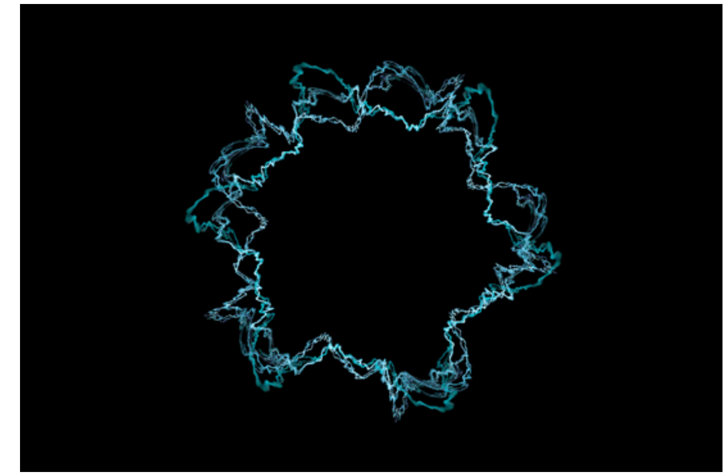
String theory in target space

based on:
- [1402.6356] with Tobias Hansen

Rutger Boels
University of Hamburg

Introduction to string theory

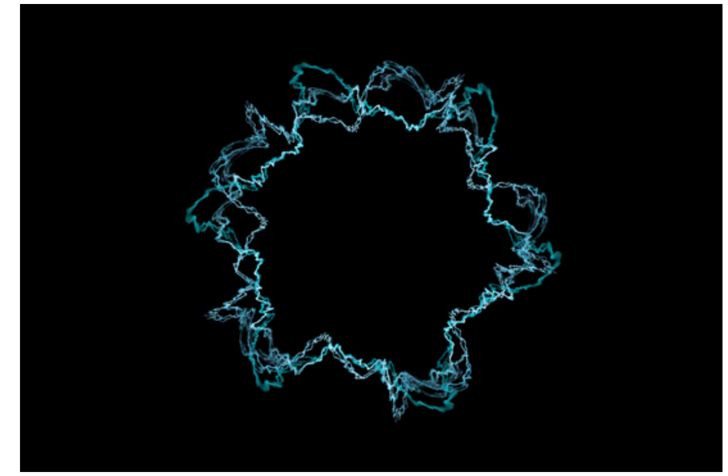


"String theory is a theory of strings"

different meaning to different people:

- a theory containing quantum gravity
- a (framework for a) "theory of everything"
- a source of toy models $\begin{cases} \text{to study } g \gg 1 \text{ QCD} \\ \text{to do condensed matter physics} \end{cases}$
- a marvellous source of mathematical conjectures
- the root of all evil
- etc, etc.

Introduction to string theory



“String theory is a theory of strings”

different meaning to different people:

- a theory containing quantum gravity
- a (framework for a) “theory of everything”
- a source of toy models $\begin{cases} \text{to study } g \gg 1 \text{ QCD} \\ \text{to do condensed matter physics} \end{cases}$
- a marvellous source of mathematical conjectures
- the root of all evil
- etc, etc.

however….

Introduction to string theory

Textbooks remain the same:
not much progress on
fundamentals of strings

Introduction to string theory

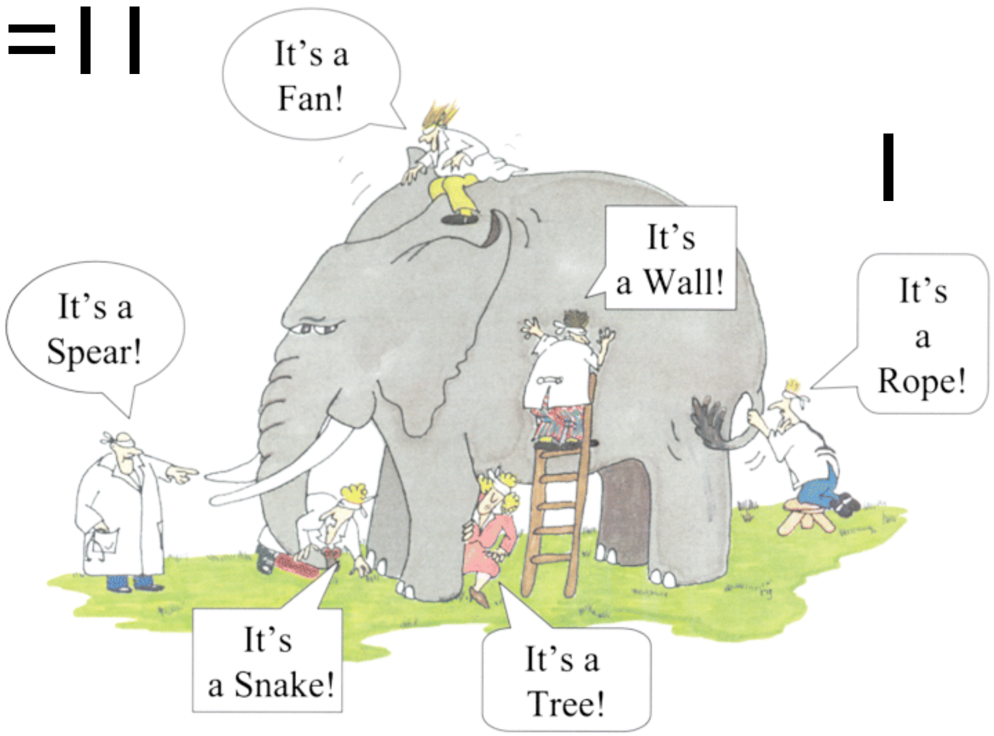
Textbooks remain the same:
not much progress on
fundamentals of strings

d=11

IIB

Het^2

I



M-branes

IIA

Introduction to string theory

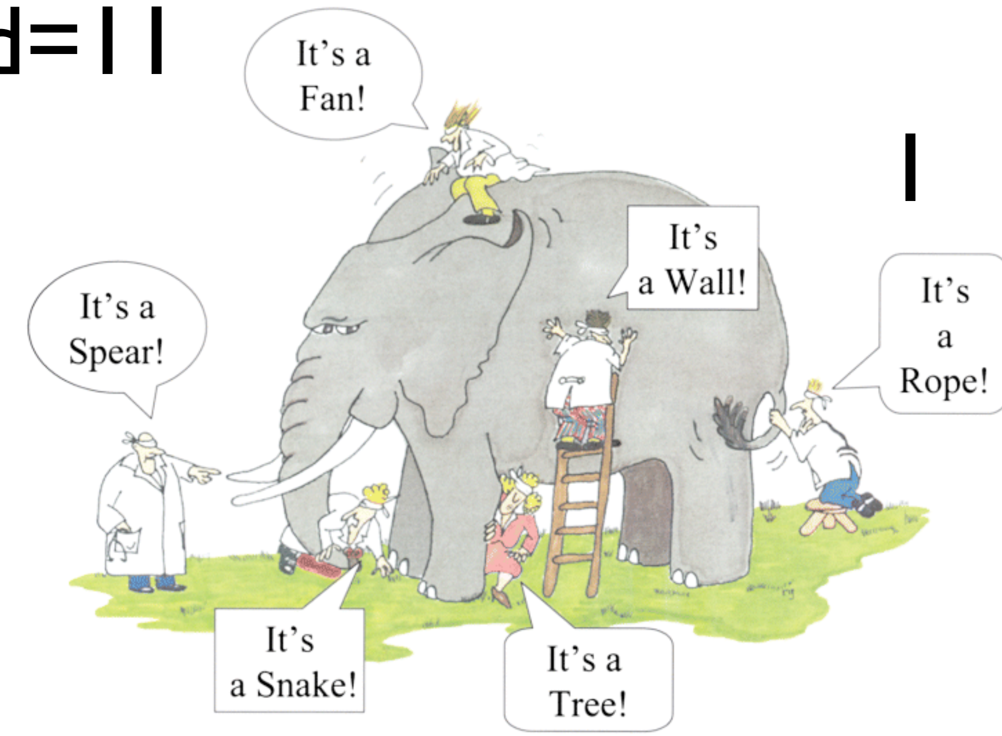
Textbooks remain the same:
not much progress on
fundamentals of strings

IIB

d=11

I

Het^2



M-branes

IIA

I think we need a new way of / perspective on
'doing string theory'

Introduction to string theory

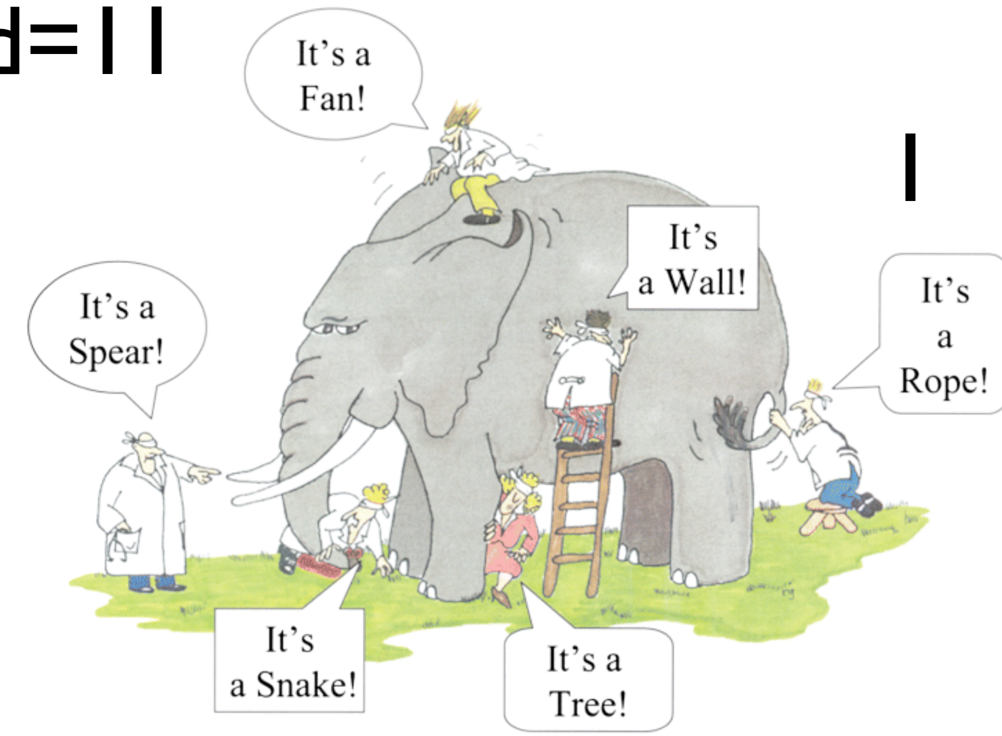
Textbooks remain the same: not much progress on fundamentals of strings

IIB

d=11

I

Het^2



M-branes

IIA

I think we need a new way of / perspective on 'doing string theory'

meanwhile....

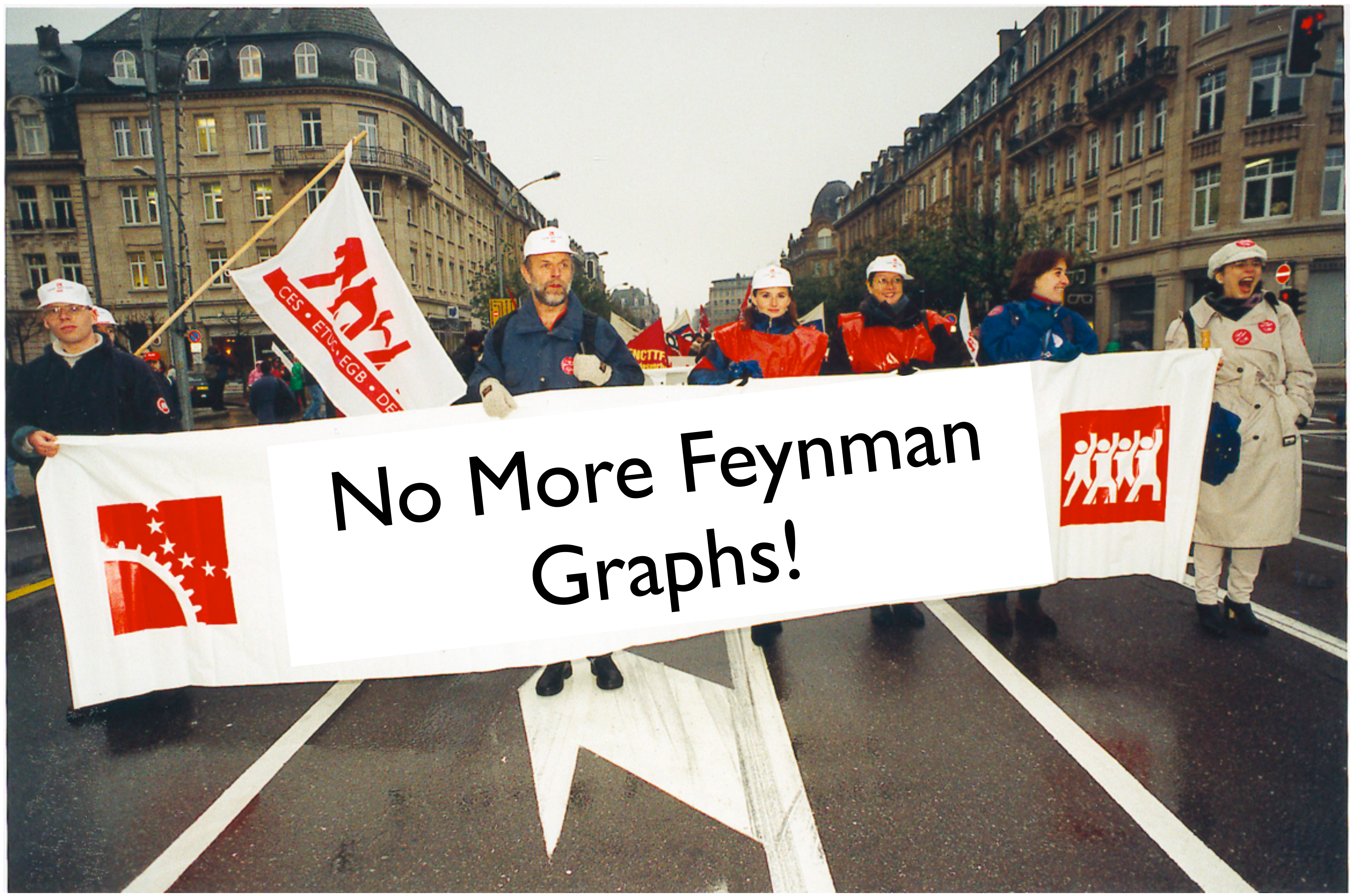
Meanwhile over in field theory…

revolution in scattering amplitudes-technology

Meanwhile over in field theory…

revolution in scattering amplitudes-technology

roughly:



Questions for string theory

Field theory: avoid Lagrangian

- e.g. BCFW [Britto-Cachazo-Feng-Witten, 04/05]
- work on foundations e.g.: [Cachazo-Benincasa 07], [Schuster-Toro 08], [Benincasa-Conde 11]
- trees mostly, loops by unitarity

Questions for string theory

Field theory: avoid Lagrangian

- e.g. BCFW [Britto-Cachazo-Feng-Witten, 04/05]
- work on foundations e.g.: [Cachazo-Benincasa 07], [Schuster-Toro 08], [Benincasa-Conde 11]
- trees mostly, loops by unitarity

→ define gauge theory S-matrix by unitarity & factorisation & spin ≤ 1 (no gauge symmetry!)

Questions for string theory

Field theory: avoid Lagrangian

- e.g. BCFW [Britto-Cachazo-Feng-Witten, 04/05]
- work on foundations e.g.: [Cachazo-Benincasa 07], [Schuster-Toro 08], [Benincasa-Conde 11]
- trees mostly, loops by unitarity

→ define gauge theory S-matrix by unitarity & factorisation & spin ≤ 1 (no gauge symmetry!)

Is there a formulation of strings which does not depend on the worldsheet (no CFT)?

Questions for string theory

Field theory: avoid Lagrangian

- e.g. BCFW [Britto-Cachazo-Feng-Witten, 04/05]
- work on foundations e.g.: [Cachazo-Benincasa 07], [Schuster-Toro 08], [Benincasa-Conde 11]
- trees mostly, loops by unitarity

→ define gauge theory S-matrix by unitarity & factorisation & spin ≤ 1 (no gauge symmetry!)

Is there a formulation of strings which does not depend on the worldsheet (no CFT)?

analytic S-matrix: back to roots of string theory!

Slogan

Revolution in understanding string theory possible?

Roughly:

Today

flat D=10 / D=26 tree level
superstrings, bosonic string

working definition of 'strings' in target space

Today

flat D=10 / D=26 tree level
superstrings, bosonic string

working definition of 'strings' in target space

(calculational method reproduces worldsheet answer)

Today

flat D=10 / D=26 tree level superstrings, bosonic string

$\boxed{\text{working definition of 'strings' in target space}}$

(calculational method reproduces worldsheet answer)

roughly:

- S-matrix which is unitary and local
- S-matrix has 'Regge-type behaviour'
- S-matrix which satisfies monodromy relations

Today

flat D=10 / D=26 tree level superstrings, bosonic string

$\boxed{\text{working definition of ‘strings’ in target space}}$

(calculational method reproduces worldsheet answer)

roughly:

- S-matrix which is unitary and local
- S-matrix has ‘Regge-type behaviour’
- S-matrix which satisfies monodromy relations

our work is a direct descendant / continuation of:

[Veneziano, 68] “Construction of a crossing - symmetric, Regge behaved amplitude for linearly rising trajectories”

BCFW for strings

BCFW for strings

"Some field theory amplitudes can be reconstructed from residues at (a subset of) their kinematic poles"

[Britto-Cachazo-Feng-Witten, 04,05]

BCFW for strings

"String theory amplitudes can be reconstructed from residues at (a subset of) their kinematic poles"

[RB-Larsen-Obers-Vonk, 08],
[Wecht-O'Connell, 09][RB-Marmiroli-Obers, 09]

BCFW for strings

"String theory amplitudes can be reconstructed from residues at (a subset of) their kinematic poles"

[RB-Larsen-Obers-Vonk, 08],
[Wecht-O'Connell, 09][RB-Marmiroli-Obers, 09]

- BCFW drives many new field theory gadgets
- pick two legs, summing over all factorisation channels with these legs on different sides
- unitarity:

$$\mathrm{res}_{s_{i\ldots j}\to m^2} A = \sum_{\mathrm{spec}@m^2} A_L A_R$$

BCFW for strings

"String theory amplitudes can be reconstructed from residues at (a subset of) their kinematic poles"

[RB-Larsen-Obers-Vonk, 08],
[Wecht-O'Connell, 09][RB-Marmiroli-Obers, 09]

- BCFW drives many new field theory gadgets
- pick two legs, summing over all factorisation channels with these legs on different sides
- unitarity:

$$\mathrm{res}_{s_{i\ldots j}\to m^2} A = \sum_{\mathrm{spec}@m^2} A_L A_R$$

- strings problem: doubly infinite summation (levels/states)
- need to know full string spectrum + 3 point amplitudes!
- 3 points in target space? (remember: no worldsheets!)

BCFW for strings

"String theory amplitudes can be reconstructed from residues at (a subset of) their kinematic poles"

[RB-Larsen-Obers-Vonk, 08],
[Wecht-O'Connell, 09][RB-Marmiroli-Obers, 09]

- BCFW drives many new field theory gadgets
- pick two legs, summing over all factorisation channels with these legs on different sides
- unitarity:

$$\mathrm{res}_{s_{i\ldots j}\to m^2} A = \sum_{\mathrm{spec}@m^2} A_L A_R$$

- strings problem: doubly infinite summation (levels/states)
- need to know full string spectrum + 3 point amplitudes!
- 3 points in target space? (remember: no worldsheets!)

some (unrelated) progress in [Chang-Feng-Fu-Lee-Wang, 12]

Key question: roots of open string amplitudes?

Veneziano amplitude:

$$A(s,t) \propto \frac{\Gamma[-s_{12}-1]\Gamma[-s_{23}-1]}{\Gamma[-s_{12}-s_{23}-2]}$$

Key question: roots of open string amplitudes?

poles of amplitude in s and t channel @ integer-spaced values, e.g.

$$s_{12} \in \{-1, 0, 1, \ldots\}$$

Veneziano amplitude:

$$A(s,t) \propto \frac{\Gamma[-s_{12}-1]\Gamma[-s_{23}-1]}{\Gamma[-s_{12}-s_{23}-2]}$$

Key question: roots of open string amplitudes?

poles of amplitude in s and t channel @ integer-spaced values, e.g.

$$s_{12} \in \{-1, 0, 1, \ldots\}$$

Veneziano amplitude:

$$A(s,t) \propto \frac{\Gamma[-s_{12}-1]\Gamma[-s_{23}-1]}{\Gamma[-s_{12}-s_{23}-2]}$$

roots of amplitude in u channel @ integer-spaced values?

$$s_{13}+s_{23} \in \{-2, -1, 0, 1, \ldots\}$$

Key question: roots of open string amplitudes?

Veneziano amplitude:

poles of amplitude in s and t channel @ integer-spaced values, e.g.

$$s_{12} \in \{-1, 0, 1, \ldots\}$$

$$A(s,t) \propto \frac{\Gamma[-s_{12}-1]\Gamma[-s_{23}-1]}{\Gamma[-s_{12}-s_{23}-2]}$$

roots of amplitude in u channel @ integer-spaced values?

$$s_{13}+s_{23} \in \{-2, -1, 0, 1, \ldots\}$$

[D' Adda, Sciuto, Auria, F. Gliozzi, 71] based on [Plahte, 70]: string monodromy relations determine location of roots

Key question: roots of open string amplitudes?

Veneziano amplitude:

poles of amplitude in s and t channel @ integer-spaced values, e.g.

$$s_{12} \in \{-1, 0, 1, \ldots\}$$

$$A(s,t) \propto \frac{\Gamma[-s_{12}-1]\Gamma[-s_{23}-1]}{\Gamma[-s_{12}-s_{23}-2]}$$

roots of amplitude in u channel @ integer-spaced values?

$$s_{13}+s_{23} \in \{-2, -1, 0, 1, \ldots\}$$

[D' Adda, Sciuto, Auria, F. Gliozzi, 71] based on [Plahte, 70]: string monodromy relations determine location of roots

rediscovered / systematized in [Bjerrum-Bohr-Damgaard-Vanhove, 09] [Stieberger, 09]

Key question: roots of open string amplitudes?

poles of amplitude in s and t channel @ integer-spaced values, e.g.

Veneziano amplitude:

$$s_{12} \in \{-1, 0, 1, \ldots\}$$

$$A(s,t) \propto \frac{\Gamma[-s_{12}-1]\Gamma[-s_{23}-1]}{\Gamma[-s_{12}-s_{23}-2]}$$

roots of amplitude in u channel @ integer-spaced values?

$$s_{13} + s_{23} \in \{-2, -1, 0, 1, \ldots\}$$

[D' Adda, Sciuto, Auria, F. Gliozzi, 71] based on [Plahte, 70]: string monodromy relations determine location of roots

rediscovered / systematized in [Bjerrum-Bohr-Damgaard-Vanhove, 09] [Stieberger, 09]

allows more modern treatment of roots

Roots of amplitudes: four point open string example

universal monodromy relations: $(k_{ij} \equiv 2\alpha' k_i \cdot k_j)$

$$A(1234) + e^{\pm i\pi k_{12}} A(2134) + e^{\pm i\pi (k_{12}+k_{13})} A(2314) = 0$$

Roots of amplitudes: four point open string example

universal monodromy relations: $(k_{ij} \equiv 2\alpha' k_i \cdot k_j)$

$$A(1234) + e^{\pm i\pi k_{12}} A(2134) + e^{\pm i\pi(k_{12}+k_{13})} A(2314) = 0$$

solve these relations: $$A(1234) = -\frac{\sin(\pi k_{\underline{13}})}{\sin(\pi k_{12})} A(\underline{13}24)$$

Roots of amplitudes: four point open string example

universal monodromy relations: $(k_{ij} \equiv 2\alpha' k_i \cdot k_j)$

$$A(1234) + e^{\pm i\pi k_{12}} A(2134) + e^{\pm i\pi (k_{12}+k_{13})} A(2314) = 0$$

solve these relations: $A(1234) = -\frac{\sin(\pi k_{\underline{13}})}{\sin(\pi k_{12})} A(\underline{13}24)$

explicit s-channel poles

Roots of amplitudes: four point open string example

universal monodromy relations: $(k_{ij} \equiv 2\alpha' k_i \cdot k_j)$

$$A(1234) + e^{\pm i\pi k_{12}} A(2134) + e^{\pm i\pi (k_{12}+k_{13})} A(2314) = 0$$

explicit u-channel roots

solve these relations: $$A(1234) = -\frac{\sin(\pi k_{\underline{13}})}{\sin(\pi k_{12})} A(\underline{13}24)$$

explicit s-channel poles

Roots of amplitudes: four point open string example

universal monodromy relations: $(k_{ij} \equiv 2\alpha' k_i \cdot k_j)$

$$A(1234) + e^{\pm i\pi k_{12}} A(2134) + e^{\pm i\pi(k_{12}+k_{13})} A(2314) = 0$$

explicit u-channel roots

solve these relations: $$A(1234) = -\frac{\sin(\pi k_{\underline{13}})}{\sin(\pi k_{12})} A(\underline{13}24)$$

explicit s-channel poles

- location of roots and poles *balances*: half-infinite series

Roots of amplitudes: four point open string example

universal monodromy relations: $(k_{ij} \equiv 2\alpha' k_i \cdot k_j)$

$$A(1234) + e^{\pm i\pi k_{12}} A(2134) + e^{\pm i\pi (k_{12}+k_{13})} A(2314) = 0$$

explicit u-channel roots

solve these relations: $$A(1234) = -\frac{\sin(\pi k_{\underline{13}})}{\sin(\pi k_{12})} A(\underline{13}24)$$

explicit s-channel poles

- location of roots and poles *balances*: half-infinite series

- pretty: predicts all roots of Veneziano amplitude / massless four point superstring amplitude / closed four point (KLT)

Computing amplitudes: Veneziano example

$$A(1234) = -\frac{\sin(\pi k_{13})}{\sin(\pi k_{12})} A(1324)$$

monodromy predicts
roots of residues:

$$\mathrm{Res}_{s_{12}\to A-1} A(1234) = \frac{(-1)^A}{\pi} \left[\sin(\pi s_{13}) A(1324)\right]_{s_{12}=A-1}$$

Computing amplitudes: Veneziano example

$$A(1234) = -\frac{\sin(\pi k_{13})}{\sin(\pi k_{12})} A(1324)$$

monodromy predicts
roots of residues:

$$\mathrm{Res}_{s_{12}\to A-1} A(1234) = \frac{(-1)^A}{\pi} \left[\sin(\pi s_{13}) A(1324)\right]_{s_{12}=A-1}$$

- by $(A-1) + s_{23} + s_{13} = -4$ there are only A roots at integer values on right hand side → functional dependence
- maximal degree at level A from Regge behavior

Computing amplitudes: Veneziano example

$$A(1234) = -\frac{\sin(\pi k_{13})}{\sin(\pi k_{12})} A(1324)$$

monodromy predicts
roots of residues:

$$\mathrm{Res}_{s_{12}\to A-1} A(1234) = \frac{(-1)^A}{\pi} \left[\sin(\pi s_{13}) A(1324)\right]_{s_{12}=A-1}$$

- by $(A-1)+s_{23}+s_{13}=-4$ there are only A roots at integer values on right hand side → functional dependence
- maximal degree at level A from Regge behavior

$$\mathrm{Res}_{s_{12}\to A-1} A(1234) = c(s_{13}+2)\ldots(s_{13}+A+1).$$

Computing amplitudes: Veneziano example

$$A(1234) = -\frac{\sin(\pi k_{13})}{\sin(\pi k_{12})} A(1324)$$

monodromy predicts roots of residues:

$$\mathrm{Res}_{s_{12}\to A-1} A(1234) = \frac{(-1)^A}{\pi} \left[\sin(\pi s_{13}) A(1324)\right]_{s_{12}=A-1}$$

- by $(A-1) + s_{23} + s_{13} = -4$ there are only A roots at integer values on right hand side → functional dependence
- maximal degree at level A from Regge behavior

$$\mathrm{Res}_{s_{12}\to A-1} A(1234) = c(s_{13}+2)\dots(s_{13}+A+1).$$

$$\lim_{s_{13}\to -1}\left[\frac{(-1)^A}{\pi}\sin(\pi s_{13})A(1324)\right] = (-1)^{A-1} A_3(T,T,T)A_3(T,T,T)$$

$$= (-1)^{A-1} g_o^2,$$

Computing amplitudes: Veneziano example

$$A(1234) = -\frac{\sin(\pi k_{13})}{\sin(\pi k_{12})} A(1324)$$

monodromy predicts
roots of residues:

$$\mathrm{Res}_{s_{12}\to A-1} A(1234) = \frac{(-1)^A}{\pi} \left[\sin(\pi s_{13}) A(1324)\right]_{s_{12}=A-1}$$

- by $(A-1) + s_{23} + s_{13} = -4$ there are only A roots at integer values on right hand side → functional dependence
- maximal degree at level A from Regge behavior

$$\mathrm{Res}_{s_{12}\to A-1} A(1234) = c(s_{13}+2)\ldots(s_{13}+A+1).$$

$$\lim_{s_{13}\to -1}\left[\frac{(-1)^A}{\pi}\sin(\pi s_{13})A(1324)\right] = (-1)^{A-1} A_3(T,T,T) A_3(T,T,T)$$

$$= (-1)^{A-1} g_o^2,$$

→ residues fixed → use BCFW to sum → Veneziano!

Computing amplitudes: n tachyons

monodromy at n points:

$$A(\underline{1}, \beta_1, ..., \beta_s, \underline{s+2}, \alpha_1, ..., \alpha_{N-s-3}, \underline{N})$$

$$= \frac{(-1)^{\alpha'\left(m_1^2+\sum\limits_{i=1}^{s} m_{\beta_i}^2\right)}}{\sin(\pi s_{1\beta_1\ldots\beta_s})}\left[\sum_{\sigma\in OP(\{\beta_1,\ldots,\beta_s\},\{\alpha\})} \mathcal{S}_{\{\beta^T,1,s+2,\alpha,N\},\{1,s+2,\sigma,N\}} A(\underline{1}, \underline{s+2}, \sigma, \underline{N})\right.$$

$$\left.+\sum_{l=1}^{s-1}\sum_{\sigma\in OP(\{\beta_{l+1},\ldots,\beta_s\},\{\alpha\})} \mathcal{S}_{\{\beta^T,1,s+2,\alpha,N\},\{1,\beta_1,\ldots,\beta_l,s+2,\sigma,N\}} A(\underline{1}, \beta_1, ..., \beta_l, \underline{s+2}, \sigma, \underline{N})\right]$$

Computing amplitudes: n tachyons

monodromy at n points:

$$A(\underline{1}, \beta_1, \ldots, \beta_s, \underline{s+2}, \alpha_1, \ldots, \alpha_{N-s-3}, \underline{N})$$
$$= \frac{(-1)^{\alpha'\left(m_1^2 + \sum\limits_{i=1}^{s} m_{\beta_i}^2\right)}}{\sin(\pi s_{1\beta_1\ldots\beta_s})} \Bigg[\sum_{\sigma \in OP(\{\beta_1,\ldots,\beta_s\},\{\alpha\})} \mathcal{S}_{\{\beta^T,1,s+2,\alpha,N\},\{1,s+2,\sigma,N\}} A(\underline{1}, \underline{s+2}, \sigma, \underline{N})$$
$$+ \sum_{l=1}^{s-1} \sum_{\sigma \in OP(\{\beta_{l+1},\ldots,\beta_s\},\{\alpha\})} \mathcal{S}_{\{\beta^T,1,s+2,\alpha,N\},\{1,\beta_1,\ldots,\beta_l,s+2,\sigma,N\}} A(\underline{1}, \beta_1, \ldots, \beta_l, \underline{s+2}, \sigma, \underline{N}) \Bigg]$$

'sines'

Computing amplitudes: n tachyons

monodromy at n points:

$$A(\underline{1}, \beta_1, ..., \beta_s, \underline{s+2}, \alpha_1, ..., \alpha_{N-s-3}, \underline{N})$$

'sines'

$$= \frac{(-1)^{\alpha'\left(m_1^2 + \sum_{i=1}^{s} m_{\beta_i}^2\right)}}{\sin(\pi s_{1\beta_1 \ldots \beta_s})} \left[\sum_{\sigma \in OP(\{\beta_1,\ldots,\beta_s\},\{\alpha\})} \mathcal{S}_{\{\beta^T,1,s+2,\alpha,N\},\{1,s+2,\sigma,N\}} A(\underline{1}, \underline{s+2}, \sigma, \underline{N}) \right.$$

$$\left. + \sum_{l=1}^{s-1} \sum_{\sigma \in OP(\{\beta_{l+1},\ldots,\beta_s\},\{\alpha\})} \mathcal{S}_{\{\beta^T,1,s+2,\alpha,N\},\{1,\beta_1,\ldots,\beta_l,s+2,\sigma,N\}} A(\underline{1}, \beta_1, ..., \beta_l, \underline{s+2}, \sigma, \underline{N}) \right]$$

residue in 'multi-peripheral' channel → …→ basis:

$$\prod_{\substack{i,j \\ 1<i<j<N}} \binom{k_{ij}}{a_{ij}}, \qquad \text{where } a_{ij} \in \mathbb{N}_0 \wedge \sum_{\substack{1<i\leq l \\ l<j<N}} a_{ij} = A_l$$

Computing amplitudes: n tachyons

monodromy at n points:

$$A(\underline{1}, \beta_1, ..., \beta_s, \underline{s+2}, \alpha_1, ..., \alpha_{N-s-3}, \underline{N})$$

'sines'

$$= \frac{(-1)^{\alpha'\left(m_1^2+\sum_{i=1}^{s} m_{\beta_i}^2\right)}}{\sin(\pi s_{1\beta_1\ldots\beta_s})}\Bigg[\sum_{\sigma\in OP(\{\beta_1,\ldots,\beta_s\},\{\alpha\})} \mathcal{S}_{\{\beta^T,1,s+2,\alpha,N\},\{1,s+2,\sigma,N\}} A(\underline{1}, \underline{s+2}, \sigma, \underline{N})$$

$$+\sum_{l=1}^{s-1}\sum_{\sigma\in OP(\{\beta_{l+1},\ldots,\beta_s\},\{\alpha\})} \mathcal{S}_{\{\beta^T,1,s+2,\alpha,N\},\{1,\beta_1,\ldots,\beta_l,s+2,\sigma,N\}} A(\underline{1}, \beta_1, ..., \beta_l, \underline{s+2}, \sigma, \underline{N})\Bigg]$$

residue in 'multi-peripheral' channel → …→ basis:

$$\prod_{\substack{i,j\\ 1<i<j<N}} \binom{k_{ij}}{a_{ij}}, \qquad \text{where } a_{ij}\in\mathbb{N}_0 \wedge \sum_{\substack{1<i\leq l\\ l<j<N}} a_{ij} = A_l$$

coefficients are signs → BCFW → n point amplitude

Computing amplitudes: n tachyons

monodromy at n points:

'sines'

$$A(1,\beta_1,\ldots,\beta_s,s+2,\alpha_1,\ldots,\alpha_{N-s-3},N)$$
$$=\frac{(-1)^{\alpha'\left(m_1^2+\sum_{i=1}^{s}m_{\beta_i}^2\right)}}{\sin(\pi s_{1\beta_1\ldots\beta_s})}\Bigg[\sum_{\sigma\in OP(\{\beta_1,\ldots,\beta_s\},\{\alpha\})}\mathcal{S}_{\{\beta^T,1,s+2,\alpha,N\},\{1,s+2,\sigma,N\}}A(1,s+2,\sigma,N)$$
$$+\sum_{l=1}^{s-1}\sum_{\sigma\in OP(\{\beta_{l+1},\ldots,\beta_s\},\{\alpha\})}\mathcal{S}_{\{\beta^T,1,s+2,\alpha,N\},\{1,\beta_1,\ldots,\beta_l,s+2,\sigma,N\}}A(1,\beta_1,\ldots,\beta_l,s+2,\sigma,N)\Bigg]$$

residue in 'multi-peripheral' channel → …→ basis:

$$\prod_{\substack{i,j\\1<i<j<N}}\binom{k_{ij}}{a_{ij}},\qquad \text{where } a_{ij}\in\mathbb{N}_0\wedge\sum_{\substack{1<i\leq l\\l<j<N}}a_{ij}=A_l$$

coefficients are signs → BCFW → n point amplitude

(from world sheet in [Cheung-O'Connel-Wecht, 09])

Strings without strings

- can compute n point tachyon amplitudes in bosonic string from monodromy + BCFW + unitarity
- fixes amplitudes with other particles by unitarity → fixes S-matrix
- not efficient…
- (closed strings by KLT)

Strings without strings

• can compute n point tachyon amplitudes in bosonic string from monodromy + BCFW + unitarity

• fixes amplitudes with other particles by unitarity → fixes S-matrix

• not efficient…

• (closed strings by KLT)

working definition of string theory without worldsheets or CFT!

Strings without strings

- can compute n point tachyon amplitudes in bosonic string from monodromy + BCFW + unitarity

- fixes amplitudes with other particles by unitarity → fixes S-matrix

- not efficient…

- (closed strings by KLT)

working definition of string theory without worldsheets or CFT!

- superstrings mostly similarly (technical issue at $n \geq 6$)
- minimal assumptions?

- string loops?

Strings without strings

- can compute n point tachyon amplitudes in bosonic string from monodromy + BCFW + unitarity

- fixes amplitudes with other particles by unitarity → fixes S-matrix

- not efficient…

- (closed strings by KLT)

working definition of string theory without worldsheets or CFT!

- superstrings mostly similarly (technical issue at n≥6)
- minimal assumptions?
- string loops? → in progress…

Strings without strings

- can compute n point tachyon amplitudes in bosonic string from monodromy + BCFW + unitarity

- fixes amplitudes with other particles by unitarity → fixes S-matrix

- not efficient…

- (closed strings by KLT)

working definition of string theory without worldsheets or CFT!

- superstrings mostly similarly (technical issue at $n \geq 6$)
- minimal assumptions?
- string loops? → in progress…

next: can we get all worldsheet results?

Unitarity from the target space point of view

- unitarity hotly debated in early days of strings: settled by proof of no-ghost theorem (worldsheet)
- no similar strength result here…

Unitarity from the target space point of view

- unitarity hotly debated in early days of strings: settled by proof of no-ghost theorem (worldsheet)
- no similar strength result here…

…but shortest route to critical dimension

Unitarity from the target space point of view

- unitarity hotly debated in early days of strings: settled by proof of no-ghost theorem (worldsheet)
- no similar strength result here…

…but shortest route to critical dimension

unitarity+locality has two consequences:

- amplitudes factorize at kinematic poles
- 3 point amplitudes must be real

Unitarity from the target space point of view

- unitarity hotly debated in early days of strings: settled by proof of no-ghost theorem (worldsheet)
- no similar strength result here…

…but shortest route to critical dimension

unitarity+locality has two consequences:

- amplitudes factorize at kinematic poles
- 3 point amplitudes must be real

$\boxed{\rightarrow \text{Veneziano amplitude must have positive residues}}$

Unitarity from the target space point of view

- unitarity hotly debated in early days of strings: settled by proof of no-ghost theorem (worldsheet)
- no similar strength result here…

…but shortest route to critical dimension

unitarity+locality has two consequences:

- amplitudes factorize at kinematic poles
- 3 point amplitudes must be real

→ Veneziano amplitude must have positive residues

$$\mathrm{res}_{s_{i\ldots j}} A = \sum_{\mathrm{spec}@m} A_L A_R$$

Unitarity from the target space point of view

- unitarity hotly debated in early days of strings: settled by proof of no-ghost theorem (worldsheet)
- no similar strength result here…

…but shortest route to critical dimension

unitarity+locality has two consequences:

- amplitudes factorize at kinematic poles
- 3 point amplitudes must be real

$\boxed{\rightarrow \text{Veneziano amplitude must have positive residues}}$

$$\mathrm{res}_{s_{i\ldots j}} A = \sum_{\mathrm{spec}@m} A_L A_R$$

spectrum = direct sum over irreps of SO(d)

SO(d) irreps are (symmetry props) + (tracelessness)

Calculational example: Veneziano amplitude

consider $A(T^4)$ in bosonic string $\rightarrow$ factorizes into $A(T^2M)^2$

Calculational example: Veneziano amplitude

consider A(T^4) in bosonic string → factorizes into A(T^2M)^2

A(T^2M) amplitude is fixed by Poincare + locality, up to constant

$$A(T,T,M^{\alpha}) = c_{A,\alpha} \prod_{a=1}^{|\alpha|} \sqrt{\frac{\alpha'}{2}}(k_1 - k_2)^{\mu_a} \xi^{I_a}_{\mu_a}.$$

Calculational example: Veneziano amplitude

consider $A(T^4)$ in bosonic string → factorizes into $A(T^2M)^2$

$A(T^2M)$ amplitude is fixed by Poincare + locality, up to constant

$$A(T,T,M^\alpha) = c_{A,\alpha} \prod_{a=1}^{|\alpha|} \sqrt{\frac{\alpha'}{2}} (k_1 - k_2)^{\mu_a} \xi^{I_a}_{\mu_a} .$$

- decomposing into irreps doable/complicated (bird tracks)

Calculational example: Veneziano amplitude

consider A(T^4) in bosonic string → factorizes into A(T^2M)^2

A(T^2M) amplitude is fixed by Poincare + locality, up to constant

$$A(T,T,M^{\alpha}) = c_{A,\alpha} \prod_{a=1}^{|\alpha|} \sqrt{\frac{\alpha'}{2}} (k_1 - k_2)^{\mu_a} \xi^{I_a}_{\mu_a} .$$

- decomposing into irreps doable/complicated (bird tracks)

take Veneziano, take residue on pole at level A, decompose polynomial into contributions for irreps, check positivity

Calculational example: Veneziano amplitude

consider A(T^4) in bosonic string → factorizes into A(T^2M)^2

A(T^2M) amplitude is fixed by Poincare + locality, up to constant

$$A(T,T,M^{\alpha}) = c_{A,\alpha} \prod_{a=1}^{|\alpha|} \sqrt{\frac{\alpha'}{2}} (k_1 - k_2)^{\mu_a} \xi^{I_a}_{\mu_a} .$$

- decomposing into irreps doable/complicated (bird tracks)

take Veneziano, take residue on pole at level A, decompose polynomial into contributions for irreps, check positivity

- obtained irrep coefficients as an explicit sum, checked unitarity up to A = 400 in D=26

Critical dimension and intercept conditions

take Veneziano amplitude with arbitrary dimension + intercept,

$$A_4 = \frac{\Gamma(-s_{12} - \alpha_0)\Gamma(-s_{23} - \alpha_0)}{\Gamma(-s_{12} - s_{23} - 2\alpha_0)}$$

$$\lim_{s_{12} \to A - \alpha_0} A_4 = \frac{1}{A!} \frac{-1}{s_{12} - A + \alpha_0} \prod_{i=1}^{A} (s_{23} + \alpha_0 + i), \qquad A \in \mathbb{N}_0$$

Critical dimension and intercept conditions

take Veneziano amplitude with arbitrary dimension + intercept,

$$A_4 = \frac{\Gamma(-s_{12}-\alpha_0)\Gamma(-s_{23}-\alpha_0)}{\Gamma(-s_{12}-s_{23}-2\alpha_0)}$$

$$\lim_{s_{12}\to A-\alpha_0} A_4 = \frac{1}{A!}\frac{-1}{s_{12}-A+\alpha_0}\prod_{i=1}^{A}(s_{23}+\alpha_0+i), \qquad A\in\mathbb{N}_0$$

factor residues into known unitarity residues: res(A_4)=(A_3)^2

$$A=1 \quad \to \quad c^2_{\text{vector}}\left(s_{23}+\frac{3}{2}\alpha_0+\frac{1}{2}\right)+c^2_{\text{scalar}}$$

$$A=2 \quad \to \quad c^2_{\text{tensor}}\left(s_{23}^2+5s_{23}+\frac{25}{4}-\frac{25}{4}\frac{1}{D-1}\right)+c^2_{\text{vector}}\left(s_{23}+\frac{5}{2}\right)+c^2_{\text{scalar}}$$

Critical dimension and intercept conditions

take Veneziano amplitude with arbitrary dimension + intercept,

$$A_4 = \frac{\Gamma(-s_{12}-\alpha_0)\Gamma(-s_{23}-\alpha_0)}{\Gamma(-s_{12}-s_{23}-2\alpha_0)}$$

$$\lim_{s_{12}\to A-\alpha_0} A_4 = \frac{1}{A!}\frac{-1}{s_{12}-A+\alpha_0}\prod_{i=1}^{A}(s_{23}+\alpha_0+i), \qquad A\in\mathbb{N}_0$$

factor residues into known unitarity residues: $\mathrm{res}(A_4)=(A_3)^2$

$$A=1 \quad \to \quad c^2_{\text{vector}}\left(s_{23}+\frac{3}{2}\alpha_0+\frac{1}{2}\right)+c^2_{\text{scalar}}$$

$$A=2 \quad \to \quad c^2_{\text{tensor}}\left(s_{23}^2+5s_{23}+\frac{25}{4}-\frac{25}{4}\frac{1}{D-1}\right)+c^2_{\text{vector}}\left(s_{23}+\frac{5}{2}\right)+c^2_{\text{scalar}}$$

coefficients real → $D\leq 26 \quad \alpha_0\leq 1$ no ghost conditions!

Critical dimension and intercept conditions (compare [Frampton, 72])

take Veneziano amplitude with arbitrary dimension + intercept,

$$A_4 = \frac{\Gamma(-s_{12}-\alpha_0)\Gamma(-s_{23}-\alpha_0)}{\Gamma(-s_{12}-s_{23}-2\alpha_0)}$$

$$\lim_{s_{12}\to A-\alpha_0} A_4 = \frac{1}{A!}\frac{-1}{s_{12}-A+\alpha_0}\prod_{i=1}^{A}(s_{23}+\alpha_0+i), \qquad A\in\mathbb{N}_0$$

factor residues into known unitarity residues: $\mathrm{res}(A_4)=(A_3)^2$

$$A=1 \quad\to\quad c^2_{\text{vector}}\left(s_{23}+\frac{3}{2}\alpha_0+\frac{1}{2}\right)+c^2_{\text{scalar}}$$

$$A=2 \quad\to\quad c^2_{\text{tensor}}\left(s_{23}^2+5s_{23}+\frac{25}{4}-\frac{25}{4}\frac{1}{D-1}\right)+c^2_{\text{vector}}\left(s_{23}+\frac{5}{2}\right)+c^2_{\text{scalar}}$$

coefficients real → $D\leq 26 \quad \alpha_0\leq 1$ no ghost conditions!

Speculation: generalisation?

you can define the S-matrix of string theory (tree level, flat background) by:

- unitarity in some channels
- BCFW (Regge-type behavior)
- monodromy
- locality, Poincare symmetry
- KLT relations to define closed strings

Speculation: generalisation?

you can define the S-matrix of string theory (tree level, flat background) by:

- unitarity in some channels
- BCFW (Regge-type behavior)
- monodromy
- locality, Poincare symmetry
- KLT relations to define closed strings

some things generalise

- monodromy, KLT $\rightarrow$ all backgrounds ([RB-Marmiroli-Obers, 09])
- how does unitarity work?
- what replaces Poincare?
- spectrum information?

$$V_1 V_2 = V_2 V_1 R_{12}$$

Summary, outlook

- new $\left\{\begin{array}{c}\text{calculational method for}\\ \text{working definition of}\end{array}\right\}$ strings directly in target space

- main new ingredient: monodromy relations fix residues at kinematic poles

- unitarity checked in examples → poor man's no-ghost theorem

- much to learn!

Toward a revolution in understanding of strings?

Your Question Here?

Computing amplitudes: 5 tachyons

monodromy at 5 points:

$$A(12345) = \frac{1}{\sin(\pi s_{12})} \left[\sin(\pi(-s_{12} + k_{23}))A(13245) + \sin(\pi(-s_{12} + k_{23} + k_{24}))A(13425)\right]$$

Computing amplitudes: 5 tachyons

monodromy at 5 points:

$$A(12345) = \frac{1}{\sin(\pi s_{12})}\left[\sin(\pi(-s_{12}+k_{23}))A(13245) + \sin(\pi(-s_{12}+k_{23}+k_{24}))A(13425)\right]$$

s_{12} poles exposed

Computing amplitudes: 5 tachyons

monodromy at 5 points:

$$A(12345) = \frac{1}{\sin(\pi s_{12})}\left[\sin(\pi(-s_{12}+k_{23}))A(13245) + \sin(\pi(-s_{12}+k_{23}+k_{24}))A(13425)\right]$$

s_{12} poles exposed

$$\mathrm{Res}_{s_{12}\to A-1}A(12345) = \frac{1}{\pi}\left[\sin(\pi k_{23})A(13245) + \sin(\pi(k_{23}+k_{24}))A(13425)\right]_{s_{12}=A-1}$$

Computing amplitudes: 5 tachyons

monodromy at 5 points:

$$A(12345) = \frac{1}{\sin(\pi s_{12})}\left[\sin(\pi(-s_{12}+k_{23}))A(13245) + \sin(\pi(-s_{12}+k_{23}+k_{24}))A(13425)\right]$$

s_{12} poles exposed

$$\mathrm{Res}_{s_{12}\to A-1} A(12345) = \frac{1}{\pi}\left[\sin(\pi k_{23})A(13245) + \sin(\pi(k_{23}+k_{24}))A(13425)\right]_{s_{12}=A-1}$$

roots at integers for which:

$$k_{23} \geq 0\ ,$$

$$k_{24} \geq 0\ ,$$

$$k_{23} + k_{24} \leq A-1$$

Computing amplitudes: 5 tachyons

monodromy at 5 points:

$$A(12345) = \frac{1}{\sin(\pi s_{12})}\left[\sin(\pi(-s_{12}+k_{23}))A(13245) + \sin(\pi(-s_{12}+k_{23}+k_{24}))A(13425)\right]$$

s_{12} poles exposed

$$\mathrm{Res}_{s_{12}\to A-1}A(12345) = \frac{1}{\pi}\left[\sin(\pi k_{23})A(13245) + \sin(\pi(k_{23}+k_{24}))A(13425)\right]_{s_{12}=A-1}$$

roots at integers for which:

$$k_{23} \geq 0\,,\quad k_{24} \geq 0\,,\quad k_{23}+k_{24} \leq A-1$$

res $\sim \binom{k_{23}}{A-a}\binom{k_{24}}{a}$

$$0 \leq a \leq A$$

Computing amplitudes: 5 tachyons

monodromy at 5 points:

$$A(12345) = \frac{1}{\sin(\pi s_{12})}\left[\sin(\pi(-s_{12}+k_{23}))A(13245)+\sin(\pi(-s_{12}+k_{23}+k_{24}))A(13425)\right]$$

s_{12} poles exposed

$$\mathrm{Res}_{s_{12}\to A-1}A(12345) = \frac{1}{\pi}\left[\sin(\pi k_{23})A(13245)+\sin(\pi(k_{23}+k_{24}))A(13425)\right]_{s_{12}=A-1}$$

roots at integers for which:

$$k_{23}\geq 0\,,\qquad k_{24}\geq 0\,,\qquad k_{23}+k_{24}\leq A-1$$

res $\sim \binom{k_{23}}{A-a}\binom{k_{24}}{a}$, $\quad 0\leq a\leq A$

new: there is non-trivial structure → take s_{123} residue of residue

$$\sim \binom{k_{23}}{A-a}\binom{k_{24}}{a}\binom{k_{34}}{B-a}, \qquad 0\leq a\leq \min(A,B)$$

Computing amplitudes: 5 tachyons

monodromy at 5 points:

$$A(12345) = \frac{1}{\sin(\pi s_{12})}\left[\sin(\pi(-s_{12}+k_{23}))A(13245) + \sin(\pi(-s_{12}+k_{23}+k_{24}))A(13425)\right]$$

s_{12} poles exposed

$$\mathrm{Res}_{s_{12}\to A-1} A(12345) = \frac{1}{\pi}\left[\sin(\pi k_{23})A(13245) + \sin(\pi(k_{23}+k_{24}))A(13425)\right]_{s_{12}=A-1}$$

roots at integers for which:

$$k_{23} \geq 0\,, \quad k_{24} \geq 0\,, \quad k_{23}+k_{24} \leq A-1$$

res $\sim \begin{pmatrix} k_{23} \\ A-a \end{pmatrix}\begin{pmatrix} k_{24} \\ a \end{pmatrix}$ $\quad 0 \leq a \leq A$

new: there is non-trivial structure → take s_{123} residue of residue

$$\sim \begin{pmatrix} k_{23} \\ A-a \end{pmatrix}\begin{pmatrix} k_{24} \\ a \end{pmatrix}\begin{pmatrix} k_{34} \\ B-a \end{pmatrix}, \qquad 0 \leq a \leq \min(A,B)$$

coefficients turn out to be signs → BCFW → 5 point amplitude