The String Theory Universe@ JGU Mainz, Sep. 22-26 th, 2014

Entanglement Entropy (EE) and Spacetime Geometry

Tadashi Takayanagi

Yukawa Institute for Theoretical Physics (YITP), Kyoto University

YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Contents

① Introduction

String Theory ⇒ a unified theory of quantum gravity

It has been still difficult to compute quantum corrections in cosmological spacetimes like big bang, de-Sitter etc.

However, a generalization of AdS/CFT (or holography) may be able to resolve this problem:

``Quantum Gravity = Quantum Many-body Systems''

For this, we need to understand the basic mechanism of AdS/CFT. ⇒ A key concept is **quantum entanglement**.

What is the quantum entanglement ?

Consider the following states in **two spin systems**:

(i) A direct product state (unentangled state)

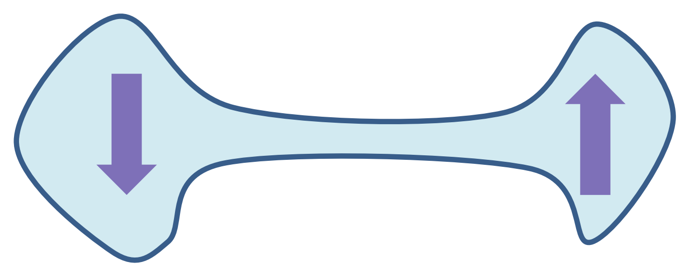
$$|\Psi\rangle = \frac{1}{2}\left[|\uparrow\rangle_A + |\downarrow\rangle_A\right] \otimes \left[|\uparrow\rangle_B + |\downarrow\rangle_B\right].$$

Independent

(ii) An entangled state (EPR pair)

$$|\Psi\rangle = \left[|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B\right] /\sqrt{2}.$$

One determines the other !



∃ Non-local correlation

A measure of quantum entanglement is known as the **entanglement entropy** defined as follows.

Divide a quantum system into two subsystems A and B.

$$H_{tot} = H_A \otimes H_B \quad .$$

Define the reduced density matrix ρ_A by

$$\rho_A = \mathrm{Tr}_B \left| \Psi \right\rangle \left\langle \Psi \right| \, .$$

The entanglement entropy S_A is now defined by

$$S_A = -\mathrm{Tr}_A \, \rho_A \log \rho_A \, . \qquad \text{(von-Neumann entropy)}$$

It is also helpful to look at **(n-th) Renyi entanglement entropy (REE)** which generalizes the EE :

$$S_A^{(n)} = \frac{1}{1-n} \cdot \log \mathrm{Tr}\left[(\rho_A)^n\right] .$$

$$\lim_{n \to 1} S_A^{(n)} = -\mathrm{Tr}[\rho_A \log \rho_A] = S_A \quad . \qquad (\mathrm{Tr}[\rho_A] = 1).$$

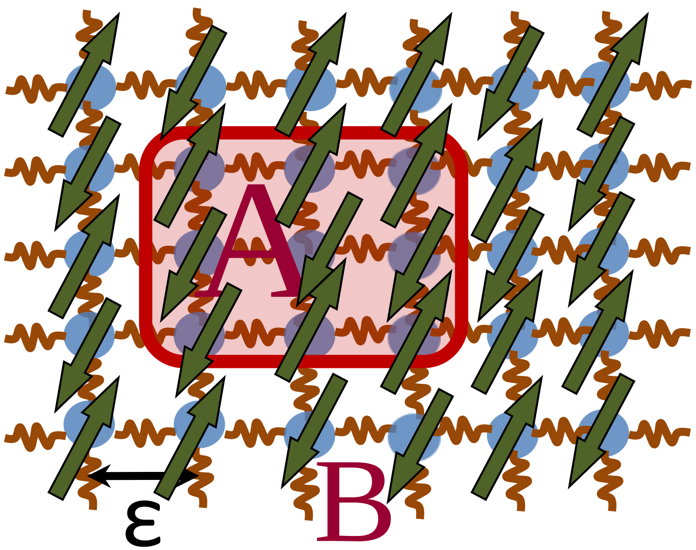
If we know all of $S_A^{(n)}$, we find all eigenvalues of ρ_A.

(so called entanglement spectrum)

② Entanglement Entropy in QFTs

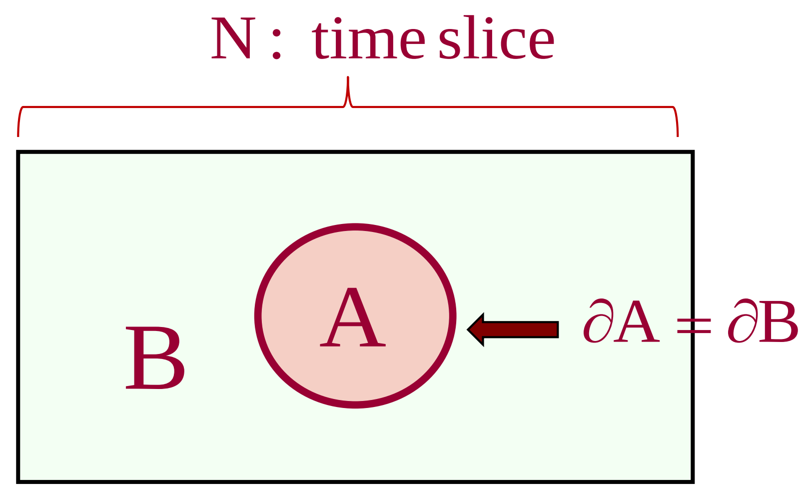
We can define the EE in QFTs by taking the continuum limit of the EE in quantum many-body systems.

Quantum Many-body Systems



Continuum Limit ε→0

Quantum Field Theories (QFTs)



$$H_{tot} = H_A \otimes H_B \ .$$

In gauge theories, since there is a gauss law constraint, the division into A and B is highly non-trivial. [Casini-Huerta 13,….]

In **QFTs**, the entanglement entropy (EE) provides us a **universal physical quantity (~order parameter)**.

For example, we can characterize the degrees of freedom of CFTs (~central charges) from the EE for ground states.

(i) 2d CFT

$$S_A = \frac{c}{3}\log\frac{l}{\varepsilon}.$$

[Holzhey-Larsen-Wilczek 94, Calabrese-Cardy 04,..]

(ii) 3d CFT

$$S_{A(=S^1)} = \gamma\cdot\frac{l}{\varepsilon} - F.$$

[F-th: Jafferis-Klebanov-Pufu-Safdi 11, Entropic proof: Casini-Huerta 12]

(iii) 4d CFT

$$S_{A(=S^2)} = \gamma\cdot\frac{l^2}{\varepsilon^2} - 4a\cdot\log\frac{l}{\varepsilon} + s.$$

[Ryu-TT 06, Solodukhin 08, Sinha-Myers 10, Casini-Huerta-Myers 11,...]

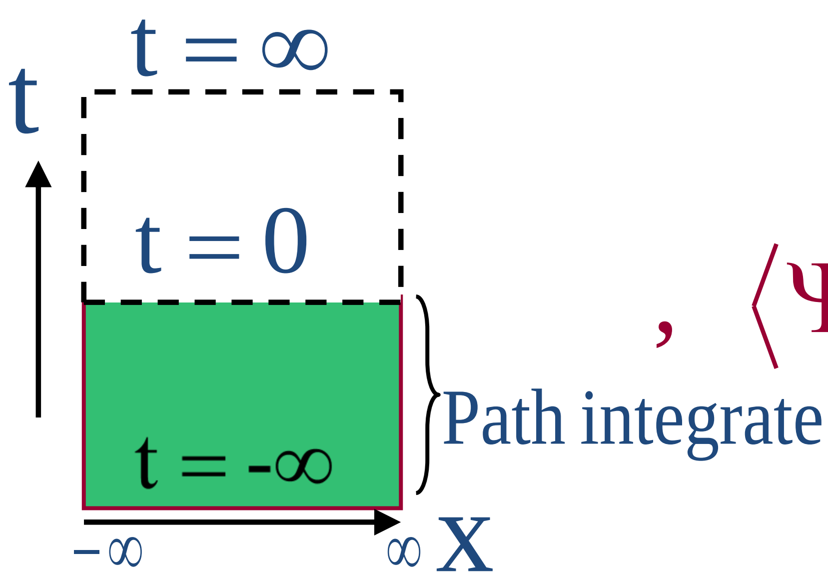
Replica method in QFT

A basic method to compute EE in QFTs is **replica method**.

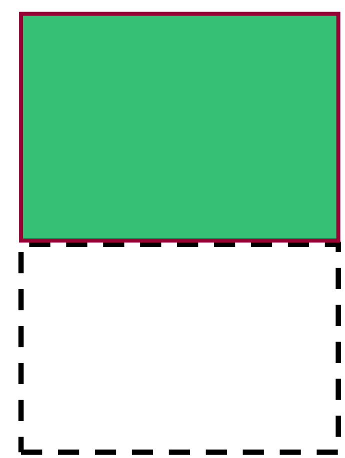
$$S_A = -\frac{\partial}{\partial n} \log \mathrm{Tr}_A (\rho_A)^n |_{n=1} \quad .$$

In the path-integral formalism, the ground state wave function $|\Psi\rangle$ (in 2d QFTs) can be expressed as follows:

$|\Psi\rangle =$

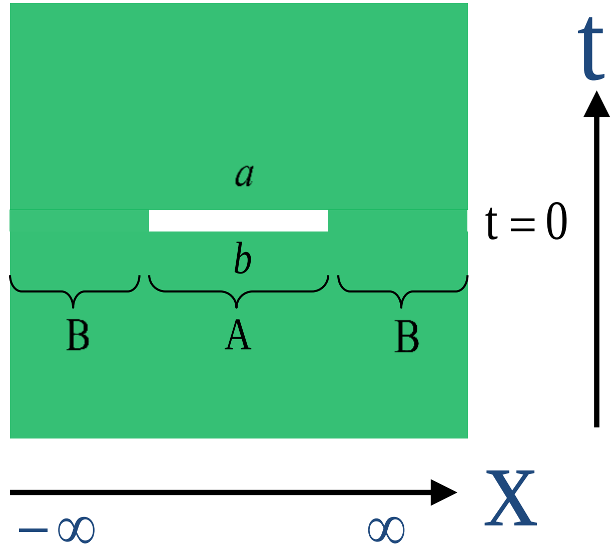


, $\langle\Psi| =$



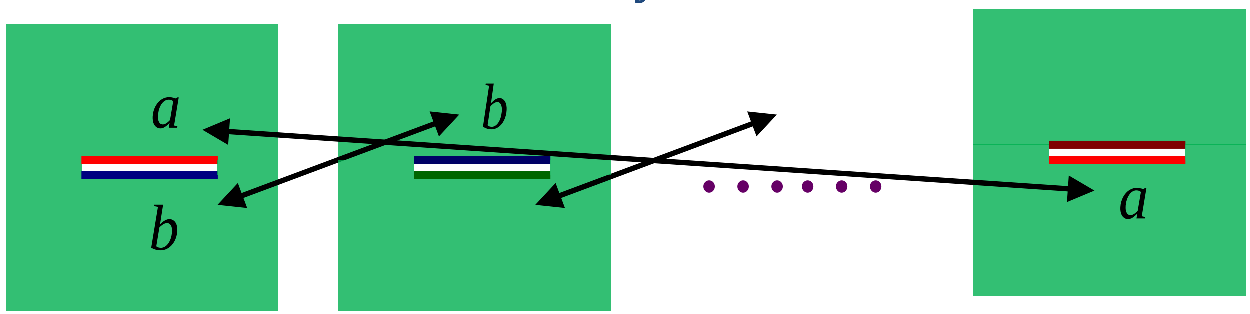
Then we can express

$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$ as follows: $[\rho_A]_{ab} =$



Glue each boundaries successively.

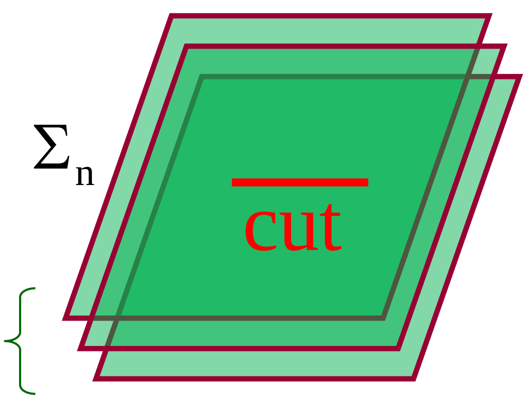
$\mathrm{Tr}\,(\rho_A)^n =$



$$= \frac{Z(\Sigma_n)}{Z(\Sigma_1)^n}.$$

n-sheeted

Riemann surface Σ_n



n sheets

Ex. **2d CFT** (A= an interval [u,v])

Twist operator

$$\mathrm{Tr}\left(\rho_A\right)^n = \frac{Z(\Sigma_n)}{(Z(\Sigma_1))^n} = \prod_{k=1}^{n} \left\langle \sigma_k(u)\bar{\sigma}_k(v) \right\rangle \propto (u-v)^{-\frac{c}{6}(n-1/n)}.$$

$$\Delta_{each\ sheet} = \frac{c}{24}\left(1-\frac{1}{n^2}\right)$$

In this way, we reproduced the EE in 2d CFT :

$$S_A = \frac{c}{3}\log\frac{l}{a} \qquad (l \equiv v-u).$$

[Holzhey-Larsen-Wilczek 94, Calabrese-Cardy 04,..]

Note: the UV cut off a is introduced such that $S_A = 0$ at $l = a$.

③ Holographic Entanglement Entropy

(3-1) AdS/CFT

AdS/CFT [Maldacena 97]

Quantum Gravity (String theory) on d+2 dim. AdS spacetime (anti de-Sitter space)	=	Conformal Field Theory (CFT) on d+1 dim. Minkowski spacetime
⬇ Classical limit		⬇ Large N limit, Strong coupling limit
General relativity with $\Lambda<0$ (Geometrical)		Strongly interacting quantum many-body systems

Basic Principle

(Bulk-Boundary relation) :

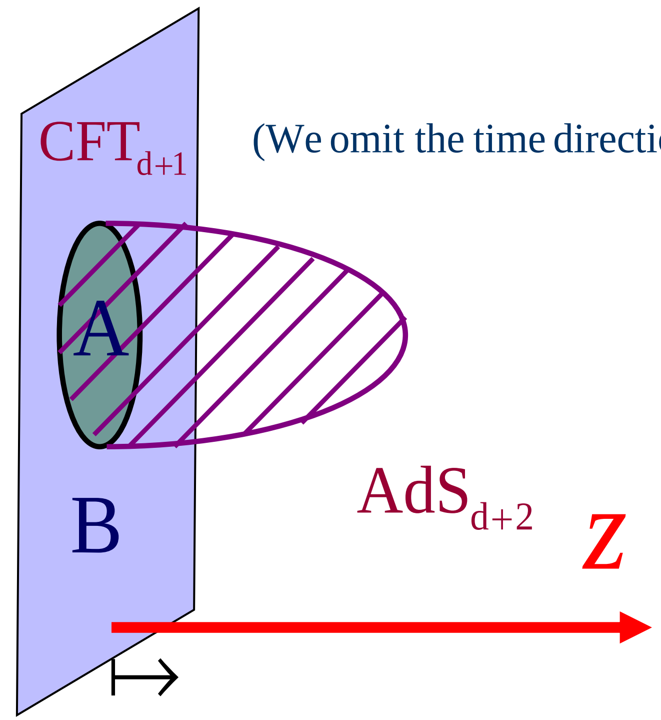
$$Z_{Gravity} = Z_{CFT}$$

(3-2) Holographic Entanglement Entropy Formula

[Ryu-TT 06]

$$S_A = \underset{\substack{\partial\gamma_A = \partial A \\ \gamma_A \approx A}}{\mathrm{Min}} \left[\frac{\mathrm{Area}(\gamma_A)}{4G_N} \right]$$

(We omit the time direction.)



$z > \varepsilon$ (UV cut off)

γ_A is the minimal area surface (codim.=2) such that

$$\partial A = \partial \gamma_A \ \text{ and } \ A \sim \gamma_A \ .$$

homologous

$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

Note: In time-dependent spacetimes, we need to take extremal surfaces.

[Hubeny-Rangamani-TT 07]

Verification of HEE

- Confirmations of basic properties:

Area law, Strong subadditivity (SSA), Conformal anomaly,….

- Direct Derivation of HEE from AdS/CFT:

(i) Pure AdS, A = a round sphere [Casini-Huerta-Myers 11]

(ii) Euclidean AdS/CFT [Lewkowycz-Maldacena 13, Faulkner 13, cf. Fursaev 06]

(iii) Disjoint Subsystems [Headrick 10, Faulkner 13, Hartman 13]

(iv) General time-dependent AdS/CFT → Not yet, but..

Hol. SSA [Evidences: Allais-Tonni 11, Callan-He-Headrick 12; A proof: Wall 13]

Causality [Headrick-Hubeny-Lawrence-Rangamani 14]

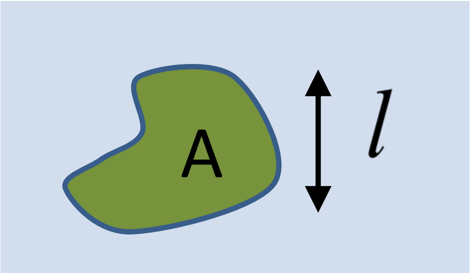
- Corrections to HEE beyond the supergravity limit:

[Higher derivatives: Hung-Myers-Smolkin 11, de Boer-Kulaxizi-Parnachev 11, , Fursaev-Patrushev-Solodukhin 13, Dong 13, Camps 13,… ; Camps' talk]

[1/N effect: Barrella-Dong-Hartnoll-Martin 13, Faulkner-Lewkowycz-Maldacena 13,..]

[Higher spin gravity: de Boer-Jottar 13, Ammon-Castro-Iqbal 13, Hijano-Kraus 14,..]

General Behavior of HEE [Ryu-TT 06]



$$S_A = \frac{\pi^{d/2} R^d}{2G_N^{(d+2)}\Gamma(d/2)}\left[p_1 \left(\frac{l}{\varepsilon}\right)^{d-1} + p_3 \left(\frac{l}{\varepsilon}\right)^{d-3} + \cdots \right.$$

$$\left. \cdots + \begin{cases} p_{d-1}\left(\frac{l}{\varepsilon}\right) + p_d & (\text{if } d+1 = \text{odd}) \\ p_{d-2}\left(\frac{l}{\varepsilon}\right)^2 + q\log\left(\frac{l}{\varepsilon}\right) & (\text{if } d+1 = \text{even}) \end{cases} \right],$$

Area law divergence

where $p_1 = (d-1)^{-1}, p_3 = -(d-2)/[2(d-3)],....$

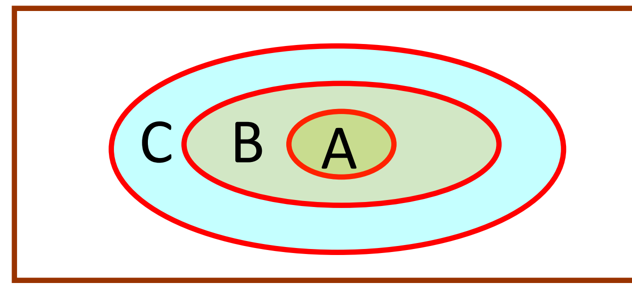
..... $q = (-1)^{(d-1)/2}(d-2)!!/(d-1)!!$.

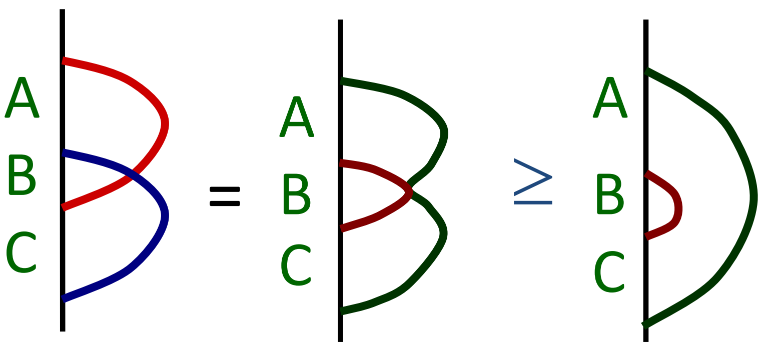
A universal quantity (**F**) which characterizes **odd dim. CFT**.
⇒A proof of c-theorem in 3 dim. (F-theorem). [Casini-Huerta 12, Liu-Mezei 12, Myers-Singh 12, …]

Agrees with **conformal anomaly** (central charge) in **even dim. CFT**
[Calabrese-Cardy 04, Solodukhin 08, Hung-Myers-Smolkin 11 …]

Holographic Strong Subadditivity [Headrick-TT 07]

We can easily derive the strong subadditivity, which is the most important inequality satisfied by EE. [Lieb-Ruskai 73]



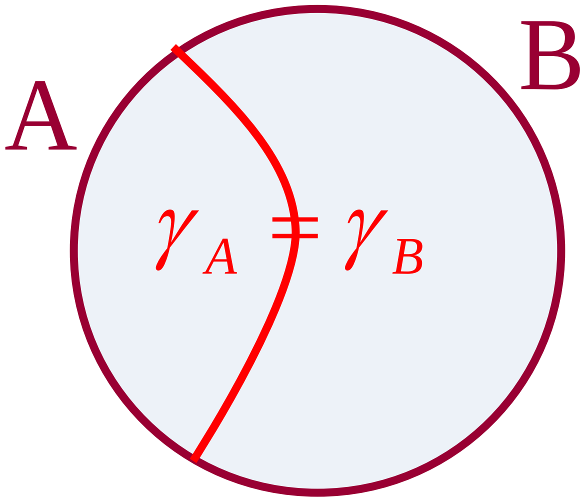


$$\Rightarrow S_{A+B} + S_{B+C} \geq S_{A+B+C} + S_B$$

Note: This proof can be applied if $S_A = \underset{\gamma_A}{\mathrm{Min}}\left[H(\gamma_A)\right]$, for any functional H.

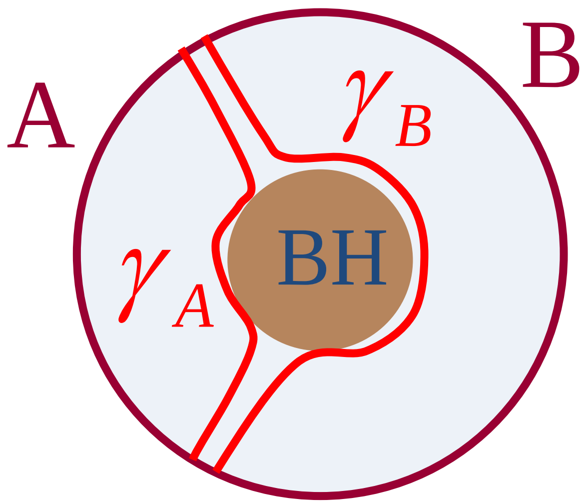
⇒ higher derivative corrections.

Pure VS Mixed State



(Pure) AdS $\Leftrightarrow$ Zero temp. CFT

ρ_{tot} is pure $\Leftrightarrow$ $S_A = S_B$.



AdS BH $\Leftrightarrow$ Finite temp. CFT

ρ_{tot} is not pure $\Leftrightarrow$ $S_A \neq S_B$.

Note: the HEE formula can be regarded as a generalization of Bekenstein-Hawking formula of black hole entropy:

$$S_{BH} = \frac{\text{Area of BH}}{4G_N}.$$

A Killing horizon (time independent black holes)

⇔ All components of extrinsic curvature are vanishing.

$$\cap$$

A minimal surface (or extremal surface)

⇔Traces of extrinsic curvature are vanishing.

Possible Generalizations of HEE

(i) Any closed surfaces in AdS → Differential entropy

[de Boer's talk; Balasubramanian-Chowdhury-Czech-de Boer-Heller 13,…]

(ii) Minimal surfaces which divide internal manifolds (eg.S^5).

$$\gamma_A = [AdS_5 \supset H_4\,(t=0)] \times [S^5 \supset S^4]$$

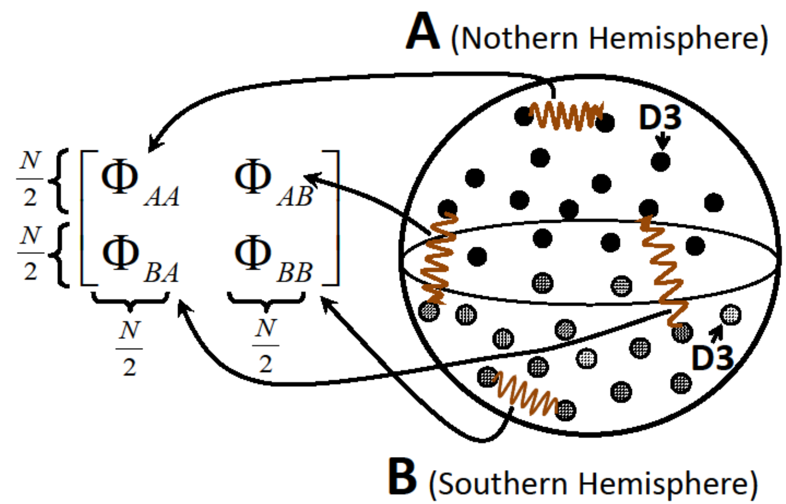
$\Rightarrow$ Area(γ_A)/4G_N = EE between two CFTs

$\sim N^2 \cdot l^d \cdot \varepsilon^{-d}$ [Volume law]

SU(N) Yang-Mills

⇒ SU(N/2) YM × SU(N/2) YM

[Mollabashi-Shiba-TT 14, Taylor's talk]



④ Entanglement Entropy for (locally) Excited States

The entanglement entropy is also a useful quantity to characterize **excited states**.

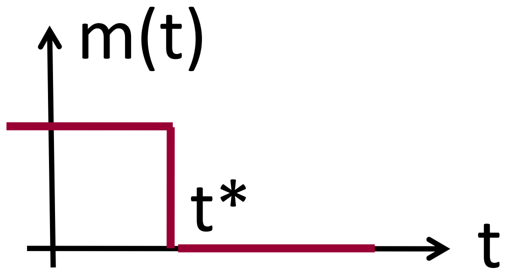
Instantaneous excitations

Well-studied examples are **quantum quenches**:

[CFT: Calabrese-Cardy 05, 07, ..; HEE: Arrastia-Aparicio-Lopez 10,...]

(a) Global quantum quenches in 2d CFTs

$$S_A \propto c \cdot (t/\varepsilon) \quad .$$



(b) Local quantum quenches in 2d CFTs

$$S_A^{2d} \propto c \cdot \log(t/\varepsilon) \, .$$

Here we want to focus on more elementary excited states (which give very instructive results):

(c) Local operator insertions at a time

⇒ Excited states are defined by local operators O(x):

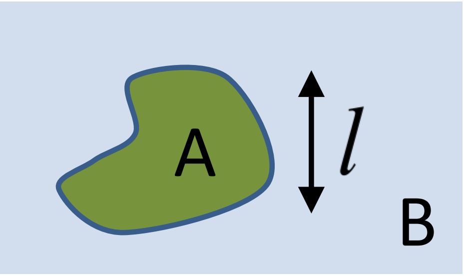
$$\left|O(x)\right\rangle \equiv O(x)\left|0\right\rangle.$$

We study

$$\Delta S_A^{(n)} \equiv S_A^{(n)}\left[\left|O(x)\right\rangle\right] - S_A^{(n)}\left[\left|0\right\rangle\right].$$

$\Delta S_A^{(n)}$ ~ `degrees of freedom' of operator O(x).

(4-1) Two limits of Subsystem A



[1] $l \to 0$ limit ($\approx$ small energy limit)

In this case, we find a property analogous to the first law of thermodynamics:

$$\Delta S_A^{(n)}\left[\,|O\rangle\right] \propto \Delta E_A$$

[Bhattacharya-Nozaki-Ugajin-TT 12, Blanco-Casini-Hung-Myers 13, Wong-Klich-Pando Zayas-Vaman 13]

[2] $l \to \infty$ limit ($\approx$ large energy limit)

This leads to a very `entropic' quantity !

⇒ We will choose this limit below.

[Nozaki-Numasawa-TT 14, He-Numasawa-Watanabe-TT 14, Caputa-Nozaki-TT 14]

(4-2) Replica Method for Excited States

We want to calculate $\mathrm{Tr}(\rho_A)^n$ for

$$\rho_A(t,x) = e^{-iHt}e^{-\varepsilon H}O(x)|0\rangle\langle 0|O(x)e^{-\varepsilon H}e^{iHt}$$
$$= O(\tau_e, x)|0\rangle\langle 0|O(\tau_l, x),$$
$$(\tau_e \equiv -\varepsilon - it, \qquad \tau_l \equiv -\varepsilon + it),$$

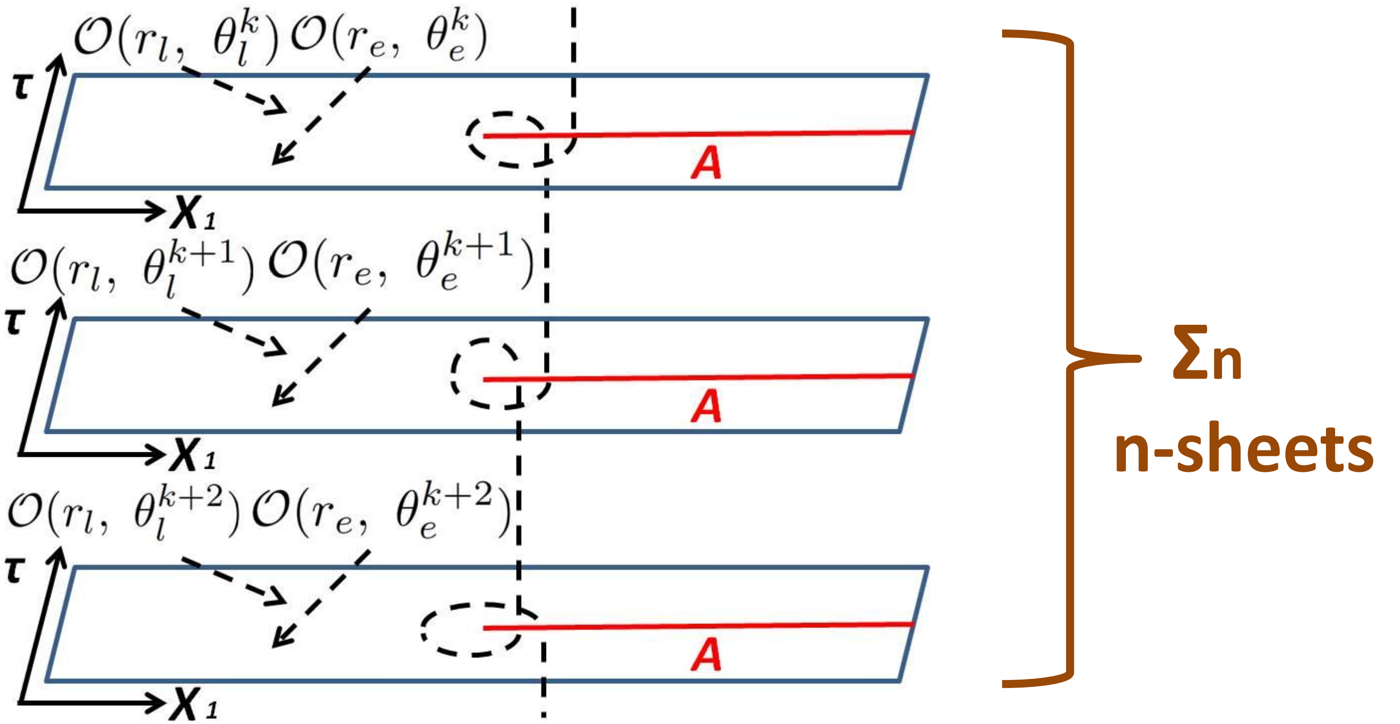
where ε is the UV regulator for the operator.

Here we consider a $d+1$ dim. CFT on R^{d+1}.

$(\tau, x_1, x_2, \cdots, x_d) \in \mathrm{R}^{d+1} \Rightarrow$ We set $x_1 + i\tau = re^{i\theta}$.

In this way, the Renyi EE can be expressed in terms of correlation functions (2n-point function etc.) on Σn :

$$\Delta S_A^{(n)} = \frac{1}{1-n} \cdot \Big[\log \left\langle O(r_l, \theta_l^n) O(r_e, \theta_e^n) \cdots O(r_l, \theta_l^1) O(r_e, \theta_e^1) \right\rangle_{\Sigma_n} - n \cdot \log \left\langle O(r_l, \theta_l) O(r_e, \theta_e) \right\rangle_{\Sigma_1} \Big].$$



(4-3) Free scalar CFTs in any dimensions

[Numasawa-Nozaki-TT 14]

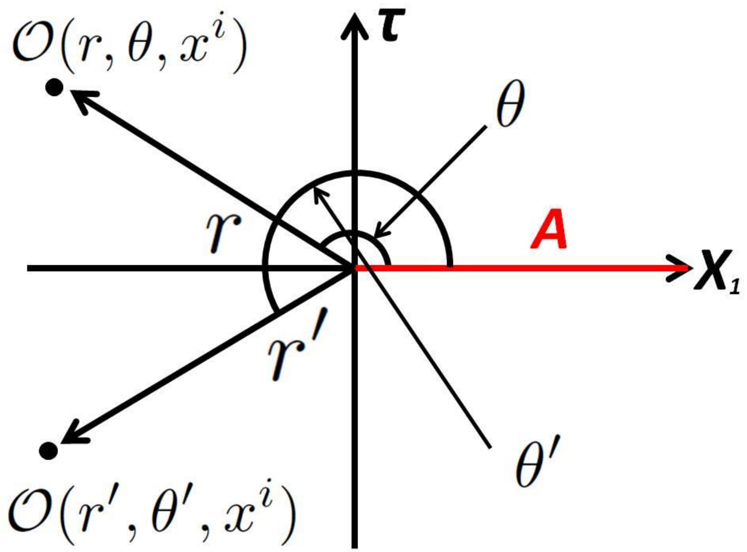
We focus on the free massless scalar field theory on Σn

$$S=\int d^{d+1}x\left[\partial_\mu\phi\partial^\mu\phi\right]$$

and calculate 2n-pt functions using the Green function:

$$G^{d=3}_{\Sigma n}[(r,\theta,\vec{x});(s,\varphi,\vec{y})]=\frac{1}{4n\pi^2 rs(a-1/a)}\cdot\frac{a^{1/n}-a^{-1/n}}{a^{1/n}+a^{-1/n}-2\cos((\theta-\varphi)/n)},$$

where $$\frac{a}{1+a^2}\equiv\frac{rs}{|\vec{x}-\vec{y}|^2+r^2+s^2}.$$



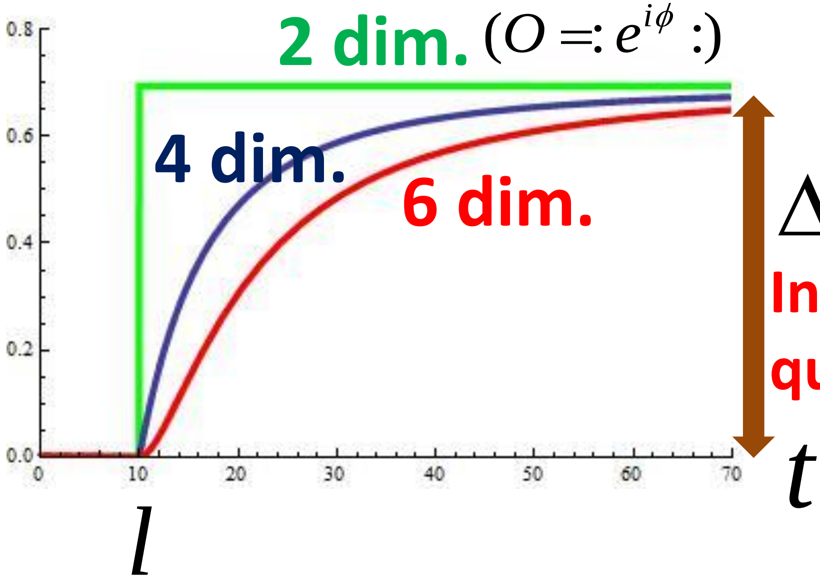
The operator O is chosen as

$$O_k=:\phi^k: \quad .$$

Time evolution in free massless scalar theory

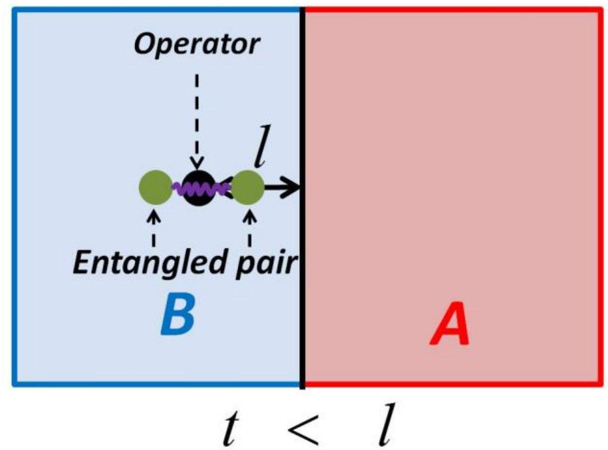
$\Delta S_A^{(2)}$ for $O =: \phi :$ (i.e. $k=1$)

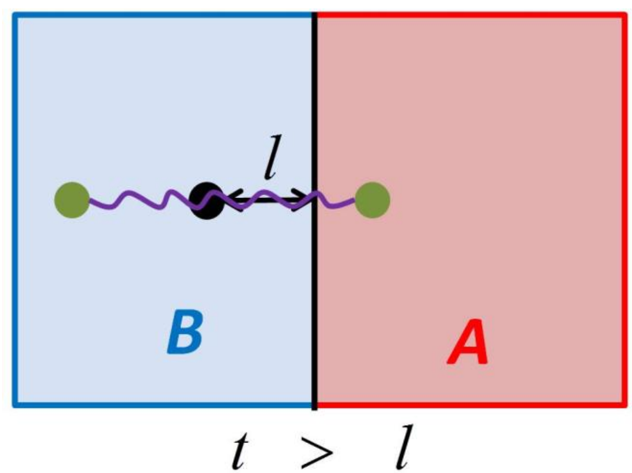
$\left(\begin{array}{l} \text{We chose } x_1 = -l \text{ with } l = 10 \\ \text{and } x_2 = \cdots = x_d = 0. \end{array} \right)$



$\Delta S_A^{(n)f}$

Interested quantities !





$$E.g. \quad \Delta S_{A(4\,\mathrm{dim})}^{(2)} = \log\left(\frac{2t^2}{t^2 + l^2}\right).$$

Note :

$\Delta S_A^{(n)f}$ is 'topologically invariant' under deformations of A.

$$\Delta S_A^{(n)f} \quad \text{for} \quad O=\phi^k \quad \text{in} \quad d+1>2 \text{ dim.}$$

TABLE I. $\Delta S_A^{(n)f}$ and $\Delta S_A^f \left(= \Delta S_A^{(1)f}\right)$ for free massless scalar field theories in dimensions higher than two ($d > 1$).

	n	$k=1$	$k=2$	$\cdots$	$k=l$
$\Delta S_A^{(n)f}$ Renyi Entropy	2	$\log 2$	$\log \frac{8}{3}$	$\cdots$	$-\log\left(\frac{1}{2^{2l}}\sum_{j=0}^{l}({}_lC_j)^2\right)$
	3	$\log 2$	$\frac{1}{2}\log\frac{32}{5}$	$\cdots$	$\frac{-1}{2}\log\left(\frac{1}{2^{3l}}\sum_{j=0}^{l}({}_lC_j)^3\right)$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	m	$\log 2$	$\frac{1}{m-1}\log\frac{2^{2m-1}}{2^{m-1}+1}$	$\cdots$	$\frac{1}{1-m}\log\left(\frac{1}{2^{ml}}\sum_{j=0}^{l}({}_lC_j)^m\right)$
ΔS_A^f EE	1	$\log 2$	$\frac{3}{2}\log 2$	$\cdots$	$l\log 2 - \frac{1}{2^l}\sum_{j=0}^{l}{}_lC_j\log{}_lC_j$

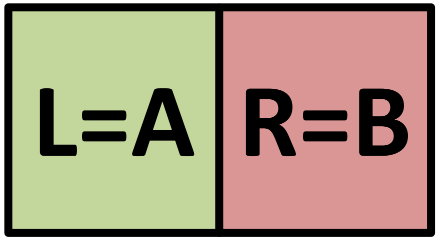
EPR state !

[For a proof: Nozaki 14]

Heuristic Explanation

First , notice that in free CFTs, there are definite (quasi) particles moving at the speed of light.

$$\Rightarrow \quad \phi \approx \underbrace{\phi_L}_{\text{left-moving}} + \underbrace{\phi_R}_{\text{right-moving}} \, .$$



$$\phi^k |\mathrm{vac}\rangle \approx \sum_{j=0}^{k} {}_kC_j \cdot (\phi_L)^j \cdot (\phi_R)^{k-j} |\mathrm{vac}\rangle$$

$$= 2^{-k/2} \sum_{j=0}^{k} \sqrt{{}_kC_j} \, |j\rangle_L |k-j\rangle_R .$$

$$\Rightarrow \quad \Delta S_A^{(n)f} = \frac{1}{1-n} \log\left[2^{-nk} \sum_{j=0}^{k} ({}_kC_j)^n\right],$$

$$\Delta S_A^{f} = k \log 2 - 2^{-k} \sum_{j=0}^{k} {}_kC_j \cdot \log[{}_kC_j].$$

Agree with replica Calculations !

(4-4) Rational 2d CFTs [He-Numasawa-Watanabe-TT 14]

We can prove the simple relation

$$\Delta S_A^{(n)f} = \log d_O$$

, where d_O is the quantity called **quantum dimension**.

[∵ n=2 → four point functions, described by the cross ratio $(z,\bar{z})$.
Time evolution = Chiral fusion transformation $(z,\bar{z}) \to (1-z,\bar{z})$]

(4-5) Free U(N) Yang-Mills at large N [Caputa-Nozaki-TT 14]

We choose $O(x) = \mathrm{Tr}[\Phi(x)^J]$. ($\Phi = N \times N$ Hermitian matrix scalar)

For example, when $J = 2$, we find the exact result:

$$\Delta S_A^{(n)f} = \frac{1}{1-n}\log\left[2^{1-2n} + \underbrace{2^{-n}\cdot N^{2(1-n)}}\right].$$

can be neglected **only if n>1**

In general, we find

$$\text{if } n>1 \Rightarrow \Delta S_A^{(n)f} = \frac{Jn-1}{n-1}\log 2 + O(N^{-2}).$$

$$\text{if } n=1 \Rightarrow \Delta S_A^{(1)f} = \frac{J}{2}\log N + O(N^{-2}).$$

Enhance at n=1

~deconfinement ?

(4-6) Holographic Results for locally excited states

$\Delta S_A^{(n>2)}$ in d dim CFTs ⇒ Holographic 2n-point function in (d+1) dim. topological AdS BH

Assuming 1<<Δ<<c we get: $\Delta S_A^{(n)} \approx \frac{4n\Delta_O}{d(n-1)} \log\left(\frac{t}{\varepsilon}\right)$.

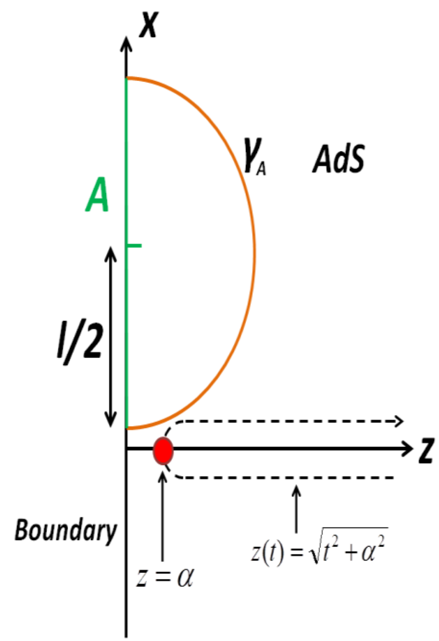
[Caputa-Nozaki-TT 14]

[This calculation is based on the \`naïve' large N limit.
Thus the n=1 limit and the late time limit t=∞ are not trustable.]

For n=1 (EE), we can employ the HEE to find $\Delta S_A^{(1)}$. [Nozaki-Numasawa-TT 13; 2d CFT derivation: Bernamonti's talk]

For 2d CFT (AdS_3/CFT_2),

$$\Delta S_A^{(1)} \approx \frac{c}{6} \log\left(\frac{t}{\varepsilon}\right) .$$



⑤ Entanglement Renormalization and AdS/CFT

(5-1) Tensor Network (TN) [See e.g. Cirac-Verstraete 09(review)]

Tensor network states

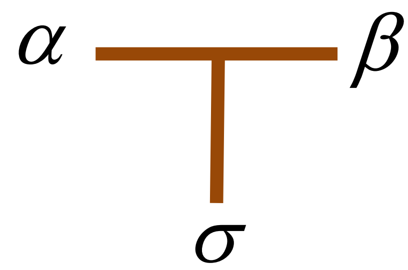
= Efficient variational ansatz for the ground state wave functions in quantum many-body systems.

[A tensor network diagram = A wave function]

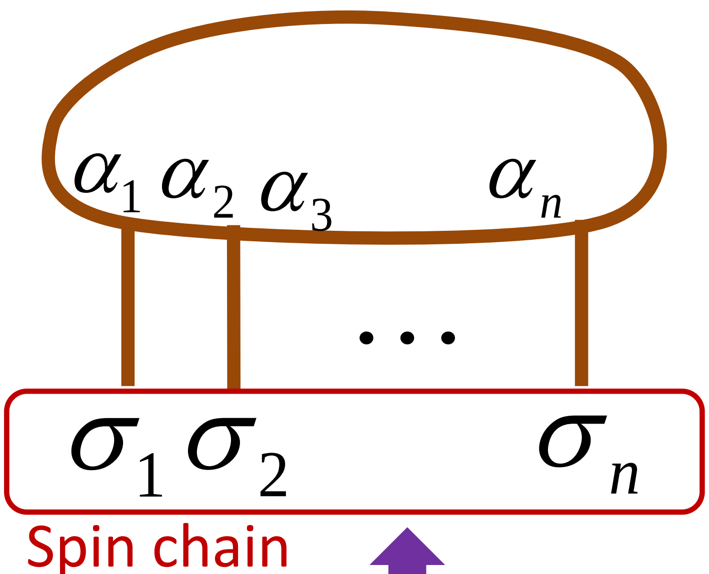
⇒ An ansatz is good if it respects the quantum entanglement of the true ground state.

Ex. Matrix Product State (MPS)

[DMRG: White 92,...,
Rommer-Ostlund 95,..]



$$\Leftrightarrow \quad M_{\alpha\beta}(\sigma)$$



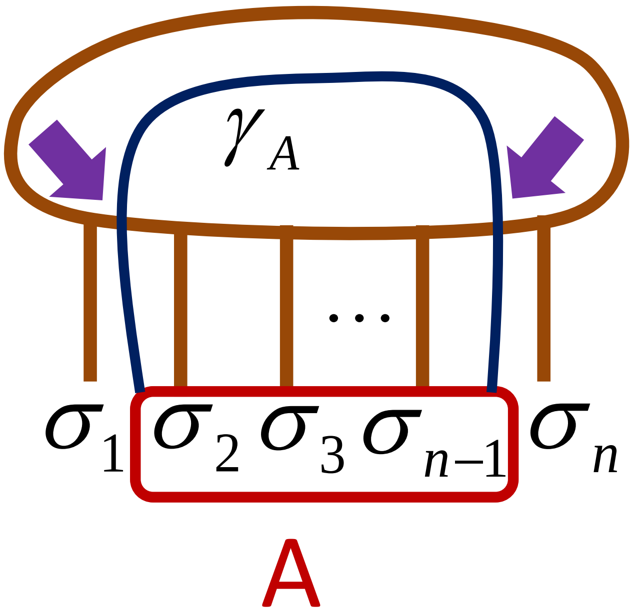
$$\alpha_i = 1, 2, \ldots, \chi,$$

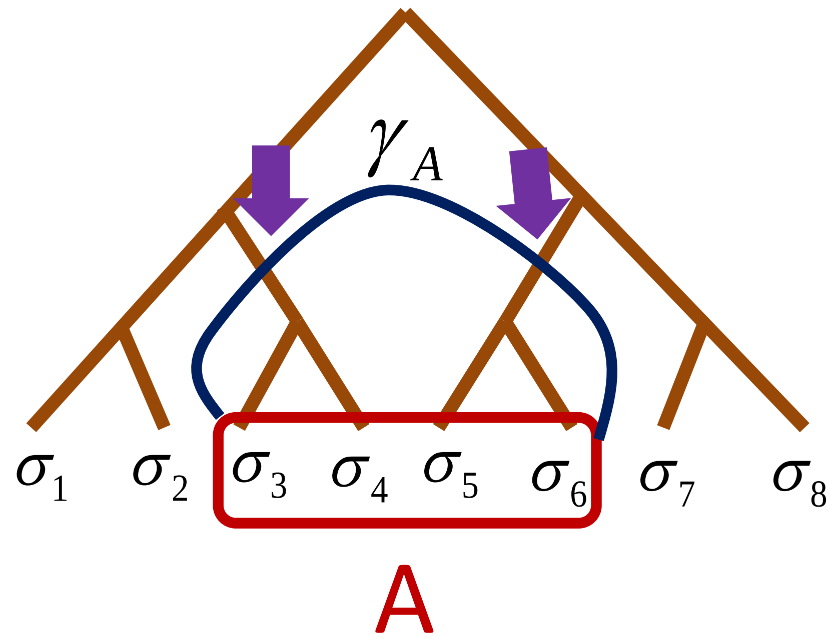
$$\sigma_i = \uparrow \text{ or } \downarrow .$$

$$|\Psi\rangle = \sum_{\sigma_1,\sigma_2,\cdots,\sigma_n} \mathrm{Tr}[M(\sigma_1)M(\sigma_2)\cdots M(\sigma_n)]\, \underbrace{|\sigma_1,\sigma_2,\cdots,\sigma_n\rangle}_{n\ Spins}$$

MPS and TTN are not good near quantum critical points (CFTs) because entanglement entropies are too small:

$$S_A \leq 2\log\chi \qquad (<< \log L \sim S_A^{CFT}).$$





In general,

$$S_A \sim N_{\text{int}} \cdot \log\chi,$$
$$N_{\text{int}} \equiv \min[\#\,\text{Intersections of } \gamma_A].$$

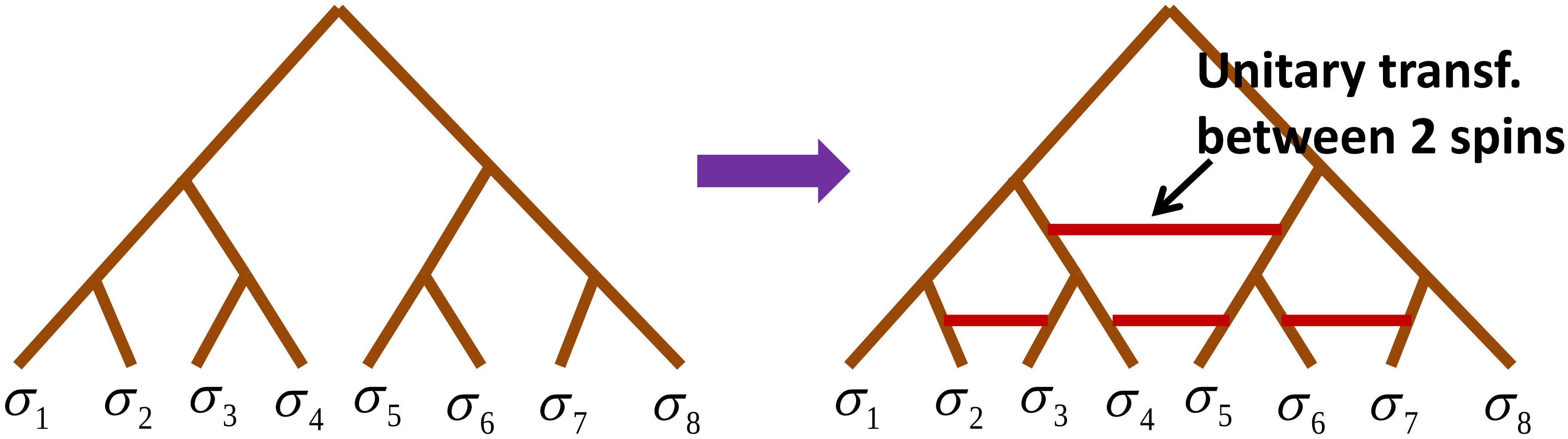
(5-2) AdS/CFT and MERA

MERA (Multiscale Entanglement Renormalization Ansatz):

⇒ An efficient variational ansatz for CFT ground states.

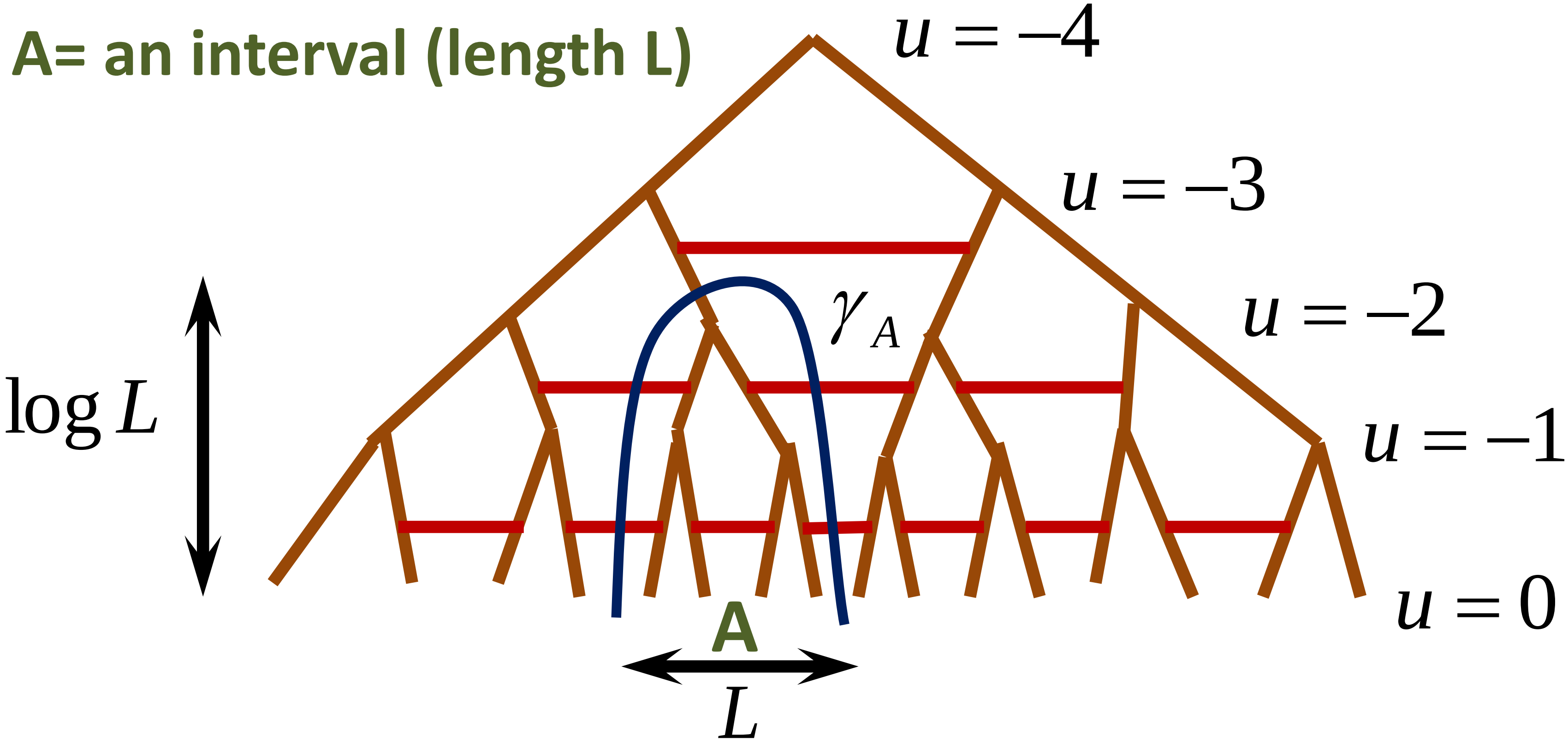
[Vidal 05]

To increase entanglement in a CFT, we add (dis)entanglers.



An Estimation of EE in 1+1 dim. MERA

A= an interval (length L)



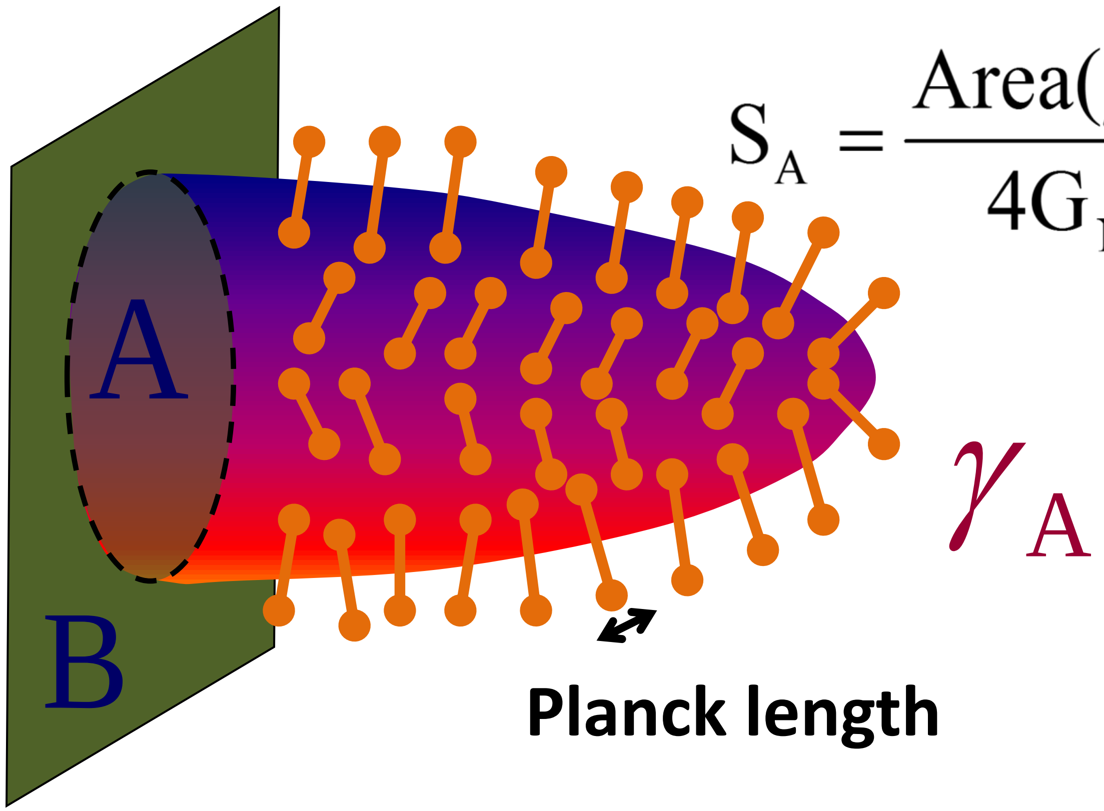
$$S_A \propto \mathrm{Min}[\#\,\mathrm{Bonds}] \propto \log L$$

$\Rightarrow$ agrees with 2d CFTs.

Indeed, the HEE also suggests that

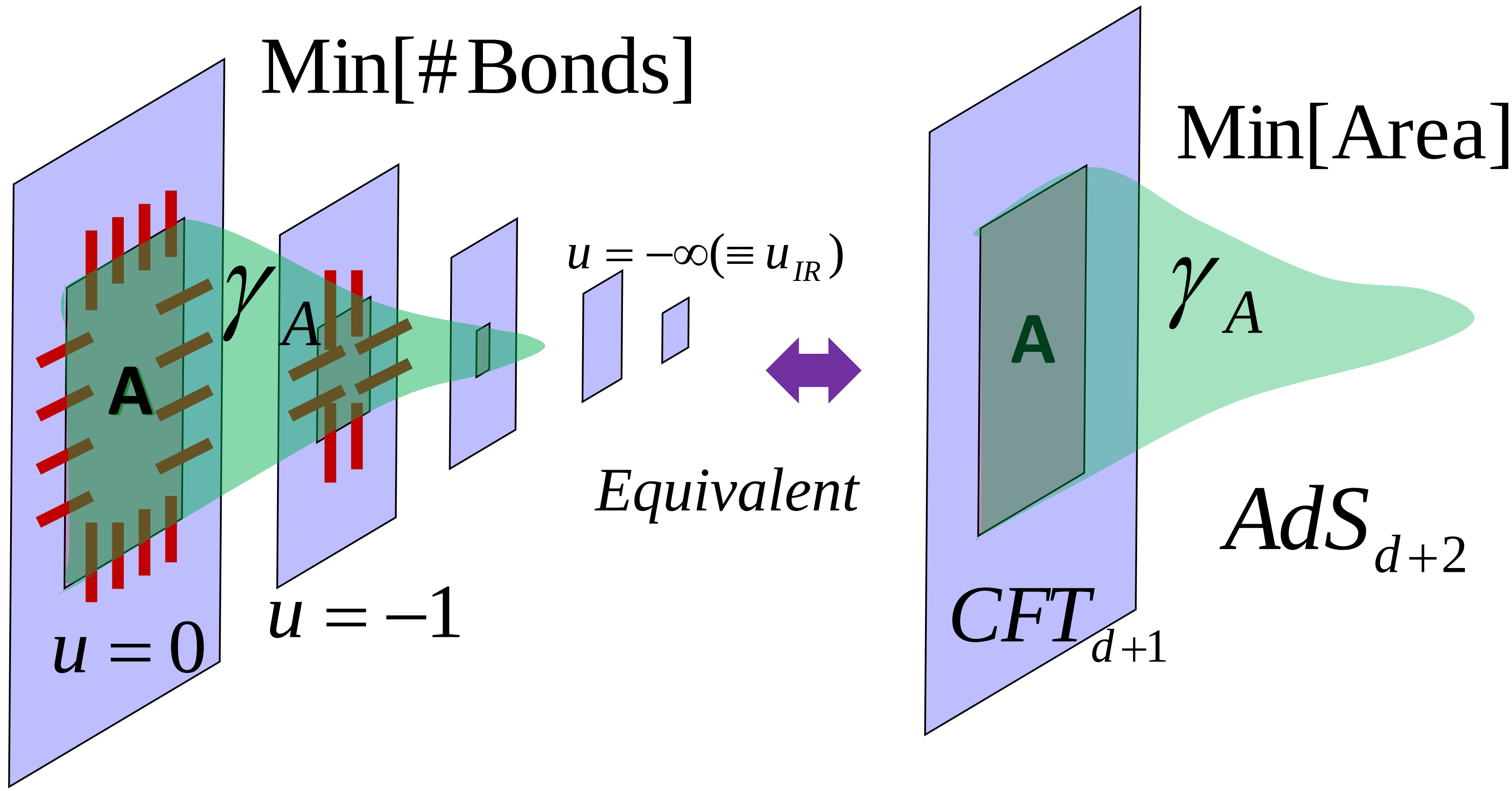
A spacetime in gravity
= Collections of bits of quantum entanglement

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N} \approx \frac{\text{Area}(\gamma_A)}{l_{pl}^2}.$$



A framework for this is the **entanglement renormalization**.

A conjectued relation to AdS/CFT [Swingle 09]



$$\text{Metric} = ds^2 + \frac{e^{2u}}{\varepsilon^2}(-dt^2 + d\vec{x}^2) = \frac{dz^2 - dt^2 + d\vec{x}^2}{z^2},$$

$$where \quad z = \varepsilon \cdot e^{-u} \quad .$$

(5-3) cMERA and Holographic Metric

To relate to the AdS/CFT, take a continuum limit of MERA:

Formulation of cMERA [Haegeman-Osborne-Verschelde-Verstraete 11]

$$\underbrace{|\Psi(u)\rangle}_{\text{State at scale u}} = P\cdot\exp\left(-i\int_{u_{IR}}^{u} ds[K(s)+L]\right)\cdot \underbrace{|\Omega\rangle}_{\text{IR state}},$$

with the UV cutoff $k \le \Lambda \ (=1/\varepsilon)$.

K(u) : disentangler, L: scale transformation

$|\Omega\rangle$: unentangled state in real space $\rightarrow S_A = 0$ for any A.

L : non-relativistic scale transformation (= coarse-graining)

s.t. $L|\Omega\rangle = 0.$

(5-4) Emergent Metric from cMERA

[Nozaki-Ryu-TT 12, Miyaji-TT work in progress]

We focus on gravity duals of translational invariant static states, which are not conformal in general.

A conjecture: holographic metric in the extra direction

$$\Rightarrow g_{uu} du^2 = N \cdot \left(1 - \left| \langle \Phi(u) | \Phi(u+du) \rangle \right|^2 \right).$$

$$N^{-1} \equiv \int dx^d \cdot \int_0^{\Lambda e^u} dk^d =$$ The total volume of phase space at energy scale u.

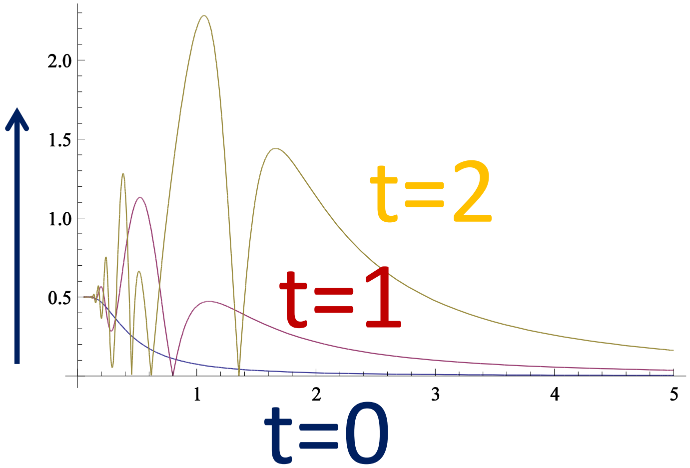
$(\Lambda = 1/\varepsilon : \text{cut off})$

$$ds^2_{Gravity} = g_{uu} du^2 + \frac{e^{2u}}{\varepsilon^2} \cdot d\vec{x}^2 - g_{tt} dt^2$$

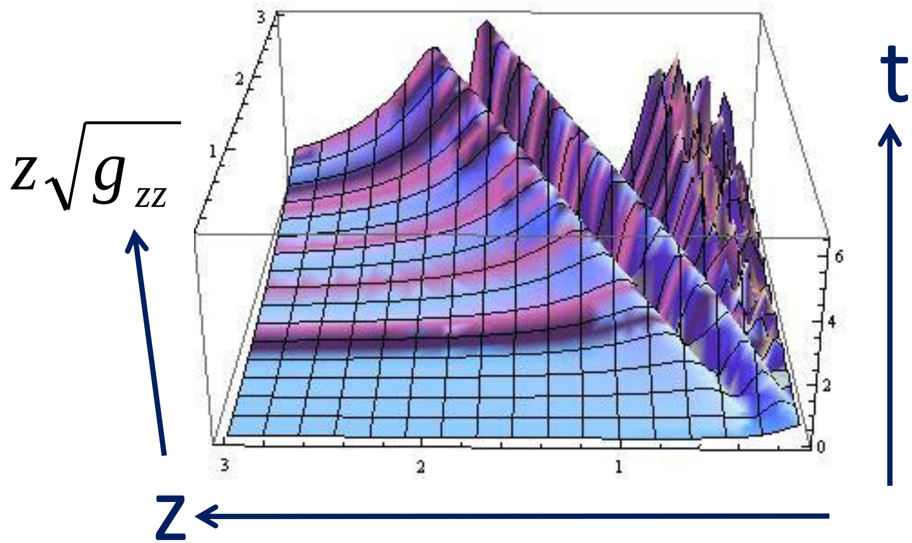
Ex. cMERA for a Quantum Quench of a free scalar

[Mollabashi-Nozaki-Ryu-TT 13]

$$z\sqrt{g_{zz}} = \chi(u,t)$$



Light cone: looks like a gravitational wave.



We can also (analytically) confirm the linear growth: SA∝t because guu∝t² at late time. This is also true in higher dim.

This is consistent with the known CFT (2d) [Calabrese-Cardy 05]. and with the holographic result (any d). [Hartman-Maldacena 13]

⑥ Conclusions

- EE for ground states of CFT describes the degrees of freedom, related to the central charges etc.

- EE for excited states describe how the system gets thermalized. If we consider locally excited states, the growth of EE can be interpreted as the degrees of freedom of the operator (=quantum dimension in 2d RCFT).

- HEE suggests a deep connection between quantum entanglement and spacetime geometry. This speculation gets manifest in the idea of entanglement renormalization (MERA).

Future Problems

- Precise definition of EE dual to HEE for extremal surfaces which divide internal manifolds. (Democratic formulation of HEE ?)

  [cf. Taylor's talk]

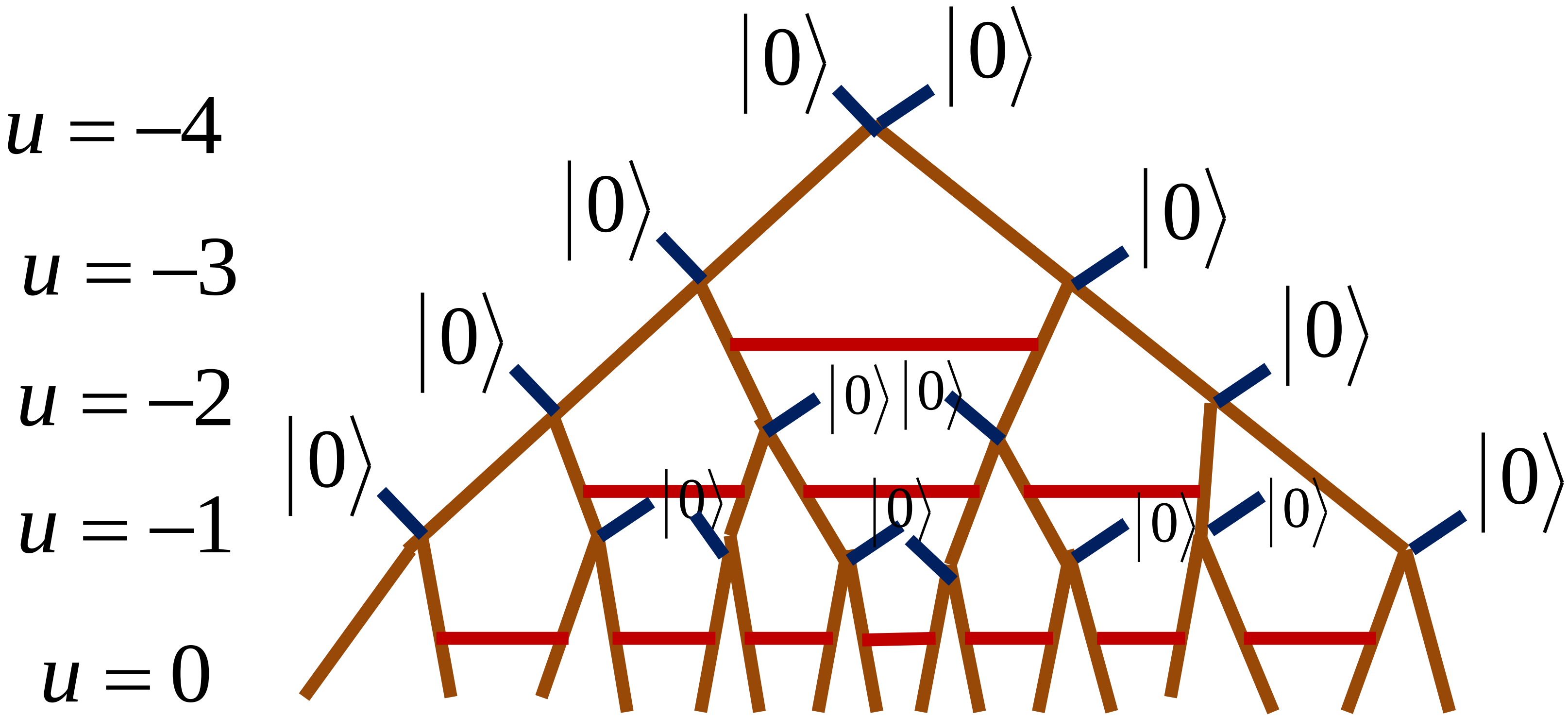
- More analysis of EE for locally excited states in higher dim. (Quantum dimension in higher dimensions ?)

- More understandings of the connection between MERA and AdS/CFT. (Construction of full metric ? , Einstein eq. ? ,....)

  [cf. Perturbative Einstein eq. from 1st law of EE: Lashkari-McDermott-Raamsdonk 13,....]

:

Appendix: Relation to (discrete) MERA



By adding dummy states, we keep the dimension of Hilbert space for any u to be the same.

⇒ We can formally describe the renormalization by a unitary transformation.

It is useful to introduce a `unrescaled state' $|\Phi(u)\rangle$:

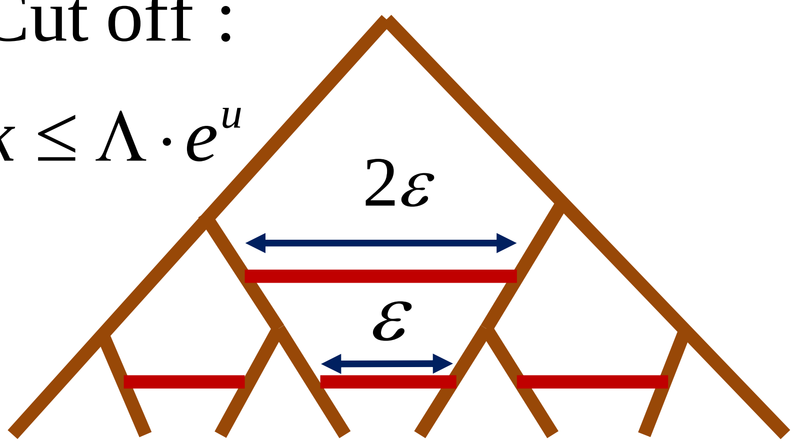
$$|\Phi(u)\rangle \equiv e^{iuL}|\Psi(u)\rangle = P \cdot e^{-i\int_{u_{IR}}^{u} \hat{K}(s)ds}|\Omega\rangle,$$

where $\hat{K}(u) \equiv e^{iuL} K(u) e^{-iuL}$ (= rescaled disentangler).

(i) Interpretation of $|\Phi(u)\rangle$

Cut off :

$k \leq \Lambda \cdot e^{u}$



(ii) Interpretation of $|\Psi(u)\rangle$

Cut off :

$k \leq \Lambda$