Anisotropic Gauge/Gravity Dualities

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Review Talk

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Outline

Status...

- Since the initial correspondence was found between the $\mathcal{N} = 4$ sYM and the superstring theory in $AdS_5 \times S^5$, there has been a lot of effort to construct gauge/gravity dualities that can be though as toy models to describe realistic systems and theories; with the (extra) hope of some universal behaviors.
- For example:

✓ Less Supersymmetry. Example: $\mathcal{N} = 1$ β deformed theories.

✓ Broken conformal symmetry, confinement. Example: D4 Witten model.

✓ Finite temperature. Example: Black hole in AdS.

✓ Inclusion of dynamical quarks in Quenched and Unquenhed approximation. Needed for: Meson Spectrum and Screening in Static Potential.

✓ Inclusion of Anisotropy. Example: In this talk

...

Why do we need anisotropic theories?

- Several physical systems are anisotropic. Eg: The expansion of the plasma along the longitudinal beam axis at the earliest times after the collision results to momentum anisotropic plasmas.
- There exist several results for observables in weakly coupled theories. Is there any relevance with the strongly coupled limit models?
- Properties of the top-down anisotropic supergravity solutions.
- Striking New Features? Several Universality Relations are violated! New universal properties depending on the shape of the geometry.

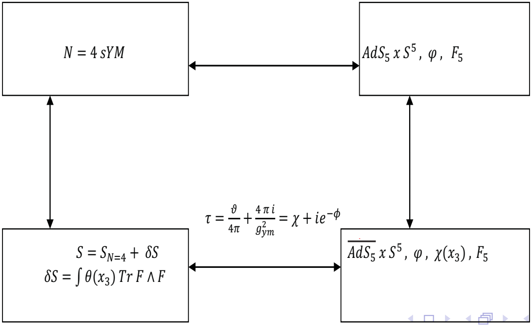
How does Anisotropy is introduced? An Example

- Introduction of additional branes: Lifhsitz-like Supergravity solutions.

(Azeyanagi, Li, Takayanagi, 0905.0688 jhep)

	x_0	x_1	x_2	x_3	u	S^5
D3	x	x	x	x		
D7	x	x	x			x

- Which equivalently leads to the following deformation diagram.



An anisotropic background

The metric in string frame *(Mateos, Trancanelli, 1105.3472 prl, 1106.1637 jhep)*

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}\mathcal{B}\, dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{H} d{x_3}^2 + \frac{du^2}{\mathcal{F}}\right) + \mathcal{Z}\, d\Omega_{S^5}^2 \,.$$

The functions $\mathcal{F}, \mathcal{B}, \mathcal{H}$ depend on the radial direction u and the anisotropy. The anisotropic parameter is α with units of inverse length ($\chi = \alpha x_3$) and $P_{x_3} < P_{x_1 x_2}$.
In sufficiently high temperatures, $T \gg \alpha$, and imposed boundary conditions the Einstein equations can be solved analytically:

$$\mathcal{F}(u) = 1 - \frac{u^4}{u_h^4} + \alpha^2 \frac{1}{24 u_h^2}\left[8u^2(u_h^2 - u^2) - 10u^4 \log 2 + (3u_h^4 + 7u^4)\log\left(1 + \frac{u^2}{u_h^2}\right)\right]$$

$$\mathcal{B}(u) = 1 - \alpha^2 \frac{u_h^2}{24}\left[\frac{10u^2}{u_h^2 + u^2} + \log\left(1 + \frac{u^2}{u_h^2}\right)\right], \quad \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}}$$

The isotropic limit $\alpha \to 0$ reproduce the well know result of the isotropic D3-brane solution (dual to $\mathcal{N} = 4$ finite sYM solution).

But let us work with generic anisotropic theories!

Write the anisotropic metric as

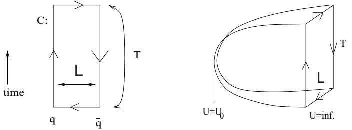
$$ds^2 = g_{00}dx_0^2 + g_{11}dx_1^2 + g_{22}dx_2^2 + g_{33}dx_3^2 + g_{uu}du^2 + \text{internal space}$$

Notation:

$x_{1,2} =: x_\perp$ transverse direction to anisotropy, $g_{11} = g_{22}$
$x_3 =: x_\parallel$ parallel direction to anisotropy.
The observables Q:
$Q_\parallel := Q_{x_3} = Q_{anisotropic}$
$Q_\perp := Q_{x_1 \ or \ x_2}$

Warm up Example: Static Potential

- Consider the string world-sheet (τ, σ) with orthogonal boundary shape:



- Lets name x_p the direction where the pair is aligned.
  The solution to Nambu-Goto action

$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\tilde{g}}$$

  is a catenary shape w-s with u_0 being the turning point.

Results depend on direction x_p. In general the length of the two endpoints of the string on the boundary is given by

$$L_p = 2\int_{\infty}^{u_0} \frac{du}{u'} = 2\int_{u_0}^{\infty} du \sqrt{\frac{-g_{uu} c_0^2}{(g_{00} g_{pp} + c_0^2) g_{pp}}} \,.$$

Which should be inverted as $u_0(L)$. The normalized energy of the string is

$$2\pi\alpha' V_p \quad = \quad c_0 L_p + 2\left[\int_{u_0}^{\infty} du \sqrt{-g_{uu} g_{00}} \left(\sqrt{1 + \frac{c_0^2}{g_{pp} g_{00}}} - 1\right) - \int_{u_h}^{u_0} du \sqrt{-g_{00} g_{uu}}\right] \,.$$

(Sonnenschein, hep-th 0003032, review; D.G 1202.4436 jhep)

Applied in the axion deformed anisotropic theory we get screening:

• $V_{\parallel} < V_{\perp} < V_{iso}$.

(D.G 1202.4436 jhep; Rebhan, Steineder 1205.4684 jhep; Chernicoff, Fernandez, Mateos, Trancanelli, 1208.2672 jhep, D.G 1306.1404 review;)

• Note: To get the complete picture of the quarkonium, the analysis of the Imaginary Potential and the Thermal Width can be made by fluctuating the same string configuration.

(Bitaghsir, D.G, Soltanpanahi, 1306.2929 jhep)

Another observable: The jet Quenching

• Parameter of momentum broadening along the transverse direction of the quark's motion.
• Going to the light-cone coordinates and calculate the on-shell action, canceling the divergences we obtain

$$\hat{q}_{p\ (k)} = \frac{\sqrt{2}}{\pi}\left(\int_0^{u_h} \frac{1}{g_{kk}}\sqrt{\frac{g_{uu}}{g_{--}}}\right)^{-1} .$$

(D.G 1202.4436 jhep)

where $g_{--} = 1/2(g_{00} + g_{pp})$.
The index p denotes the direction along the motion of the quark and k the direction along which the momentum broadening happen.
Eg:Motion along the anisotropy: $p = 3$ and momentum broadening in the transverse space $k = 2$.

$\hat{q}$	x_p	x_k	Energetic parton along	Momentum broadening along
$\hat{q}_{\perp(\parallel)}$	$x_\perp$	$x_\parallel$	$x_\perp$	$x_\parallel$
$\hat{q}_{\parallel(\perp)}$	$x_\parallel$	$x_\perp$	$x_\parallel$	$x_\perp$
$\hat{q}_{\perp(\perp)}$	$x_{\perp,1}$	$x_{\perp,2}$	$x_{\perp,1}$	$x_{\perp,2}$

(Chernicoff, Fernandez, Mateos, Trancanelli, 1203.056 jhep;)

Generic Remark that needs further attention: Certain Properties of the observables depend on the shape of the anisotropic geometry (prolate, oblate) than its exact details. Universal features?

Universality Relations

The Shear Viscosity over Entropy density ratio universal low value prediction

$$\frac{\eta}{s} \gtrsim \frac{1}{4\pi} \ .$$

In anisotropic theories has been found to be clearly violated!
(Rebhan, Steineder 1110.6825 prl; Jain, Kundu, Sen, Sinha, Trivedi 1406.4874)
Reason:

$$\frac{\eta}{s} \propto \frac{g_{11}(u_h)}{g_{33}(u_h)} \frac{1}{4\pi} \ .$$

All prolate deformed geometries violate the bound!

- Other universality relations which are violated? Yes.

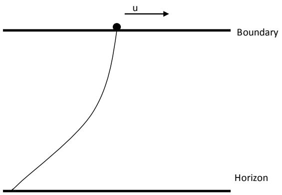
Drag Force

The dynamics of the quark can be described by

$$\frac{dp}{dt} = F_{drag} + F(t) \ .$$

The single quark moving in the anisotropic plasma can be represented by a trailing string from the boundary where the probe quark moves with the constant speed, to the horizon of the black hole.

(Herzog, Karch, Kovtun, Kozcaz, Yaffe hep-th/0605158 jhep; Gubser, hep-th/0605182 prd)



The quark moves along the direction p.
At $u = u_0$ there is horizon of the induced worldsheet metric given by

$$\left(g_{uu}(g_{00} + g_{pp}v^2)\right)\,|_{u=u_0} = 0 \;.$$

Calculating the momentum flowing from the boundary to the bulk we can find the drag force

$$F_{drag,p} = -\sqrt{\lambda}\frac{\sqrt{-g_{00}g_{pp}}}{(2\pi)}\Big|_{u=u_0} \;.$$

The "effective world-sheet temperature" is

$$T_{ws}^2 = \left|\frac{1}{16\pi^2}\frac{1}{g_{00}g_{uu}}(g_{00}\,g_{pp})'\left(\frac{g_{00}}{g_{pp}}\right)'\right|\Big|_{u=u_0} \;.$$

(D.G, Soltanpanahi, 1310.6725 prd)

In near horizon Dp black brane geometries $T_{ws} < T$.

(Nakamura, Ooguri 1309.4089 prd)

In Anisotropic Theories $T_{ws} \gtrless T$.

(D.G, Soltanpanahi, 1312.7474 jhep)

Momentum Broadening

The $F(t)$ is the factor that causes the momentum broadening, which leads to

$$\frac{\left\langle p_{L,T}^2 \right\rangle}{\mathcal{T}} = 2\kappa_{L,T}$$

$\kappa =$ Mean Squared Momentum Transfer per Time.

- The index L refers to the direction along the motion of quark, the index T is the direction transverse to the velocity of quark.
- In strong coupling limit the coefficients are obtained from fluctuations to the Wilson line.

For a quark moving along the p direction and the transverse direction of broadening is taken to be k:

$$\kappa_T = \frac{1}{\pi} g_{kk}\Big|_{u=u_0} T_{ws} \;, \qquad \kappa_L = 16\,\pi \left.\frac{|g_{00}|\, g_{uu}}{g_{pp}\left(\frac{g_{00}}{g_{pp}}\right)'^2}\right|_{u=u_0} T_{ws}^3 \;.$$

(D.G,Soltanpanahi 1310.6725 prd)

Their ratio can be simplified to

$$\frac{\kappa_L}{\kappa_T} = \frac{1}{g_{pp} g_{kk}} \frac{(g_{00} g_{pp})'}{(g_{00}/g_{pp})'}\Bigg|_{u=u_0}$$

Example: $p = 3$ and $k = 1$: Quark moves along the anisotropic direction x_3 and the transverse broadening direction is x_1.

•For any isotropic theory it can be proved that $\kappa_L > \kappa_T$.
This is a Universal Inequality independent of the background used!
(Gursoy, Kiritsis, Mazzanti, Nitti 1006.3261 jhep; D.G, Soltanpanahi 1310.6725 prd)

• Only possibility to have it violated is in the anisotropic theories! A particular theory that the violation happen has been found.

• Only anisotropic theories allow negative κ_L coefficient!
In search for a concrete example of such theory theory.

(D.G, Soltanpanahi 1310.6725 prd, 1312.7474 jhep)

Conclusions

Working with generic anisotropic theories:

- Several observables have been studied Static Potential, the Drag Force, the Jet Quenching...
- Universal Relations are violated in the Anisotropic theories. Surprisingly depending on the shape of the geometry. The Langevin coefficients inequality $\kappa_L > \kappa_T$ proved to hold for isotropic backgrounds is violated for the anisotropic theories!

Related progress:

- Non-Integrability of the anisotropic spaces and possible appearance of chaos. *(D.G, Sfetsos 1403.2703 jhep)*

Work in progress:

- Anisotropic k-string configurations. Challenging due to broken isotropy.
- Thermalization!

Thank you