D-branes in non-abelian gauged linear sigma models

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Outline

CYs and GLSMs

D-branes in GLSMs

Conclusions

CYs and GLSMs

- A Calabi-Yau space (CY) can be realized as the low energy configuration of a supersymmetric $\mathcal{N} = (2, 2)$ gauge theory in two dimensions - the gauged linear sigma model (GLSM).

  [Witten 93]
- The choice of gauge groups and matter content determines the CY.
- Hypersurfaces and complete intersections in toric ambient spaces are realized by GLSMs with gauge groups $G = U(1)^k$.
- Non-abelian gauge groups lead to exotic CYs (e.g. Pfaffian CYs).
- Phase: by tuning coupling constants (FI parameters) in the gauge theory, we can probe the (enlarged) Kähler moduli space of a CY.

GLSM Data

- Gauge group $G = \frac{U(1)^l \times H}{\{\pm 1, \pm \mathbf{1}\}}$.
  - This talk: $H = USp(k)$ or $H = O(k)$, $l = 1$.
- Matter
  - Chiral fields
    - p^i, $i = 1, \ldots, M$, $U(1)^l$-charges $Q^l_{p^i}$, trivial rep. of H
    - x^a_i, $i = 1, \ldots, N$, $a = 1, \ldots, k$, $U(1)^l$-charges $Q^l_{x_i}$, fundamental rep. of H
  - Twisted chiral fields
    - σ_k, take values in the maximal torus of G.
- Superpotential W

$$H = SU(2): \qquad W = \sum_{i,j=1}^{N} A^{ij}(p)[x_i x_j] = \sum_{i,j=1}^{N} \sum_{a,b=1}^{2} A^{ij}(p) x_i^a \varepsilon_{ab} x_k^b$$

$$H = O(2): \qquad W = \sum_{i,j=1}^{N} S^{ij}(p)(x_i x_j) = \sum_{i,j=1}^{N} \sum_{a,b=1}^{2} S^{ij}(p) x_i^a \delta_{ab} x_k^b$$

Equations of motion I

- The vacua, i.e. solutions to the equations of motion, determine the CY.
- On the Higgs branch $\sigma = 0$.
- D-terms

$$
\begin{aligned}
U(1)^l: &\qquad \sum_{i=1}^{M} Q^l_{p^i}|p^i|^2 + \sum_{j=1}^{N} Q^l_{x_j}||x_j||^2 = r_l \\
SU(2): &\qquad xx^\dagger - \frac{1}{2}||x||^2 \mathbf{1}_2 = 0 \\
O(2): &\qquad xx^\dagger - (xx^\dagger)^T = 0.
\end{aligned}
$$

- $r_l \in \mathbb{R}$ are the FI parameters. Together with the θ-angle they can be identified with the complexified Kähler modulus of the CY: $t = r - i\theta$.
- By changing the value of the r_l we can probe the Kähler moduli space of the CY.

Equations of motion II

- E.o.ms continued...
  - F-terms

$$\frac{\partial W}{\partial p^i} = 0 \qquad \frac{\partial W}{\partial x_i^a} = 0.$$

- Phases of the GLSM
  - Different FI parameter regions have different solutions of the e.o.ms.
  - Gauge group gets broken to a subgroup in different phases. $\Rightarrow$ strong coupling effects if broken to continuous subgroup
  - CYs in phases are not necessarily birational, yet share the same Kähler moduli space.
  - Conjecture: The derived categories associated to the CYs in the various phases are equivalent.

CY data

- CY condition

$$\sum_{i=1}^{M} Q^{l}_{p^i} + k \sum_{j=1}^{N} Q^{l}_{x_j} = 0 \quad \mathrm{rk} H = k$$

- Dimension:

$$\dim = M - 1 - \frac{k(k \pm 1)}{2} \quad + \ldots USp(k), \, - \ldots O(k)$$

- Hodge number estimate
  - $h^{1,1} \Leftrightarrow$ number of FI-parameters
  - $h^{2,1} \Leftrightarrow$ number of monomials in W modulo reparametrization
- Quantum corrections on the Coulomb branch determine the conifold point(s).

Non-abelian Duality

- There is a duality between models with different non-abelian gauge groups. [Hori 11]

	original	dual
G	$G = U(1)^l \times USp(k)$	$\tilde{G} = U(1)^l \times USp(N-k-1)$
fields	$\begin{array}{cc} p^i & x_i \\ \hline Q^l_{p^i} & Q^l_{x_i} \end{array}$	$\begin{array}{ccc} p^i & \tilde{x}^i & a_{ij} = -a_{ji} \\ \hline Q^l_{p^i} & -Q^l_{x_i} & Q_{x_i} + Q_{x_j} \end{array}$
W	$W = \sum_{i,j=1}^N A^{ij}(p)[x_i x_j]$	$\tilde{W} = \sum_{i,j=1}^N [\tilde{x}^i \tilde{x}^j] a_{ij} + A^{ij}(p) a_{ij}$

- Similar structure for orthogonal gauge groups with

$$\begin{array}{ccc} O(k) & \leftrightarrow & SO(N-k+1) \\ s_{ij} & = & s_{ji} \end{array}$$

- Strong/weak coupling duality: Certain properties of the CY are more easily derived in the dual theory.

Non-abelian examples

- Notation: Models A_q^k and $S_q^{k,\bullet}$
  - A_q^k: $H = USp(k)$, with charges q
  - $S_q^{k,\bullet}$: $H = SO(k)$ $(\bullet = 0)$ or $H = O(k) \simeq SO(k) \rtimes \mathbb{Z}_2$ $(\bullet = \pm)$ with charges q
- Rodland Model $A^2_{-2^7,1^7}$ [Rodland '98][Hori-Tong '06]
  - $r \gg 0$: Complete intersection of codim. 7 in $G(2,7)$
  - $r \ll 0$: Pfaffian CY $\mathrm{rk} A(p) = 4$ (strongly coupled)
- Hosono-Takagi $S^{2,+}_{-2^5,1^5}$ [Hosono-Takagi][Hori '11]
  - $r \gg 0$: Complete intersection of codim.5 in $(\mathbb{P}^4 \times \mathbb{P}^4)/\mathbb{Z}_2$
  - $r \ll 0$: Determinantal variety $\mathrm{rk} S(p) \leq 4$ (strongly coupled)
- Further interesting new one-parameter examples. [Hori-JK, '13]
  - CYs with different Hodge numbers $h^{2,1}$ but same Kähler moduli space.

Matrix Factorizations

- D-branes are gauge-invariant, R-invariant matrix factorizations of the GLSM potential. [Herbst-Hori-Page '08][Hond-Okuda,Hori-Romo '13]
  - Take a square matrix Q with polynomial entries such that
  $$Q^2 = W\mathbf{1}$$
  - Gauge invariance:
  $$\rho(g)^{-1}Q(g\phi)\rho(g) = Q(\phi)$$
  - R-invariance:
  $$\lambda^{\mathbf{r}_*}Q(\lambda^R\phi)\lambda^{-\mathbf{r}_*} = \lambda Q(\phi)$$
- GLSM branes map to (trivial or non-trivial) branes in the individual phases.

Elementary matrix factorizations I

- To construct matrix factorizations for the Rodland and HT-examples we start with a toy model with gauge group $U(2)$.
  - Superpotential
  $$W = p[x_1 x_2] = p(x_1^1 x_2^2 - x_1^2 x_2^1).$$
  - Gauge transformations with $g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in U(2)$:
  $$\begin{aligned} p &\rightarrow (ad-bc)^{-1} p \\ x_i^1 &\rightarrow a x_i^1 + b x_i^2 \\ x_i^2 &\rightarrow c x_i^1 + d x_i^2 . \end{aligned}$$
  - R-charge: can choose $[p] = 2$, $[x_i^a] = 0$.

Elementary matrix factorizations II

- Simplest one:

$$Q = \begin{pmatrix} 0 & p \\ x_1^1 x_2^2 - x_1^2 x_2^1 & 0 \end{pmatrix} \qquad \rho = \begin{pmatrix} 1 & 0 \\ 0 & \det g \end{pmatrix}$$

- Using a basis of 4×4 Clifford matrices $\{\eta_i, \bar{\eta}_j\} = \delta_{ij}$: $Q = x_1^1 \eta_1 + p x_2^2 \bar{\eta}_1 + x_1^2 \eta_2 - p x_2^1 \bar{\eta}_2$

$$Q = \begin{pmatrix} 0 & 0 & x_1^1 & x_1^2 \\ 0 & 0 & -p x_2^1 & -p x_2^2 \\ p x_2^2 & x_1^2 & 0 & 0 \\ -p x_2^1 & -x_1^1 & 0 & 0 \end{pmatrix} \quad \rho = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\det g} & 0 & 0 \\ 0 & 0 & \frac{d}{\det g} & -\frac{c}{\det g} \\ 0 & 0 & -\frac{b}{\det g} & \frac{a}{\det g} \end{pmatrix}$$

- For the models of interest we can obtain matrix factorizations by taking tensor products of these two elementary constructions.

D-brane charges

- Quantum corrected D-brane central charges are computed by the hemisphere partition function. [Honda-Okuda,Hori-Romo '13]

$$Z_{D^2}(\mathcal{B}) = C \int_\gamma d^{l_G}\tau \prod_{\alpha>0} \alpha(\tau) \sinh(i\pi\alpha(\tau)) \prod_i \Gamma\left(Q_i(\tau) + \frac{R_i}{2}\right) e^{t_R(\tau)} f_{\mathcal{B}}(\tau)$$

  - $\alpha > 0$ positive roots
  - $\tau = \in \mathfrak{t}_{\mathbf{C}}$ with $\mathfrak{t}_{\mathbf{C}} = \mathrm{Lie}(T_G) \otimes \mathbb{C}$ (maximal torus)
  - $l_G = \mathrm{rk}(G)$
  - R_i ... R-charges, Q_i ... weights under the maximal torus
  - $t_R = r - i\theta$ ... complexified FI (Kähler) parameter
  - γ ... integration contour (s.t. integral is convergent)
- Brane factor

$$f_{\mathcal{B}}(\tau) = \mathrm{tr}_M \left((-1)^{\mathbf{r}_*} e^{-2\pi i \rho_*(\tau)} \right)$$

  - M ... Chan-Paton space
  - The brane input is obtained by restricting the matrices $\rho(g)$ and $\lambda^{\mathbf{r}_*}$ to the maximal torus.

Structure Sheaf

- In geometric phases Z_{D^2} reduces to

$$Z_{D^2}(\mathcal{B}) = \int_X \hat{\Gamma}(X) e^{B+\frac{i}{2\pi}\omega} \mathrm{ch}(\mathcal{B}).$$

- The structure sheaf encodes the topological data of the CY.

$$\begin{aligned} Z(\mathcal{O}_X) &= \frac{H^3}{3!}\left(\frac{it}{2\pi}\right)^3 + \left(\frac{it}{2\pi}\right)\frac{c_2 H}{24} + i\frac{\zeta(3)}{(2\pi)^3}\chi(X) + O(e^{-t}) \\ &\overset{mirror}{=} \frac{H^3}{3!}\varpi_3 + \frac{c_2 H}{24}\varpi_2 + i\frac{\zeta(3)\chi(X)}{(2\pi)^3}\varpi_0 \end{aligned}$$

  - Different topological data for different (geometric) phases in the GLSM.
  - Identify the brane factor/matrix factorization associated to the structure sheaf.

Structure Sheaf II

- For Rodland and HT matrix factorizations discussed above lead to a brane factors of type

$$f_{\mathcal{B}}(a,b) = (1-e^{-2\pi i(\tau_1+\tau_2)})^a(1-e^{-2\pi i\tau_1})^b(1-e^{-2\pi i\tau_2})^b \qquad a,b\in\mathbb{Z}_{\geq 0}$$

- In $r \gg 0$-phases the structure sheaf is given by the matrix factorization with brane factor $f(N,0)$

$$Q(N,0) = \sum_{i=1}^{N} p^i\eta_i + \frac{\partial W}{\partial p^i}\bar{\eta}_i$$

- In the strongly-coupled $r \ll 0$-phases no simple matrix factorization has been found. However,
  - There are brane factors that give the right charges, e.g. for Rodland:

$$f_{\mathcal{B}} = 2f(5,1) - 2f(3,2) + f(6,1) - 2f(4,2) - f(5,2)$$

  - In the weakly-coupled dual theory the structure sheaf is described by a simple matrix factorization.

Conclusions

- We constructed matrix factorizations in examples of one-parameter non-abelian GLSMs.
- Using the hemisphere partition function we computed quantum-corrected D-brane charges.
  - We identified the structure sheaf.
  - We found many examples of lower-dimensional branes in all phases.
- These methods are very general.
  - Not restricted to hypersurfaces/complete intersections in toric spaces.
  - No mirror symmetry required.
  - No Landau-Ginzburg point required for matrix factorizations.

Outlook

- D-brane transport
  - By choosing integration contours such that Z_{D^2} converges one can derive a grade restriction rule. [HHP '08]
  - This reproduces mathematical results. [Addington-Donovan-Segal '14]
  - For strongly coupled phases, branes have to be grade restricted deep inside the phase.
- How do construct matrix factorizations for the structure sheaf in strongly coupled phases?
- How to classify matrix factorizations in (non-)abelian GLSMs?
- Interesting phenomenology on these CYs?