Cosmological Solutions in Dilaton-Gauss-Bonnet Theory

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The String Theory Universe

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Introduction to Cosmology

The usual starting point is the following action functional

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa^2} + \mathcal{L}_X^i \right),$$

where R is the Ricci scalar, and $\mathcal{L}_X^i$ the source terms for all the "ingredients" present in the universe: $(\rho_r, \rho_m, \rho_{dm}, \rho_{de})$, as dictated by the current observations

Also, the complete theory should account for the presence of a fifth ingredient, the inflaton field

$$\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + V(\phi)$$

with an as-yet-unspecified potential, that cures the problems of the 'old' Cosmological Model

Introduction to Cosmology

There are a lot of open questions in Cosmology today - is perhaps time to change Einstein's theory of gravity?

Over the years, the generalised, or modified, theories of gravity have attracted a lot of attention

$$S = \int d^4x \sqrt{-g}\left[f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}) + \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + V(\phi)\right]$$

Of particular interest is the Gauss-Bonnet (GB) combination

$$R^2_{GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

Although a higher-derivative – quadratic – term, the GB term is ‘similar’ to the Ricci term: it leads to field equations with up to second-order derivatives and they work side-by-side to create interesting, black-hole or cosmological, solutions

The Einstein-Scalar-Gauss-Bonnet Theory

The GB term is also naturally present in the action of the effective low-energy heterotic superstring theory

In the context of the latter theory with moduli corrections included, singularity-free cosmological solutions were found (Antoniadis, Rizos & Tamvakis, 1994)

But, non-singular solutions were found also in the context of the much simpler theory (Kanti, Rizos & Tamvakis, 1999)

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} + \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + \frac{1}{8} f(\phi) \, R^2_{GB} \right]$$

when the coupling function retains some basic characteristics of the stringy one: it is an even function with a global minimum that tends to $+\infty$ as $\phi \to \pm\infty$

The Einstein-Scalar-Gauss-Bonnet Theory

In the context of this Einstein-Scalar-GB theory, the scalar and gravitational equations read

$$\frac{1}{\sqrt{-g}}\,\partial_\mu\left[\sqrt{-g}\,\partial^\mu\phi\right]=\frac{1}{8}\,\frac{df}{d\phi}\,R^2_{GB}$$

$$R_{\mu\nu}-\frac{1}{2}\,g_{\mu\nu}\,R=-\partial_\mu\phi\,\partial_\nu\phi+\frac{1}{2}\,g_{\mu\nu}\,\partial_\rho\phi\,\partial^\rho\phi+K_{\mu\nu}\,,$$

with

$$K_{\mu\nu}=\frac{1}{8}\,(g_{\mu\rho}g_{\nu\lambda}+g_{\mu\lambda}g_{\nu\rho})\,\eta^{\kappa\lambda\alpha\beta}\,D_\gamma\left(\tilde{R}^{\rho\gamma}{}_{\alpha\beta}D_\kappa f\right)$$

In 4D, the GB is a total derivative and, if $f=const.$, it drops out. On the other hand, R^2_{GB} provides a potential for the scalar field

Thus, the dilaton field and GB term are supporting each other. Is the presence of the Ricci term really necessary?

The Einstein-Scalar-Gauss-Bonnet Theory

For the Friedmann-Robertson-Walker line-element

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right],$$

the equations take the explicit form

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\,\dot{\phi} = \frac{df}{d\phi}\,\frac{3\ddot{a}}{a^3}\,(k+\dot{a}^2)$$

$$\frac{3\,(k+\dot{a}^2)}{a^2}\,(\underline{1} + \dot{f}\,\frac{\dot{a}}{a}) - \frac{\dot{\phi}^2}{2} = 0$$

$$\frac{(k+\dot{a}^2)}{a^2}\,(\underline{1} + \ddot{f}) + \frac{2\ddot{a}}{a}\,(\underline{1} + \dot{f}\,\frac{\dot{a}}{a}) + \frac{\dot{\phi}^2}{2} = 0$$

The scalar field is totally unaffected as it doesn't couple to R....
But the gravitational field equations do change...

The Einstein-Scalar-Gauss-Bonnet Theory

Let's consider a toy model in which we impose the constraint $R^2_{GB} = 0$. Then:

$$R^2_{GB} = \frac{24\,\ddot{a}}{a^3}\,(k + \dot{a}^2) \equiv 0 \;\Rightarrow\; a(t) = At + B$$

and

$$\ddot{\phi} + 3\dot{\phi}\,\frac{\dot{a}}{a} = 0 \;\Rightarrow\; \dot{\phi}(t) = \frac{C}{a^3(t)}$$

If R is present, the system of equations closes only for

$$f(\phi) = f_1\,\phi + \frac{f_2}{\phi} + f_3$$

where A, B, C, and f_i are integration constants. If R is absent, then $f(\phi)$ is as above but with $f_2 = 0$.

The Einstein-Scalar-Gauss-Bonnet Theory

We now restore the GB term and look for viable solutions. We will assume that $f(\phi) = \lambda\, \phi^n$, where λ a constant and $n \geq 1$

- For the case of $n = 1$, the dilaton equation can be written as

$$\frac{d(\dot\phi\, a^3)}{dt} = 3\lambda \ddot{a}\,(k + \dot{a}^2) \;\Rightarrow\; \dot\phi = \frac{C}{a^3} + \frac{\lambda \dot{a}\,(3k + \dot{a}^2)}{a^3}$$

Let us first assume that $C = 0$. Then, Einstein's equations lead to

$$3(k + \dot{a}^2)^2 \left[4 + \lambda^2\, \frac{3\ddot{a}}{a^3}\,(k + \dot{a}^2)\right] + \lambda^2\, \frac{\ddot{a}\dot{a}^2}{a^3}\,(3k + \dot{a}^2)^2 = 0$$

The above has no dependence on ϕ, and can be integrated to give

$$3a^4 = -\lambda^2 \left(\frac{13k\dot{a}^2}{2} + \frac{5\dot{a}^4}{2} + \frac{2k^3}{k + \dot{a}^2}\right) + c_1$$

The Einstein-Scalar-Gauss-Bonnet Theory

For a flat universe ($k=0$), we eventually find the relation

$$a(t)\, F\left[\frac{1}{4},\frac{1}{4},\frac{5}{4};\frac{3a^4(t)}{c_1}\right]=\left(\frac{2c_1}{5\lambda^2}\right)^{1/4}(t+t_0)$$

with $F(a,b,c;x)$ the hypergeometric function. For $a\to 0$, we find again $a(t)\sim t+t_0$ (in accordance to Kanti, Rizos & Tamvakis, 1999)

If $C\neq 0$, then, the derived equation is again difficult to solve, so one may study particular limits of it. Overall, no non-singular solutions were found – afterall, $f(\phi)=\lambda\phi$ is not an even function

For higher values of n, the equations are not decoupled and any analytic treatment is very difficult

The Dilaton-Gauss-Bonnet Theory

Now, we ignore the presence of the Ricci term in the theory. Then, the gravitational constraint is re-written as

$$(k+5\dot{a}^2)\,\ddot{a}+\frac{12\dot{a}^2}{a^3}\,(k+\dot{a}^2)\,\lambda n(n-1)\,\phi^{n-2}=0$$

The case $n=1$ leads again (much-much easier!) to the singular solution $a(t)=At+B$ derived previously.

On the other hand, for $n=2$, we integrate once and obtain

$$\frac{12\lambda}{a^2}=-\frac{1}{k}\,\ln\left(\frac{\sqrt{k+\dot{a}^2}}{\dot{a}}\right)-\frac{2}{(k+\dot{a}^2)}+C_1$$

In order to proceed, we have to set again $k=0$, and then find

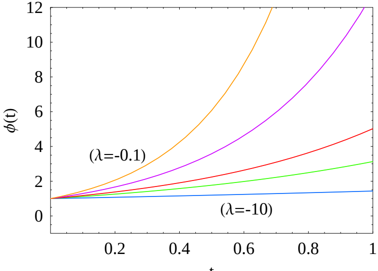
$$\frac{5}{2\dot{a}^2}=C_1-\frac{12\lambda}{a^2}$$

The Dilaton-Gauss-Bonnet Theory

- If $C_1 = 0$ and $\lambda < 0$, we easily obtain the solution

$$a(t) = a_0 \exp\left(\sqrt{\frac{5}{24|\lambda|}}\, t\right), \quad \phi = \phi_0 \exp\left(\frac{5}{4}\sqrt{\frac{5}{6|\lambda|}}\, t\right)$$

This may be an alternative for the usual inflation with the GB term providing a potential for ϕ - the potential becomes flatter the larger λ gets



But, it may also describe a late-time accelerating phase for the universe in the absence of a cosmological constant or dark energy

The Dilaton-Gauss-Bonnet Theory

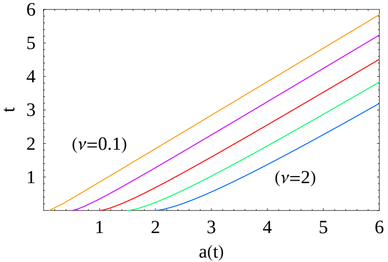
- For $C_1 > 0$ and $\lambda > 0$, after integrating, we obtain

$$\sqrt{a^2 - \nu^2} - \nu \arccos\left(\frac{\nu}{a}\right) = \pm\sqrt{\frac{5}{2C_1}}\,(t + t_0)$$

where $\nu^2 \equiv 12\lambda/C_1$. The constraint $a^2 \geq \nu^2$ should always hold, therefore the scale factor never vanishes and no singularity appears. Close to its minimum value, we find the approximate form

$$a(t) \simeq \nu\,[1 + (At + B)^{2/3}]$$

For the dilaton field, we obtain an implicit expression that is always regular



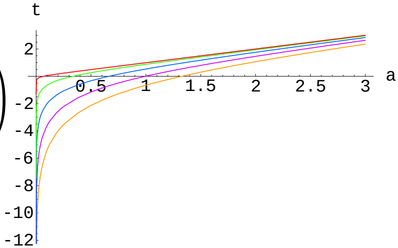
The Dilaton-Gauss-Bonnet Theory

- For $C_1 > 0$ and $\lambda < 0$, we similarly find the solution

$$\sqrt{a^2 + \tilde{\nu}^2} + \tilde{\nu} \ln \left(\frac{\sqrt{a^2 + \tilde{\nu}^2} - \tilde{\nu}}{a} \right) = \pm \sqrt{\frac{5}{2C_1}} (t + t_0)$$

where $\tilde{\nu}^2 \equiv 12|\lambda|/C_1$. A vanishing value of $a(t)$ can be attained but for larger and larger negative values of time as $\tilde{\nu}$ increases. In the limit $a(t) \to 0$, we find

$$a(t) \simeq a_0 \exp \left(\sqrt{\frac{5}{24|\lambda|}} (t + t_0) \right)$$



Thus, this solution is expanding exponentially at past infinity and linearly at large times

Conclusions

- Even in the absence of the Ricci term, the Gauss-Bonnet term, in conjunction with a scalar field, seems to encode all important information for the existing solutions in the theory
- By ignoring R, we have found very easily analytical non-singular cosmological solutions - for a quadratic coupling function, in accordance to previous suggestions
- We have also found, in the absence of Λ, purely de Sitter-like solutions or solutions that asymptote to such a solution at past infinity
- We still need to add some type of fluid in the theory and check whether these solutions survive or how they are modified