THE STRING THEORY UNIVERSE, MITP

SPONTANEOUS HOLOGRAPHIC CHECKERBOARDS

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FINITE CHARGE DENSITY

Interested in modelling charged phases of matter with 2 spatial dimensions. A minimal set of holographic ingredients:

$$S = \int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F^2\right)$$

Turn on chemical potential μ
1-parameter family of AdS-RN black brane solutions

at T=0 these solutions interpolate between UV AdS_4 and and IR $\mathrm{AdS}_2 \times \mathrm{R}^2$ solution.

Adding more ingredients (e.g. scalars) results in instabilities, which can be diagnosed using the AdS_2 factor in the IR. e.g. superfluids [Hartnoll et. al.] spatial modulation [Donos, Gauntlett]

Stabilise at finite T. Marks a new branch of solutions emerging at that T, and a potential new contribution to the phase diagram.

SPONTANEOUS VS EXPLICIT

Spatial modulation can be added manually using boundary conditions. e.g. [Horowitz, Santos, Tong].

Can exploit design freedom to simplify/gain analytic control, e.g.
— 5d helical [Donos, Hartnoll]
— phase of a complex scalar / axions [Donos,Gauntlett][Andrade,BW]
see also talk by Gauntlett, [Gouteraux], [Taylor, Woodhead] and massive gravity theories [Vegh][Davison][Blake,Tong]

This talk: **spontaneous**.
The design freedom doesn't exist - have to solve PDEs.
Here - want to break all continuous spatial symmetries.

MOTIVATIONS

Possible that such instabilities are generic in holography at finite density at low enough temperatures. Can’t (easily) control the outcome. Natural to seek phases with no surviving continuous spatial symmetries.

Modulation observed in condensed matter

— what is possible holographically?

— what is dominant?

A step towards determining the ground states

MODEL

4D Gravity + U(1) gauge field + pseudoscalar [Donos, Gauntlett]

$$S = \int d^4x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{\tau(\phi)}{4}F^2 - V(\phi)\right) - \int \frac{\vartheta(\phi)}{2}F \wedge F.$$

take a 2-parameter model, n, c_1

$$\tau(\phi) = \mathrm{sech}\frac{\sqrt{n}}{2\sqrt{3}}\phi, \quad V(\phi) = -6\cosh\frac{\phi}{\sqrt{3}}, \quad \vartheta(\phi) = \frac{c_1}{6\sqrt{2}}\tanh\sqrt{3}\phi$$

ϕ has a mass $m^2 = -2$

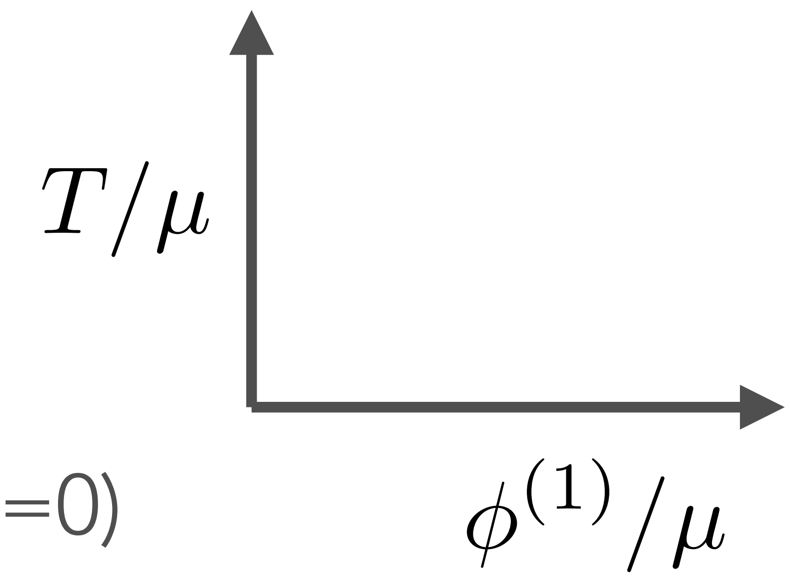
$$\phi(z, x^\mu) = \phi^{(1)}z + \phi^{(2)}(x^\mu)z^2 + O(z)^3$$

for $\phi^{(1)} = 0$ with μ normal
phase is simply RN

also deform with
operator dual to ϕ,
by turning on a
spatially constant $\phi^{(1)}$

NORMAL PHASE

2 parameter family of normal phase BHs



- $\phi^{(1)}/\mu = 0$
  corresponds to RN (IR AdS_2 factor at T=0)
- $\phi^{(1)}/\mu \neq 0$
  solutions have IR HSV factor at T=0 in the cases studied

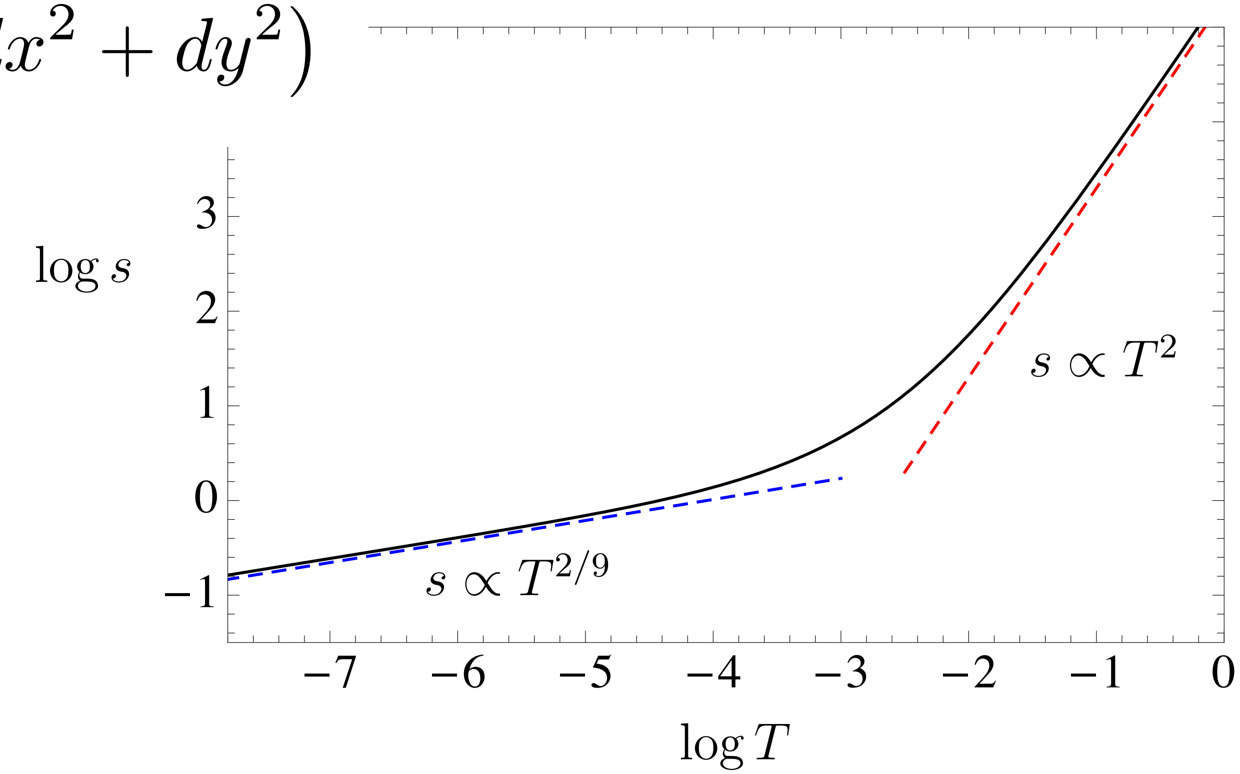
An example: n=0 model. Has HSV solutions:

$$ds^2 = -z^{11/2}dt^2 + \frac{11e^{\frac{\sigma}{\sqrt{3}}}}{4}\frac{dz^2}{z} + \sqrt{z}\left(dx^2 + dy^2\right)$$

$$\phi = \sqrt{3}\log z + \sigma \qquad A = 2\sqrt{\frac{5}{11}}z^{11/4}dt$$

expect e.g. entropy scaling

$$s \propto T^{2/9}$$



MARGINAL MODES

about RN ($\phi = 0$)

$$\delta\phi = \lambda(z)\cos kx \quad \delta g_{ty} = h_{ty}(z)\sin kx \quad \delta A_y = a_y(z)\sin kx$$

[Donos, Gauntlett]

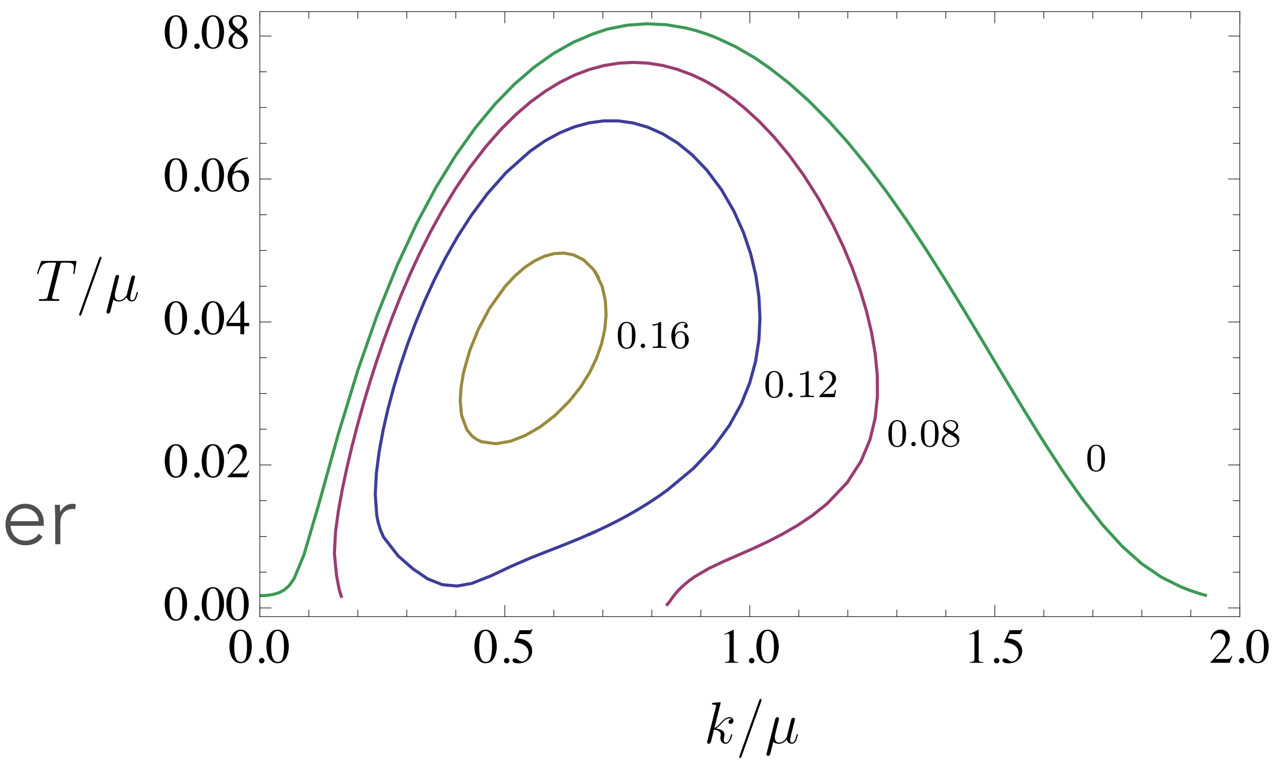
with A_t modulated at higher orders in this expansion.

Current in translationally invariant direction.

about the numerical $\phi \neq 0$ backgrounds a consistent set of fluctuations now include also:

$$\delta A_t(z,x) = a_t(z)\cos kx$$
$$\delta g_{ii}(z,x) = h_{ii}(z)\cos kx$$
$$\delta g_{zz}(z,x) = h_{zz}(z)\cos kx.$$

counting indicates two parameter families, e.g. $k/\mu \quad \phi^{(1)}/\mu$



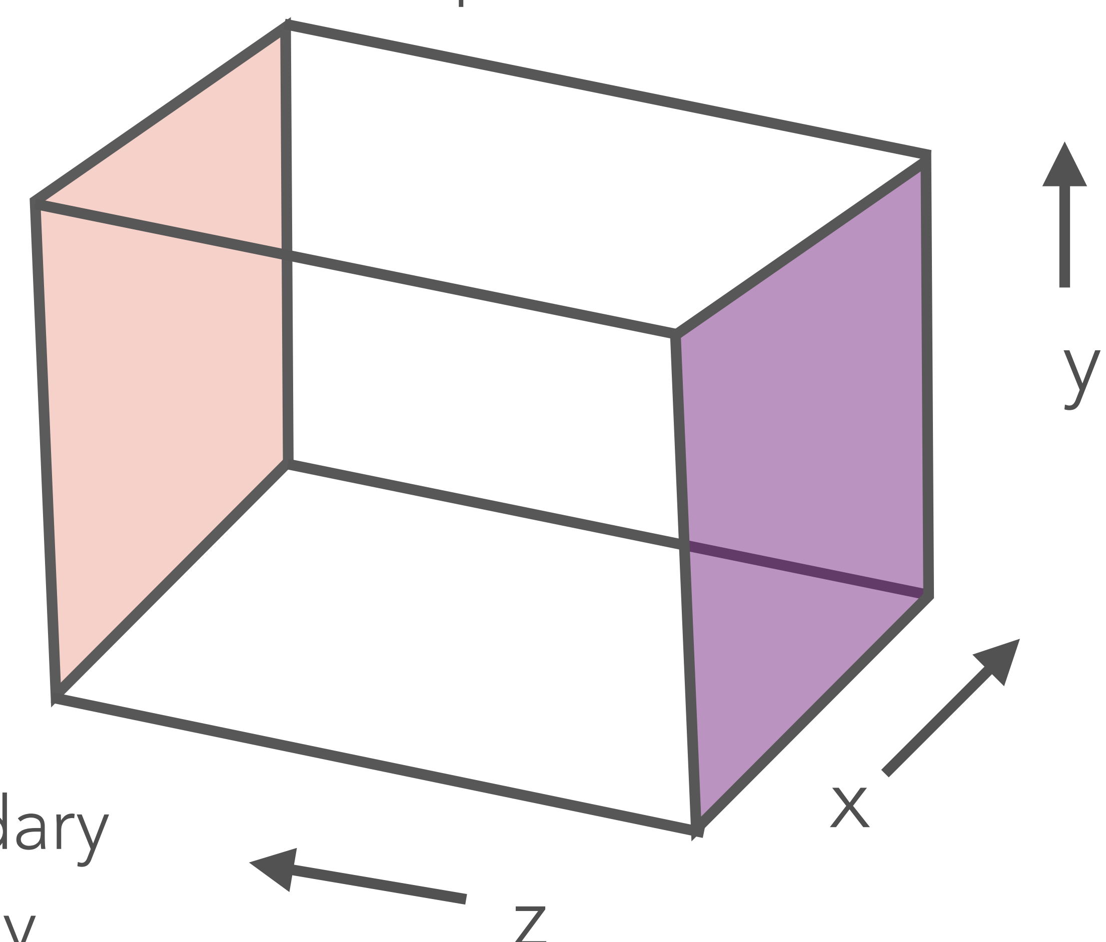
The marginal modes indicate branches of spatially modulated black branes.

STRIPES nonlinearly modulated black brane branch continuously connecting to a single marginal mode, k, preserving translations $\perp k$ [Rozali et. al][Donos][B.W.]

CHECKERBOARDS branch connects to two marginal modes e.g. $k_1 \perp k_2$

stationary solutions of Einstein equations in a box:

Regular horizon



UV Dirichlet conditions: normalisable + constant sources ($\phi^{(1)}$, μ)

Periodic boundary conditions in x,y

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Use the `Harmonic Einstein' / DeTurck approach

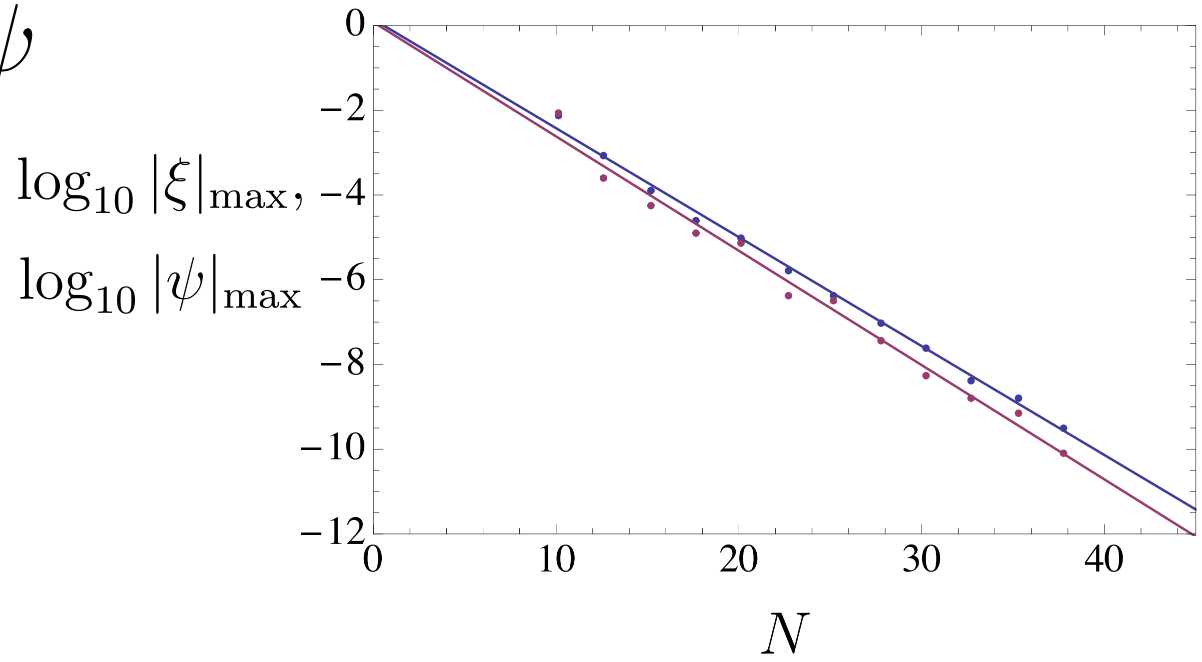
[Headrick, Kitchen, Wiseman]

$$R_{MN} \rightarrow R^H_{MN} = R_{MN} - \nabla_{(M}\xi_{N)}$$

$$\xi^M = g^{NP}(\Gamma^M_{NP} - \tilde{\Gamma}^M_{NP})$$

similar trick for U(1) redundancy: ψ

work with this (modified) system and checking that ξ^M, ψ vanish on any solution.



leads to: 15 fields, F_I, (all metric + gauge field components, plus the scalar) and equations $E_I(F_J) = 0$

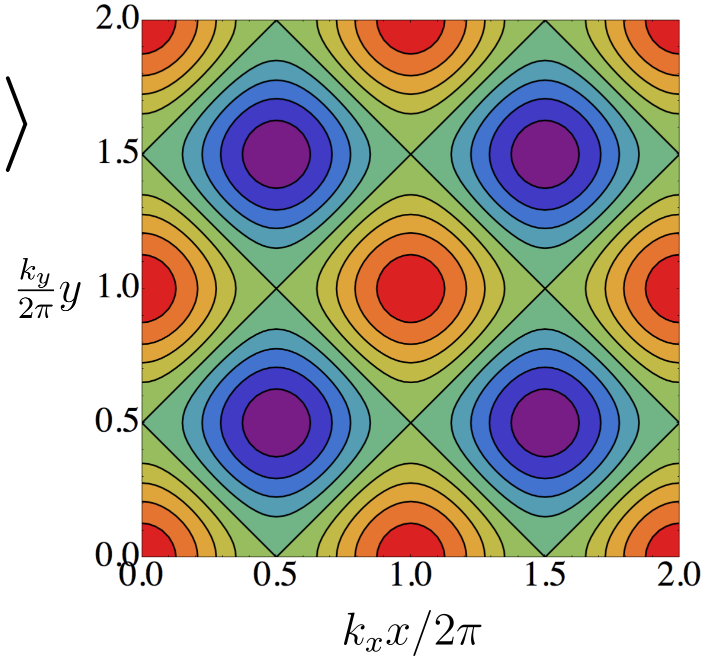
Discritise and iteratively solve using Newton-Raphson method.

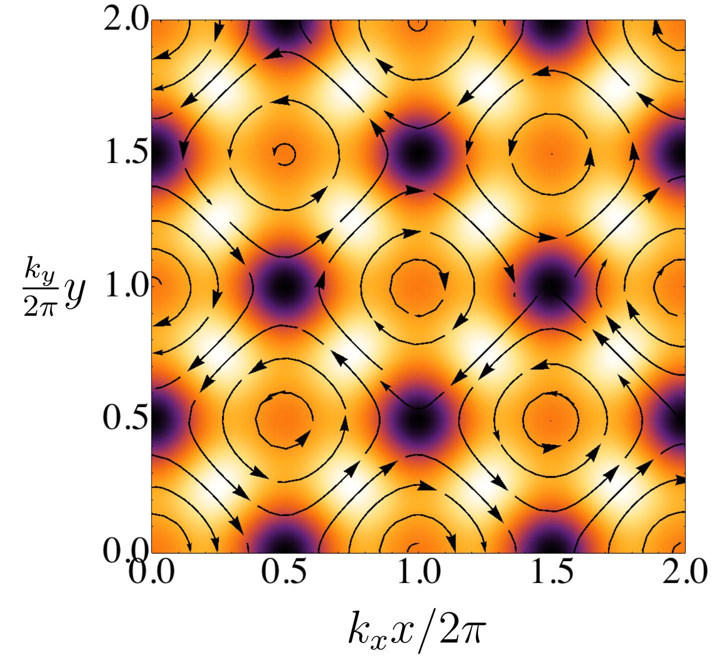
CHECKERBOARDS

black brane branch which continuously connects to two marginal modes with momentum k_x k_y

$k_x = k_y$ case (40^3, spectral, T=0.55Tc, $\phi^{(1)}/\mu = 0$)

$\langle O_\phi \rangle$

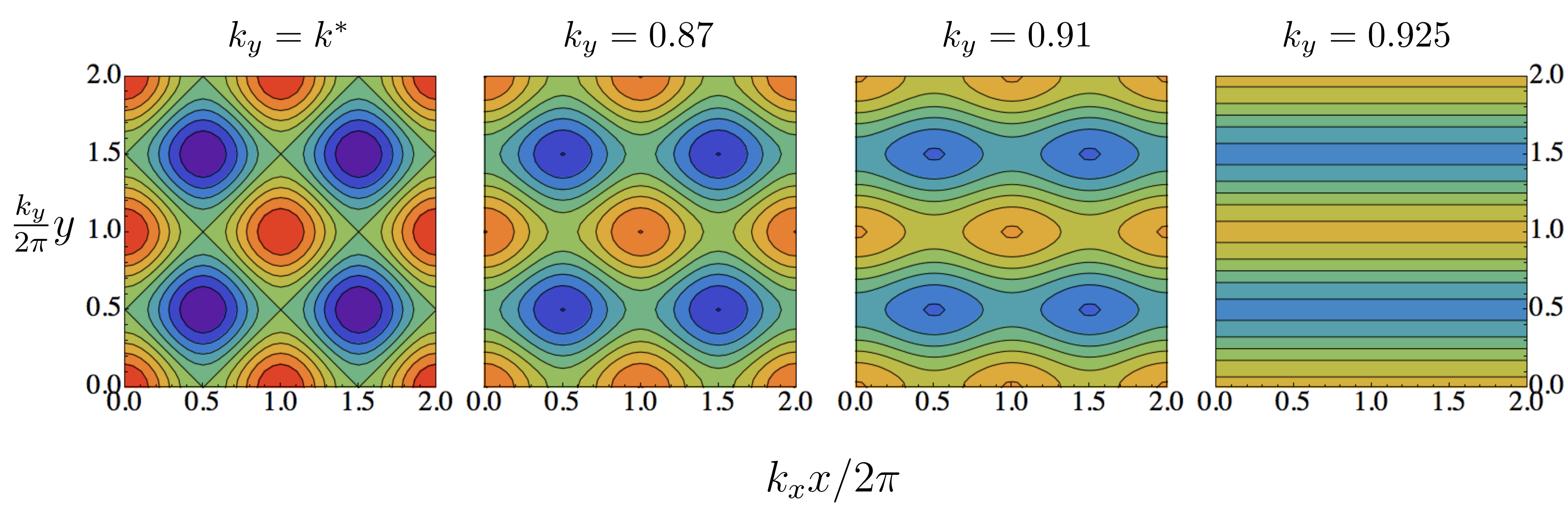




$\langle J_t \rangle$

integral curves of $\langle J_i \rangle$

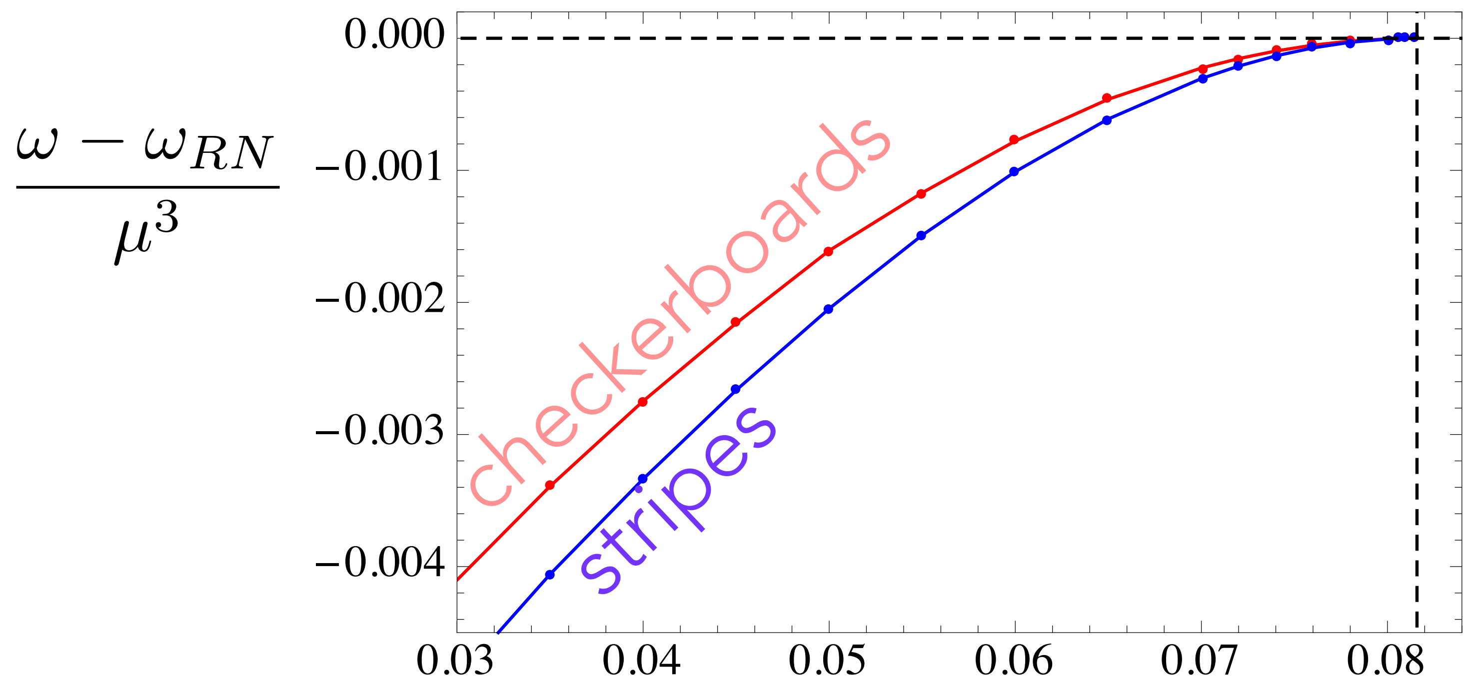
a large space of solutions, e.g.



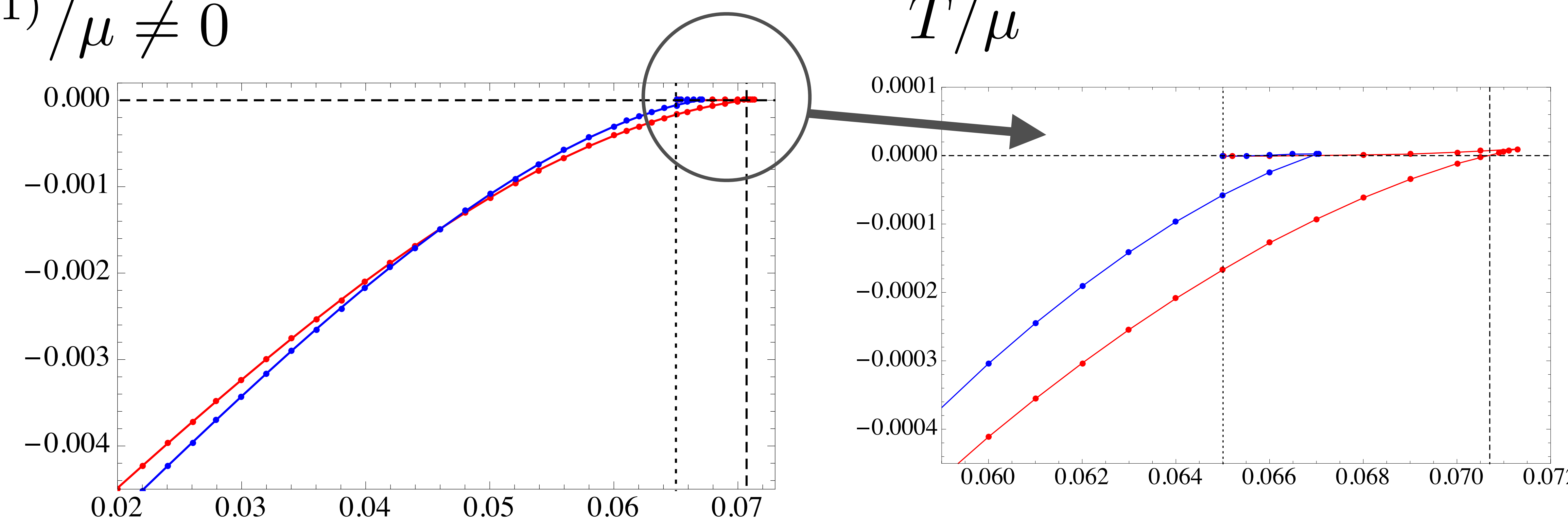
FREE ENERGY

minimise free energy w.r.t. $k_x = k_y$ at fixed T

$$\phi^{(1)}/\mu = 0$$

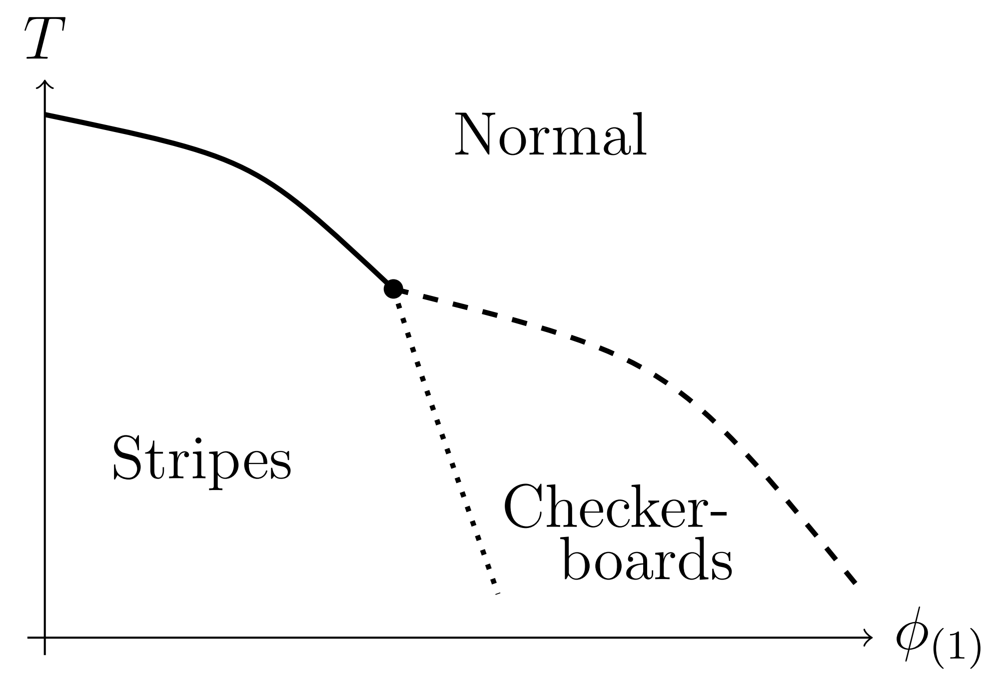


$$\phi^{(1)}/\mu \neq 0$$



SUMMARY

- Spontaneously modulated phases appear in holography, natural to seek those which break all continuous spatial symmetries.
- Distinct physical (and technical) problem to explicit modulation
- Constructed stationary, cohomogeneity-three black holes describing checkerboard phases
- Stripe-to-checkerboard first order phase transitions. Schematic phase diagram:



- Questions:
  Low temperatures?
  The larger space of solutions e.g. triangular (c.f. [Erdmenger et. al.])
  Lattice symmetry breaking

THANK YOU