Large Field Inflation and Moduli Stabilization

Ralph Blumenhagen

Max-Planck-Institut für Physik, München

(RB, Daniela Herschmann, Erik Plauschinn, appeared today)

Introduction

Introduction

The still controversial BICEP2 results indicate a large tensor-to-scalar ratio $r = 0.2 - 0.1$, a mass scale of inflation of $M_{\rm inf} \sim 10^{16}$ GeV and an inflaton mass of $m_\theta \sim 10^{13}$ GeV. The Lyth bound

$$\frac{\Delta\phi}{M_{\rm pl}} = O(1)\sqrt{\frac{r}{0.01}}$$

implies a rolling of the inflaton ϕ over trans-Planckian distances $\Delta\phi > M_{\rm pl}$.

- Makes it important to control Planck suppressed operators (eta-problem)
- Invoking a symmetry like the shift symmetry of axions helps

Introduction

Introduction

Axions are ubiquitous in string theory so that many scenarios have been proposed

- Natural inflation with a potential

$$V(\theta) = V_0(1 - \cos(\theta/f))\,.$$

  Hard to realize in string theory, as $f > 1$ lies outside perturbative control

- Aligned inflation with two axions, $f_{eff} > 1$, (talk by H.P Nilles)
- N-flation with many axions and $f_{eff} > 1$
- Monodromy inflation: Shift symmetry is broken by branes or fluxes unwrapping the compact axion $\rightarrow$ polynomial potential for θ, (talk by G. Shiu)

Introduction

Introduction

Proposal: Realize axion monodromy inflation via the F-term scalar potential induced by background fluxes.

(Marchesano.Shiu,Uranga)

Advantages

- Avoids the explicit supersymmetry breaking of models with the monodromy induced by branes
- Supersymmetry is broken spontaneously by the very same effect by which usually moduli are stabilized
- Generic in the sense that the potential for the the axions arise from the type II Ramond-Ramond field strengths $F_{p+1} = dC_p + H \wedge C_{p-2}$ involving the gauge potentials C_{p-2} explicitly.

Introduction

Introduction

Recently, a couple of a priori possible string realizations have been discussed. To name a few, the inflaton was given by:

- Wilson line and (B_2, C_2) modulus with potential generated by geometric flux (Marchesano.Shiu,Uranga)
- The universal axion c in type IIB flux compact. $\rightarrow$ natural reheating mechanism (Bhg, Plauschinn)
- D7-brane deformation modulus in the large complex structure limit (Hebecker, Kraus, Wittkowski)
- Higgs inflation (Ibanez, Valenzuela)

More proposals by Mc Allister, Gao, Grimm, Ibanez, Li, Long, Mc Guirk, Shukla, Silverstein, Valenzuela, Westphal,..

Introduction

Introduction

Learned from talks by (Ibanez, Kallosh, Nilles, Shiu, Zavala) that, for a consistent inflationary scenario, all moduli need to be stabilized in the window $H < m_\sigma < M_{\rm inf}$.

Aim: Systematic study of realizing single-field fluxed F-term axion monodromy inflation, taking into account the interplay with moduli stabilization.

Note:

- There exist a no-go theorem for having an unconstrained axion in supersymmetric minima of $N = 1$ supergravity models (Conlon)
- In the LVS scenario, the (Kähler) axion a_b is massless with all other moduli massive

The set-up

The set-up

Investigate whether the landscape of minima of the flux-induced scalar potential admits solutions with the following properties:

1. All moduli are stabilized such that a single axion is parametrically lighter than the other moduli and the axion admits a shift symmetry.
2. For this inflaton candidate, the tree-level scalar potential in the trans-Planckian regime still realizes large-field inflation.

Work in the large complex structure limit $\mathrm{Im}\,\mathcal{U} = v \to \infty$ so that axion (like) fields

- universal axion c
- real parts of complex structure moduli $\mathrm{Re}\mathcal{U}_i = u_i$

Review: flux induced potential

Review: flux induced potential

3-form flux in Type IIB orientifolds

$$G_3 = F_3 + \tau H_3 \,,$$

with the axio-dilaton

$$\tau = C_0 + i\, e^{-\phi} = c + i\, s \,.$$

In terms of symplectic basis $(\alpha_\Lambda, \beta^\Lambda)$ of $H^3(X, \mathbb{Z})$, the covariantly constant $(3,0)$-form can be expanded as

$$\Omega_3 = X^\Lambda\, \alpha_\Lambda - F_\Lambda\, \beta^\Lambda,$$

where the periods X^Λ and F_Λ are functions of the complex structure moduli $\mathcal{U}^i$, with $i = 1, \ldots, h^{2,1}$.

Review: flux induced potential

Review: flux induced potential

3-form flux can be written as

$$\frac{1}{(2\pi)^2\alpha'}\,G_3 = e_\Lambda\,\beta^\Lambda + m^\Lambda\,\alpha_\Lambda\,, \qquad \begin{aligned} e_\Lambda &= \tau\,h_\Lambda + f_\Lambda\,,\\ m^\Lambda &= \tau\,\overline{h}^\Lambda + \overline{f}^\Lambda\,, \end{aligned}$$

The superpotential

$$W = \int \Omega_3 \wedge G_3 = X^\Lambda e_\Lambda + F_\Lambda m^\Lambda$$

induces the no-scale scalar potential

$$V_F = -\frac{M_{\rm pl}^4}{4\pi}\,\frac{1}{\mathcal{V}^2\,{\rm Im}\,\tau}\,(e + m\overline{\mathcal{N}})({\rm Im}\mathcal{N})^{-1}(\overline{e} + \mathcal{N}\,\overline{m})\,.$$

Review: flux induced potential

Review: flux induced potential

$\mathcal{N}_{\Lambda\Sigma}$ is called the period matrix, and with $F_{\Lambda\Sigma} = \partial F_\Lambda / \partial X_\Sigma$ it is defined as

$$\mathcal{N}_{\Lambda\Sigma} = \overline{F}_{\Lambda\Sigma} + 2i \, \frac{\mathrm{Im}(F_{\Lambda\Gamma}) X^\Gamma \, \mathrm{Im}(F_{\Sigma\Delta}) X^\Delta}{X^\Gamma \, \mathrm{Im}(F_{\Gamma\Delta}) X^\Delta} \, .$$

Therefore, minima of V are at

$$e_\Lambda + m^\Sigma \overline{\mathcal{N}}_{\Sigma\Lambda} = 0 \, ,$$

(equivalent to $D_\Lambda W = 0$).

Review: flux induced potential

Review: flux induced potential

In the large complex-structure regime, the prepotential has the simple form

$$F = \frac{\kappa_{ijk} X^i X^j X^k}{X^0} ,$$

with κ_{ijk} denoting the triple intersection numbers of the mirror Calabi-Yau manifold.
The complex structure moduli $\mathcal{U}^i \equiv u^i + i\,v^i$ are defined via

$$X^0 = 1 , \qquad F_0 = -\kappa_{ijk} \mathcal{U}^i \mathcal{U}^j \mathcal{U}^k ,$$

$$X^i = \mathcal{U}^i , \qquad F_i = 3\kappa_{ijk} \mathcal{U}^j \mathcal{U}^k .$$

Review: flux induced potential

Review: flux induced potential

Tree-level Kähler potential for the complex structure moduli

$$K_{\rm cs} = -\log\left(-i\int_{\mathcal{X}} \Omega_3 \wedge \overline{\Omega}_3\right) = -\log\left(\kappa_{ijk} v^i\, v^j\, v^k\right),$$

featuring the continuous shift symmetry $u^i \to u^i + c^i$.
The period matrix becomes

$$\begin{aligned}
{\rm Im}\,\mathcal{N}_{ij} &= 4\,\kappa\, G_{i\bar{j}}\,, & {\rm Re}\,\mathcal{N}_{ij} &= 6\,\kappa_{ijk}\, u^k\,,\\
{\rm Im}\,\mathcal{N}_{i0} &= -4\,\kappa\, G_{i\bar{j}}\, u^j\,, & {\rm Re}\,\mathcal{N}_{i0} &= -3\,\kappa_{ijk}\, u^j u^k\,,\\
{\rm Im}\,\mathcal{N}_{00} &= \kappa\left(1 + 4\, G_{i\bar{j}}\, u^i u^j\right), & {\rm Re}\,\mathcal{N}_{00} &= 2\,\kappa_{ijk}\, u^i u^j u^k\,,
\end{aligned}$$

where the Kähler metric reads

$$G_{i\bar{j}} = -\frac{3}{2}\,\frac{\kappa_{ij}}{\kappa} + \frac{9}{4}\,\frac{\kappa_i\kappa_j}{\kappa^2}\,,$$

Search principle

Search principle

For parametrically controlling the mass of a massless field ϕ we proceed as follows

- First, one identifies fluxes so that (only) θ is unconstrained
- Then one identifies fluxes also constraining θ. Then, these fluxes are the order parameters for the mass of θ
- Express the scalar potential for the canonically normalized field θ after integrating out the heavy fields.

General question: Which parameters in the LEEA can be dialed small/large by choosing hierarchical fluxes?

Problem: A hierarchical choice $f_1 \gg f_2$ can be compensated by an induced hierarchy between the VEVs $m_1 \ll m_2$ so that e.g. $a\, f_1 m_1 + b\, f_2 m_2 = O(1)$.

Examples

Examples

Findings by explicit computation for many models:

- In the paper we present concrete examples where this program works. In a simple example with 4 moduli, the canonically normalized field θ can be expressed as

$$V_{\rm val}(\theta) = \frac{3\,M_{\rm pl}^4}{\pi}\,\frac{\kappa}{\mathcal{V}^2}\,\frac{f_1\,\overline{f}_1}{f_0\,\overline{f}_0}\,\sinh^2\left(\frac{\theta}{\sqrt{6}}\right),$$

- Here, θ is a combination of axions and saxions. Can in principle give small field inflationary models.
- It turns out to be hard to find a model where an axionic direction is the only unconstrained field

Conclusion

Conclusion

We conclude:

> **Proposition**: For realizing F-term monodromy inflation, the inflaton should be a linear combination of only axions.

In that situation, the shift symmetry is intact, guaranteeing that the above η-problem is absent and that the effective scalar potential is of polynomial form.

General structure

General structure

Notoriously hard to approach, as

- We are dealing with non-linear, coupled equations in many variables
- This is the string landscape

No-scale scalar potential

$$V = M_{\rm pl}^4 \, e^K \left[G^{i\bar{j}} \, D_i W \, D_{\bar{j}} \overline{W} + G^{\tau\bar{\tau}} \, D_\tau W \, D_{\bar{\tau}} \overline{W} \right] ,$$

where the indices i, j run over all complex-structure moduli $\mathcal{U}^i = u^i + i v^i$ with $i, j = 1, \ldots, N$. Minkowski minima at

$$\partial_I W(\mathcal{U}) = -\partial_I K(v) \, W(\mathcal{U}) \, .$$

General structure

General structure

Require: axion $\theta = a_0\, c + a_i\, u^i$ unconstrained

$K(v)$ independent of axion θ and $W_0 \neq 0$

$$\Rightarrow \ \partial_\Theta W \equiv 0\,, \qquad \partial_\Theta K = 0$$

- Condition on the fluxes $\rightarrow$ two types of fluxes: $f_{\rm ax} = 0$ and $f_{\rm mass} \neq 0$.
- Analyze whether $f_{\rm mass}$ are sufficient to freeze all the remaining moduli σ_α inside the physical domain
- If not $\Rightarrow$ no-go theorem
- If yes, proceed getting parametric control over inflaton mass by $f_{\rm mass} \neq 0$

General structure

General structure

The superpotential in this case can be written as

$$W = f_{\rm mass}\, W_{\rm mass}(\tilde{\mathcal{U}}_I) + f_{\rm ax}\, W_{\rm ax}(\Theta, \tilde{\mathcal{U}}_I)$$

- Scale $f_{\rm mass} \to \lambda f_{\rm mass}$. At leading order in λ^{-1}, ignore backreaction of the second term on stabilization of σ_α
- The scalar potential is

$$V = \lambda^2 V_{\rm mass}(\sigma_\alpha) + f_{\rm ax}^2 V_{\rm ax}(\theta, \sigma_\alpha)\,.$$

- Integrating out σ_α moduli, for $\lambda \gg f_{\rm ax}^2$, we get

$$\frac{m_\theta^2}{m_{\sigma_\alpha}^2} \sim \left(\frac{f_{\rm ax}}{\lambda}\right)^2\,.$$

Case A: no-go theorem

Case A: no-go theorem

Distinguish two cases

- Case A: θ contains the universal axion c, i.e. $a_0 \neq 0$
- Case B: θ is a combination of u_i only, i.e. $a_0 = 0$

For case A one can prove the following no-go theorem:

> **Theorem**: The type IIB flux-induced no-scale scalar potential does not admit non-supersymmetric Minkowski minima, where a single axion involving c is unfixed while all remaining axions and saxions are stabilized inside the physical domain.

As a consequence, there cannot exist minima in this setting with an axion parametrically lighter than all the remaining moduli.

Case B

Case B

Can assume $\theta = u_N$ so that $\partial_\Theta W \equiv 0$ implies the constraints

$$f_N = h_N = 0\,, \qquad \overline{f}_0 = \overline{h}_0 = 0$$

$$\sum_{j=1}^{N} \kappa_{Nij}\, \overline{f}^j = \sum_{j=1}^{N} \kappa_{Nij}\, \overline{h}^j = 0\,, \quad \text{for all } i \in \{1, \ldots N\}$$

With $\mathcal{A}_{ij} = \kappa_{Nij}$ this means that

$$\overline{f}, \overline{h} \in \ker(\mathcal{A})$$

Case B

Case B

- $\mathrm{rk}(\mathcal{A}) = N$: $\overline{f}_i = \overline{h}_i = 0$ trivial
- $\mathrm{rk}(\mathcal{A}) = N - 1$: Can prove a no-go theorem as in Case A, i.e. not all remaining moduli can be stabilized
- $\mathrm{rk}(\mathcal{A}) = 0, 1$: $\partial_\Theta K = 0$ implies $\det(G) = 0$
- $\mathrm{rk}(\mathcal{A}) = N - 2$: In a deep numerical analysis we found a model with all remaining moduli stabilized inside the physical domain

Details of example

Details of example

Prepotential

$$F(X_0, X_1, X_2, X_3, X_4) = (X_3^3 + X_1 X_2 X_3 + X_3 X_4^2)/X_0 \,.$$

Solving $e_\Lambda + \overline{\mathcal{N}}_{\Lambda\Sigma} m^\Sigma = 0$ we managed to iteratively fix $\{u_1, u_2, u_3, c, s, v_1\}$ in terms of $\{v_2, v_3\}$.
The remaining two relations are

$$f(v_2, v_3) = 27 v_2^8 - 72 v_2^6 v_3^2 + 294 v_2^4 v_3^4 - 784 v_2^2 v_3^6 + 48 v_2^4 v_3^6 \\ + 343 v_3^8 - 32 v_2^2 v_3^8 + 112 v_3^{10} = 0$$

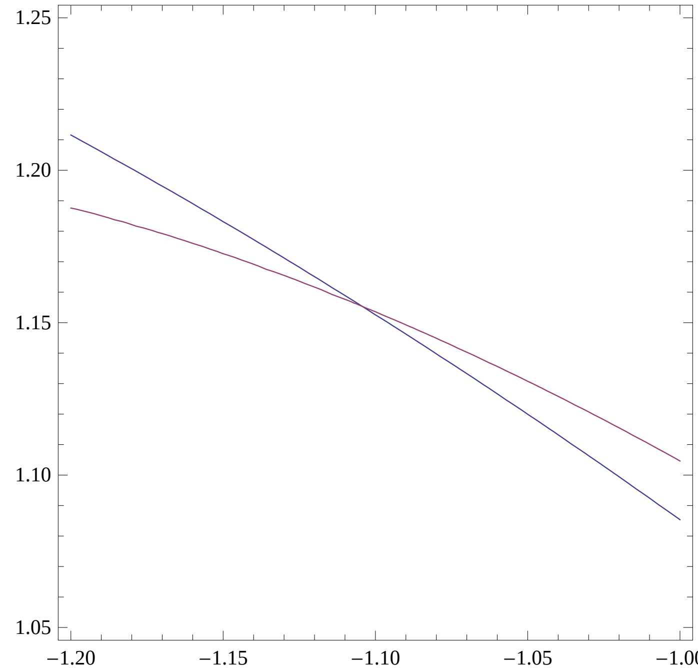
and

$$g(v_2, v_3) = -729 v_2^{15} + 2430 v_2^{13} v_3^2 - 891 v_2^{12} v3^3 + 6237 v_2^{11} v_3^4 + \ldots$$

Details of example

Details of example

A contour plot reveals the solutions



The values of the moduli are $\partial_\Theta K = v3\, v4 = 0$

$$u_1 = 0.492\,, \quad u_2 = -0.371\,, \quad u_3 = -0.065\,, \quad c = 1.041\,,$$
$$s = 0.932\,, \quad v_1 = 3.775\,, \quad v_2 = -1.104\,, \quad v_3 = 1.155\,.$$

Details of example

Details of example

Turning on e.g. $f_4 \neq 0$, at leading order we get

$$V_{\text{eff}}(\theta) = \frac{M_{\text{pl}}^4}{4\pi \mathcal{V}^2}\, f_4^2 \Big(a + \frac{b}{c}\theta^2 \Big)$$

- Problem: The flux f_4 cannot make the mass of the inflaton smaller than

$$m_{\text{ax}} = O(1)\,\frac{M_{\text{pl}}}{\mathcal{V}}$$

  With $\mathcal{V} = 10^2 - 10^3$ one gets $m_{\text{ax}} > m_{\text{inf}} \sim 10^{13} GeV$
- Hierarchy to mass of Kähler moduli is not controlled.
- Future: Investigate in a model scan the regime of the parameters b, c

Conclusions

Conclusions

- Started a honest technical investigation whether the flux induced scalar potential admits minima with parametrically light axions
- For the flux landscape with proved a no-go theorem for the case that the inflaton involves the universal axion
- For the inflaton being an axion-like complex structure modulus, we found a model with all remaining moduli stabilized
- Important to generalize this investigation to other proposals

Case A: Proof of no-go theorem

Case A: Proof of no-go theorem

As a consequence the minimum conditions become

$$\mathcal{P}_0 = (f_0 + ch_0) - \frac{1}{2} u^i \,\mathrm{Re}\mathcal{N}_{ij}\, (\overline{f}^j + c\overline{h}^j) - u^i \,\mathrm{Im}\mathcal{N}_{ij}\, s\overline{h}^j + \frac{1}{3} u^i u^j \mathrm{Re}\mathcal{N}_{ij} \overline{f}^0 = 0$$

$$\mathcal{Q}_0 = sh_0 - \frac{1}{2} u^i \,\mathrm{Re}\mathcal{N}_{ij}\, s\overline{h}^j + u^i \,\mathrm{Im}\mathcal{N}_{ij}\, (\overline{f}^j + c\overline{h}^j) - (\kappa + u^i u^j \mathrm{Im}\mathcal{N}_{ij}) \overline{f}^0 = 0$$

$$\mathcal{P}_i = (f_i + ch_i) + \mathrm{Re}\mathcal{N}_{ij}\, (\overline{f}^j + c\overline{h}^j) + \mathrm{Im}\mathcal{N}_{ij}\, s\overline{h}^j - \frac{1}{2} u^j \,\mathrm{Re}\mathcal{N}_{ij} \overline{f}^0 = 0$$

$$\mathcal{Q}_i = sh_i + \mathrm{Re}\mathcal{N}_{ij}\, s\overline{h}^j - \mathrm{Im}\mathcal{N}_{ij}\, (\overline{f}^j + c\overline{h}^j) + u^j \,\mathrm{Im}\mathcal{N}_{ij} \overline{f}^0 = 0\,.$$

Case A: Proof of no-go theorem

Case A: Proof of no-go theorem

Case A1: $\kappa_{NNi} \neq 0$ for at least one $i \in \{1, \ldots, N\} \Rightarrow \overline{f}^0 = 0$ and $\kappa_{ijk}\overline{h}^k = 0$.

Case A: Proof of no-go theorem

Case A1: $\kappa_{NNi} \neq 0$ for at least one $i \in \{1, \ldots, N\} \Rightarrow \overline{f}^0 = 0$ and $\kappa_{ijk}\overline{h}^k = 0$.

The set of equations simplifies

$$
\begin{aligned}
\mathcal{P}_0 &= (f_0 + ch_0) - \frac{1}{2}u^i \,\mathrm{Re}\mathcal{N}_{ij}\,\overline{f}^j = 0 \\
\mathcal{Q}_0 &= sh_0 + u^i \,\mathrm{Im}\mathcal{N}_{ij}\,\overline{f}^j = 0 \\
\mathcal{P}_i &= (f_i + ch_i) + \mathrm{Re}\mathcal{N}_{ij}\,\overline{f}^j = 0 \\
\mathcal{Q}_i &= sh_i - \mathrm{Im}\mathcal{N}_{ij}\,\overline{f}^j = 0\,.
\end{aligned}
$$

Case A: Proof of no-go theorem

Case A1: $\kappa_{NNi} \neq 0$ for at least one $i \in \{1, \ldots, N\} \Rightarrow \overline{f}^0 = 0$ and $\kappa_{ijk}\overline{h}^k = 0$.

Due to $\mathcal{Q}_0 + \sum_i u_i \mathcal{Q}_i = h_0 + u^i\, h_i = 0$, these $2N + 2$ relations split into

- $N + 2$ relations depending only on the $N + 1$ axions $\{c, u^i\}$
- and N relations depending on the $N + 1$ saxions $\{s, v^i\}$.

Therefore, at least one saxionic direction must remain unconstrained.

Case A: Proof of no-go theorem

Case A: Proof of no-go theorem

Case A2: $\kappa_{NNi} = 0$ for all $i \in \{1, \ldots, N\}$

First, let us consider the conditions $\mathcal{Q}_i$

$$\mathcal{Q}_i = 6s\kappa_{Nij}\left(\overline{f}^j + u^j\overline{f}^0\right) - \sum_{j=1}^{N-1} \mathrm{Im}\mathcal{N}_{ij}\left(\overline{f}^j - u^j\overline{f}^0\right) - \mathrm{Im}\mathcal{N}_{iN}\left(\overline{f}^N - (u^N - c)\overline{f}^0\right) = 0\,.$$

These are N conditions which fix the axions as

$$u^N - c = \frac{\overline{f}^N}{\overline{f}^0}\,, \qquad u^i = \frac{\overline{f}^i}{\overline{f}^0}\,, \quad i \in \{1, \ldots N-1\}$$

Case A: Proof of no-go theorem

Case A: Proof of no-go theorem

Taking into account $\partial_\Theta K = 0$ which implies $s = -\kappa/\kappa_N$, after some algebra one obtains

$$\mathcal{P}_i = \frac{1}{\overline{f}^0}\left(\overline{f}^0 f_i + 3\kappa_{ijk}\overline{f}^j\overline{f}^k + s\left(\overline{f}^0\right)^2 \mathrm{Im}\mathcal{N}_{iN}\right) = 0 \quad \text{(-33)}$$

and for $\mathcal{Q}_0 + u^i\mathcal{Q}_i = 0$

$$\frac{s}{\overline{f}^0}\left(h_0\overline{f}^0 + h_i\overline{f}^i + 3\kappa_{Nij}\overline{f}^i\overline{f}^j + \overline{f}^0\kappa_N\right) = 0\,.$$

where for $\kappa_{NNi} = 0$ the factors κ_N and $s\mathrm{Im}\mathcal{N}_{iN}$ do not depend on v^N (QED).