Holographic Lattices, Metals and Insulators

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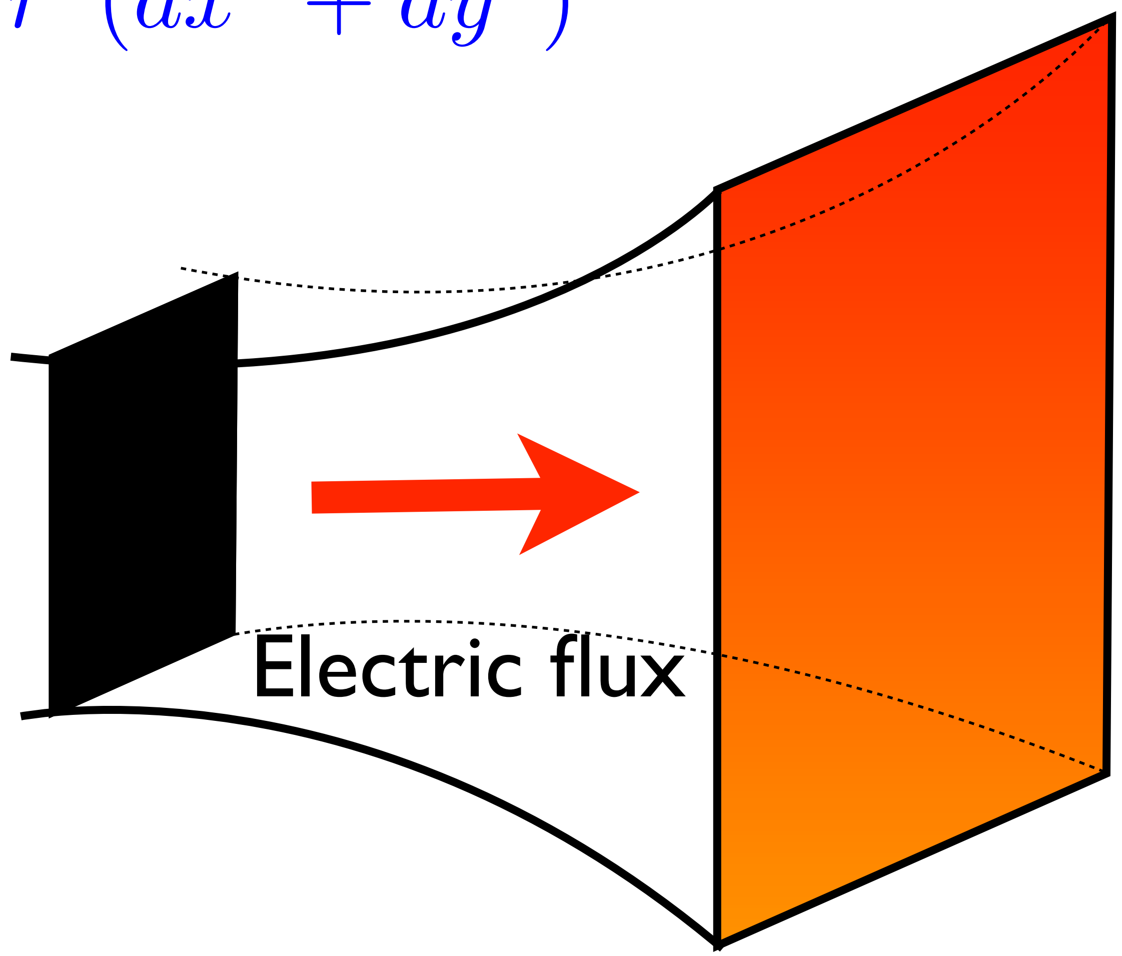
Electrically charged AdS-RN black hole (brane)

Describes holographic matter at finite charge density that is translationally invariant

$$ds^2 = -U dt^2 + \frac{dr^2}{U} + r^2(dx^2 + dy^2)$$

$$A_t = \mu\left(1 - \frac{r_+}{r}\right)$$



d=3 CFT

$\mu \quad T$

T=0 limit: $AdS_2 \times \mathbb{R}^2 \longleftarrow AdS_4$

IR UV

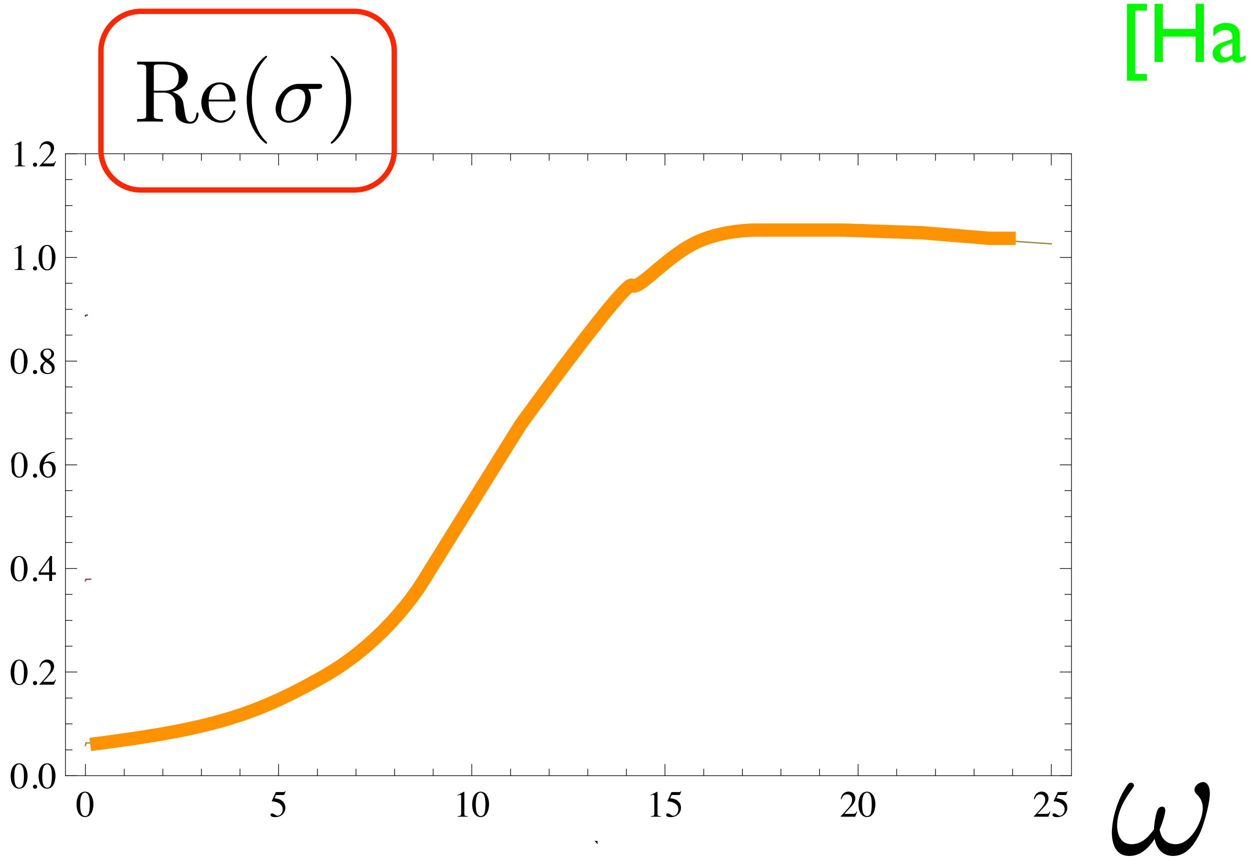
Conductivity calculation

$$\sigma(\omega) = -i \frac{G_{J_x J_x}(\omega)}{\omega}$$

$$\delta A_x = e^{-i\omega t} a_x(r)$$

$$\delta g_{tx} = e^{-i\omega t} h_{tx}(r)$$

[Hartnoll]



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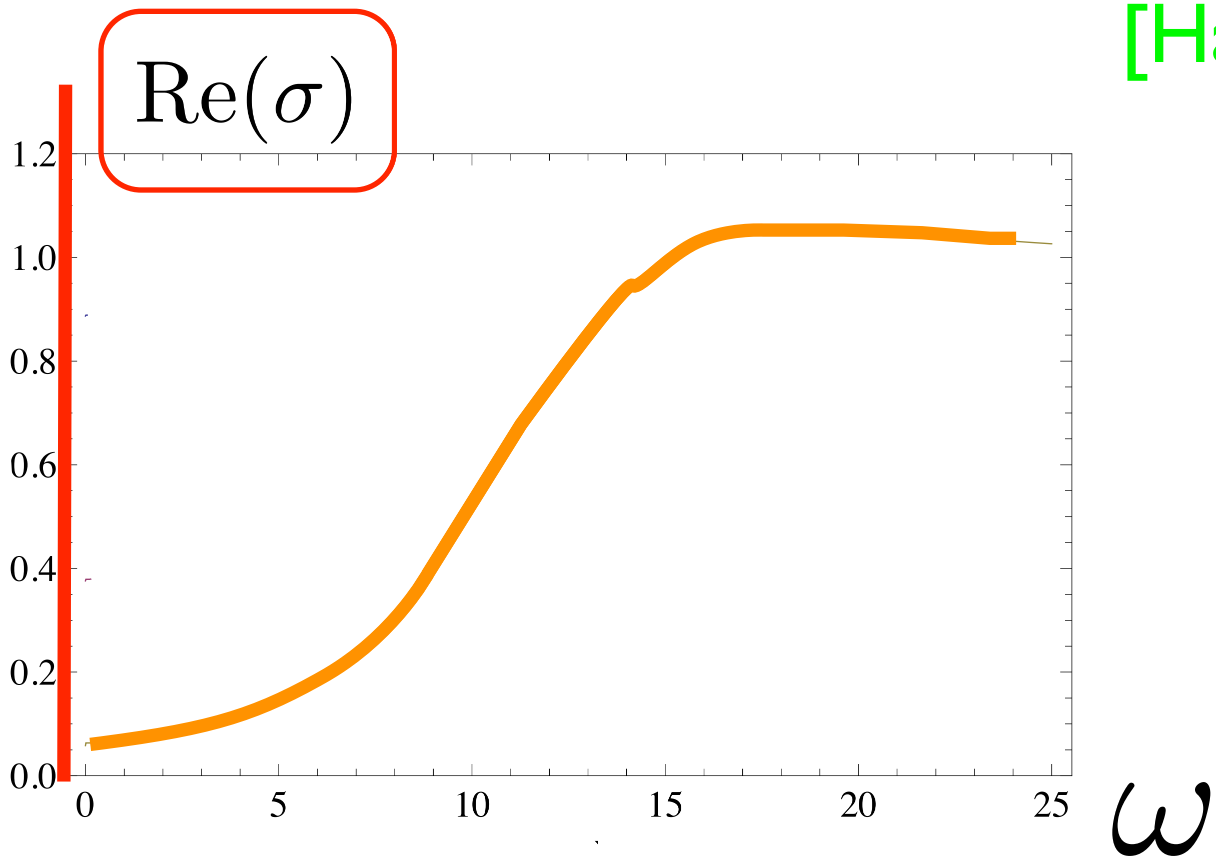
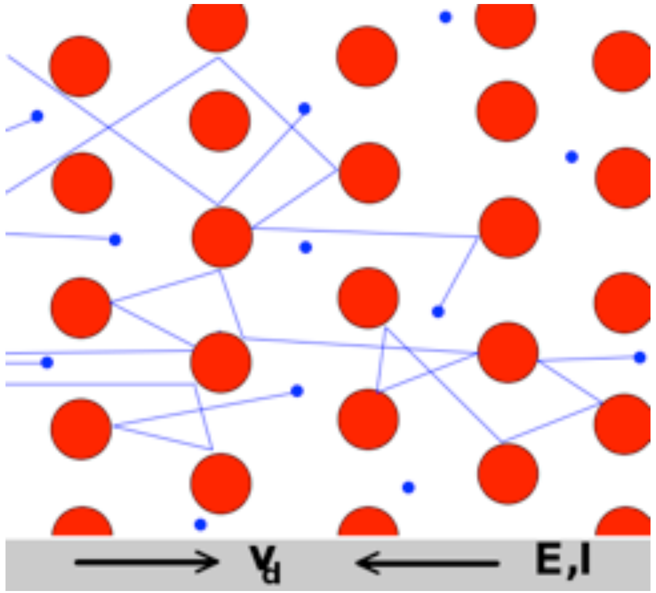
[Hartnoll]



More precisely $\sigma(\omega) \sim \delta(\omega) + \frac{i}{\omega}$ near $\omega \sim 0$

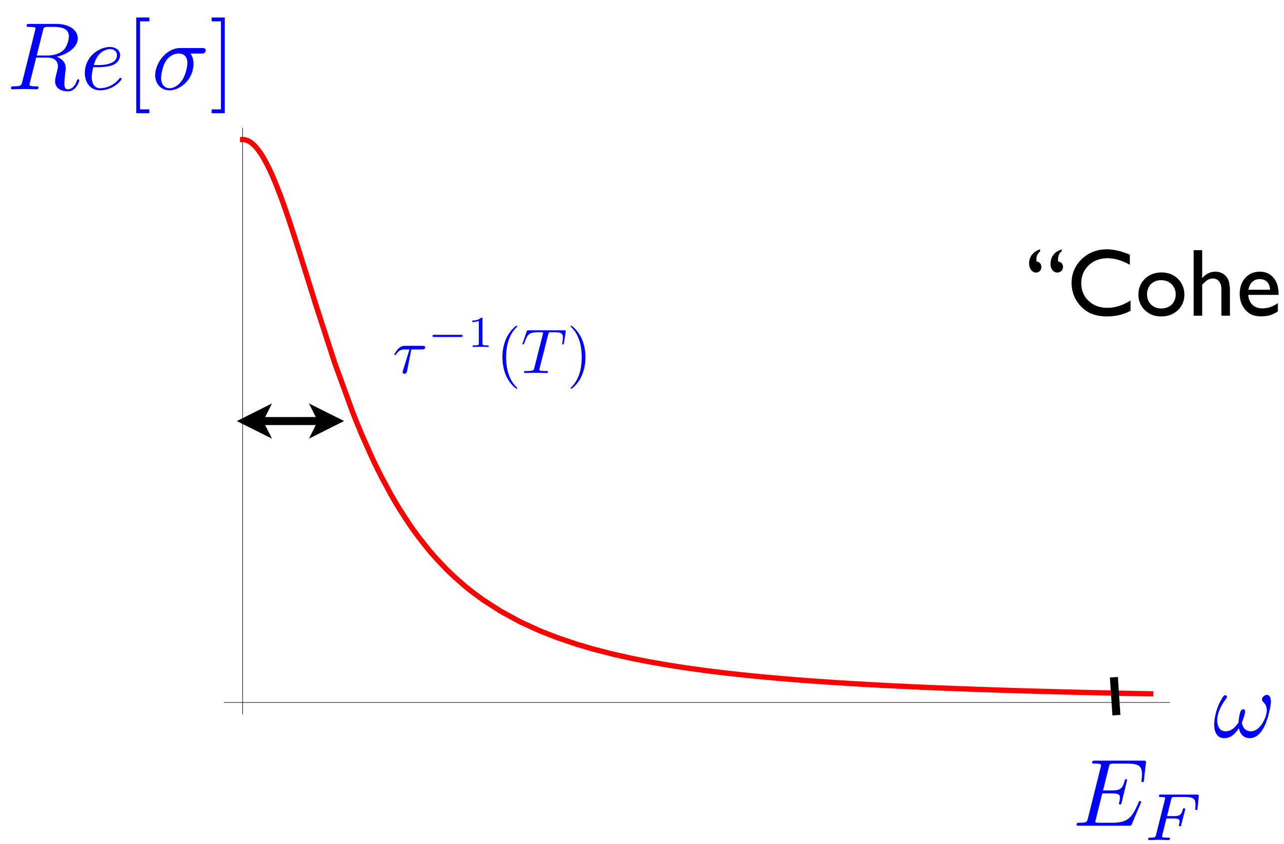
$\sigma_{DC} = \infty$ arises because translation invariance implies there is no momentum dissipation

Drude Model of transport in a metal
e.g. quasi-particles and no interactions



$$J(\omega) = \sigma(\omega)E(\omega)$$

$$\sigma(\omega) = \frac{\sigma_{DC}}{1 - i\omega\tau} \qquad \sigma_{DC} = \frac{nq^2\tau}{m}$$



"Coherent" or "good" metal

Note: $\tau \to \infty$ $\quad \sigma(\omega) \sim \delta(\omega) + \frac{i}{\omega}$

- Drude physics doesn't require quasi-particles

  Arises when momentum is nearly conserved

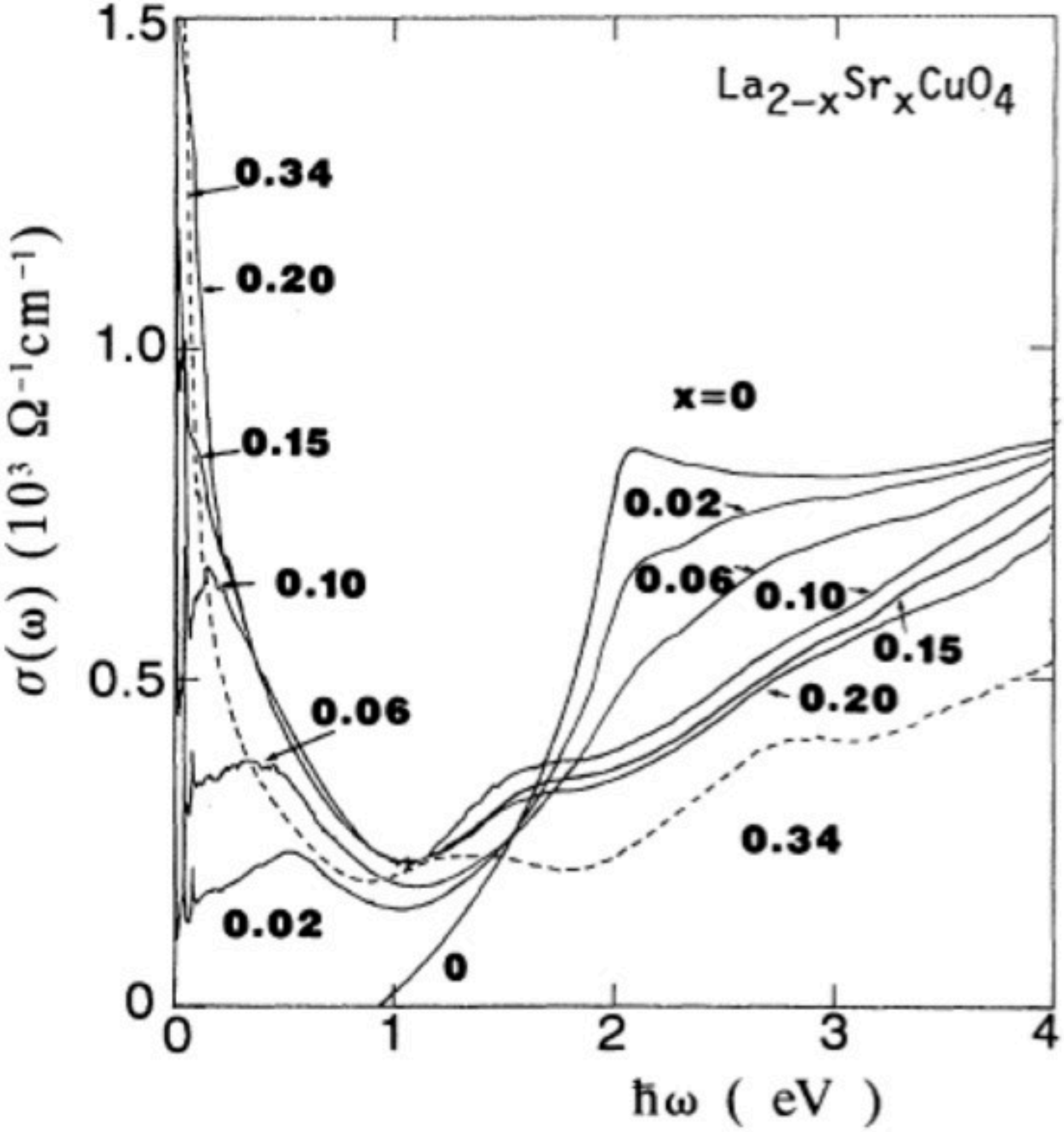
  "Coherent" metals

- There are also "incoherent" metals without Drude peaks

- Insulators with $\sigma_{DC} = 0$

- Metal-insulator transitions involve dramatic reorganisation of degrees of freedom

Want to study these within holography



Interaction driven and strongly coupled

Holographic Lattices and metals

To realise more realistic metals and/or insulators we want to construct charged black holes that explicitly break translations using a deformation of the CFT [Horowitz, Santos, Tong]

E.g In Einstein-Maxwell theory consider:

$$\mu(x) = \mu + A\cos kx$$

E.g. add a real scalar field to Einstein-Maxwell and consider

$$\phi(r,x) \sim \frac{\lambda\cos(kx)}{r^{3-\Delta}} + \dots$$

Need to solve PDEs

Can we simplify? Find some agreement and some differences

Plan

• Holographic Q-lattices - solve ODEs

• Calculation of thermoelectric DC conductivity σ_{DC} , α_{DC} , $\bar{\kappa}_{DC}$ in terms of black hole horizon data

Analogous to $\eta = \frac{s}{4\pi}$ [Policastro,Kovtun,Son,Starinets]

For σ_{DC} c.f. [Iqbal,Liu][Davison][Blake,Tong,Vegh][Andrade,Withers]

• Q-lattices can give coherent metals, incoherent metals and insulators and transitions between them.

• Comments on $\mu(x)$ lattices in Einstein-Maxwell theory

Holographic Q-lattices

- Illustrative D=4 model

$$\mathcal{L} = R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi(\phi)(\partial\chi)^2\right] + V(\phi) - \frac{Z(\phi)}{4}F^2$$

- Choose Φ, V, Z so that we have an AdS_4 vacuum and that AdS-RN is a solution at $\phi = 0$

- Particularly interested in cases where χ is periodic.
  eg if it is the phase of a complex scalar field $\varphi = \phi e^{i\chi}$ with $\Phi = \phi^2$

  Analysis covers cases when χ is not periodic e.g.
  [Azeneyagi,Takayanagi,Li][Mateos,Trancanelli][Andrade,Withers]

- The model has a gauge $U(1)$ and a global $U(1)$ symmetry
  Exploit the global bulk symmetry to break translations

Ansatz for fields

$$ds^2 = -U dt^2 + U^{-1} dr^2 + e^{2V_1} dx_1^2 + e^{2V_2} dx_2^2$$

$$A = a(r) dt$$

$$\chi = k x_1, \qquad \phi = \phi(r)$$

UV expansion:

$$U = r^2 + \ldots, \qquad e^{2V_1} = r^2 + \ldots \qquad e^{2V_2} = r^2 + \ldots$$

$$a = \mu + \frac{q}{r} \ldots, \qquad \phi = \frac{\lambda}{r^{3-\Delta}} + \ldots$$

IR expansion: regular black hole horizon

Homogeneous and anisotropic and periodic holographic lattices

UV data: $T/\mu \qquad \lambda/\mu^{3-\Delta} \qquad k/\mu$

Analytic result for DC in terms of horizon data

Apply electric fields and thermal gradients and find linear response

Generalised Ohm/Fourier Law:

$$\begin{pmatrix} J \\ Q \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E \\ -(\nabla T)/T \end{pmatrix}$$

J^a Electric current

$Q^a = T^{ta} - \mu J^a$ Heat current

For Q-lattice black holes the DC matrices $\sigma, \alpha, \bar{\alpha}, \bar{\kappa}$ diagonal

- Calculating σ and $\bar{\alpha}$

  Switch on constant electric field perturbation

  $$A_x = \boxed{-Et} + \delta a_x(r) \quad \text{plus} \quad \delta g_{tx}(r) \quad \delta g_{rx}(r) \quad \delta\chi(r)$$

  Gauge equation of motion:

  $$\nabla_\mu (Z(\phi) F^{\mu\nu}) = 0 \quad \Rightarrow \quad \partial_r(\sqrt{-g} Z(\phi) F^{rx}) = 0$$

  $$\Rightarrow \quad J = -e^{V_2 - V_1} Z(\phi) U \delta a'_{x_1} + q e^{-2V_1} \delta g_{tx_1} \quad \text{constant}$$

  Use Einstein equations and regularity at the black hole horizon to relate J and E to get σ

  Similar analysis relates Q and E to get $\bar{\alpha}$

- Calculating α and $\bar{\kappa}$

  Consider a source for heat currents

$$\mathcal{L} = R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi(\phi)(\partial\chi)^2\right] + V(\phi) - \frac{Z(\phi)}{4}F^2$$

$$ds^2 = -U dt^2 + U^{-1} dr^2 + e^{2V_1} dx_1^2 + e^{2V_2} dx_2^2$$

$$\chi = k x_1 \qquad A = a dt$$

$$\alpha_{DC} = \bar{\alpha}_{DC} = -\left[\frac{4\pi q}{k^2 \Phi(\phi)}\right]_{r=r_+} \qquad \bar{\kappa}_{DC} = \left[\frac{4\pi s T}{k^2 \Phi(\phi)}\right]_{r=r_+}$$

$$\sigma_{DC} = \left[e^{-V_1+V_2} Z(\phi) + \frac{q^2 e^{-V_1-V_2}}{k^2 \Phi(\phi)}\right]_{r=r_+}$$

First term in σ is finite for AdS-Schwarzschild [Iqbal,Liu]

"Pair evolution" term. In general it is $(J/E)_{Q=0} \equiv \sigma - \alpha^2 \bar{\kappa}^{-1} T$

Second term "Dissipation" term

Different ground states can be dominated by first or second term

- Some general results

- $$\bar{L} \equiv \frac{\bar{\kappa}}{\sigma T} \leq \frac{s^2}{q^2}$$

Bound is saturated for dissipation dominated systems

c.f. Wiedemann-Franz Law.

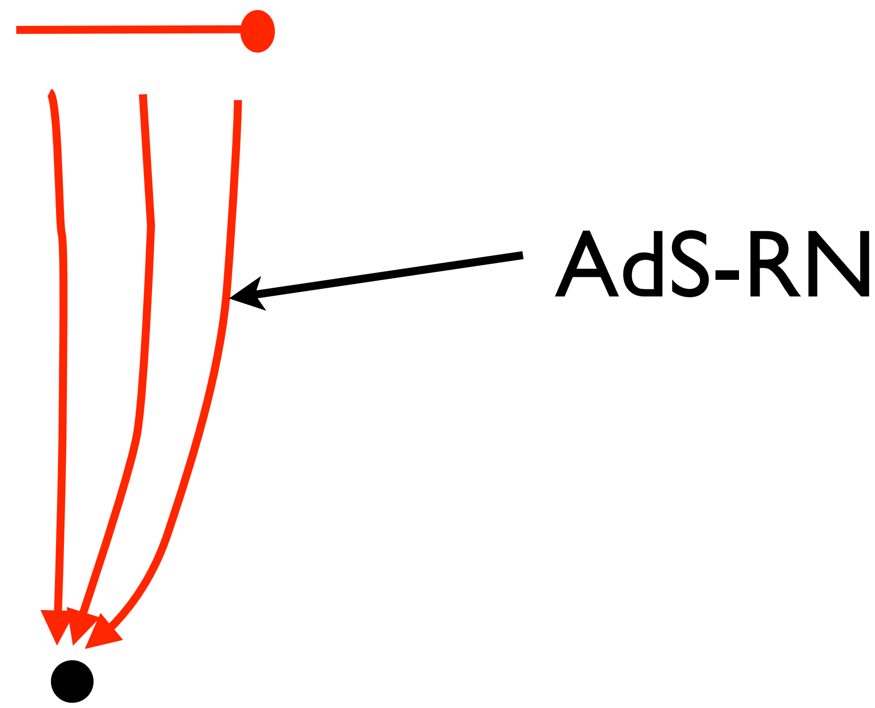
- $$\frac{\bar{\kappa}}{\alpha} = -\frac{Ts}{q}$$

Coherent metal phases

UV data

$k/\mu \quad \lambda/\mu$

T=0



IR fixed point $\quad AdS_2 \times \mathbb{R}^2$

At T=0 the black holes approach $AdS_2 \times \mathbb{R}^2$ in the IR

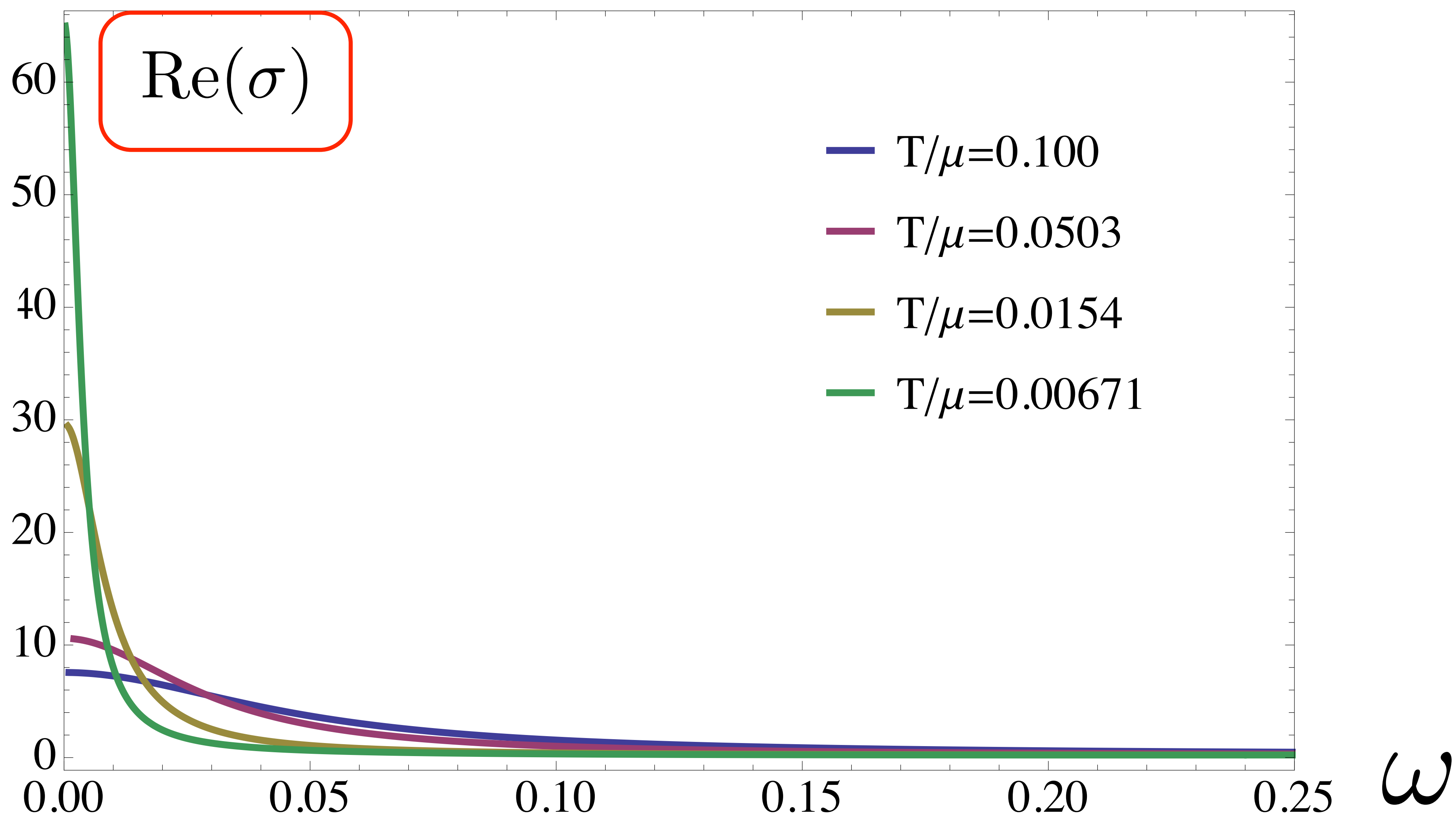
perturbed by irrelevant operator with $\Delta(k_{IR}) \geq 1$

Low T DC conductivity is dissipation dominated: $\sigma \sim T^{2-2\Delta(k_{IR})}$

[Hartnoll, Hoffman]

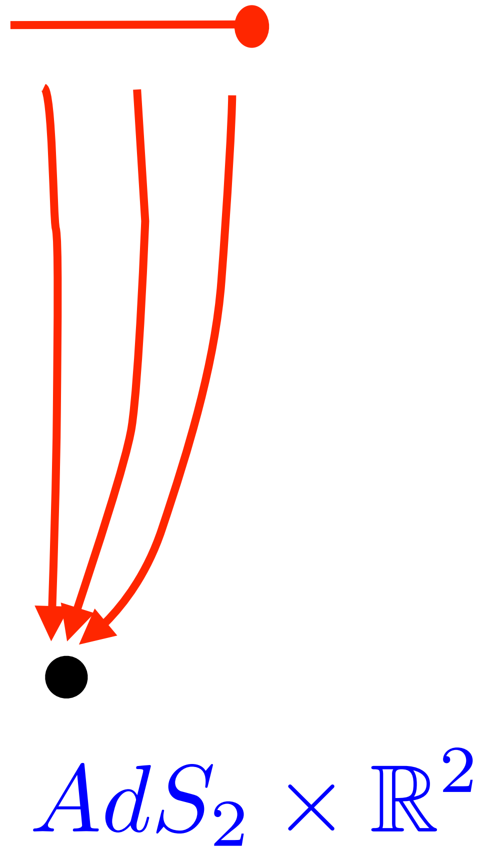
Note: k_{IR} depends on RG flow

Drude peaks at finite T



Similar to what was seen for different lattices in [Horowitz,Santos,Tong]

Insulating phases



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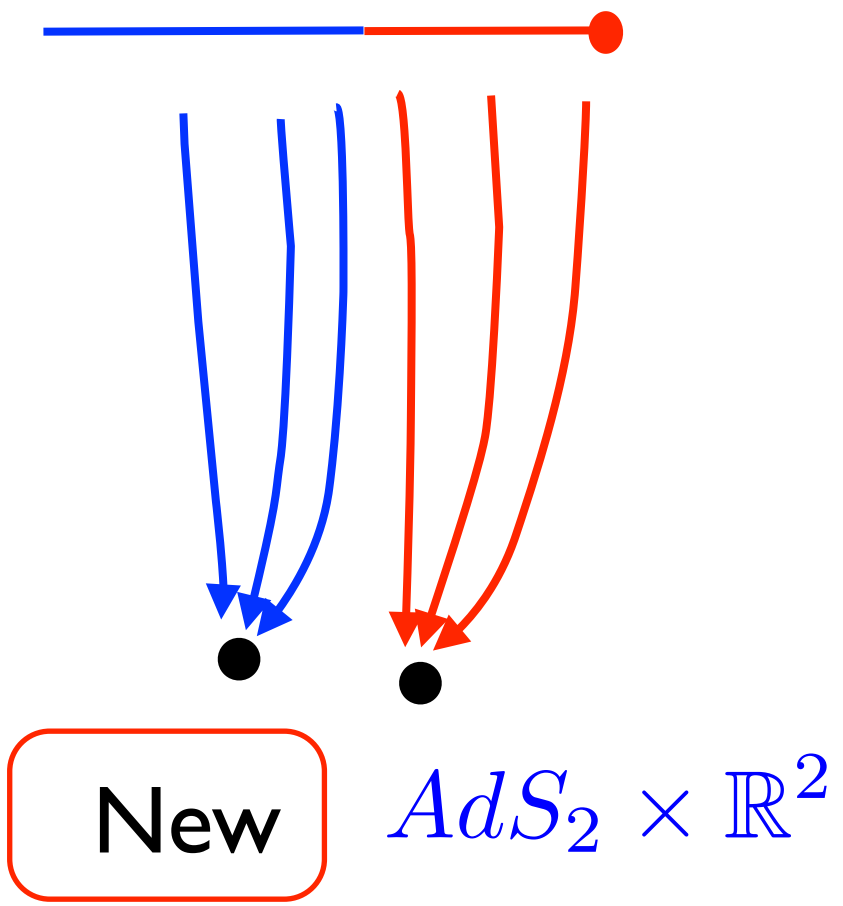

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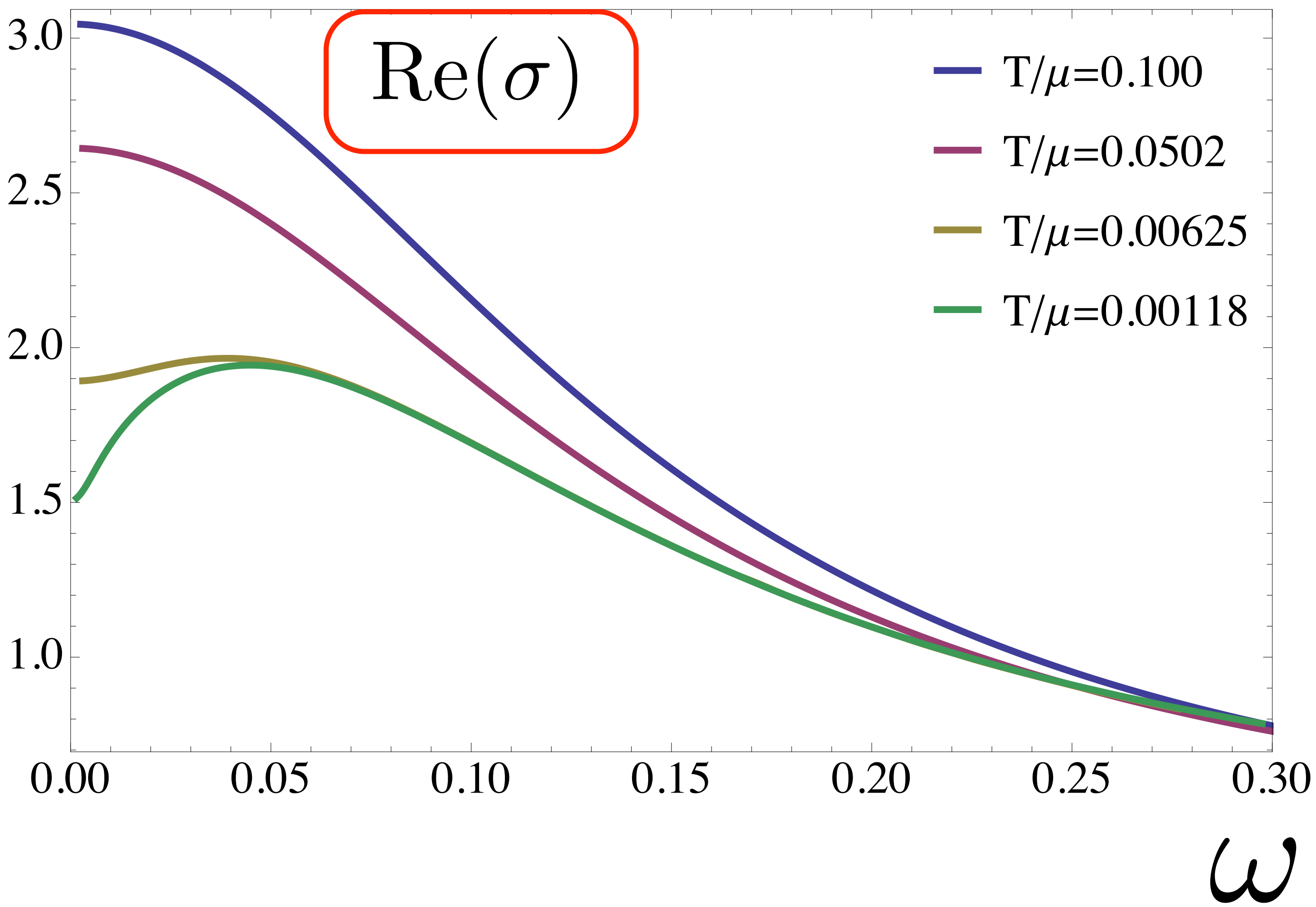

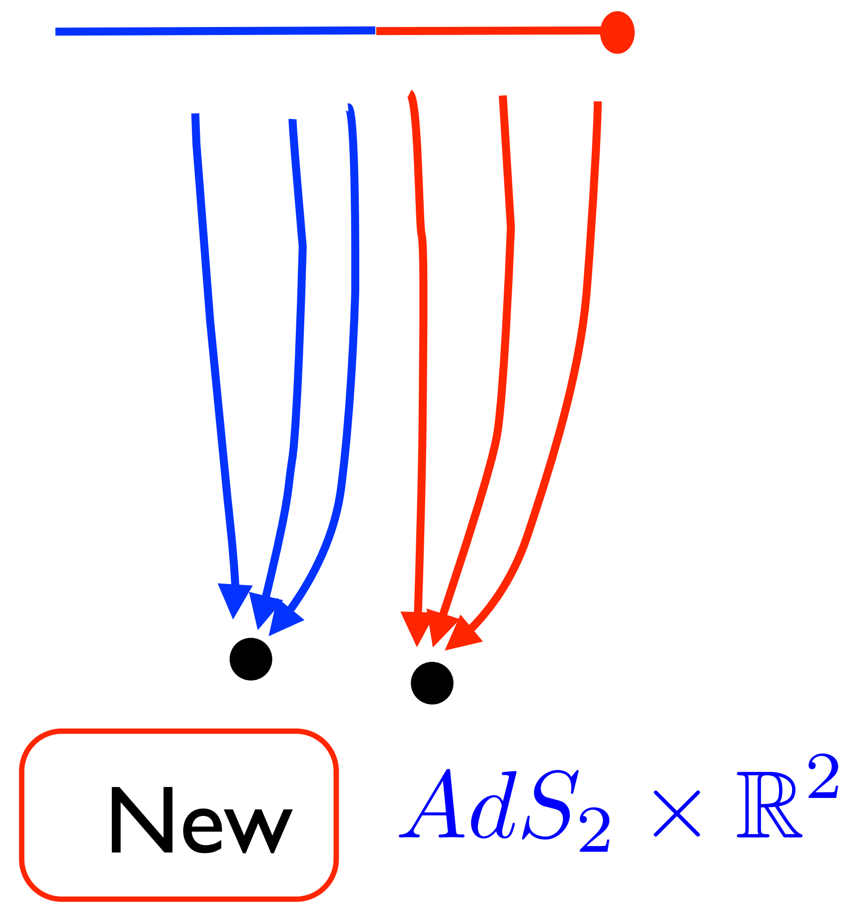

Appearance of a mid-frequency hump.
Spectral weight is being transferred, consistent with sum rule (see also [Donos,Hartnoll])

What are the T=0 insulating ground states??

Focus on specific models (see also [Gouteraux])

New Insulating and Metallic ground states - Anisotropic

$$\mathcal{L} = R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi(\phi)(\partial\chi)^2\right] + V(\phi) - \frac{Z(\phi)}{4}F^2$$

Focus on models and T=0 ground states which are solutions with $\phi \to \infty$ as $r \to 0$

and $$\mathcal{L} \to R - \frac{3}{2}\left[(\partial\phi)^2 + e^{2\phi}(\partial\chi)^2\right] + e^{\phi} - \frac{e^{\gamma\phi}}{4}F^2$$

IR "fixed point" solutions

$$ds^2 \sim -r^u dt^2 + r^{-u} dr^2 + r^{v_1} dx_1^2 + r^{v_2} dx_2^2$$

$$e^{\phi} \sim r^{-\phi_0} \qquad A \sim r^a dt \qquad \chi = k x_1$$

with exponents fixed by γ

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- Calculate DC conductivity $\sigma_{DC} \sim T^{b(\gamma)}$

For $T << \mu$ the scaling is obtained from the IR fixed point solutions

$b > 0$ New type of insulating ground states

$b < 0$ Novel incoherent metallic ground states not associated with Drude physics

$b = 0$ Novel metallic ground states with finite conductivity at T=0

Metallic ground states are all thermal insulators $\bar{\kappa} \rightarrow 0$

New Insulating and Metallic ground states - Isotropic

Use a model with scalar fields ϕ and χ_1, χ_2
with

$$\chi_1 = kx_1$$

$$\chi_2 = kx_2$$

Can have coherent metals and transitions to

- Insulators
- Incoherent metals

  Some have peaks in AC conductivity that are not associated with Drude physics

Summary

- Holographic Q-lattices are simple and illuminating
- Analytic result for DC conductivity in terms of horizon data.

  Can be generalised to inhomogeneous lattices $\mu(x)$

- Coherent metallic phases with Drude peaks
  - No intermediate 2/3 scaling in AC conductivity

    We find it to be absent in inhomogeneous lattices $\mu(x)$
    Absent in another recent example [Taylor,Woodhead]

- Also find novel metallic phases and insulating phases

  Metal-Insulator and Metal-Metal transitions

• Lattices are a good way to look for new holographic ground states

(but no "floppy" ground states reported by [Hartnoll,Santos] for the lattices with $\mu(x)$)