The String Theory Universe, Mainz 2014

F-Theory Compactification with Abelian Sector

Mirjam Cvetič

The String Theory Universe, Mainz 2014

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Based on:
arXiv:1303.6970 [hep-th]: M. C., Denis Klevers, Hernan Piragua
arXiv:1306.0236 [hep-th]: M. C., Antonella Grassi, D. Klevers, H. Piragua
arXiv:1307.6425 [hep-th]: M. C., D. Klevers, H. Piragua
arXiv:1310.0463 [hep-th]: M.C., D. Klevers, H. Piragua, Peng Song
arXiv:1410....[hep-th]: M.C., D. Klevers, H. Piragua, Wati Taylor
& work in progress (also w/ P. Langacker)

Penn
UNIVERSITY of PENNSYLVANIA

CAMTP
University of Maribor

F-theory & U(1) Gauge Symmetries

MOTIVATION

c.f., Fernando Marchesano’s nice review

Why F-theory?

F-theory (12dim) = Type IIB String Theory (10dim)

- on elliptically fibered Calabi-Yau manifold

- w/ back-reacted (p,q) 7-branes
- regions with finite/infinite g_s on non-Calabi-Yau space

Through chain of dualities related to:

M-theory (w/ limit 11dim Supergravity) [via a limiting T-duality]

Heterotic String Theory (10dim) [via ``stable degeneration'']

Why F-theory Compactification?

A broad domain of non-perturbative string theory landscape with promising particle physics & cosmology [gauge symmetry, matter repres. & couplings-non-pert./exceptional symm. groups]

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Focus on [SU(5)] GUT:

Local model building: [Donagi,Wijnholt;Beasley,Heckman,Vafa;... Font,Ibanez;... Hayashi,Kawano,Tsuchiya,Watari,Yamazaki;...Dudas,Palti;... Cecotti,Cheng,Heckman,Vafa; ...Marchesano,Martucci;...]

Global model building: [Blumenhagen,Grimm,Jurke,Weigand; Marsano,Saulina,SchäferNameki;Grimm,Krause,Weigand;... M.C.,Halverson,Garcia-Etxebarria;...] SM [Lin,Weigand;...]

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[Vafa; Vafa,Morrison,...]

Employing geometric techniques for elliptically fibered Calabi-Yau manifolds and/or dualities to determine

Primarily discrete data:

Gauge symmetries, matter repres .& multiplicities,Yukawa couplings,…

Why Abelian Symmetries in F-theory?

Physics: important ingredient of the Standard Model and beyond

➡ Multiple U(1)'s desirable (both local & global)

Formal developments: new CY elliptic fibrations related to Mordell-Weil group

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A lot of recent progress: [Grimm,Weigand;Esole,Fullwood,Yau; Morrison,Park; M.C.,Grimm,Klevers; Braun,Grimm,Keitel; Lawrie,Schäfer-Nameki; Borchmann,Mayrhofer,Palti,Weigand; M.C.,Klevers,Piragua; Grimm,Kapfer,Keitel;Braun,Grimm,Keitel; MC,Grassi,Klevers,Piragua;Borchman,Mayrhofer,Palti,Weigand; M.C.,Klevers,Piragua; M.C.,Klevers,Piragua,Song; Braun,Collinucci,Valandro; Morrison,Taylor; Kuntzler,Schäfer-Nameki]

Torsion part: [Morrison,Vafa; Aspinwall,Morrison;...Morrison,Till,Weigand]

Outline & Summary

Systematic Construction of Abelian sector of F-theory

1. Engineering rank n Abelian sector $U(1)^n$ of global F-theory compactification:

construction of elliptically fibered Calabi-Yau manifolds with

rank n Mordell-Weil (MW) group

- Exemplify for rank 2 MW [$U(1)^2$]

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2. Develop techniques:

- Matter representations, multiplicities in D=6
- Yukawa couplings, chirality (G_4 - flux) in D=4 (not in this talk)

Two-fold advances: <u>geometry</u> & M-/F-theory duality

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   Two-fold advances: geometry & M-/F-theory duality

3. Applications:

   - Construction of rank 3 [$U(1)^3$] - complete intersection CY in $\mathbb{P}^3$(only results)
   - D=4 GUT's w/ $SU(5)xU(1)^2$: all Yukawa couplings
   - Study of moduli space of $U(1)^2$ with non-Abelian enhancement (un-Higgsing)
   -

Type IIB perspective

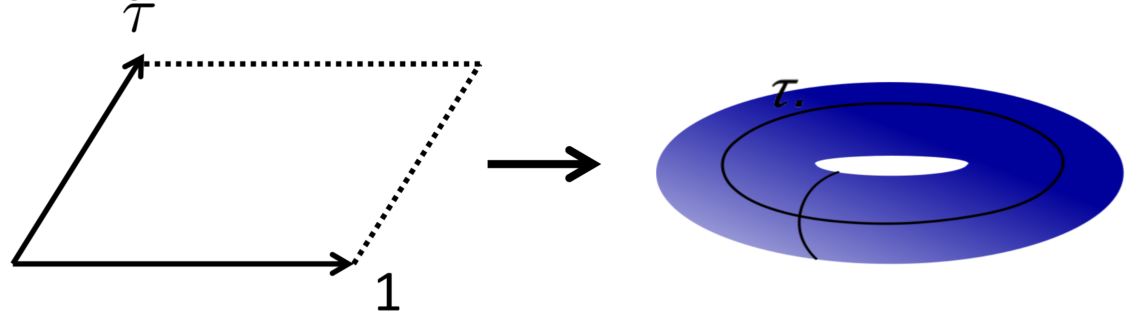
F-THEORY BASIC INGREDIENTS

F-theory Compactification: Basic Ingredients

F-theory geometrizes the (Type IIB) string coupling (axio-dilaton):

$\tau \equiv C_0 + i g_s^{-1}$ as a modular parameter of two-torus $T^2(\tau)$

SL(2,Z)-symm.



[Vafa]

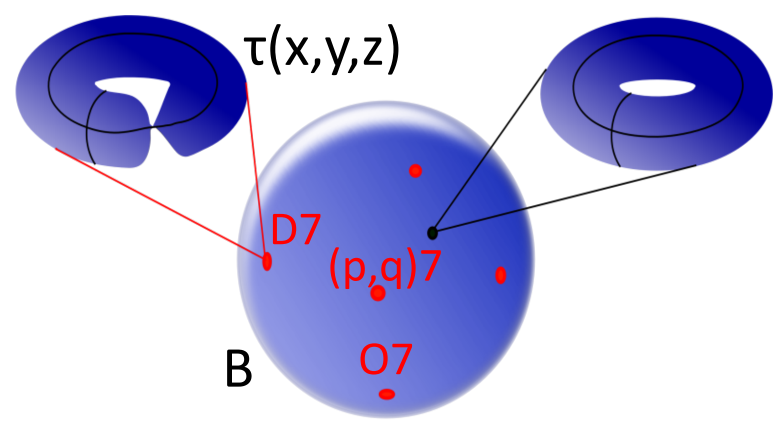
Compactifcation is a two-torus $T^2(\tau)$-fibration over a compact base space B:

Weierstrass form:

$$y^2 = x^3 + f x z^4 + g z^6$$

f, g- function fields on B

[z:x:y] homog. coords on $\mathbf{P}^2(1,2,3)$



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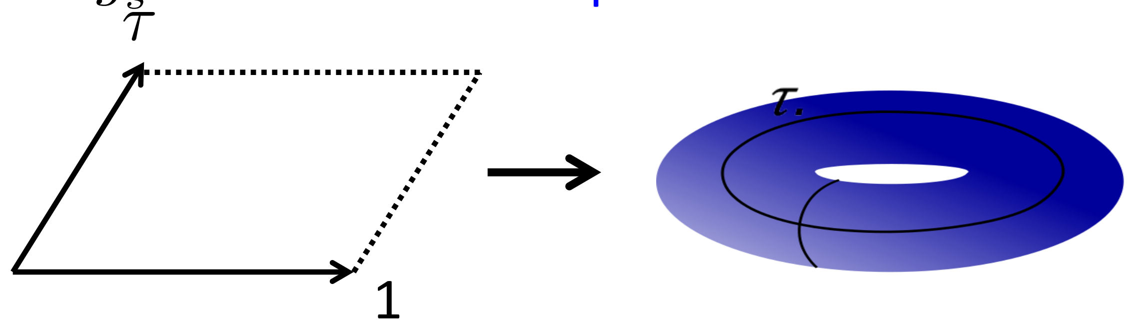

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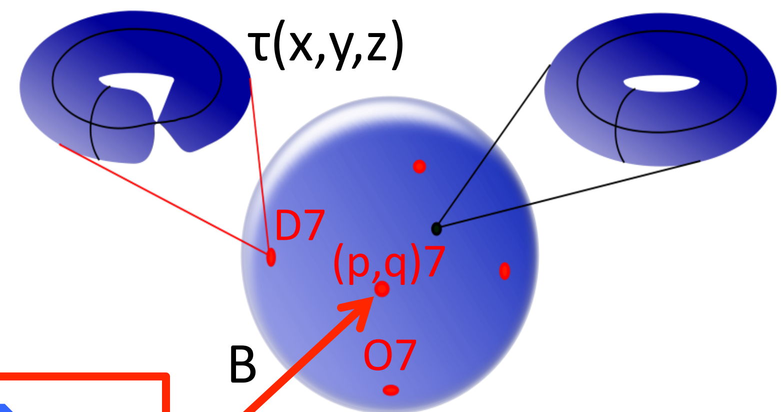


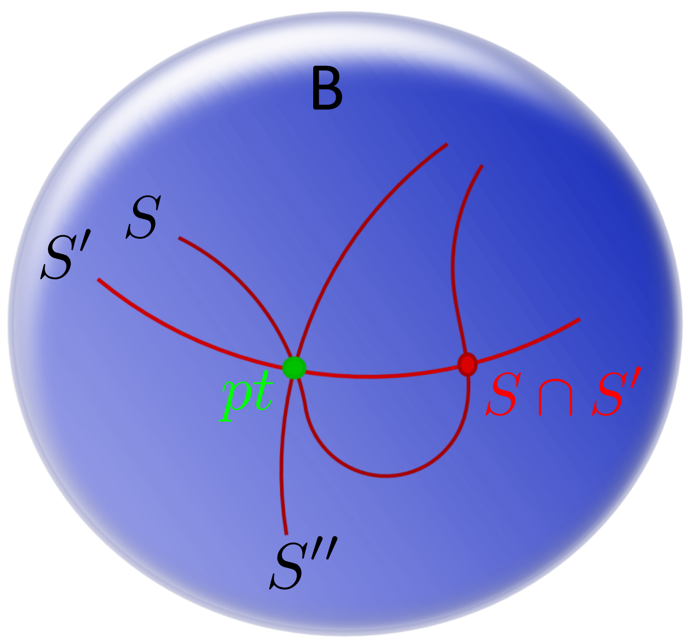
singular $T^2(\tau)$-fibr. → $g_s \to \infty$
location of 7-branes

F-theory Compactification: Basic Ingredients

- Total space of $T^2(\tau)$-fibration: singular elliptic Calabi-Yau manifold X

  D=4, SUSY vacua: fourfold X_4 [D=6, SUSY vacua: threefold X_3]

- X_4-singularities encode complicated set-up of intersecting 7-branes:



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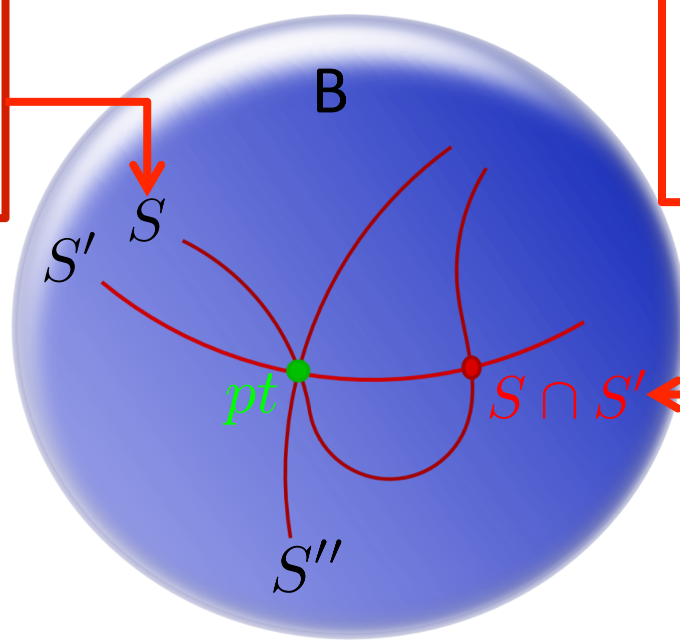

Matter in co-dim.
two in B_3 [Katz,Vafa]
(intersecting 7-branes)

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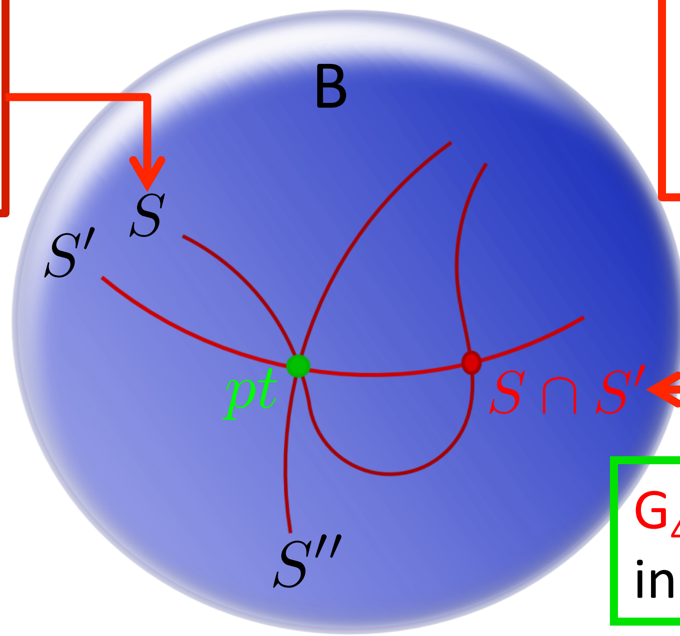

Matter in co-dim. two in B_3 [Katz,Vafa] (intersecting 7-branes)

G_4-flux at intersection induces chiral 4D matter

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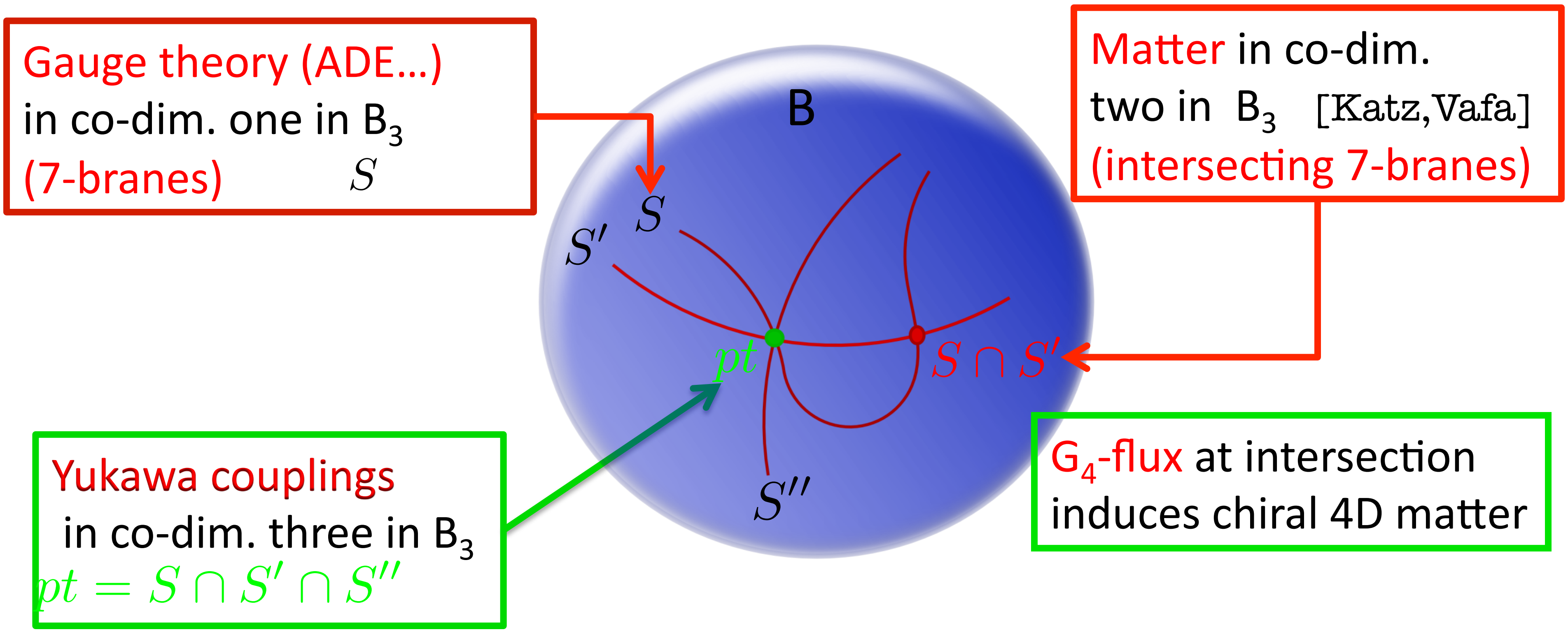


Constructing elliptic fibrations with higher rank Mordell-Weil groups

U(1)-SYMMETRIES IN F-THEORY

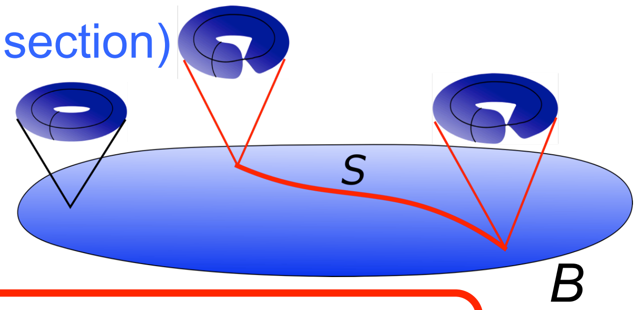
Recall: Non-Abelian Gauge Symmetry

[Kodaira;Tate;Vafa;Morrison,Vafa; Bershadsky,Intriligator,Kachru,Morrison,Sadov,Vafa;...]

1. Weierstrass form for elliptic fibration of X (w/ zero section)

$$y^2 = x^3 + fxz^4 + gz^6$$

2. Severity of singularity along divisor S in B:



$[ord_S(f), ord_S(g), ord_S(\Delta)]$ ⟷ Singularity type of fibration of X

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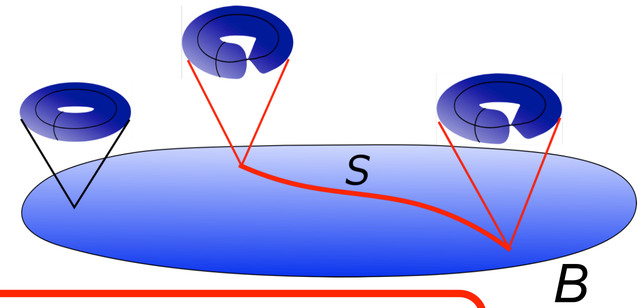
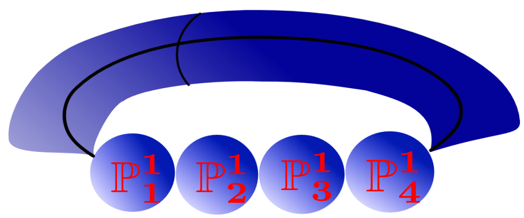


$[ord_S(f), ord_S(g), ord_S(\Delta)]$ ⟷ Singularity type of fibration of *X*

3. Resolution: singularity type ⟷ structure of a tree of $\mathbb{P}^1$'s over *S*

I_n-singularity:



Recent refinements: [Esole,Yau;...;Hayashi,Lawrie,Schäfer-Nameki,Morrison;...]

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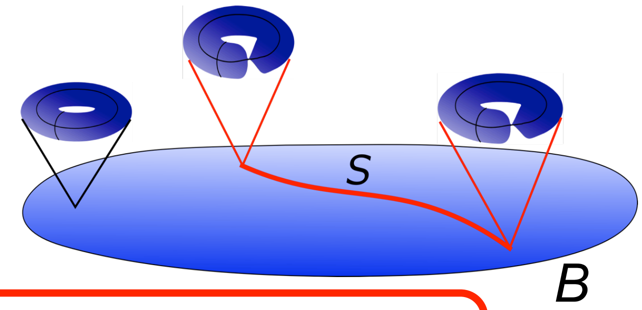

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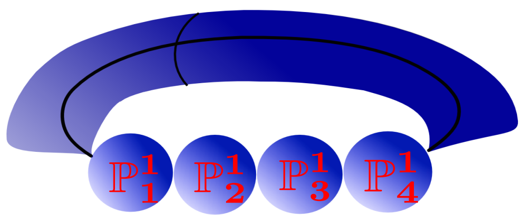

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- Cartan generators for A^i gauge bosons: in M-theory via Kaluza-Klein (KK) reduction of C_3 forms along (1,1)-forms $\omega_i \leftrightarrow \mathbb{P}^1_i$ on X

$$C_3 \supset A^i \omega_i$$

- Non-Abelian generators: light M2-brane excitations on $\mathbb{P}^1$'s [Witten]

U(1)'s-Abelian Symmetry

- U(1)'s gauge bosons A^m should also arise via KK-reduction $C_3 \supset A^m \omega_m$.

(1,1)-forms on X

- Forbid non-Abelian enhancement (by M2's wrapping $\mathbb{P}^1$'s): only I_1-fibers

U(1)'s-Abelian Symmetry & Rational Sections

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(1,1) - form ω_m ⟺ rational section

[Morrison,Vafa II]

U(1)'s-Abelian Symmetry &Mordell-Weil Group

Rational sections of ellip. fibration ➡ Rational points of ellip. curve

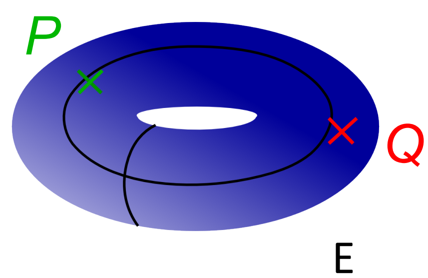
U(1)'s-Abelian Symmetry &Mordell-Weil Group

Rational sections of ellip. fibration ➡ Rational points of ellip. curve

1. Rational point Q on elliptic curve E with zero point P
   - is solution (x_Q, y_Q, z_Q) in field K of Weierstrass form

$$y^2 = x^3 + fxz^4 + gz^6$$

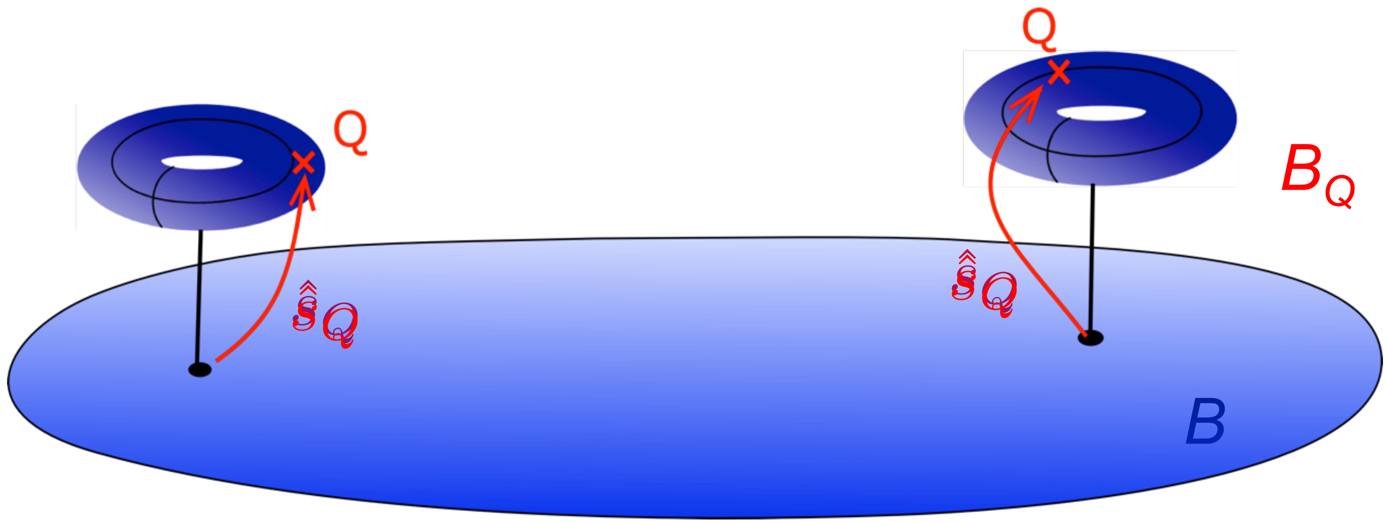
   - Rational points form a group (under addition) on E



➡ Mordell-Weil group of rational points

U(1)'s-Abelian Symmetry &Mordell-Weil Group

2. *Q* on *E* induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration



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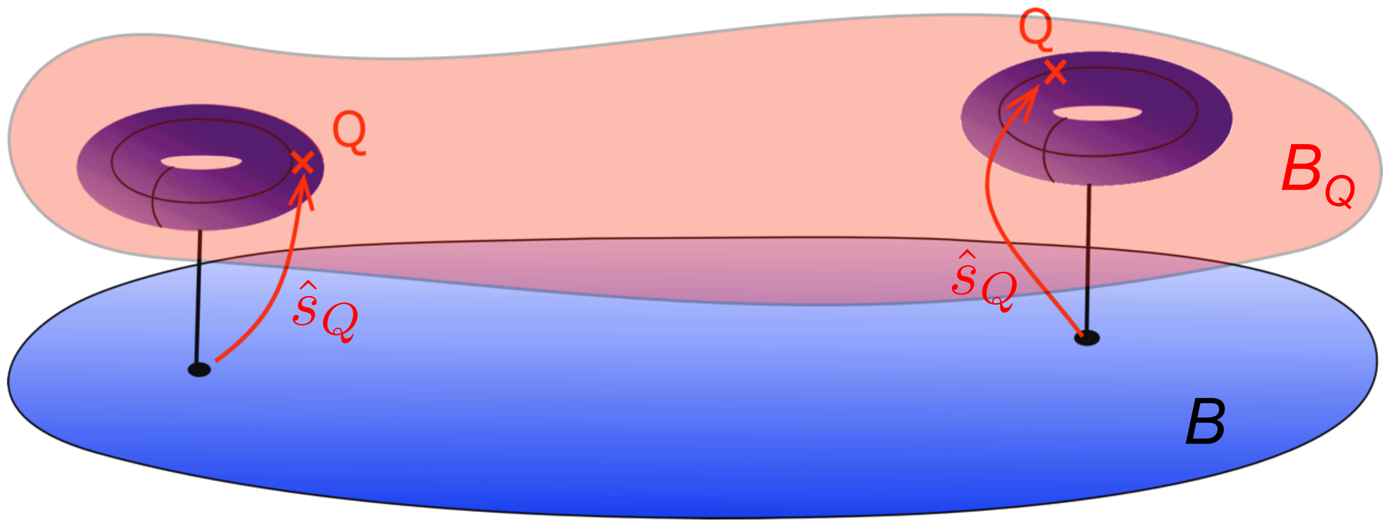

➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

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➡ (1,1)-form ω_m constructed from divisor B_Q (Shioda map)

indeed (1,1) - form ω_m ⟺ rational section

Structure of Elliptic Fibrations with Rational Sections

Consequences for Weierstrass form (WSF) w/ rat. point Q= $[x_Q : y_Q : z_Q]$

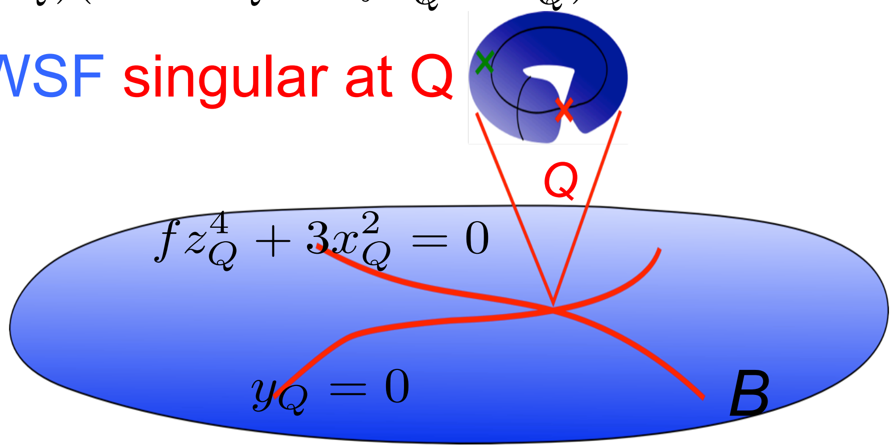
1. Implies constraint relation between *f, g*

$$g z_Q^6 = y_Q^2 - x_Q^3 - f x_Q z_Q^4$$

2. Implies singularity at codimension two in *B*:

-Factorization: $(y - y_Q)(y + y_Q) = (x - x_Q)(x^2 + x_Q x + f z_Q^4 + x_Q^2)$

- Singularity: $y_Q = f z_Q^4 + 3x_Q^2 = 0$ ➡ WSF singular at Q



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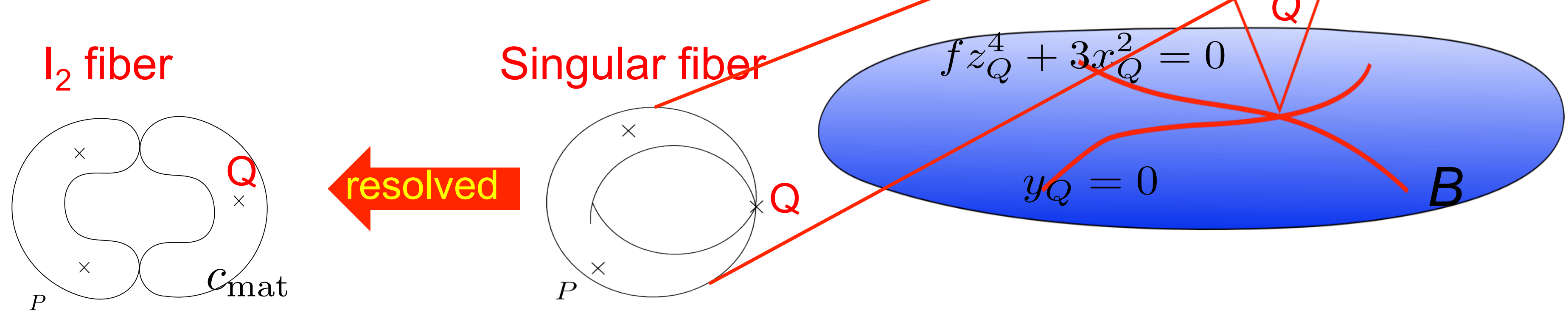

w/isolated (matter) curve

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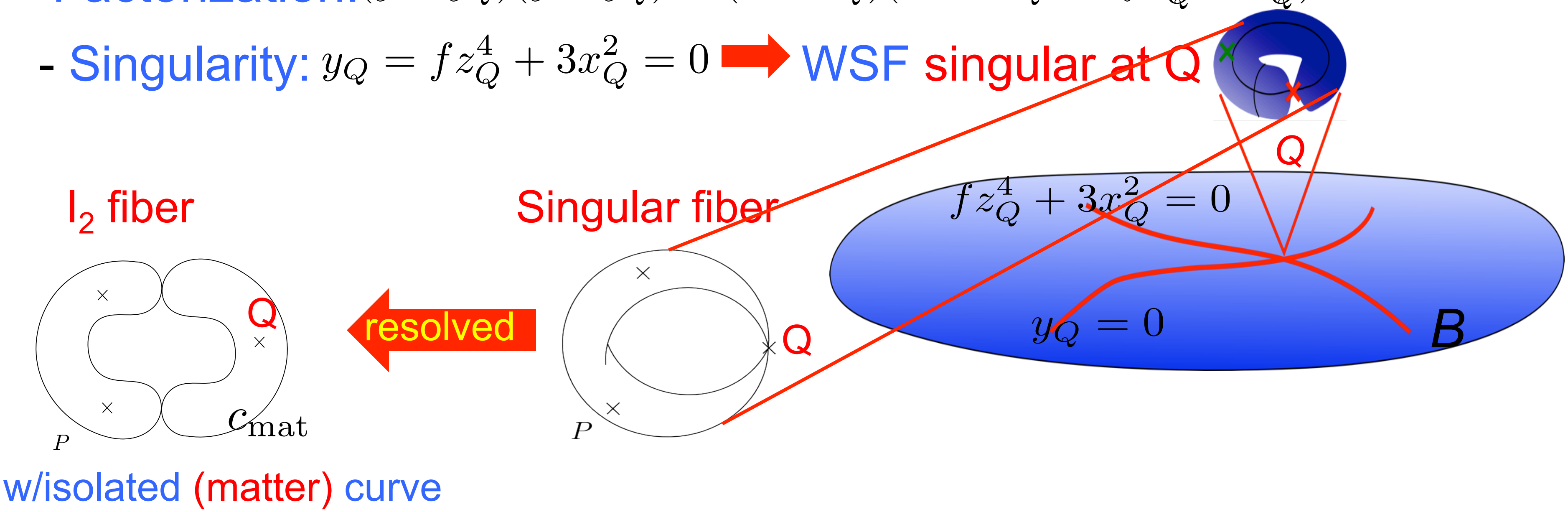


➡ Section $\hat{s}_Q$ implies U(1)-charged matter, though only I_1-fiber in codim. one

Elliptic curves with rank-n Mordell-Weil group

ELLIPTIC FIBRATIONS WITH n RATIONAL SECTIONS

Elliptic Curves E with Rational Points

Elliptic curve with zero point P and *n* rational points Q_i

1. Consider line bundle $M=O(P+Q_1+\ldots+Q_n)$ of degree $n+1$ on E:

   1) $H^0(M)=\langle x_1,\ldots,x_{n+1}\rangle$, $n+1$ sections

   2) $H^0(M^k)= k(n+1)$ sections, $r := \binom{n+k}{k}$ sections known (deg. k monomials in x_i)

   - $r < k(n+1)$: need to introduce new sections in M^k ($n=0, 1$)
   - $r > k(n+1)$: $r-k(n+1)$ relations between sections: E embeddable in $W\mathbb{P}^m$

2. Existence of rational points $Q_1,\ldots, Q_n$:

   E non-generic Calabi-Yau one-fold in $W\mathbb{P}^m$ $=$

   generic Calabi-Yau in blow-up of $W\mathbb{P}^m$ at rational points Q_i

Elliptic Curves E with Rational Points

Elliptic curve with zero point P and n rational points Q_i

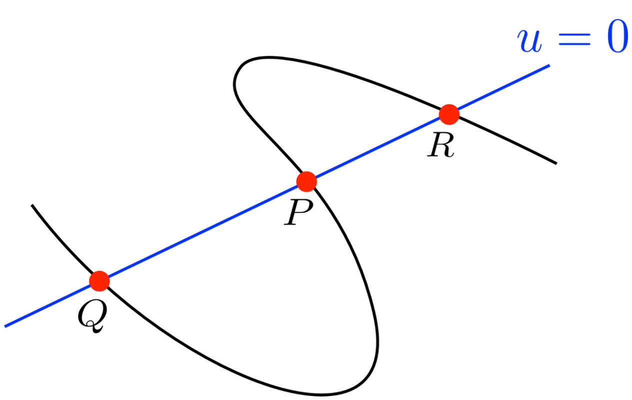
has canonical embedding as Calabi-Yau one-fold in $W\mathbb{P}^m$

Example n=2: points *P, Q, R*

1. *M=O(P+Q+R)* degree three: three sections (*u,v,w*) = $\mathbb{P}^2$-coordinates

   → *E* is cubic curve in $\mathbb{P}^2[u:v:w]$

2. Existence of points *P, Q, R:* *E* non-generic cubic has to factorize as



$$uf_2(u,v,w) + \prod_{i=1}^{3}(a_i v + b_i w) = 0$$

Degree two polynomial $f_2(u,v,w)$

E generic Calabi-Yau in blow-up of $\mathbb{P}^2$ at points *Q, R* = dP_2

Explicit Examples

n=0: Tate form in $\mathbb{P}^2(1,2,3)$

n=1: E with *P, Q* is generic CY in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$ [Morrison,Park]

n=2: E with *P, Q, R* is generic CY in dP_2 [Borchmann,Mayerhofer,Palti,Weigand; M.C.,Klevers,Piragua]

n=3: E with *P, Q, R, S* is CICY in $\mathrm{Bl}_3\mathbb{P}^3$ [M.C.,Klevers,Piragua,Song]

n=4 is determinantal variety in $\mathbb{P}^4$ work in progress: [M.C.,Klevers,Piragua,Song]

higher *n*, not clear…

Illustration: dP_2-elliptic fibrations with two rational sections

ENGINEERING F-THEORY WITH $U(1)^2$

Elliptic curve with rk(MW)=2: concrete example

[M.C., Klevers,Piragua]

Elliptic curve E with two rational points Q, R

Consider line bundle M=O(P+Q+R) of degree 3 on E (non-generic cubic in **P**^{2})

➡ natural representation as hypersurface p=0 in del Pezzo dP_2

$$p = u(s_1 u^2 e_1^2 e_2^2 + s_2 u v e_1 e_2^2 + s_3 v^2 e_2^2 + s_5 u w e_1^2 e_2 + s_6 v w e_1 e_2 + s_8 w^2 e_1^2) + s_7 v^2 w e_2 + s_9 v w^2 e_1$$

$[u:v:w:e_1:e_2]$ –homogeneous coordinates of dP_2

(blow-up of **P**^{2} w/ [u':v':w'] at 2 points: $u'=ue_1e_2$, $v'=ve_2$, $w'=we_1$)

$$\begin{array}{lllllll} & & u & v & w & e_1 & e_2 \\ P: & E_2 \cap p = [& -s_9 : & s_8 : & 1 : & 1 : & 0] \,, \\ Q: & E_1 \cap p = [& -s_7 : & 1 : & s_3 : & 0 : & 1] \,, \\ R: & D_u \cap p = [& 0 : & 1 : & 1 : & -s_7 : & s_9] \,. \end{array}$$

Points represented by intersections of different divisors in dP_2 with p

Classification of dP_2 elliptic fibrations

[M.C., Klevers, Piragua; M.C.,Grassi,Klevers,Piragua]

I. Ambient space:

– dP_2 fibration determined by two divisors $\mathcal{S}_7$ and $\mathcal{S}_9$ (loci of $s_7=0, s_9=0$)

$$dP_2 \longrightarrow dP_2^B(\mathcal{S}_7, \mathcal{S}_9) \\ \downarrow \\ B$$

II. Calabi-Yau hypersurface X:

– cuts out E in dP_2

– coefficients s_i in CY-equation get lifted to sections of the base B (only s_7, s_9 dependent)

– coordinates $[u:v:w:e_1:e_2]$ lifted to sections

$$\hat{E} \subset dP_2 \longrightarrow X \\ \downarrow \\ B$$

sections $\hat{s}_P, \hat{s}_Q, \hat{s}_R$

Classification of dP_2 elliptic fibrations

Construction of general elliptic fibrations:

section	bundle
u'	$\mathcal{O}(H - E_1 - E_2 + \mathcal{S}_9 + [K_B])$
v'	$\mathcal{O}(H - E_2 + \mathcal{S}_9 - \mathcal{S}_7)$
w'	$\mathcal{O}(H - E_1)$
e_1	$\mathcal{O}(E_1)$
e_2	$\mathcal{O}(E_2)$

section	bundle
s_1	$\mathcal{O}(3[K_B^{-1}] - \mathcal{S}_7 - \mathcal{S}_9)$
s_2	$\mathcal{O}(2[K_B^{-1}] - \mathcal{S}_9)$
s_3	$\mathcal{O}([K_B^{-1}] + \mathcal{S}_7 - \mathcal{S}_9)$
s_5	$\mathcal{O}([2K_B^{-1}] - \mathcal{S}_7)$
s_6	$\mathcal{O}([K_B^{-1}])$
s_7	$\mathcal{O}(\mathcal{S}_7)$
s_8	$\mathcal{O}([K_B^{-1}] + \mathcal{S}_9 - \mathcal{S}_7)$
s_9	$\mathcal{O}(\mathcal{S}_9)$

– CY-condition: $\mathcal{S}_7$ and $\mathcal{S}_9$ fixed

Engineer non-Abelian groups: make s_i non-generic

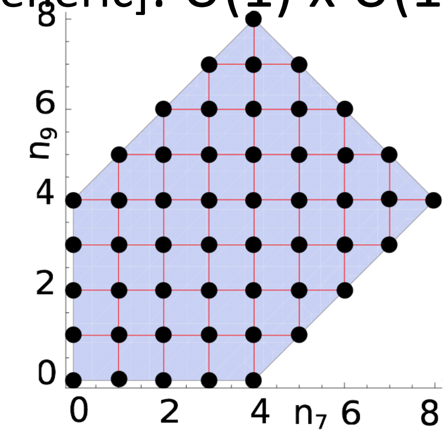
Construction of CY Elliptic Fibrations

Classify all vacua with fixed E in dP_2 & chosen base B in D=6 and D=4

Example: D=4, $B = \mathbb{P}^3$

1. X generic [all s_i exist, generic]: U(1) x U(1)

$$\mathcal{S}_7 = n_7 H_{\mathbb{P}^3}$$
$$\mathcal{S}_9 = n_9 H_{\mathbb{P}^3}$$

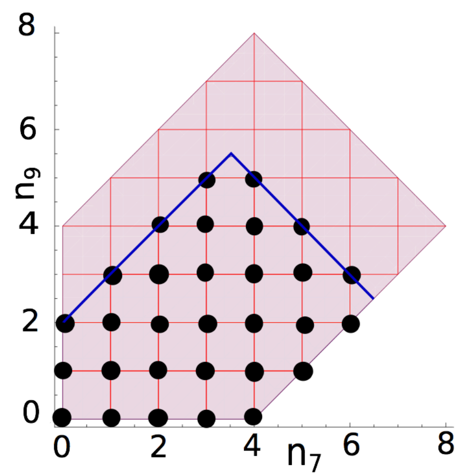


Can construct and study entire family of CY's explicitly

:

2. X non-generic [$s_{i,}$ realize SU(5) at t=0]: SU(5) x U(1) x U(1)

$$s_1 = t^3 s_1'$$
$$s_2 = t^2 s_2'$$
$$s_3 = t^2 s_3'$$
$$s_5 = t s_5'$$



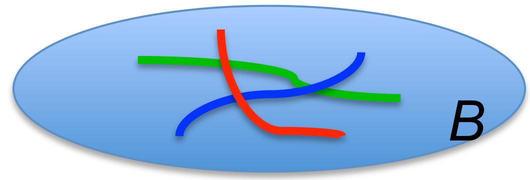
Codimension two singularities

MATTER IN F-THEORY WITH U(1)'s

Matter in General Geometries with U(1)'s

[M.C.,Klevers,Piragua;M.C.,Grassi,Klevers,Piragua]

Charged matter at codim-two singularities in B



Matter locus : $y_Q = f z_Q^4 + 3x_Q^2 = 0$

- Highly reducible variety:

Individual matter loci ⟺ irreducible components = associated prime ideals

- Ideal techniques: irreducible components ⟺ prime ideals (of polys y_Q, f-high degree)

Matter in General Geometries with U(1)'s

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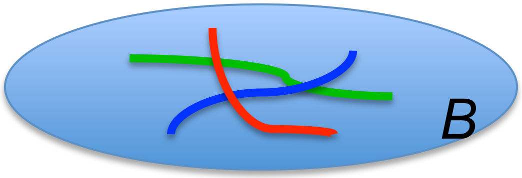

Matter locus : $y_Q = f z_Q^4 + 3x_Q^2 = 0$

- Highly reducible variety:

Individual matter loci ⟺ irreducible components = associated prime ideals

- Ideal techniques: irreducible components ⟺ prime ideals (of polys y_Q, f-high degree)

Identify matter at distinguished loci:

Type A: matter at loci in *B* where sections are ill-defined

Type B: matter at loci characterized by additional constraints

➡ matter with multiple U(1)-charges

- Leads to all prime ideals for dP_2-fibrations w/ SU(5) [M.C.,Klevers,Langacker,Piragua]

Illustration: Codimension two singularities of dP_2-elliptic fibrations

MATTER FOR F-THEORY VACUA: $U(1)^2$

Summary of Matter Representations

	$U(1) \times U(1)$
Type B	$(1,0)$ $(0,1)$ $(1,\ \cdot 1)$
Type A	$(-1,1)$ $(0,2)$ $(-1,-2)$

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Apply analogous techniques to determine matter representation

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Apply analogous techniques to determine matter representation

Specific example:

$$\begin{aligned} s_1 &= t^3 s_1' \\ s_2 &= t^2 s_2' \\ s_3 &= t^2 s_3' \\ s_5 &= t s_5' \end{aligned}$$

w/ SU(5) at t=0

Summary of Matter Representations

	$U(1)\times U(1)$	$SU(5)\times U(1)\times U(1)$
Type B	$(1,0)\ (0,1)\ (1,\ \cdot 1)$	$(\mathbf{5},-\frac{2}{5},0)\ (\mathbf{5},\frac{3}{5},0)\ (\mathbf{5},-\frac{2}{5},-1)$
Type A	$(-1,1)\ (0,2)\ (-1,-2)$	$(\mathbf{5},-\frac{2}{5},1)\ (\mathbf{5},\frac{3}{5},1)\ (\overline{\mathbf{10}},-\frac{1}{5},0)$

X non-generic → realize SU(5) x U(1) x U(1)

Apply analogous techniques to determine matter representation

Specific example:

$$\begin{aligned} s_1 &= t^3 s_1' \\ s_2 &= t^2 s_2' \\ s_3 &= t^2 s_3' \\ s_5 &= t s_5' \end{aligned}$$

w/ SU(5) at t=0

D=6 Matter Spectrum & Multiplicities

Closed formula for D=6 matter multiplicities for entire class of F-theory vacua over any base *B*

Example: B=$\mathbb{P}^2$

	(q_1, q_2)	Multiplicity
Type B	$(1,0)$	$54 - 15n_9 + n_9^2 + (12 + n_9)\, n_7 - 2n_7^2$
Type B	$(0,1)$	$54 + 2\left(6n_9 - n_9^2 + 6n_7 - n_7^2\right)$
Type B	$(1,1)$	$54 + 12n_9 - 2n_9^2 + (n_9 - 15)\, n_7 + n_7^2$
Type A	$(-1,1)$	$n_7\,(3 - n_9 + n_7)$
Type A	$(0,2)$	$n_9 n_7$
Type A	$(-1,-2)$	$n_9\,(3 + n_9 - n_7)$

Integers n_7, n_9 specify all dP_2-fibration over $\mathbb{P}^2$

Full spectrum and multiplicities also with SU(5)xU(1)x(1) group

Consistency check: spectrum cancels D=6 anomalies in quantum field theory

D=4: Matter surfaces, G_4-flux

MATTER SPECTRUM IN D=4

No details here

Construction of G_4 flux – $H_V^{(2,2)}(X. Z/2)$ → determine chiralities
First explicit construction of G_4 for B= $\mathbb{P}^3$; Matter curves
Constraints from D=3 M-/F-theory duality

D=4 Spectrum

Example B= $\mathbb{P}^3$ with U(1) x U(1): most general solution for G_4-flux [a_i]

$$G_4 = a_5 n_9 \left(4 - n_7 + n_9\right) H_B^2 + 4 a_5 H_B S_P + a_3 H_B \sigma(\hat{s}_Q) + a_4 H_B \sigma(\hat{s}_R) + a_5 S_P^2$$

(q_1, q_2)	D=4 chiralities
$(1,0)$	$\frac{1}{4}\left[a_5 n_7 n_9 \left(4 - n_7 + n_9\right) + a_3 \left(2n_7^2 - \left(12 - n_9\right)\left(8 - n_9\right) - n_7 \left(16 + n_9\right)\right)\right]$
$(0,1)$	$\frac{1}{2}\left[a_5 n_9 \left(4 - n_7 + n_9\right)\left(12 - n_9\right) - a_4 \left(n_7 \left(8 - n_7\right) + \left(12 - n_9\right)\left(4 + n_9\right)\right)\right]$
$(1,1)$	$\frac{1}{4}\left[2a_5 n_9 (4 - n_7 + n_9)(12 - n_9) - (a_3 + a_4)(n_7^2 + n_7(n_9 - 20) + 2(12 - n_9)(4 + n_9))\right]$
$(-1,1)$	$\frac{1}{4}\left(a_3 - a_4\right) n_7 \left(4 + n_7 - n_9\right)$
$(0,2)$	$\frac{1}{4} n_7 n_9 (-2a_4 + a_5(4 - n_7 + n_9))$
$(-1,-2)$	$-\frac{1}{4} n_9 (n_7 - n_9 - 4)(a_3 + 2a_4 + a_5(n_7 - 2n_9))$

All D=4 anomalies cancelled - employing [Hayashi,Grimm;Grimm,M.C.,Klevers]

D=4 Spectrum

Example B= $\mathbb{P}^3$ with U(1) x U(1): most general solution for G_4-flux [a_i]

$$G_4 = a_5 n_9 \left(4 - n_7 + n_9\right) H_B^2 + 4 a_5 H_B S_P + a_3 H_B \sigma(\hat{s}_Q) + a_4 H_B \sigma(\hat{s}_R) + a_5 S_P^2$$

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Same methods for SU(5)xU(1)xU(1) applied:

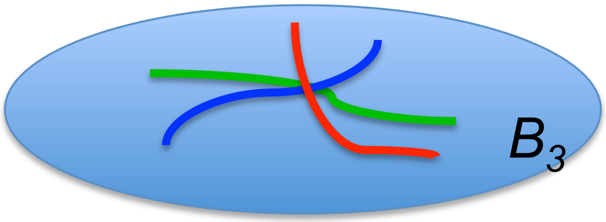
G_4-flux has 7 parameter; all D=4 chiralities determined; anomalies checked;

Chirality checked against [Type B] matter geometric calculations

Yukawa Couplings in GUT's & U(1)'s

[M.C., Klevers, Langacker, Piragua]-to appear

D=4 Yukawa couplings at codimension three:



Intersections of matter curves ⟺ Calculations of prime ideals

Results for $U(1)^2$ & specific $SU(5)xU(1)^2$:

All gauge invariant couplings geometrically realized!

➡ Miraculous structure at co-dimension three of fibration

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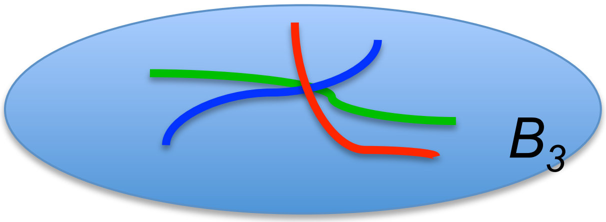

Intersections of matter curves ⟺ Calculations of prime ideals

Results for $U(1)^2$ & specific $SU(5)xU(1)^2$:

All gauge invariant couplings geometrically realized!

➡ Miraculous structure at co-dimension three of fibration

Application: Exploration of gauge symmetry enhancement (un-Higgsing)

specific $SU(5)xU(1)^2$ → rank-preserving non-Abelian symmetry

D=4 examples of spectrum and couplings, fit into SU(6)xSU(2);SO(10)xU(1);$\subset E_6$

[but NOT into SU(7)] Primarily field theory analysis; further study of geometry

[M.C.,Klevers,Piragua,Langacker]

Application: $U(1)^3$

Rank 3 MORDEL-WEIL ELLIPTIC FIBRATIONS

E as a complete intersection CY in $\mathrm{Bl}_3\mathbb{P}^3$

Classify Calabi-Yau elliptic fibrations of *E* over given base B

Matter multiplicity involved: determinantal varieties

No details here

D=6 Matter & Multiplicity for $U(1)^3$

[Calabi-Yau elliptic fibrations of E over given base B classified]

Charges	Multiplicities
(1,1,-1)	$[s_8]\cdot[s_{18}]$
(0,1,2)	$[s_9]\cdot[s_{19}]$
(1,0,2)	$[s_{10}]\cdot[s_{20}]$
(-1,0,1)	$[\tilde{s}_3]\cdot\tilde{\mathcal{S}}_7-[s_8]\cdot[s_{18}]$
(0,-1,1)	$[\hat{s}_3]\cdot\hat{\mathcal{S}}_7-[s_8]\cdot[s_{18}]$
(-1,-1,-2)	$[\tilde{s}_8]\cdot\mathcal{S}_9-[s_{10}]\cdot[s_{20}]$
(0,0,2)	$\tilde{\mathcal{S}}_7\cdot\mathcal{S}_9-[s_{19}][s_9]$
tri-fundamental (non-pert.)! (1,1,1)	$4[K_B^{-1}]^2-3([p_2]^b)^2-2[K_B^{-1}]\hat{\mathcal{S}}_7-3([p_2]^b)\hat{\mathcal{S}}_7-2[K_B^{-1}]\tilde{\mathcal{S}}_7-3([p_2]^b)\tilde{\mathcal{S}}_7$ $-2\hat{\mathcal{S}}_7\tilde{\mathcal{S}}_7+2[K_B^{-1}]\mathcal{S}_9+9([p_2]^b)\mathcal{S}_9+5\hat{\mathcal{S}}_7\mathcal{S}_9+5\tilde{\mathcal{S}}_7\mathcal{S}_9-8\mathcal{S}_9^2$
(1,1,0)	$2[K_B^{-1}]^2+3([p_2]^b)^2+2[K_B^{-1}]\hat{\mathcal{S}}_7+3([p_2]^b)\hat{\mathcal{S}}_7+2[K_B^{-1}]\tilde{\mathcal{S}}_7+3([p_2]^b)\tilde{\mathcal{S}}_7$ $+\hat{\mathcal{S}}_7\tilde{\mathcal{S}}_7-3[K_B^{-1}]\mathcal{S}_9-9([p_2]^b)\mathcal{S}_9-4\hat{\mathcal{S}}_7\mathcal{S}_9-4\tilde{\mathcal{S}}_7\mathcal{S}_9+7\mathcal{S}_9^2$
(1,0,1)	$2[K_B^{-1}]^2+3([p_2]^b)^2+2[K_B^{-1}]\hat{\mathcal{S}}_7+3([p_2]^b)\hat{\mathcal{S}}_7-3[K_B^{-1}]\tilde{\mathcal{S}}_7+3([p_2]^b)\tilde{\mathcal{S}}_7$ $+2\hat{\mathcal{S}}_7\tilde{\mathcal{S}}_7+\tilde{\mathcal{S}}_7^2+2[K_B^{-1}]\mathcal{S}_9-9([p_2]^b)\mathcal{S}_9-5\hat{\mathcal{S}}_7\mathcal{S}_9-4\tilde{\mathcal{S}}_7\mathcal{S}_9+6\mathcal{S}_9^2$
(0,1,1)	$2[K_B^{-1}]^2+3([p_2]^b)^2-3[K_B^{-1}]\hat{\mathcal{S}}_7+3([p_2]^b)\hat{\mathcal{S}}_7+\hat{\mathcal{S}}_7^2+2[K_B^{-1}]\tilde{\mathcal{S}}_7$ $+3([p_2]^b)\tilde{\mathcal{S}}_7+2\hat{\mathcal{S}}_7\tilde{\mathcal{S}}_7+2[K_B^{-1}]\mathcal{S}_9-9([p_2]^b)\mathcal{S}_9-4\hat{\mathcal{S}}_7\mathcal{S}_9-5\tilde{\mathcal{S}}_7\mathcal{S}_9+6\mathcal{S}_9^2$
(1,0,0)	$4[K_B^{-1}]^2-3([p_2]^b)^2-2[K_B^{-1}]\hat{\mathcal{S}}_7-3([p_2]^b)\hat{\mathcal{S}}_7+2[K_B^{-1}]\tilde{\mathcal{S}}_7-3([p_2]^b)\tilde{\mathcal{S}}_7$ $-\hat{\mathcal{S}}_7\tilde{\mathcal{S}}_7-2\tilde{\mathcal{S}}_7^2-2[K_B^{-1}]\mathcal{S}_9+9([p_2]^b)\mathcal{S}_9+4\hat{\mathcal{S}}_7\mathcal{S}_9+5\tilde{\mathcal{S}}_7\mathcal{S}_9-6\mathcal{S}_9^2$
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(0,0,1)	$4[K_B^{-1}]^2-4([p_2]^b)^2+2[K_B^{-1}]\hat{\mathcal{S}}_7-4([p_2]^b)\hat{\mathcal{S}}_7-2\hat{\mathcal{S}}_7^2+2[K_B^{-1}]\tilde{\mathcal{S}}_7-4([p_2]^b)\tilde{\mathcal{S}}_7$ $-2\hat{\mathcal{S}}_7\tilde{\mathcal{S}}_7-2\tilde{\mathcal{S}}_7^2+2[K_B^{-1}]\mathcal{S}_9+12([p_2]^b)\mathcal{S}_9+6\hat{\mathcal{S}}_7\mathcal{S}_9+6\tilde{\mathcal{S}}_7\mathcal{S}_9 s-10\mathcal{S}_9^2$

Application: un-Higgsing of $U(1)^2$

NON-ABELIAN GAUGE ENHANCMENT of U(1)'s

Non-Abelian Gauge Enhancement

Elliptic fibrations with higher rank Mordell-Weil group crucial for understanding the moduli space of F-theory compactifications

➡ Study un-Higgsing in complex structure moduli space:

enhancement of U(1)'s → to non-Abelian symmetry

Rank 1 case understood: [Morrison, Taylor]

D=6 F-theory with single U(1) un-Higgses to SU(2)

U(1) ⇄ SU(2) on Riemann surface Σ_g

(→ tuning comp. str. moduli; ← adjoint matter VEV)

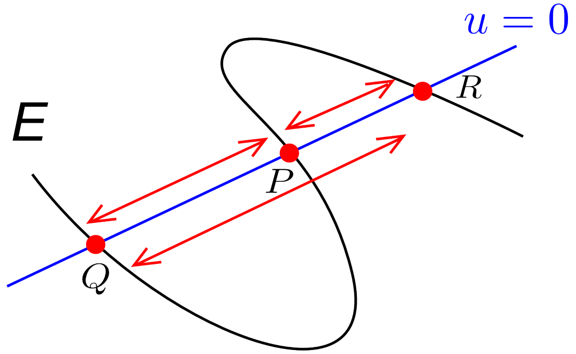
Geometric: transition of vertical divisor into rational section

Non-Abelian Gauge Enhancement:U(1)^2

[M.C.,Klevers,Piragua,Taylor]

Enhancement of U(1)xU(1): richer structure

Reduce MW-rank to zero by merging rational points *Q, R* with zero *P*



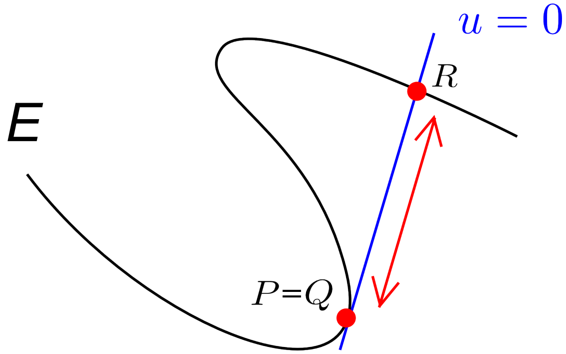
$$u f_2(u,v,w) + \prod_{i=1}^{3}(a_i v + b_i w) = 0$$

Non-Abelian Gauge Enhancement:U(1)^2

[M.C.,Klevers,Piragua,Taylor]

Enhancement of U(1)xU(1): richer structure

Reduce MW-rank to zero by merging rational points *Q, R* with zero *P*



$$uf_2(u,v,w)+\lambda_1(a_1v+b_1w)^2(a_3v+b_3w)=0$$

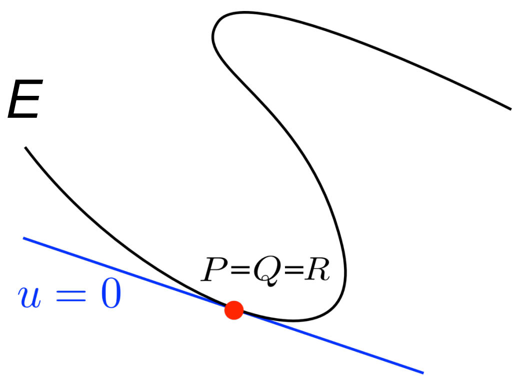
- rk(MW)=2 → 1 as $\overline{PQ}$ → 0

Non-Abelian Gauge Enhancement:U(1)^2

[M.C.,Klevers,Piragua,Taylor]

Enhancement of U(1)xU(1): richer structure

Reduce MW-rank to zero by merging rational points *Q, R* with zero *P*



$$uf_2(u,v,w) + \lambda_1\lambda_2(a_1 v + b_1 w)^3 = 0$$

- rk(MW)=2 → 1 as $\overline{PQ}$ → 0
- rk(MW)=1 → 0 as $\overline{PR}$ → 0

Tuned fibration with codimension one singularity built in:

1. U(1)*x*U(1) → SU(3): set $\lambda_i = 1$ at locus $f_2(0,-b_1,a_1) = 0$ in *B*

   ➡ I_3-singularity at P

2. U(1)*x*U(1) → SU(2)*x*SU(2): set $f_2(0,-b_1,a_1) = 1$

   ➡ I_2-fiber at $\lambda_i = 0$ in *B*: $uf_2(u,v,w) = 0$

3. General case not rank preserving: U(1)^2 → SU(3)*x*SU(2)^2

Summary

- Systematic construction of elliptic fibrations w/ Mordel-Weil groups
  (explicit n=2,3 w/ general: U(1)x U(1) and U(1)xU(1)xU(1) [& w/ SU(5)]

- Develop techniques (general):
  - D=6 matter presentations, multiplicity
  - D=4 Yukawa couplings & chirality (G_4 flux $-H_V^{(2,2)}(X,Z/2)$; constraints)

  From geometry [Determinantal variety techniques]:

  miraculous structure of codim. 2 and 3 singularities: tri-fund. reps., couplings,…

- Applications:
  - explicit rank 2 (hypersurface in dP_2): $U(1)^2$ [& w/ SU(5)]
  - explicit rank 3 (complete intersections in $\mathrm{Bl}_3(\mathbb{P}^3)$): $U(1)^3$
  - study of un-Higgsing of $U(1)^2$ [w/ SU(5)] → non-Abelian gauge symmetries

Outlook

- D=4 global SM/GUT models w/ U(1): general base; SUSY conditions, quantization of G_4 flux,... ➡ Particle Physics Implications

  [M.C.,Klevers,Langacker,Piragua] - in progress

- n>3: explicit construction for n=4 [M.C.,Klevers,Piragua,Song]- in progress
- Comprehensive study of moduli space for un-Higgsing:

  $U(1)^n \rightarrow$ non-Abelian gauge theory enhancement

  [M.C.,Klevers,Piragua,Taylor] - in progress

- Study of heterotic duals of F-theory with U(1)'s

  [M.C.,Grassi,Klevers,Piragua,Song]- in progress

Outlook

- F-theory without zero section → Discrete symmetries (Tate-Shafarevich Group)

- [Braun,Morrison],[Morisson,Taylor],[Anderson,Garcia-Extebarria Grimm], [Klevers,Pena,Oehlmann,Piragua,Reuter] (beyond Z_2) **c.f., Klever's talk;** [Garcia-Extebarria, Grimm,Keitel],[Mayrhofer,Palti,Till,Weigad] (Z_2) **c.f., Mayrhofer's talk**

→ work in progress

Mordell-Weil meets Tate-Shafarevich !