: q_I / p^I Electric/magnetic flux: h_I / h^I

Dall'Agata, Gnecchi 2010

Barisch, Cardoso, Nampuri, Haack, Obers 2011

Black objects in gauged supergravity

First order dynamical system $\longleftrightarrow$ Search for fixed points

Fixed points (Attractors):

$$\frac{\partial}{\partial r}\phi^i = 0 \quad \Longrightarrow \quad AdS\text{-factor}$$

Often correspond to black extremal horizons.

There are solutions interpolating to AdS_d

$$AdS_5 \longrightarrow AdS_2 \times \mathbb{R}^3$$
$$AdS_5 \longrightarrow AdS_3 \times \Sigma_k^2$$
$$AdS_4 \longrightarrow AdS_2 \times \Sigma_k^2$$

Barisch, Cardoso, Nampuri, Haack, Klemm, Obers
Gnecchi, Toldo, Katmadas, Hristov, AVO,...

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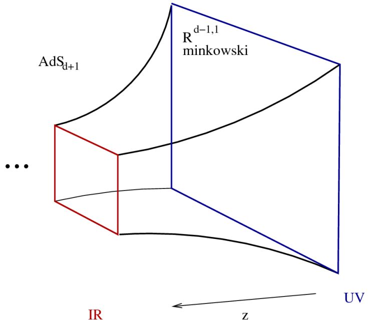
Attractor RG flows

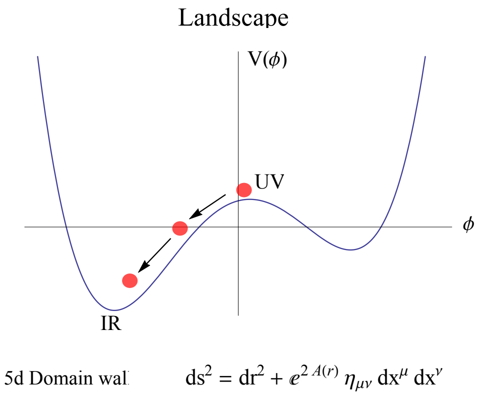
Holographic RG

Quantum gravity in AdS $\longleftrightarrow$ Gauge theory



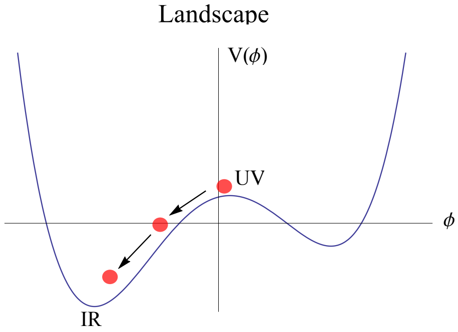
$$\boxed{\frac{L_{AdS}}{L_s} = (4\pi g_s N)^{1/4}}$$

Holographic c-functions



Girardello, Petrini, Porrati, Zaffaroni, 1998
Freedman, Gubser, Pilch, Warner, 1999

Holographic c-functions



5d Domain wal

$$ds^2 = dr^2 + e^{2A(r)}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu$$

Holographic "c-theorems" in many dimensions

Myers, Sinha 2010; Liu, Sabra, Zhao, 2010

Attractive holographic *c*-functions

What makes attractors different?

- Two independent warp factors $a(r)$ and $b(r)$.
- Break Poincaré invariance.
- Flow between *AdS* spaces with different dimensions.
- Non-vanishing (bulk) gauge fields are present.
- Black object $\longleftrightarrow$ Ensemble

Early attempt:

$$\mathcal{C}(r) = \text{Folliation area}$$

Goldstein, Jena, Mandal, Trivedi 2005

Attractive holographic c-functions

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Early attempt:

Regretfully it is divergent in the UV

Attractive holographic c-functions (AdS_3 IR)

Five-dimensional line elements

$$ds^2 = a(r)^2\left(-dt^2 + dz^2\right) + a(r)^{-2}dr^2 + b(r)^2\sum_{i=1}^{d-2} dx_i^2$$

Fixed points $\phi' = 0$:

AdS_5 UV	$AdS_3 \times \Sigma_k$ IR
$a(r) \sim r \quad b(r) \sim r$	$a(r) \sim r \quad b(r) \sim r^0$

$$\text{Scalar kinetic term} = -a(r)^2\left(\frac{a''(r)}{a(r)} + (d-2)\frac{b''(r)}{b(r)}\right) \geq 0$$

Attractive holographic c-functions (AdS_3 IR)

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$$\text{Scalar kinetic term} = -a(r)^2\left(\frac{a''(r)}{a(r)} + (d-2)\frac{b''(r)}{b(r)}\right) = 0 \ \ \text{(F.P.)}$$

Attractive holographic c-functions (AdS_3 IR)

Proposal for attractive c-function

$$\mathcal{C}'_{AdS_3}(r) = \frac{\text{Kin}}{\mathcal{F}(r)}$$

$\mathcal{F}(r)$ positive regular function

Take $\mathcal{F}(r) = a^2(r)$:

$$\boxed{\mathcal{C}_{AdS_3}(r) = \lambda + \kappa\, \mathcal{H}(r)}$$

$$\mathcal{H}(r) = \left(\frac{a'}{a} + (d-2)\frac{b'}{b}\right) + \int^r dr' \left(\left(\frac{a'}{a}\right)^2 + (d-2)\left(\frac{b'}{b}\right)^2\right)$$

The constants

$$\lambda = \frac{\mathcal{H}_{UV} c_{IR} - \mathcal{H}_{IR} c_{UV}}{\mathcal{H}_{UV} - \mathcal{H}_{IR}} \qquad \kappa = \frac{c_{UV} - c_{IR}}{\mathcal{H}_{UV} - \mathcal{H}_{IR}}$$

Attractive holographic c-functions (AdS_2 IR)

Proposal for attractive c-function

$$\mathcal{C}'_{AdS_3}(r) = \frac{\text{Kin}}{\mathcal{F}(r)} \qquad \mathcal{F}(r) \text{ positive regular function}$$

Take $\mathcal{F}(r) = a^2(r) b^{-1}(r)$:

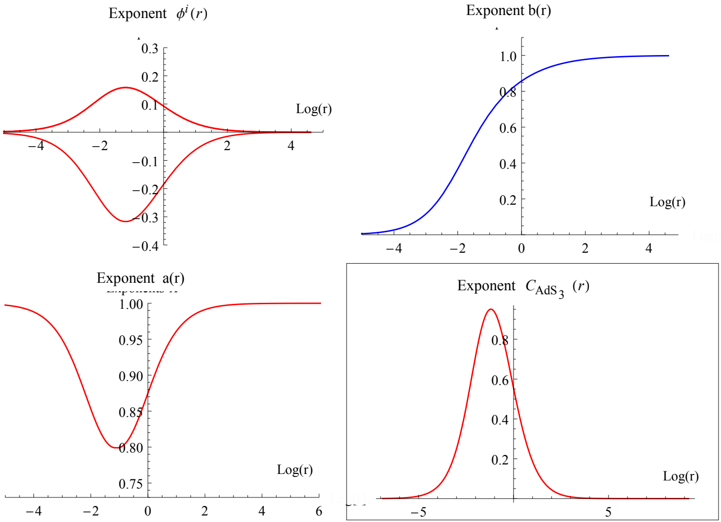
$$\boxed{\mathcal{C}_{AdS_3}(r) = \lambda + \kappa\, \mathcal{H}(r)}$$

$$\mathcal{H}(r) = -(d-1)\, b'(r)$$

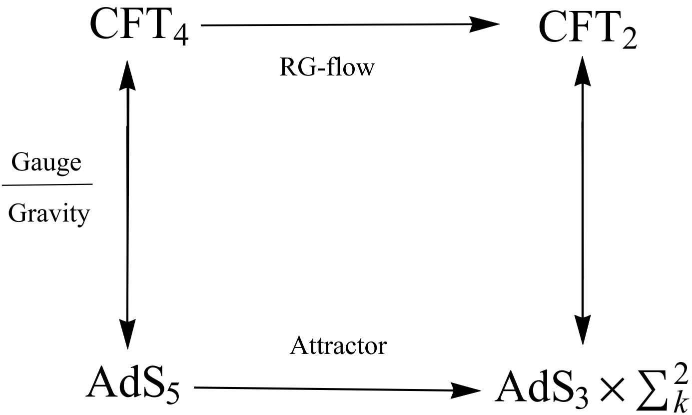
The constants

$$\lambda = \frac{\mathcal{H}_{UV} c_{IR} - \mathcal{H}_{IR} c_{UV}}{\mathcal{H}_{UV} - \mathcal{H}_{IR}} \qquad \kappa = \frac{c_{UV} - c_{IR}}{\mathcal{H}_{UV} - \mathcal{H}_{IR}}$$

Magnetic $AdS_5 \to AdS_3$ brane



Attractive RG



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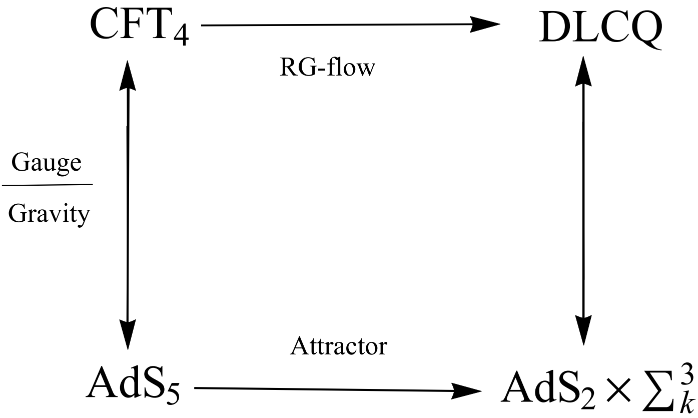


Epilogue

Solodukhin's decomposition

Define four-dimensional field theory on

$$\mathcal{M}_\alpha \xrightarrow{\textit{locally}} \Sigma \times C_\alpha \quad \text{(Surface} \times \text{Cone)}$$

Two point function $T(x) = T^\mu_\nu(x)$

$$\langle T(x)T(y)\rangle^{(4)}_\alpha = -\frac{a\pi}{4}(1-\alpha)\partial^2_\Sigma \delta(x-y)\delta_\Sigma$$

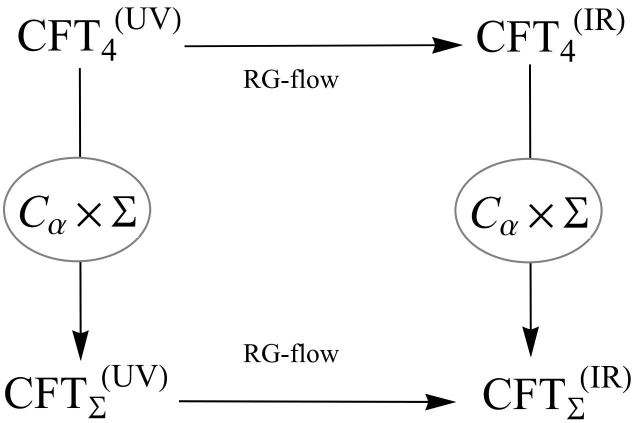
Compare with 2d CFT

$$\langle T(x)T(y)\rangle^{(2)} = -\frac{c}{12\pi}\partial^2 \delta^{(2)}(x-y)$$

4d CFT a-coefficient $\longleftrightarrow$ 2d central charge CFT on Σ

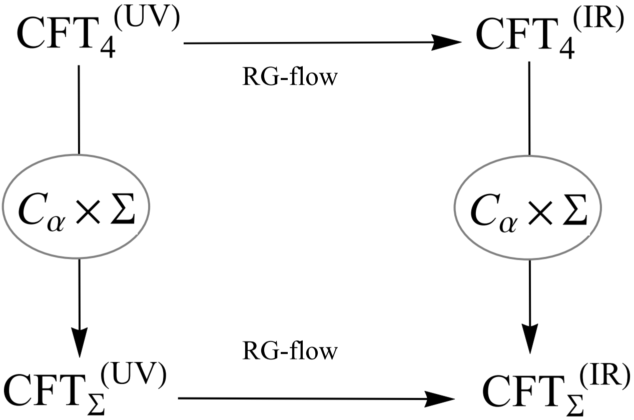
Solodukhin 2013, 2014

An interpretation for the attractor RG (?)



In general is not commutative

An interpretation for the attractor RG (?)



Claim: For ARG it commutes!!!

Thank you very much for your attention