

Achilleas Passias | [Milano-Bicocca University](#)



# Advances in $\text{AdS}_6/\text{CFT}_5$ correspondence

based on [arXiv:1209.3267 \[hep-th\]](#) and [arXiv:1406.0852 \[hep-th\]](#)  
in collaboration with [F. Apruzzi](#), [M. Fazzi](#), [D. Rosa](#) and [A. Tomasiello](#)

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## First Period of Activity: 1996-2000

[Seiberg '96] First examples of [five-dimensional supersymmetric fixed points](#) with global symmetry  $E_{N_f+1}$ <sup>1</sup>,  $N_f \leq 7$ , obtained in the limit of infinite bare coupling of an  $\mathcal{N} = 1$  supersymmetric gauge theory with  $Sp(1)$  gauge group and  $N_f$  “quark” hypermultiplets in the fundamental representation.

This theory appears as the effective field theory on the worldvolume of a [D4-brane](#) near  $N_f$  D8-branes on top of a single O8 orientifold plane in type I' string theory.

In the case of multiple D4-branes the gauge group becomes  $Sp(N)$  and there is an extra massless hypermultiplet in the anti-symmetric representation.

These theories also appear as effective field theories on a [web of  \$\(p, q\)\$  5- and 7-branes](#) in type IIB string theory [DeWolfe, Hanany, Iqbal, Katz '99]

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<sup>1</sup> $E_5 = Spin(10)$ ,  $E_4 = SU(5)$ ,  $E_3 = SU(3) \times SU(2)$ ,  $E_2 = SU(2) \times U(1)$ ,  $E_1 = SU(2)$

## First Period of Activity: 1996-2000

[Ferrara, Kehagias, Zaffaroni '98] The fixed points correspond to  $F(4)$  gauged supergravity coupled to matter vector multiplets<sup>2</sup> in the adjoint representation of  $E_{N_f+1}$ .<sup>3</sup> Evidence for a **D4-D8-brane** configuration in massive type IIA supergravity, with an **AdS<sub>6</sub> near horizon geometry**, preserving 8 supercharges.

[Brandhuber, Oz '99] A **warped AdS<sub>6</sub> × S<sup>4</sup> solution** of massive type IIA supergravity is found, arising as the near horizon limit of a localised D4-D8-brane configuration.

[Cvetic, Lu, Pope '99]  $F(4)$  gauged supergravity is obtained upon **Kaluza-Klein reduction** of massive IIA supergravity **on S<sup>4</sup>**.

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<sup>2</sup> $F(4)$  gauged supergravity coupled to vector multiplets was constructed in [D'Auria, Ferrara, Vaulá '00].

<sup>3</sup> $F(4)$  is the unique anti-deSitter superalgebra in six dimensions.

## The Brandhuber-Oz solution

Near horizon geometry of a localised D4-D8-brane configuration with  $N_f$  D8-branes located to one O8 plane and  $16 - N_f$  D8-branes to the other O8 plane.

$$ds^2 = e^{-\frac{\phi}{10}} N^{\frac{3}{5}} C^{-\frac{2}{5}} \left( \frac{9}{4} ds_{\text{AdS}_6}^2 + d\alpha^2 + \cos^2 \alpha d\Omega_3^2 \right)$$

$$F_4 = \frac{5}{12} N^{\frac{9}{10}} C^{-\frac{3}{5}} e^{-\frac{2}{5}\phi} \cos^3 \alpha d\alpha \wedge \text{vol}_{S^3}, \quad e^\phi = N^{-\frac{1}{4}} C^{\frac{1}{6}} (3F_0 \sin \alpha)^{-\frac{5}{6}}$$

- The space has a boundary at  $\alpha = 0$  which corresponds to the location of the O8 orientifold plane. The dilaton and the curvature diverge at the boundary.
- The  $SO(5)$  isometry of  $S^4$  is reduced to  $SO(4) \simeq SU(2) \times SU(2)$  corresponding to the  $SU(2)_R$  R-symmetry and to an  $SU(2)$  global symmetry of the dual field theory.

## Second Period of Activity 2012 -

[Bergman, Rodríguez-Gómez '12] D4-branes at orbifold singularities; supersymmetric [quiver gauge theories](#) (the gauge group is a product of symplectic and unitary groups) dual to  $AdS_6 \times S^4/\mathbb{Z}_p$  backgrounds of massive type IIA supergravity.

[Jafferis, Pufu '12]  $S^5$  [partition function](#) of large  $N$  superconformal field theories (localization); [free energy](#) of the dual gravity background (entanglement entropy); [agreement](#) between the two sides and reproduction of the  $N^{\frac{5}{2}}$  scaling.

large  $N$  limit of vevs of [Wilson loops](#) [Assel, Estes, Yamazaki '12] and [superconformal indices](#) [Kim, Kim, Lee '12],[Bergman, Rodríguez-Gómez, Zafrir '13] on both sides of the  $AdS_6/CFT_5$  correspondence.

RG flows [Karndumri '12],[Pini, Rodríguez-Gómez '14], giant gravitons in  $AdS_6$  [Bergman, Rodríguez-Gómez '12] ...

# Supersymmetric AdS<sub>6</sub> Solutions

## type IIA (massive)

Brandhuber-Oz solution and orbifolds thereof; proven to be unique. [Passias '12]

## M-theory

no solutions. [Apruzzi, Fazzi, Passias, Rosa, Tomasiello '14]

## type IIB

- T-dual of the Brandhuber-Oz solution along the Hopf fiber in  $S^3 \in S^4$ ; additional singularity at  $\alpha = \frac{\pi}{2}$ . [Cvetic, Pope, Vázquez-Poritz '00], [Lozano, Ó Colgáin, Rodríguez-Gómez, Sfetsos '12]
- non-Abelian T-dual of the Brandhuber-Oz solution with respect to the  $SU(2) \in SU(2) \times SU(2)_R$ ; non-compact; additional singularity at  $\alpha = \frac{\pi}{2}$ . [Lozano, Ó Colgáin, Rodríguez-Gómez, Sfetsos '12]

# Supersymmetric AdS<sub>6</sub> Solutions

We would like to perform a **systematic search** for supersymmetric AdS<sub>6</sub> × M<sub>4</sub> backgrounds of type IIB supergravity.

**Most general Ansatz:** a warped product of AdS<sub>6</sub> and a Riemannian manifold M<sub>4</sub>

$$ds_{10}^2 = e^{2A} ds_{\text{AdS}_6}^2 + ds_{M_4}^2$$

**SO(5, 2) symmetry** dictates:

- **warp factor A** and **dilaton φ** functions on M<sub>4</sub>
- $F_5 = 0$ ;  $H, F_1, F_3$  forms on M<sub>4</sub>.

**Supersymmetry:** There exists a **Spin(9, 1) Weyl spinor**  $\epsilon = \epsilon_1 + i\epsilon_2$  such that the gravitino and dilatino supersymmetry variations,  $\delta_\epsilon \psi$  and  $\delta_\epsilon \lambda$ , vanish ( $\epsilon_1$  and  $\epsilon_2$  are Majorana-Weyl spinors of the same chirality).

## Supersymmetric AdS<sub>6</sub> Solutions

$\epsilon_1$  and  $\epsilon_2$  define a  $G$ -structure on  $M_{10}$  characterized by a set of differential forms, defining the metric and the fluxes, which can be summarised by the bispinor  $\Phi = \epsilon_1 \otimes \bar{\epsilon}_2$ , via the Clifford map  $\gamma^{i_1 \dots i_n} \rightarrow dx^{i_1} \wedge \dots \wedge dx^{i_n}$ .

The supersymmetry constraints translate to a system of differential equations obeyed by  $\Phi$ . The precise form of this system was derived in [Tomasiello '11].

Spin(5, 1)  $\times$  Spin(4) decomposition of  $\epsilon_1, \epsilon_2$  yields a set of Spin(4) Weyl spinors

$$\eta_{\pm}^1, \quad \eta_{\pm}^2$$

on  $M_4$ . Each  $\eta^{1,2}$  defines an identity structure. The two identity structures can be parametrized by a vielbein  $e^a$  and a set of functions encoding the map between the two identity structures.

Reduction of the system of [Tomasiello '11] constraints  $e^a$  and the functions.



# Supersymmetric AdS<sub>6</sub> Solutions

We have been able to determine the [local form of the metric](#)

*S<sup>2</sup>-fibration over a two-dimensional space  $\Sigma_2$*

- The isometry of  $S^2$  corresponds to the  $SU(2)$  R-symmetry
- The fluxes are determined in terms of the geometry. The equations of motion and Bianchi identities are automatically satisfied.
- The warp factor and the dilaton depend on  $\Sigma_2$ ; they satisfy [two first-order PDEs](#).
- We have recovered the abelian and non-abelian T-duals of the Brandhuber-Oz solution.

## Conclusions and Outlook

- We have reduced the problem of supersymmetric  $\text{AdS}_6$  solutions in type IIB to [two first-order PDEs](#) for the warp factor and the dilaton.
- We expect solutions arising as [near-horizon geometries of of  \$\(p, q\)\$  five- and seven-brane webs](#).