Cardy Formulae in SUSY Theories

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1407.6061 with Zohar Komargodski

Introduction

- Recently: exact results on path integral of SUSY theories on curved spaces (localization)
- What are the model-independent properties? Any universal behavior?
- Anomalies

Introduction

- Generalization of Witten index: $\mathrm{Tr}_{\mathcal{H}(\mathcal{M}_3)}((-1)^F e^{-\beta(H-\mu_a Q_a)})$ counting of short rep
- Path integral on $\mathcal{M}_3 \times S^1_\beta$ with periodic fermions

$$Z_{\mathcal{M}_3\times S^1}(\beta,\mu_a)$$

Introduction

- Superconformal case

$$\mathcal{H}(S^3) \equiv \{\mathcal{O}\}$$

$$Z_{S^3 \times S^1} = \sum_{\mathcal{O}} (-1)^F e^{-\beta(\Delta + 1/2R)}$$

- Superconformal index: counts BPS operators

Kinney et al; Romelsberger

Result

- Universal formula in the limit $\beta \to 0$

$$Z_{\mathcal{M}_3 \times S^1} \underset{\beta \to 0}{\simeq} e^{\left[-\frac{\pi^2 L_{\mathcal{M}_3}}{\beta}\mathrm{Tr}R - \frac{\pi^2 L'_{\mathcal{M}_3,a}}{\beta}\mathrm{Tr}Q_a\right]}$$

Asymptotics fixed by gravitational anomalies

Result

- Superconformal case

$$\mathrm{Tr} R \sim a - c$$

$$\sum_{\mathcal{O}} (-1)^F e^{-\beta(\Delta + \frac{R}{2})} \underset{\beta \to 0}{\simeq} e^{-\frac{16\pi^2}{3}(a-c)\frac{r_3}{\beta}}$$

Similar to Cardy formula in 2d CFT

$$Z_{T^2} = \sum_{\mathcal{O}} e^{-\beta\Delta} \underset{\beta \to 0}{\simeq} e^{-\frac{\pi^2}{3} c \frac{r_1}{\beta}}$$

Derivation

- QFT with a $U(1)$ symmetry: couple to bkgd metric and gauge field

$$g_{\mu\nu}dX^\mu dX^\nu = (dX^4 + a_i dx^i)^2 + h_{ij}dx^i dx^j$$
$$A_\mu dX^\mu = (dX^4 + a_i dx^i)A_4 + \mathcal{A}_i dx^i$$

$$Z_{\mathcal{M}_3\times S^1}[g_{\mu\nu}, A_\mu] \underset{\beta\to 0}{\simeq} Z_{\mathcal{M}_3}[a_i, h_{ij}, A_4, \mathcal{A}_i]$$

Derivation

- Thermal case: tower of m.ve KK states (gap $\sim \beta^{-1}$)
- $\log Z_{\mathcal{M}_3}$ is a local functional
- Classify possible terms order by order

Banerjee, Bhattacharya, Bhattacharyya, Jain, Minwalla, Sharma; Jensen, Loganayagam, Yarom

Derivation

$$\log Z_{\mathcal{M}_3} = \int d^3x\sqrt{h}\,\Big[\beta^{-3}c_0$$

$$+\beta^{-1}\left(c_1\,\mathcal{R} + c_2\,\epsilon^{ijk}\mathcal{A}_i\partial_j a_k + c_3\,f_{ij}f^{ij}\right)\Big]$$

$+$ subleading

$c_0 \longrightarrow$ not universal

Gubser, Klebanov, Tseytlin

Derivation

$$\log Z_{\mathcal{M}_3} = \int d^3x\sqrt{h}\,\big[\beta^{-3}c_0$$

$$+\beta^{-1}\left(c_1\,\mathcal{R} + c_2\,\epsilon^{ijk}\mathcal{A}_i\partial_j a_k + c_3\,f_{ij}f^{ij}\right)\big]$$

$+$ subleading

$c_2 \longrightarrow$ fixed by $\mathrm{Tr}U(1)$ anomaly!

Landsteiner, Megias, Melgar, Pena-Benitez

Derivation

$$\log Z_{\mathcal{M}_3} = \int d^3x\sqrt{h}\,[\beta^{-3} c_0$$

$$+\beta^{-1}\left(c_1\,\mathcal{R} + c_2\,\epsilon^{ijk}\mathcal{A}_i\partial_j a_k + c_3\,f_{ij}f^{ij}\right)]$$

+ subleading

SUSY: take $U(1) \to U(1)_R$

- Key to universality: existence of $\mathcal{A} \wedge da$ + SUSY

SUSY Actions

$$-\log Z \underset{\beta\to 0}{\simeq} \frac{\pi^2 L_{\mathcal{M}_3}}{\beta}\mathrm{Tr}R + \frac{\pi^2 L'_{\mathcal{M}_3,a}}{\beta}\mathrm{Tr}Q_a$$

- $(h_{ij}, \mathcal{A}_i^{(R)}, a_i, H)$

  3d supergravity multiplet

- SUSY EH term Kuzenko et al.

$$L_{\mathcal{M}_3} = \frac{1}{24\pi^2}\int_{\mathcal{M}_3} d^3x\sqrt{h}\left(\frac{1}{2}\mathcal{R} - H^2 - \frac{1}{2}f_{ij}f^{ij} - 2i\epsilon^{ijk}\mathcal{A}_i^{(R)}\partial_j a_k\right)$$

SUSY Actions

$$-\log Z \underset{\beta\to 0}{\simeq} \frac{\pi^2 L_{\mathcal{M}_3}}{\beta}\mathrm{Tr}R + \frac{\pi^2 L'_{\mathcal{M}_3,a}}{\beta}\mathrm{Tr}Q_a$$

- $(\mathcal{A}_i, \sigma, D)$

  3d vector multiplet

- FI term Closset et al.

$$L'_{\mathcal{M}_3,a} = \frac{1}{12\pi^2}\int_{\mathcal{M}_3} d^3x\sqrt{h}\Big(D^{(a)} + i\epsilon_{ijk}\mathcal{A}^{(a)}\partial_j a_k - \sigma^{(a)} H\Big)$$

Outlook

- Higher order terms in β : other anomalies, universality of Casimir energy terms
- Generalization to 6d: find the off-shell SUSY actions
- Thermal FT: non-pert. argument for $A \wedge da$ Yarom et al.

Thank you!