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New perspectives on black holes in gauged Supergravity

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based on work in collaboration with
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Outline

1. Supersymmetry: BPS flow
2. Finite temperature: non extremal first order flow
3. Holographic analysis and mass in AdS

Motivations: Black holes in Supergravity

Black holes as a Quantum Gravity lab

- Microscopic origin of BH entropy
- Holographic dual theories

Two main areas of investigations

1. String/M-theory origin of SUGRA black holes in asymptotically flat space
2. Black holes in Anti de Sitter space

Motivations: Gauged Supergravity

Holography requires to go beyond Minkowski asymptotics

- The gauging procedure introduces a potential that behaves as a cosmological constant

$$\mathcal{L}_g = \int d^4x\sqrt{-g}\ V(\varphi) \neq 0$$

of which AdS_4 is a vacuum at spatial infinity

$$\partial_\varphi V|_\infty = 0$$

Extend the analysis to black holes with general asymptotics

- Gauged Supergravity is a theory of charged particles: $\psi_\mu^A \sim (\widetilde{g}^\Lambda\ , g_\Lambda)$
- What's the String/M-theory interpretation of a black hole in a Minkowski vacuum of a gauged SUGRA?

AdS black holes in Supersymmetric theories

Duff & Liu, Nucl.Phys. B554 (1999)
Cvetic et al. Nucl.Phys. B558 (1999)
Embedding in String Theory/M-theory

$\mathcal{N}=8$ Supergravity contains the *stu* model as an $\mathcal{N}=2$ subsector.
Truncation of the gauge group

$$SO(8) \qquad \rightarrow \qquad U(1)^4$$

Scalars couple to the vector field strengths

$$S=\int d^4x\left(-\frac{R}{2}+g_{i\bar{\jmath}}\partial_\mu z^i\partial^\mu\bar{z}^{\bar{\jmath}}+\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^\Lambda_{\mu\nu}F^{\Lambda\,\mu\nu}+\right.$$
$$\left.+\frac{1}{2\sqrt{-g}}\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}\epsilon^{\mu\nu\rho\sigma}F^\Lambda_{\mu\nu}F^\Sigma_{\rho\sigma}-V_g\right)$$

scalar potential

$$V=-4g^2\left(\cosh\phi^{(12)}+\cosh\phi^{(13)}+\cosh\phi^{(14)}\right)$$

Supersymmetric solutions with spherical horizons have naked singularities

Finite horizon Supersymmetric solutions

Stationary solutions of the bosonic action of $\mathcal{N} \geq 2$ Supergravity

Extremal solutions $\Leftrightarrow \quad T = 0$

- Bertotti-Robinson near horizon geometry $AdS_2 \times S^2$
- Solve BPS equations in a zero fermions, zero hypermultiplets background

$$\delta_\epsilon \psi_\mu^A = 0 \;, \qquad \delta_\epsilon \lambda_A^i = 0 \;.$$

- Attractor equations at the horizon are algebraic constraint on the scalars

  [Ferrara-Gibbons-Kallosh-Strominger,'96, AG-Dall'Agata 2010]

$$\partial_i V|_h = 0 \qquad \Leftrightarrow (z^i)'|_h = 0 \;,$$

- Enhancement of Supersymmetry at the horizon and at asymptotic infinity

Finite horizon Supersymmetric solutions

[Cacciatori-Klemm, 2009]

- Gaugingof the abelian factor

$$U(1)_R \subset SU(2)_R \subseteq \mathcal{QM} \qquad\qquad \mathcal{L}_{g=0} \to \mathcal{L}_{g=0} + g^2 V(z, \bar{z}, \xi_\Lambda)$$

- What changes with gauging

$$\begin{array}{ccc} AdS_2 \times S^2 & \Longleftrightarrow & AlAdS_4 \\ \to \quad D_i \mathcal{Z} = 0 & & \to \quad \partial_i V = 0 \end{array}$$

- Magnetic BPS black hole p^Λ, electrically charged gravitini $g_\Lambda = g\xi_\Lambda$.
- Double attractors, at ∞ and at the horizon
- $R_{AdS_2} \neq R_{S^2}$ the supersymmetry at the horizon is only $\mathcal{N} = 1$
  [de Wit-Van Zalk, '11]
- mAdS: also at infinity only partial enhancement of SUSY to $\mathcal{N} = 1$
  [Hristov,Toldo,Vandoren, '11]
- The BPS flow is 1/4-BPS

Finite horizon Supersymmetric solutions

[Cacciatori-Klemm, 2009]

The metric ansatz

$$ds^2 = -e^K f(r)dt^2 + e^{-K}\left(\frac{dr^2}{f(r)} + r^2 d\Omega^2\right)$$

$$r^2 f(r) = (r^2 - r_h^2)^2 \qquad e^{-K} = \sqrt{H^0(H^1)^3}$$

The problem is still solved by harmonic functions $H^i = 1 + \frac{Q^i}{r}$, the scalar fields are

$$z^i = z^i_\infty \frac{H^i}{H^0}\ , \qquad \partial_i V(z^i, \xi_\Lambda)\big|_\infty = 0$$

for magnetic gauge fields $A^\Lambda = p^\Lambda \cos\theta d\phi\ ,\ \ p^\Lambda g_\Lambda = \kappa$.

- Magnetic BPS black hole p^Λ, electrically charged gravitini $g_\Lambda = g\xi_\Lambda$.

First order BPS flow

In terms of the combinations of warp factors

$$e^{\psi} = r\sqrt{f(r)}\ , \qquad e^{-2U} = \sqrt{H^0 H^1 H^2 H^3}/f(r)$$

BPS flow equations

$$\begin{aligned} U' &= -e^{U-2\psi}\mathrm{Re}(e^{-i\alpha}\mathcal{Z}) + e^{-U}\mathrm{Im}(e^{-i\alpha}\mathcal{L}) \\ \psi' &= 2e^{-U}\mathrm{Im}(e^{-i\alpha}\mathcal{L}) \\ \dot{z}^i &= -e^{i\alpha}g^{i\bar{\jmath}}(e^{U-2\psi}\overline{D_{\bar{\jmath}}\mathcal{Z}} + ie^{-U}\overline{D_{\bar{\jmath}}\mathcal{L}}) \end{aligned}$$

- For Fayet-Iliopoulos gauging define a symplectic scalar function, analogous to the central charge

$$\mathcal{Z} \equiv \langle Q, \mathcal{V}\rangle = (L^{\Lambda}q_{\Lambda} - M_{\Lambda}p^{\Lambda}) \qquad V_{BH} = |D\mathcal{Z}|^2 + |\mathcal{Z}|^2$$

$$\mathcal{L} = \langle \mathcal{G}, \mathcal{V}\rangle = (L^{\Lambda}g_{\Lambda} - M_{\Lambda}\widetilde{g}^{\Lambda}) \qquad V_g = g^{i\bar{\jmath}}D_i\mathcal{L}D_{\bar{\jmath}}\mathcal{L} - 3|\mathcal{L}|^2$$

- BPS first order flow is driven by the superpotential

$$\mathcal{W} = e^{U}|\mathcal{Z} - ie^{2(\psi-U)}\mathcal{L}|$$

Finite temperature black holes

[Klemm, Vaughan '12, Toldo, Vandoren, '12]

Fix a frame of electric gaugings, consider the t^3 truncation of *stu* model with prepotential $F = -2i\sqrt{X^0(X^1)^3}$. Simply deform the perfect square in $f(r)$ as

$$f(r) = 1 + \frac{c_1}{r} + \frac{c_2}{r^2} + r^2 \ell_{AdS}^{-2} e^{-2K} \ ,$$

$$H_\Lambda = 1 + \frac{Q_\Lambda}{r} \ , \qquad \ell_{AdS} = \left(g \frac{\sqrt{2}\xi_0^{1/4}\xi_1^{3/4}}{3^{3/4}} \right)^{-1}$$

$$z^{i'} = -\frac{e^{K/2}}{r} g^{ij} \partial_j W$$
$$(r\, e^{-K/2})' = W$$

for real scalar fields supplemented by a Hamiltonian constraint fixing the parameters c_1, c_2 in terms of black hole and gravitini charges.

Superpotential and duality

The defining equation for the superpotential

$$V_g(z) = \left(\partial_i W \partial_{\bar{\jmath}} W \, g^{i\bar{\jmath}} - 3W^2\right)$$

- Electric: q_Λ, g_Λ

$$W_{el} = |\mathcal{L}| = g_\Lambda L^\Lambda$$

- Magnetic: p^Λ, g_Λ

$$W_{mag} = |g_\Lambda \mathcal{I}_\infty^{\Lambda\Sigma} M_\Sigma|$$

The symplectic transformation $S \in Sp(4, \mathbb{R})$

$$S = \begin{pmatrix} 0 & -\mathcal{I}_\infty^{\Lambda\Sigma} \\ \mathcal{I}_{\infty\,\Lambda\Sigma} & \end{pmatrix}$$

generates a duality transformation on the symplectic sections $\mathcal{V} \to S\mathcal{V}$. The rotation $g_\Lambda \to \mathcal{I}_\infty^{\Lambda\Sigma} g_\Sigma$ leaves the action invariant but not the SUSY equations

$$\delta\psi_{\mu\,A} = D_\mu \epsilon_A + \varepsilon_{AB}\, T^-_{\mu\nu}\, \gamma^\nu\, \epsilon^B + \frac{i}{2}\, \mathcal{L}\, \delta_{AB}\, \gamma^\nu\, \eta_{\mu\nu}\, \epsilon^B$$

BPS limit

Solution with magnetic charges admits an extremal BPS limit, when the function $f(r)$ has a double pole

$$r^2 f_0(r) = \frac{\beta^2}{\ell_{AdS^2}}(r^2 - r_h^2)^2$$

plus additional restrictions on the parameters c_1 and c_2. The 1/4-BPS solution satisfies the Supersymmetric first order upon identification of the warp factors

$$e^{\psi} = r\sqrt{f_0(r)}\ , \qquad e^{U} = e^{K/2}\sqrt{f_0(r)}$$

driven by the superpotential

$$\mathcal{W} = e^{U}|\mathcal{Z} - ie^{2(\psi-U)}\mathcal{L}|$$

On-shell, the magnetic superpotential and the BPS one are identical functions of r, as expected

$$e^{-\psi(r)}\mathcal{W}(r) \equiv -\beta\ell_{AdS}W_{mag}(r)\ .$$

BPS black holes or supersymmetric domain walls ?

BPS black branes to DW

Supersymmetric black brane

$$p^\Lambda g_\Lambda = 0 \ ,$$

the $p^\Lambda \to 0$ limit is well defined, and independent on g_Λ. The first order flow then reduces to

$$\begin{aligned} U'(r) &= e^{-U}\mathrm{Im}(e^{-i\alpha}\mathcal{L}) \ , \\ \psi'(r) &= 2e^{-U}\mathrm{Im}(e^{-i\alpha}\mathcal{L}) \ , \\ \dot{z}^i &= ie^{i\alpha}g^{i\bar{\jmath}}e^{-U}\overline{D_{\bar{\jmath}}\mathcal{L}} \ , \end{aligned}$$

with phase $e^{i\alpha} = \pm i e^{i\alpha_{\mathcal{L}}}$, thus yielding $\psi' = 2U'$ and

$$ds^2_{p=0} = e^{-2U}dr^2 + e^{2U}\left(-dt^2 + dx^2 + dy^2\right) \ ,$$

which is the metric for a domain wall with BPS flow

$$\begin{aligned} U'(r) &= \pm e^{-U}|\mathcal{L}| \ , \\ \dot{z}^i &= \mp e^{i\alpha_{\mathcal{L}}}e^{-U}g^{i\bar{\jmath}}\overline{D_{\bar{\jmath}}\mathcal{L}} \ , \end{aligned}$$

governed by the superpotential $\mathcal{W}_{DM} = e^{2U}|\mathcal{L}|$.

Holographic analysis

Action for the real scalar

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{2} + \frac{1}{2}\partial_\mu \varphi(r) \partial^\mu \varphi(r) - \left(\frac{3\xi_0}{\xi_1}\right)^{3/2} e^{\sqrt{6}\varphi} F^0_{\mu\nu} F^{0\,\mu\nu} + \right.$$
$$\left. -3\left(\frac{3\xi_0}{\xi_1}\right)^{-1/2} e^{-\sqrt{2/3}\varphi} F^1_{\mu\nu} F^{1\,\mu\nu} - V(\phi) \right)$$

with potential

$$V(\varphi) = -\frac{3}{\ell^2_{AdS}} \mathrm{Cosh}\left(\sqrt{\frac{2}{3}}\varphi\right) \ ,$$

The field φ is a massive scalar field with

$$m^2_\varphi = -\frac{2}{\ell^2_{AdS}} \ ,$$

Dual operator conformal dimensions are $\Delta_- = 1$, $\Delta_+ = 2$.
Breitenlohner-Friedman bound $m^2_\varphi \ell^2_{AdS} \geq -9/4$, the mass is in the window $-9/4 \leq m^2_\varphi \ell^2_{AdS} \leq -9/4 + 1$ which allows for mixed boundary conditions.

Marginal multitrace deformation

The field expansion from the asymptotic boundary

$$\varphi \sim e^{-\Delta_- \widetilde{r}/\ell}(\varphi_-(x) + ...) + e^{-\Delta_+ \widetilde{r}/\ell}(\varphi_+(x) + ...) \ .$$

Where r and $\widetilde{r}$ are related by $\frac{r}{\ell} = e^{\widetilde{r}/\ell}$
Multitrace deformation when

$$\varphi_+ = \lambda \varphi_-^2$$

In the case of black hole solutions presented $\lambda = \frac{\epsilon}{\sqrt{6}}$ and $\epsilon_{mag} = +1\,,\ \epsilon_{el} = -1.$

The solutions of electric and magnetic black holes are dual to marginal multitrace deformations

Holographic renormalization and a well defined variational principle [Papadimitriou, 2007]

Counterterm action

$$I_{ct,can} = \int_{\partial \mathcal{M}_0} d^3x \sqrt{h} \left(W(\varphi) + W_0 \mathcal{R} \right)$$

The Hamilton Jacobi equations imply the superpotential constraint

$$V(\phi) = \frac{1}{2} \left(\partial_\phi W^2 - \frac{3}{2} W^2 \right)$$

A 1-parameter class of superpotentials exist for $V(\varphi) = -\frac{3}{\ell_{AdS}^2} \mathrm{Cosh}\left(\sqrt{\frac{2}{3}}\varphi\right)$ that differ at cubic order in the fields expansion

$$W_\nu(\varphi) = -\frac{2}{\ell} \left(1 + \frac{\varphi^2}{4} + \frac{\nu}{6\sqrt{6}} \varphi^3 + \mathcal{O}(\varphi^4) \right)$$

For any finite value of $\nu \geq -1$, the coefficient of the quadratic term in φ is $-\Delta_- / (2\ell_{AdS})$.

Holographic renormalization and a well defined variational principle

- Only one value of the parameter ν is compatible with the variational principle, and it's

$$\nu(\lambda) \,, \qquad \varphi_+ = \lambda \varphi_-^2 \ .$$

- For the solutions presented here:

$$\nu = \sqrt{6}\lambda$$

  which is precisely the value selecting in the class of $\mathcal{W}$ the superpotential driving the first order flow of the solution!

- The cubic term is a finite term that affects the conserved charges, like the mass. The correct finite term comes from the superpotential driving the first order flow giving

$$Mass = -\frac{c_1}{2}$$

  c_1 is a free parameter and negative $c_1 < 0$, except for the BPS case when it is constraint to $c_1 = \frac{8Q_1^3}{\ell^2}$.

Summary of results

- New supersymmetric solutions
- Also finite temperature solutions satisfy a first order flow
- Superpotential of the flow $\mathcal{W}$ determines the mass from holographic renormalization

What's next?

- Embedding in String/M-theory
- Understand the holographic dual theories
- New dynamics in presence of charged particles?