A Holographic Quantum Hall Ferromagnet and its Conductivity Tensor

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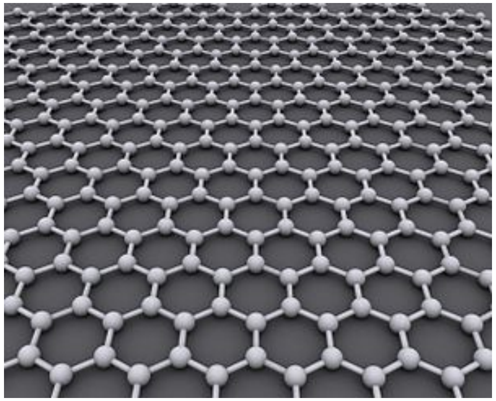
Based on:

- C. K. & G. Semenoff, arXiv:1212.5609 [hep-th], JHEP 1306 (2013) 048
- C. K., R. Pourhasan & G. Semenoff, arXiv:1311.6999 [hep-th], JHEP 1402 (2014) 097
- J. Hutchinson, C. K. & G. Semenoff, arXiv:1408.3320

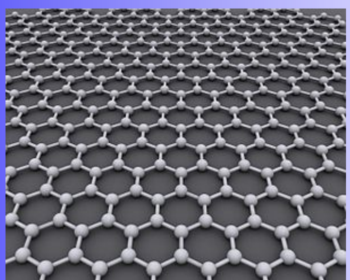
Mainz, September 23, 2014

A certain probe D-brane model which might be of relevance for graphene



A dual model of graphene (?)

Graphene



Field theory
(Gordon Semenoff)

String-theory

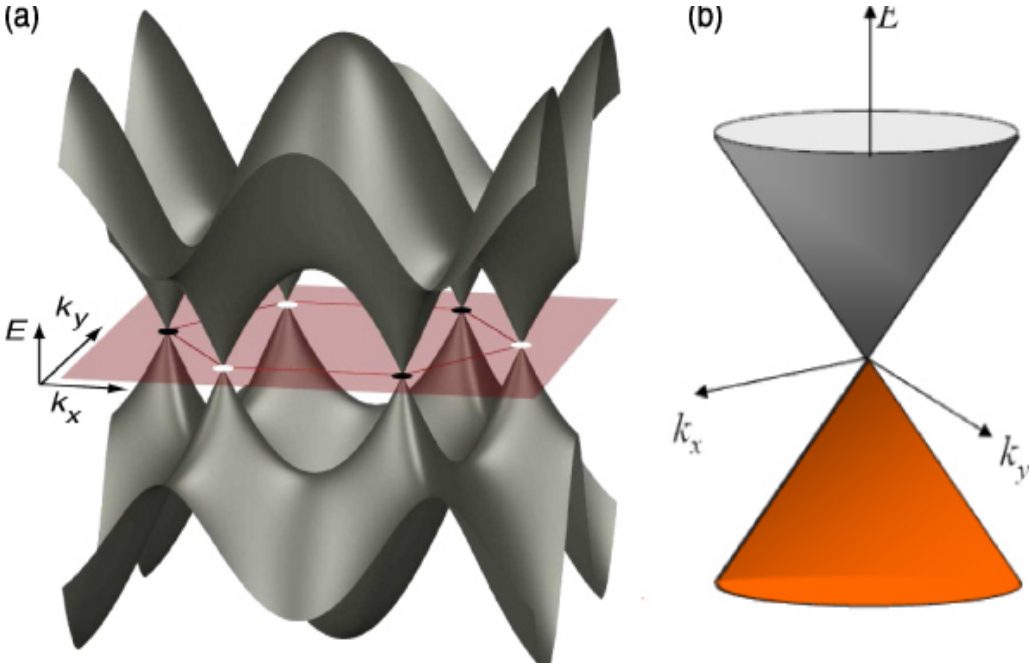
?

Complexity, field theory →

← Complexity, string theory

Particle description of graphene

1. Relativistic dispersion relation



$$E = \pm v_F |\hbar k|$$

$$v_F = \frac{c}{300}$$

2. Strong coupling

In an electromagnetic field: $\frac{e^2}{4\pi\hbar c} \to \frac{e^2}{4\pi\hbar v_F} \approx \frac{300}{137}$

2 valleys and 2 spin states $\longrightarrow$ SU(4) symmetry

The three-dimensional world of graphene

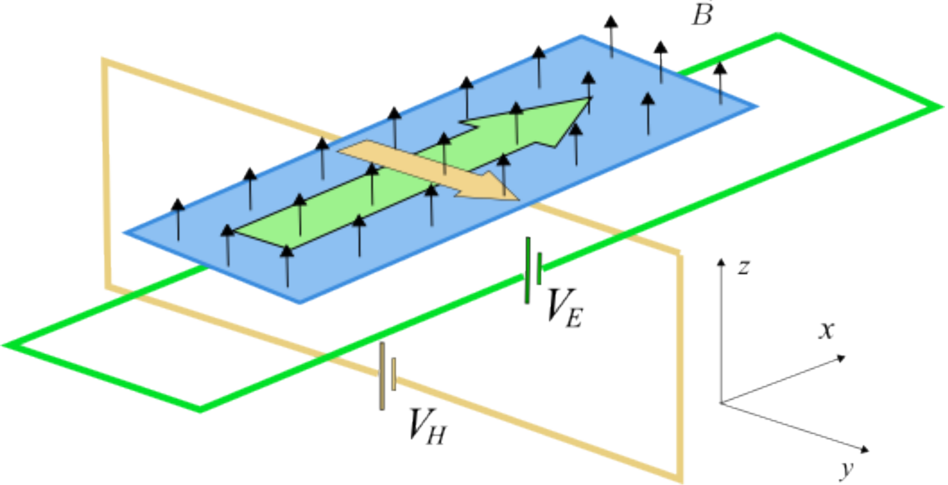
	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
$D3$	$\times$	$\times$	$\times$	$\times$						
$D5$	$\times$	$\times$	$\times$		$\times$		$\times$	$\times$		
$D7$	$\times$	$\times$	$\times$		$\times$		$\times$	$\times$	$\times$	$\times$

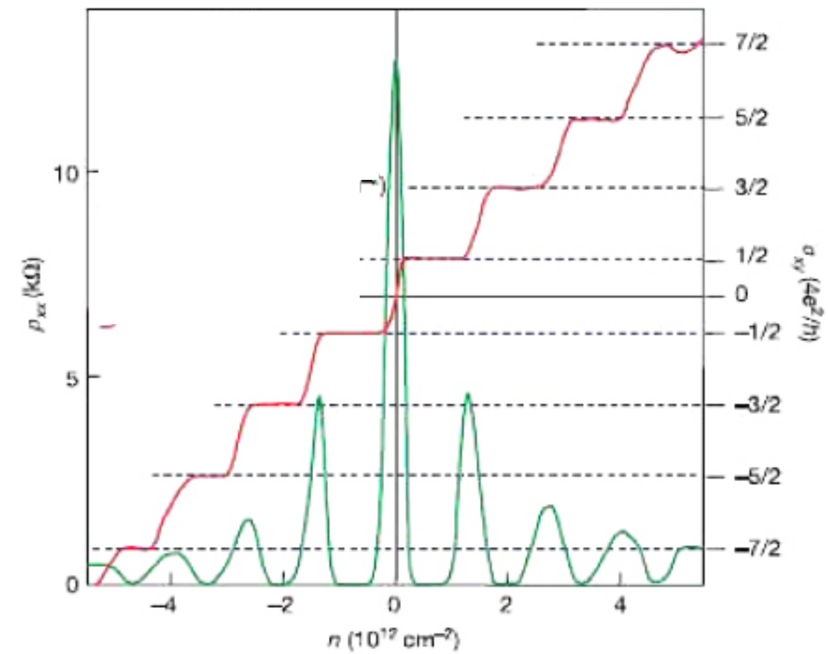
D3-D7: Only fermionic excitations in 3D

D3-D5: Fermions as well as bosons in 3D

Our model: D3-D5 brane type but some D5 branes can blow up to D7-branes

Quantum Hall effect in graphene





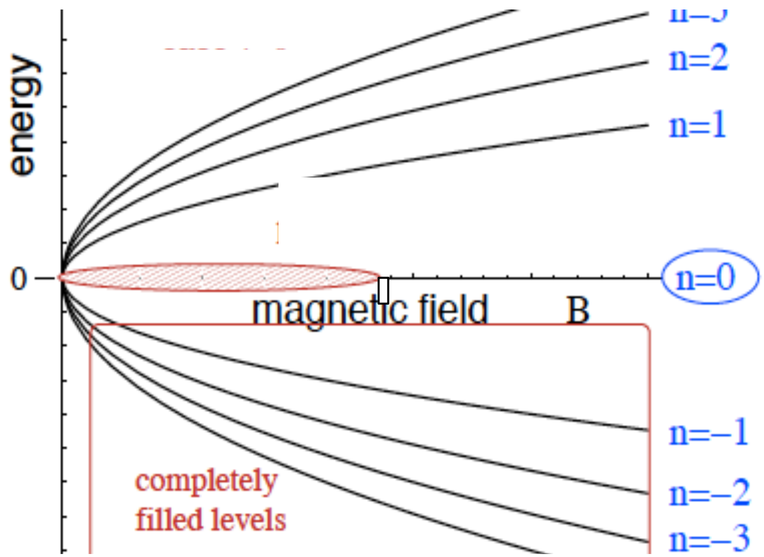
Standard QHE: The middle of a plateau corresponds to filling fraction n integer

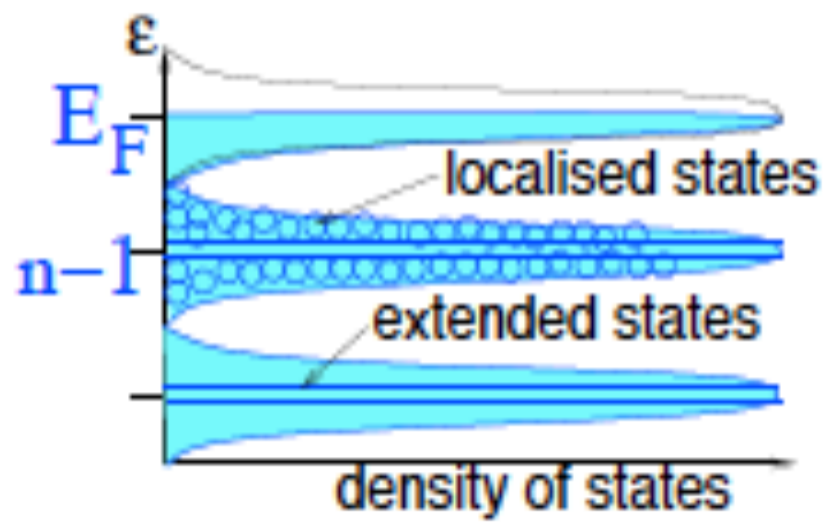
QHE for graphene: $\sigma_{xy} = 4\left(n + \frac{1}{2}\right)\frac{e^2}{h}$

Standard quantum Hall effect explanation

Landau levels = Gapped states

Localization





$$E_n = \sqrt{2nB}, \quad n = 0, \pm 1, \pm 2, \ldots$$

Filling fraction: $n = \dfrac{n_e}{n_L} = \dfrac{e^2}{h}\dfrac{\rho}{B}$

Landau levels in the string theory picture

Need: Charge density ρ / Magnetic field B } Introduce background gauge field

Filling fraction $\nu = \frac{2\pi}{N}\frac{\rho}{B}$

D5 brane: $AdS_4 \times S^2$: Gapped state at $\nu = 0$

Evans, Gebauer, Kim & Magou '10

D7 brane: $AdS_4 \times S^2 \times S^2$ (Blown up D5 brane) } Gapped states at integer ν $(= 1)$

The geometric set-up

Metric of $AdS_5 \times S^5$

$$ds^2 = \sqrt{\lambda}\alpha' \Big[r^2(-dt^2 + dx^2 + dy^2 + dw^2) + \frac{dr^2}{r^2} +$$
$$+ d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2) + \cos^2\psi(d\tilde{\theta}^2 + \sin^2\tilde{\theta} d\tilde{\phi}^2) \Big]$$

	t	x	y	w	r	ψ	θ	ϕ	$\tilde{\theta}$	$\tilde{\phi}$
$D3$	$\times$	$\times$	$\times$	$\times$						
$D5$	$\times$	$\times$	$\times$		$\times$		$\times$	$\times$		
$D7$	$\times$	$\times$	$\times$		$\times$		$\times$	$\times$	$\times$	$\times$

D5: $w = const, \;\; \tilde{\theta} = const, \;\; \tilde{\phi} = const, \;\; \psi = \psi(r)$

D7: $w = const, \;\; \psi = \psi(r)$

Boundary condition: $\psi(r) \to \frac{\pi}{2}$ for $r \to \infty$

Consequence: D7-brane can be viewed as blown-up (giant) D5-brane

Parameters:

Filling fraction $\nu = \frac{2\pi}{N} \frac{\rho}{B}$, Number of D5-branes/Flux $f = \frac{2\pi N_5}{\sqrt{\lambda}}$

$$2\pi\alpha' \mathcal{F}_7 = \sqrt{\lambda}\alpha' \left[\frac{d}{dr} a(r) dr \wedge dt + b dx \wedge dy + \frac{f}{2} d\tilde{\Omega}_2 \right].$$

For given parameters, the trivial solution $\psi = \frac{\pi}{2}$,
non-trivial D5 and non-trivial D7 compete, Energies must be compared.

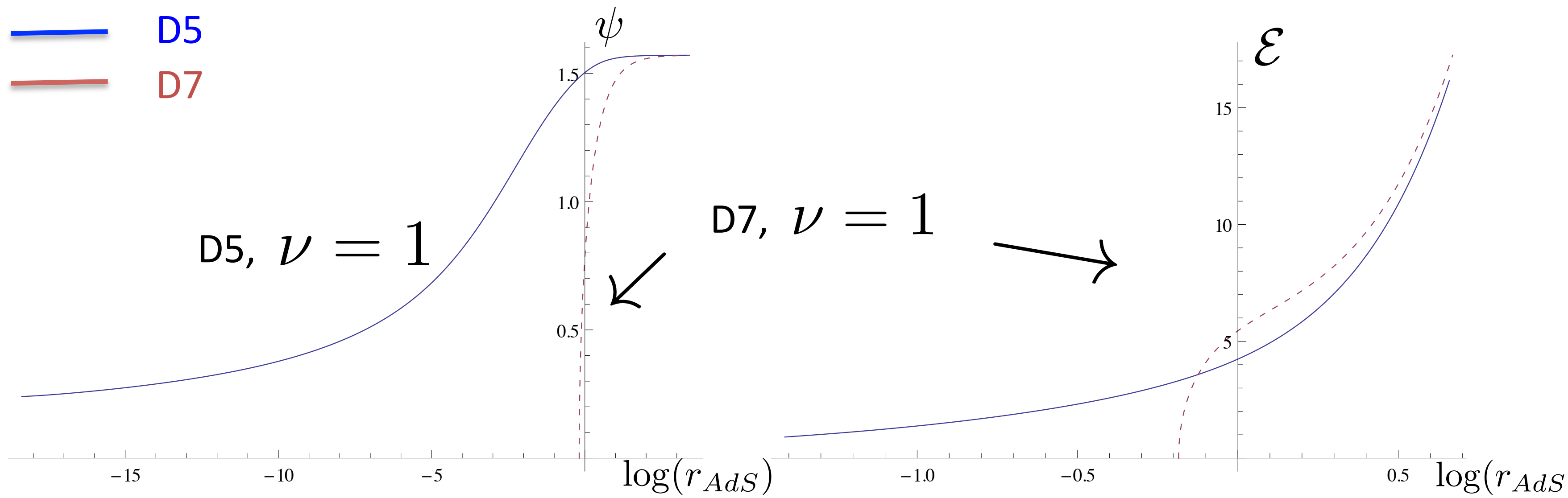
Actions and Approximations

- 't Hooft limit $N \to \infty, \lambda$ fixed
- Strong coupling $\lambda \to \infty$
- Probe approximation $N_7 = 1, (2, 3, ...) \ll N_5 \ll N$

$$S = S_{DBI} + S_{WZ}$$

WZ term does not play any role for the D5-brane

The gapped state for $\nu = 1, \ f = 1$



The gapped D7-brane is the preferred solution for $\nu = 1 \ (\text{any } f)$ (existence can be seen analytically).

Mapping out the phase diagram

Parameters:

Filling fraction $\nu = \frac{2\pi}{N} \frac{\rho}{B}$, Flux/Number of D5-branes $f = \frac{2\pi N_5}{\sqrt{\lambda}}$

Temperature $T = \frac{r_h}{\pi}$ (of $AdS_5 \times S^5$ black hole background)

$$ds^2 = \sqrt{\lambda}\alpha' \left[r^2(-h(r)dt^2 + dx^2 + dy^2 + dw^2) + \frac{dr^2}{h(r)r^2} + ds^2_{S^5} \right],$$

$$h(r) = 1 - \frac{r_h^4}{r^4}$$

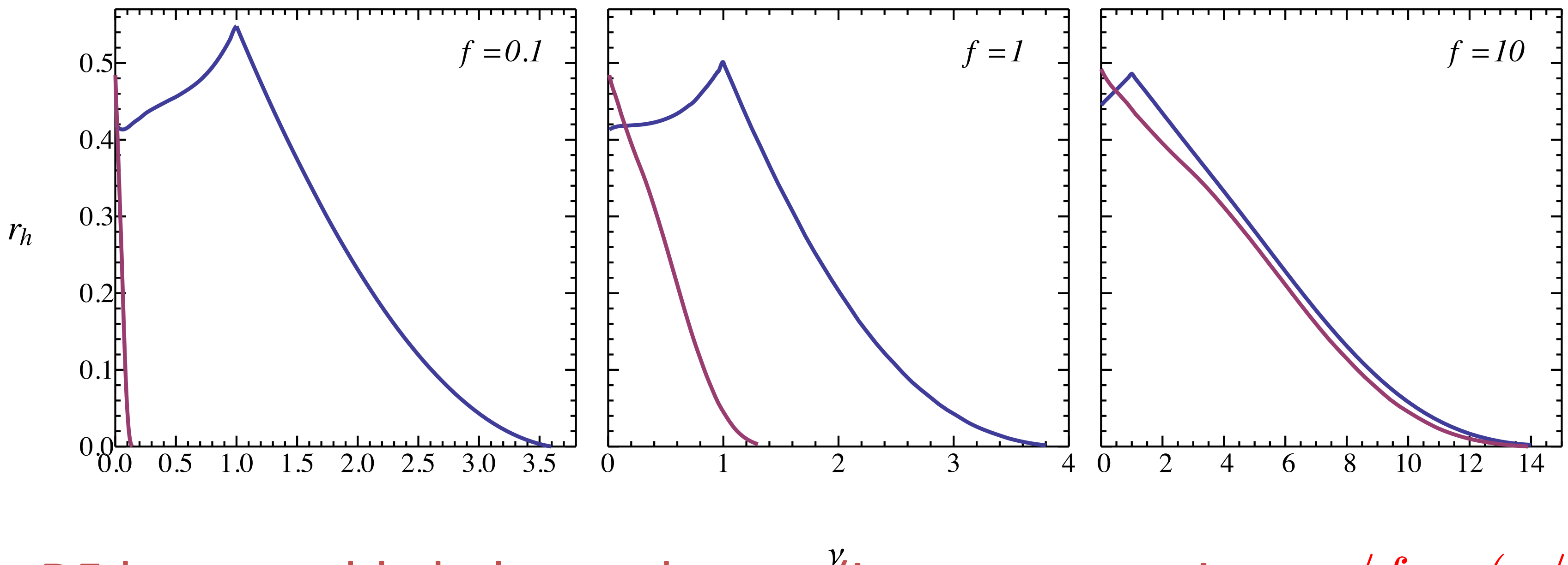
Expectation:

The constant solution $\psi = \frac{\pi}{2}$ (chirally symmetric phase)
is favoured for large values of ν and T

Analytical result for T=0:
Chiral symmetry broken for: $\nu/f < (\nu/f)_c = \frac{2\sqrt{7}}{\pi} \approx 1.68$

Jensen,Karch, Son, Thompson '11

Stability regions



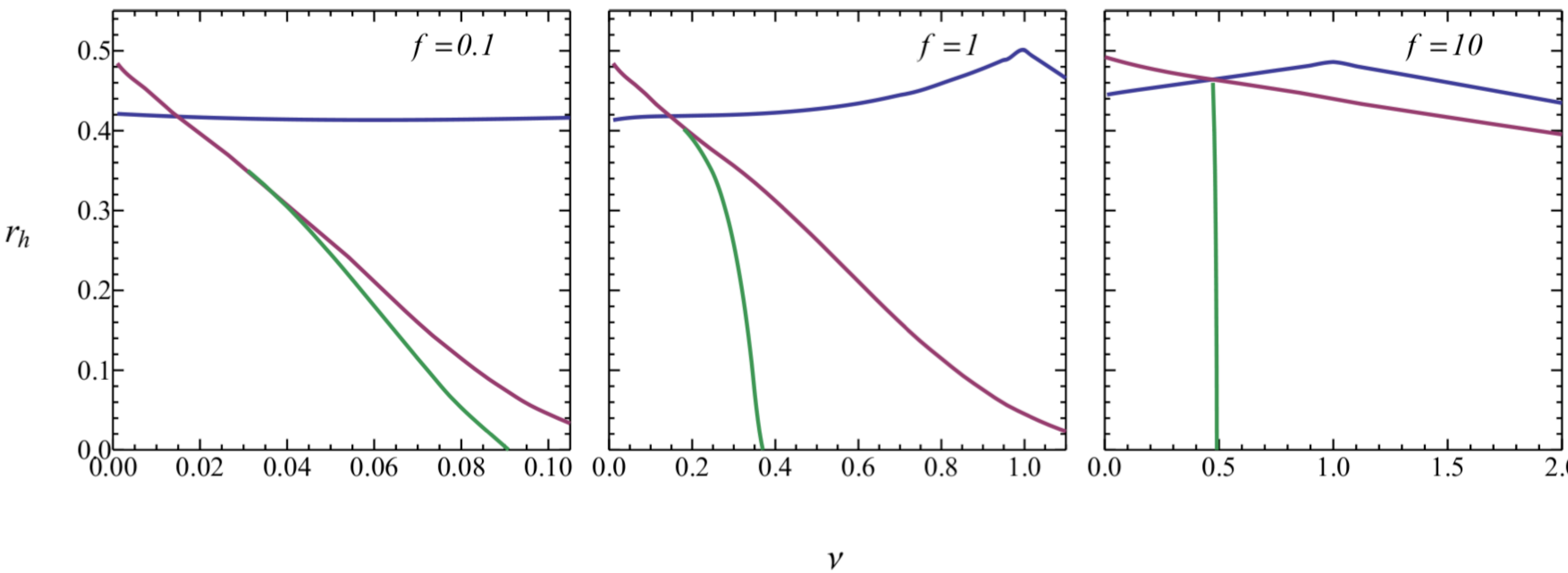
D5 brane stable below red curve (intersects x-axis at $\nu/f = (\nu/f)_c$)

D7 brane stable below blue curve, co-exist with constant solution in some region.

Notice that there is a dynamical upper bound on T

Which solution is preferred when both are stable?

Competition between D5 and D7



D7 brane preferred to the right of the green curve

OBS: Composite solutions for $\nu > 1$

The structure of the phase diagram and composite solutions

Approximate phase structure for low T and $f \to \infty$

$\nu = 0$	Gapped D5 branes
$0 < \nu < 0.5$	Ungapped D5 branes
$0.5 < \nu < 1$	Ungapped D7 brane
$\nu = 1$	One gapped D7 brane
$1 < \nu < 1.5$	One gapped D7 brane + ungapped D5 branes
$1.5 < \nu < 2$	One gapped D7 brane plus one ungapped D7 brane
$\nu = 2$	Two gapped D7 branes

a.s.o (at most until $\nu = \pm N_5$)

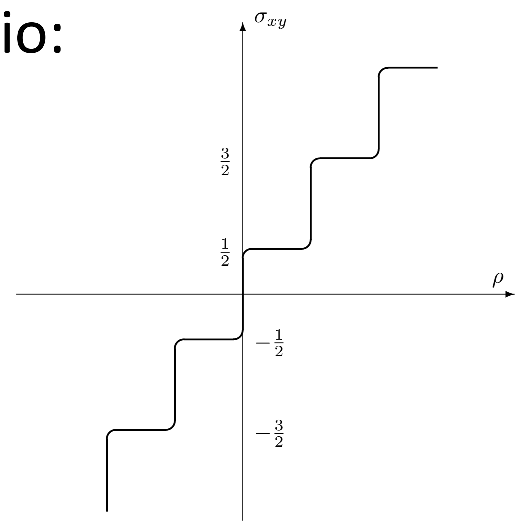
The conductivity tensor from string theory

Need to introduce E-field: Classical solution not enough

~~Fluctuations must be considered~~

$$J_y = \sigma_{xy} E_x$$

Dream scenario:



Concentrate on $\nu \in [0, 1]$

The conductivity tensor from string theory

Introducing the world volume electric and magnetic fields (Fefferman-Graham coord.)

$$2\pi\alpha'\mathcal{F}_5 = \sqrt{\lambda}\alpha' \left[\frac{d}{dz}a(z)dz \wedge dt + b\, dx \wedge dy + \frac{d}{dz} f_y(z) dz \wedge dy + \frac{d}{dz} f_x(z) dz \wedge dx - e\, dt \wedge dx \right]$$

Three conserved charges

$$q \equiv \frac{\delta S_5}{\delta a'(z)}, \qquad j_x \equiv -\frac{\delta S_5}{\delta f'_x(z)}, \qquad j_y \equiv -\frac{\delta S_5}{\delta f'_y(z)}$$

Aim: Fix q, b and e. Determine j_x and j_y

$$\sigma_{xy} = \left.\frac{j_y}{e}\right|_{e=0}, \qquad \sigma_{xx} = \left.\frac{j_x}{e}\right|_{e=0}$$

Hall conductivity

Longitudinal conductivity

The conductivity tensor from string theory

Expressing the (Laplace transformed) action in terms of conserved charges:

$$\mathcal{R}_5 = -N_5 \mathcal{N}_5 \int dz \sqrt{\frac{g_{zz}}{g_{tt}}} \frac{1}{g_{xx}} \sqrt{B \cdot C - A^2},$$

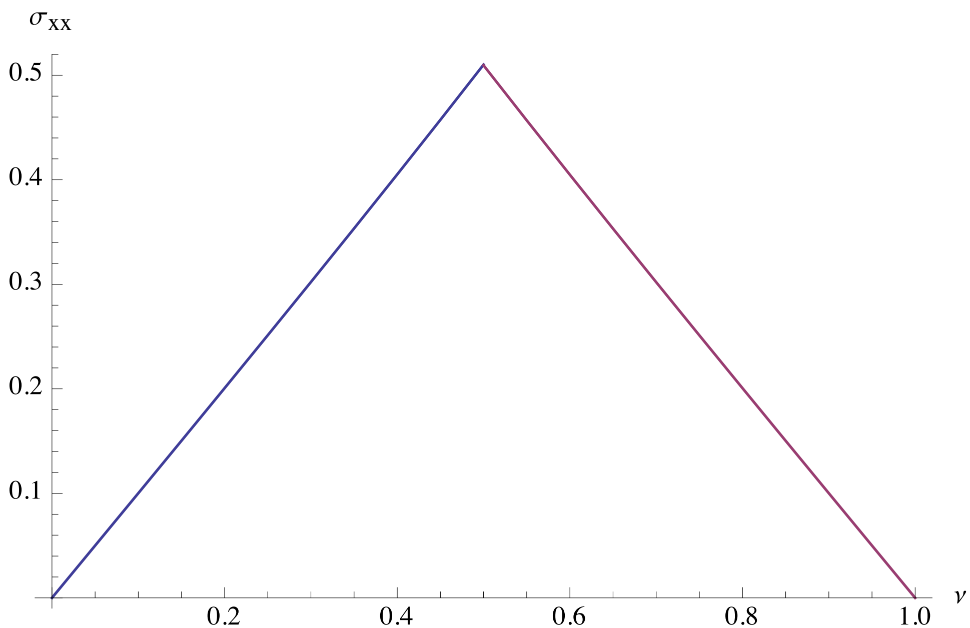
$$A = q\, b\, g_{tt} - j_y\, e\, g_{xx},$$
$$B = g_{tt}\, g_{xx}^2 + g_{tt}\, b^2 - g_{xx}\, e^2,$$
$$C = 4 \sin^4 \psi\, g_{tt}\, g_{xx}^2 + g_{tt}\, q^2 - g_{xx}\, (j_x^2 + j_y^2).$$

$$\exists\ z^* :\ 0 < z^* < z_h,\ B(z^*) = 0$$

Demanding that $B \cdot C - A^2$ is always real and positive gives $A(z^*) = C(z^*) = 0$ which determines j_x and j_y

$$j_y = \frac{b\, q}{b^2 + g_{xx}^2(z^*)}\, e, \quad j_x = \frac{g_{xx}(z^*)\, e}{b^2 + g_{xx}^2(z^*)} \sqrt{4 \sin^4 \psi(z^*)(b^2 + g_{xx}^2(z^*)) + q^2}.$$

Results for the longitudinal conductivity σ_{xx}



The longitudinal conductivity for $f = \infty$ and $\nu \in [0, 1]$

in units of $\frac{\tilde{r}_h^2}{1+\tilde{r}_h^4} \cdot \frac{N}{2\pi}$

Results for σ_{xx} for $f \to \infty$ and $\nu \in [0,1]$

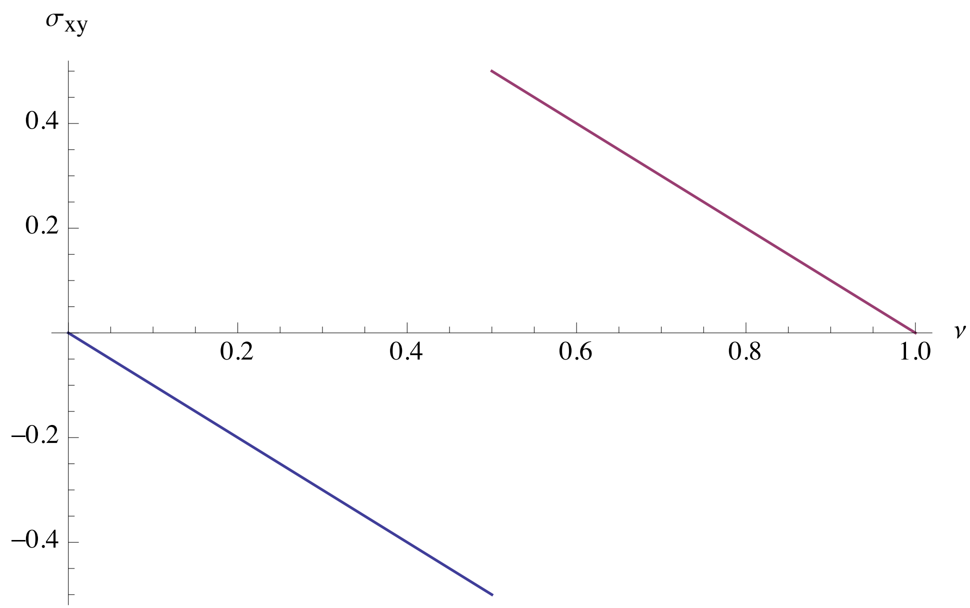
$$\sigma_{xx}^{D5} = \frac{N\nu}{2\pi}\,\frac{\hat{r}_h^2}{1+\hat{r}_h^4}, \qquad \hat{r}_h = \sqrt{\frac{\pi}{2B}}\,\lambda^{1/4}\,T$$

$$\sigma_{xx}^{D7} = \frac{N(1-\nu)}{2\pi}\,\frac{\hat{r}_h^2}{1+\hat{r}_h^4}.$$

Observations

- The longitudinal conductivity vanishes for T=0 (as expected)
- σ_{xx} vanishes for the gapped states $\nu = 0$ and $\nu = 1$
- The longitudinal conductivity is continuous at $\nu = 1/2$ where the D7 brane takes over

Results for the Hall conductivity



The deviation of the Hall conductivity from the linear from for $f = \infty$ and $\nu \in [0, 1]$ in units of $\frac{\tilde{r}_h^4}{1+\tilde{r}_h^4} \cdot \frac{N}{2\pi}$

Results for the Hall conductivity for $f \to \infty$ and $\nu \in [0,1]$

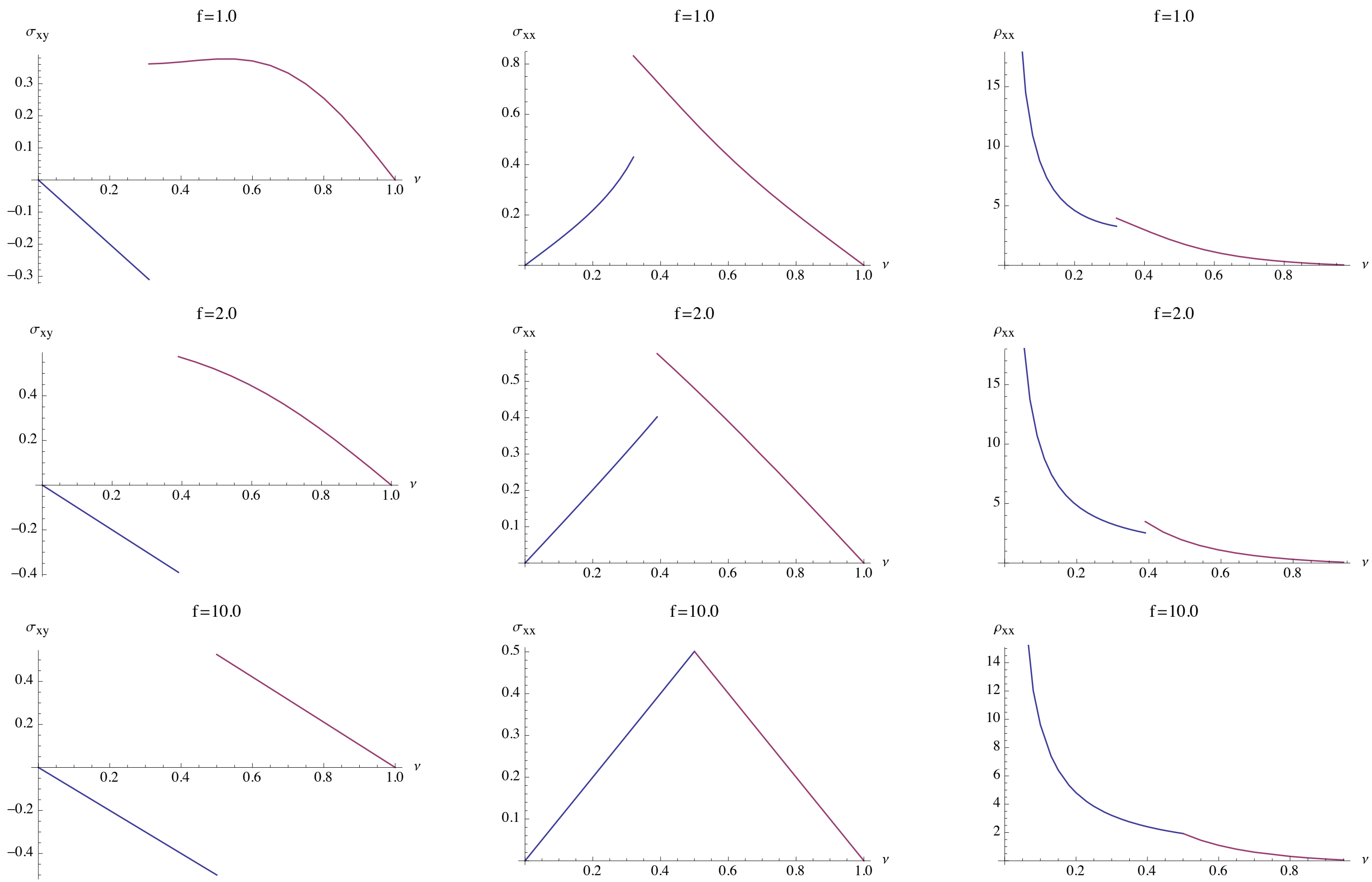
$$\sigma_{xy}^{D5} = \frac{N\nu}{2\pi} - \frac{N\nu}{2\pi}\,\frac{\hat{r}_h^4}{1+\hat{r}_h^4}, \qquad \hat{r}_h = \sqrt{\frac{\pi}{2B}}\,\lambda^{1/4}\,T$$

$$\sigma_{xy}^{D7} = \frac{N\nu}{2\pi} + \frac{N(1-\nu)}{2\pi}\,\frac{\hat{r}_h^4}{1+\hat{r}_h^4}.$$

Observations

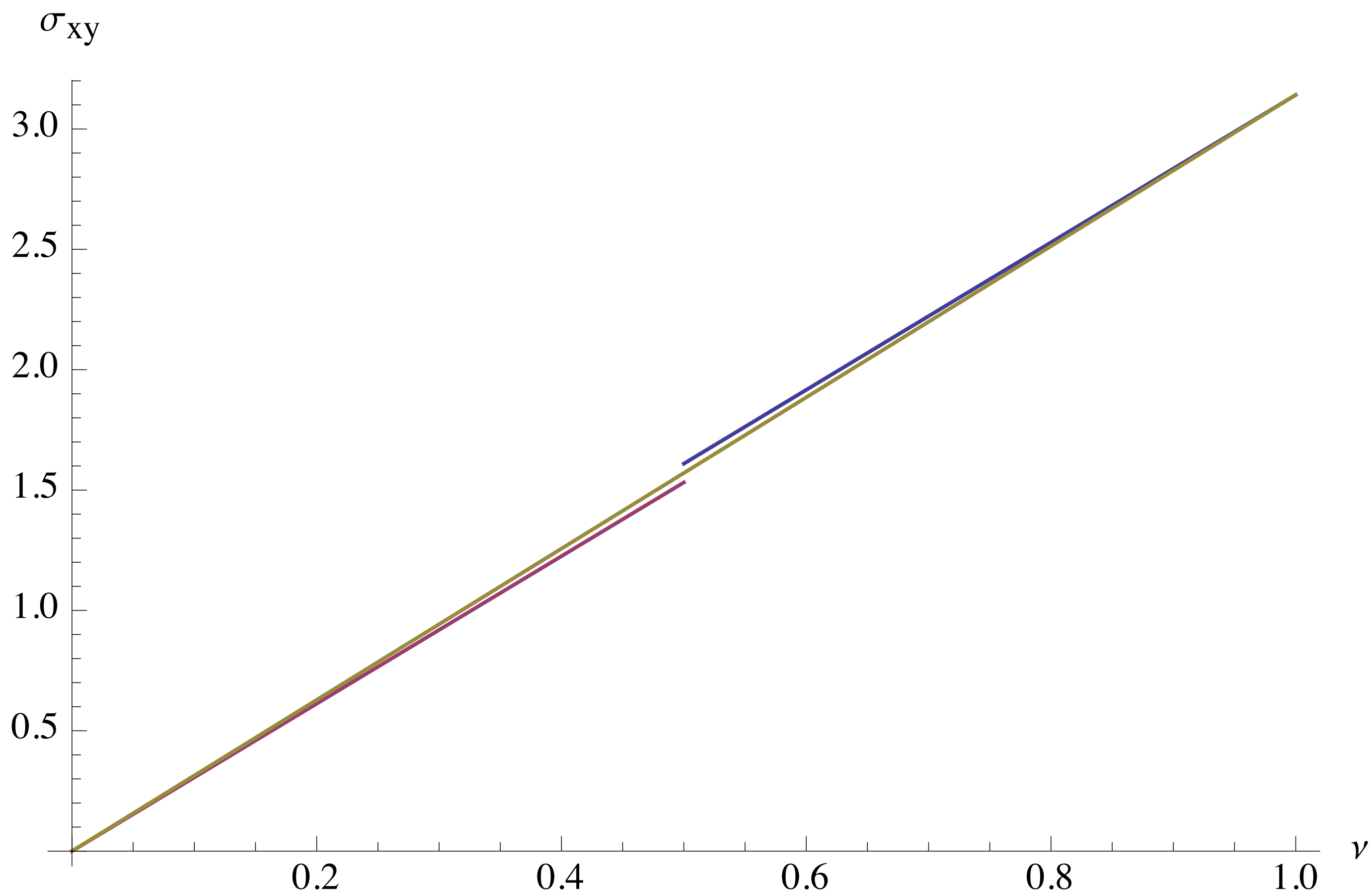
- The correction vanishes for T=0 (as expected)
- The correction vanishes for the gapped states $\nu = 0$ and $\nu = 1$
- There is a jump upwards in Hall conductivity at $\nu = 1/2$ where the D7 brane takes over
- For $T \to \infty$ one gets a perfect Hall step (but $\hat{r}_h \lesssim 0.4$)

Numerical results for the conductivity tensor for finite f



The deviation of the Hall conductivity from the linear form, the longitudinal conductivity and the longitudinal resistivity for $\nu \in [0, 1]$

Results for the Hall conductivity



$f = 10, \hat{r}_h = 0.4$ (for maximal effect)

Conclusion

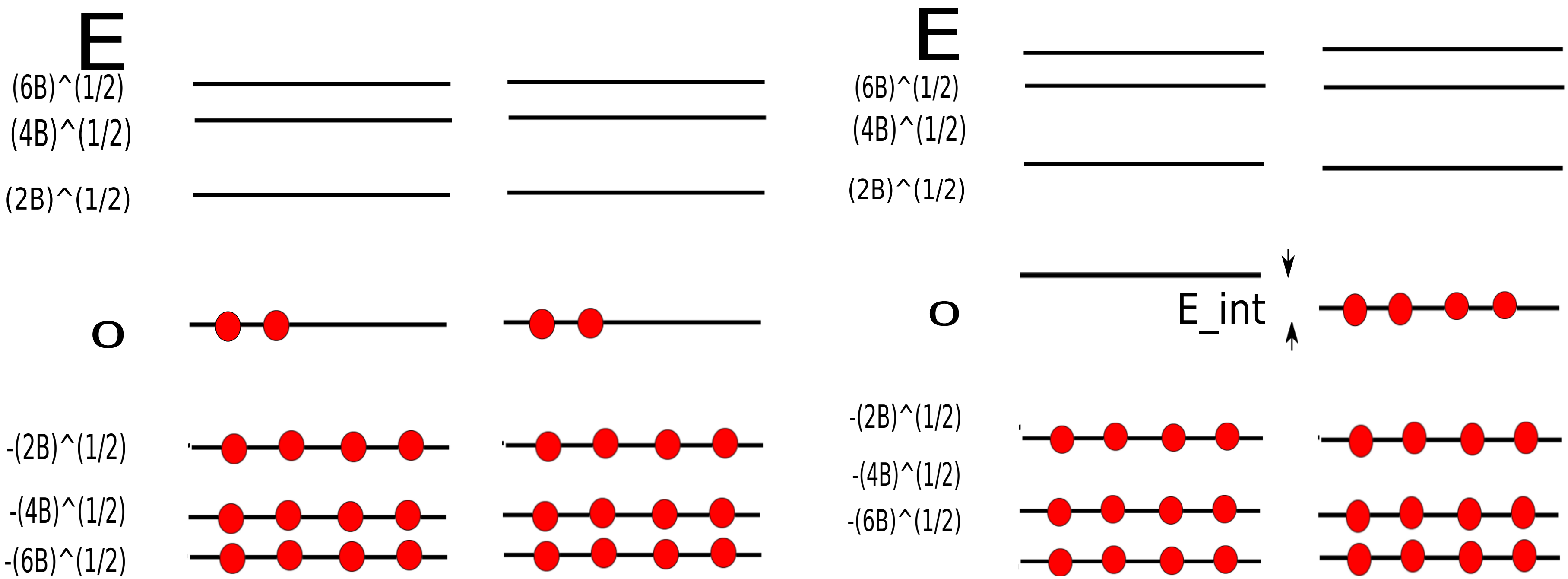
We have identified a $\nu = 1$ Hall state at strong coupling (+generalizations)

We have observed a strong coupling mechanism that gives some tendency towards plateau formation.

The observed strong coupling effect requires a finite temperature and its magnitude is small.

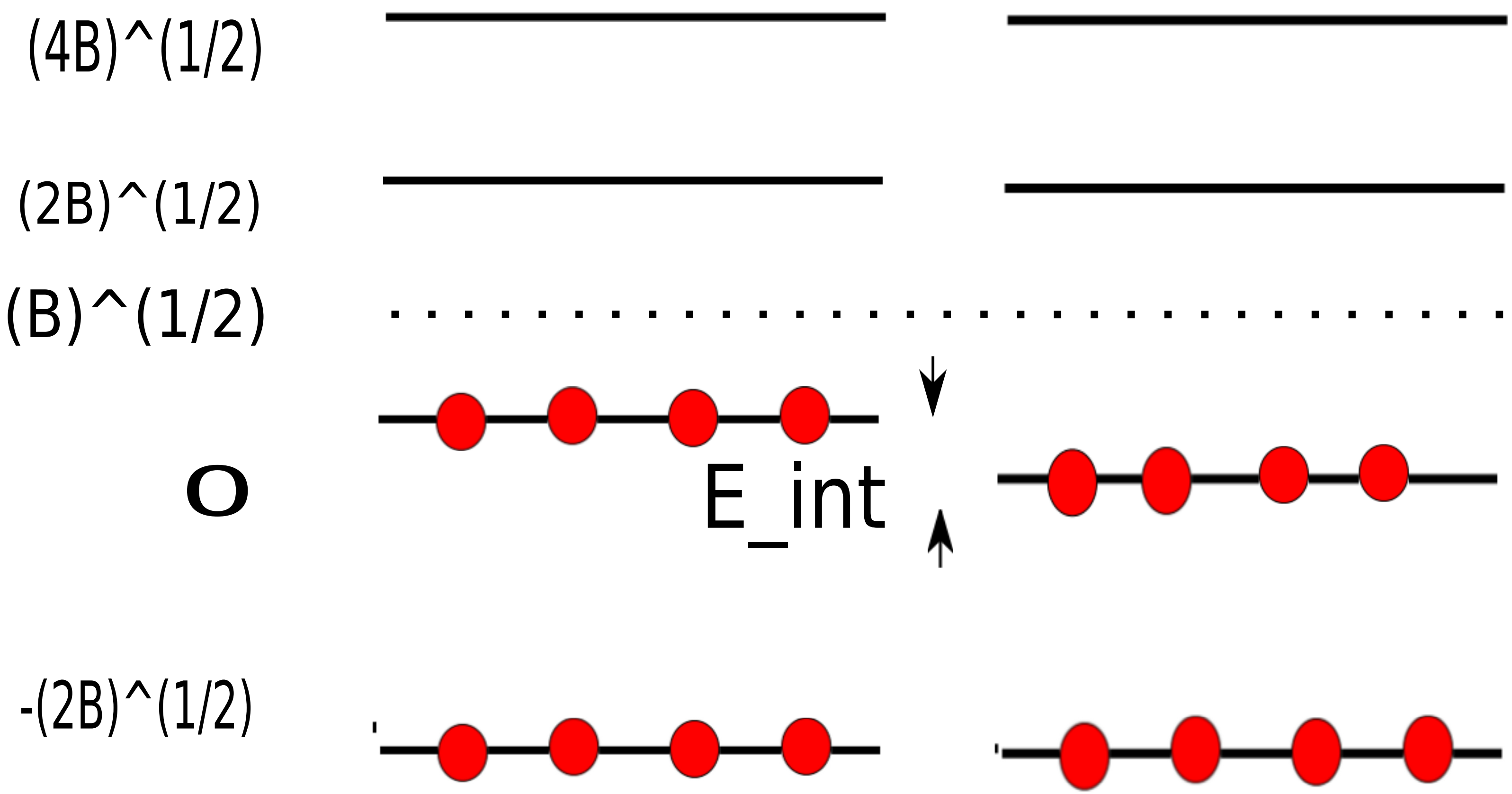
Quantum Hall Ferromagnetism --- The weak coupling story

$N_5 = 1,$ Two isospin states



$\rho = 0, B = 0$ $\rho = 0, B \neq 0$

The $\nu = 1$ quantum Hall state, $N_5 = 1$



Similar system with many quantum Hall states $(N_5 = 3)$