The Social Network of F-theory fibrations: From torsion to discrete symmetries

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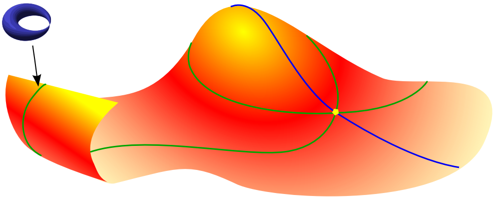
What is F-theory?



Identify the complex structure τ of an auxiliary torus to be the Type IIB axio-dilaton $\tau = C_0 + ie^{-\phi}$ that is fibered over the physical physical base space.

Information of the elliptic fiber

1. The gauge group:
   - Codimension 1 singularities: non-Abelian factors
   - Freely acting Mordell-Weil group: Abelian factors [Morrison,Park'12;Braun,Grimm,Keitel'13]
   - Mordell-Weil torsion: Quotient groups [Morrison,Palti,Till,Weigand'14]
   - Discrete $\mathbb{Z}_n$ symmetries: n-sections [Morrison,Taylor'14]
2. Charged Matter: Codimension 2 singularities
3. Yukawa Couplings: Codimension 3 singularities

Why F-theory

- Non-perturbative formulation of Type IIB strings with D7 branes fully back-reacted on the geometry New possibilities for Model building:
  - **GUT** ($SU(5)$ or E_6) gauge groups that are **localized** + add U(1)'s to forbid couplings
  - Generate top-quark Yukawa couplings that are pert. forbidden in Type IIB settings [Aparacio,Font,Ibanez,Regalado,Marchesano,Zoccarato '11-'13]

Geometric engineering of gauge groups: non-Abelian part easy but the Abelian usually hard

- Global GUTS with additional U(1)'s using Tops

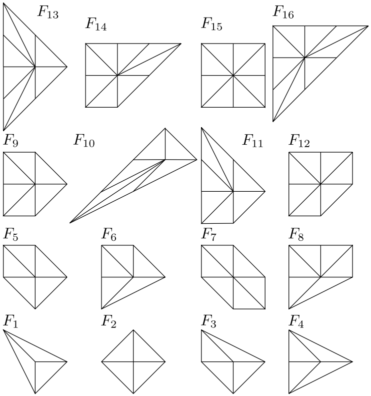
[Grimm,Braun,Keitel,Cvetic,Grassi,,Klevers,Piragua,Borchmann,Weigand,Mayerhofer,Palti '13-'14]

$$\underbrace{U(1)^2}_{\subset dP^2} \times \underbrace{SU(5)}_{Coef.Restriction}$$

Describe U(1)'s parts as hypersurfaces in toric varieties a fruitful approach.

Extend this toolbox to all 2D varieties.

16 Polyhedra



The set

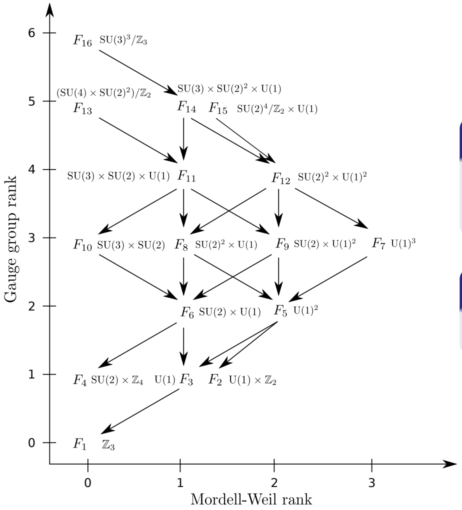
- All 16,2D toric varieties
- Genus-one fiber as hypersurface in F_i
- F_{17-i} and F_i are dual

The Program:

General genus-one fibered Calabi-Yau threefolds
$\rightarrow$ 6D $\mathcal{N} = 1$ theories

- computed gauge group
- compute matter & multiplicity
- check full 6D gauge & gravity anomaly cancellation
- compute all transitions

The Network



Toric Higgs Effect

The Geometry Side

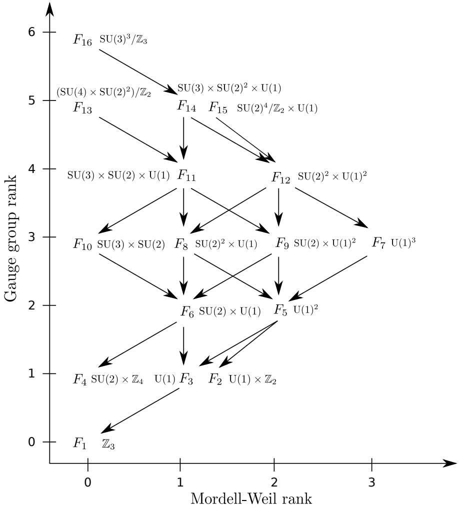
Extremal transitions of the polyhedra used for fiber i.e. blow-ups

The Field Theory Side

The 6D Higgs effect

→ Another powerful cross-check for geometry and field theory properties

The Network

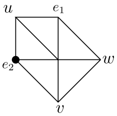


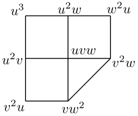
The Highlights

1. Mordell-Weil torsion
   [Mayrhofer,Morrison,Till,Weigand'14]
   - F_{16} : Trinification
   - F_{13} : Pati-Salam
2. F_{11} : The Standard Model
3. F_3 : $q = 3$ $U(1)$ charged matter
4. Multi sections:
   - discrete symmetries
     [Anderson,Garcia-Etxebarria, Grimm, Keitel'14]
     [Mayrhofer,Palti,Till,Weigand '14]
5. Mirror-Symmetry
   $\rightarrow$ torsion $\overset{mirrordual}{\longleftrightarrow}$ discrete symmetries?
6. The Sum Rule:
   $\text{Rank}(F_{17-i} + \text{Rank}(F_i) = 6$

Toric Higgs Effect

The Higgsing of F_5: $U(1)^2$





Describe F_5 fiber by hyperplane equation:

$$
\begin{aligned}
p_{F_5} = &s_1 e_2^2 e_1^2 u^3 + s_2 e_2^2 e_1 u^2 v + s_3 e_2^2 u v^2 \\
&+ s_5 e_2 e_1^2 u^2 w + s_6 e_2 e_1 u v w + s_7 e_2 v^2 w \\
&+ s_8 e_1^2 u w^2 + s_9 e_1 v w^2
\end{aligned}
$$

- s_i are sections of the base
- Three rational points
  $\rightarrow U(1)^2$ gauge symmetry

Toric Higgs Effect

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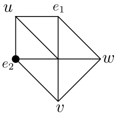

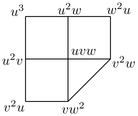

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\end{aligned}
$$

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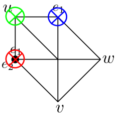
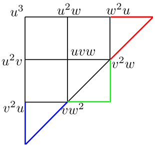
The charged matter: Fiber factorization into an I_2 fiber at codim. 2:

F_5 States	$\mathbf{1}_{(-1,1)}$	$\mathbf{1}_{(-1,-2)}$	$\mathbf{1}_{(0,2)}$	...
location	$s_3 = s_7 = 0$	$s_8 = s_9 = 0$	$s_7 = s_9 = 0$	...
multiplicity	$S_7([K_B^{-1}] + S_7 - S_9)$	$S_9([K_B^{-1}] - S_7 + S_9)$	$S_7 S_9$	...

Toric Higgs Effect

The Higgsing of F_5: $U(1)^2$





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- Three rational points $\rightarrow U(1)^2$ gauge symmetry

Choose a Higgs

F_5 States	$\mathbf{1}_{(-1,1)}$	$\mathbf{1}_{(-1,-2)}$	$\mathbf{1}_{(0,2)}$	...
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Toric Higgs Effect

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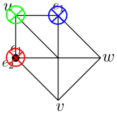

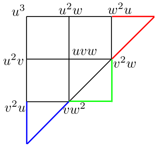

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&+ s_8 e_1^2 u w^2 + s_9 e_1 v w^2
\end{aligned}
$$

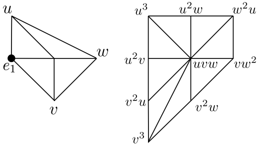
- s_i are sections of the base
- Three rational points $\rightarrow U(1)^2$ gauge symmetry

- Higgs: $\langle \mathbf{1}_{(-1,1)} \rangle$ breaks $U(1)^2 \rightarrow U(1)$
- Unbroken charge: $Q' = Q_1 + Q_2$

F_5 State	$\mathbf{1}_{(-1,1)}$	$\mathbf{1}_{(-1,-2)}$	$\mathbf{1}_{(0,2)}$	...
F_3 State	$\mathbf{1}_0$	$\mathbf{1}_{(-3)}$	$\mathbf{1}_{(2)}$	...
location	$s_3 = s_7 = 0$	$s_8 = s_9 = 0$	$s_7 = s_9 = 0$	...
multiplicity	$S_7([K_B^{-1}] + S_7 - S_9)$	$S_9([K_B^{-1}] - S_7 + S_9)$	$S_7 S_9$	...

Toric Higgs Effect

Match geometry: The F_3 Fibration



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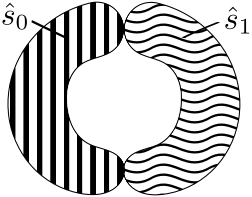
&+ s_8 e_1^2 u w^2 + s_9 e_1 v w^2 + {\color{blue} s_4 v^3 e_1^2}
\end{aligned}
$$

- s_i sections of the base

- **subtlety:** Get the non-toric U(1) from a section that is the intersection with the fiber and a double intersection with the zero section
- Lets have a look at a codimension 2

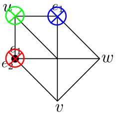


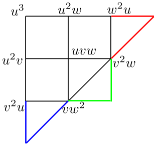
- The zero section wraps first $\mathbb{P}^1$
- The second section wraps other $\mathbb{P}^1$

F_3 State	$\mathbf{1}_3$
location	$s_8 = s_9 = 0$
multiplicity	$S_9([K_B^{-1}] - S_7 + S_9)$

Toric Higgs Effect

The Higgsing of F_5: $U(1)^2$





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\end{aligned}
$$

- s_i are sections of the base

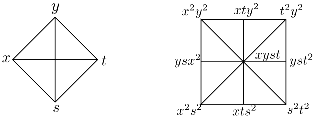
Have chosen $\langle \mathbf{1}_{(0,2)} \rangle$ that breaks $U(1)^2$ to $U(1) \times \mathbb{Z}_2$

The Higgsed Spectrum: New Charge operator: $Q' = Q_1,\ Q_{\mathbb{Z}_2}$

F_5 State	$\mathbf{1}_{(-1,1)}$	$\mathbf{1}_{(-1,-2)}$	$\mathbf{1}_{(0,2)}$	...
F_2 State	$\mathbf{1}_{(-1,-)}$	$\mathbf{1}_{(1,+)}$	$\mathbf{1}_{(0)}$	...
location	$s_3 = s_7 = 0$	$s_8 = s_9 = 0$	$s_7 = s_9 = 0$	...
multiplicity	$S_7([K_B^{-1}] + S_7 - S_9)$	$S_9([K_B^{-1}] - S_7 + S_9)$	$S_7 S_9$	...

Toric Higgs Effect

Matching geometry: The F_2 fiber



Describe F_2 fiber by hyperplane

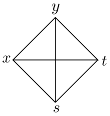
$$p_{F_2} = (b_1 y^2 + b_2 sy + b_3 s^2)x^2 + (b_5 y^2 + b_6 sy + b_7 s^2)xt + (b_8 y^2 + b_9 sy + b_{10} s^2)t^2$$

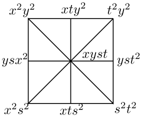
- b_i are sections of the base

- F_2 The biquadric has **no section**, but two, two-sections $\hat{\sigma}_0$ and $\hat{\sigma}_1$
- Map biquadric to the cubic and match coefficients
  $\rightarrow$ gives a zero-section (Jacobian fibration) [Braun, Morrison'14]

Toric Higgs Effect

Matching geometry: The F_2 fiber





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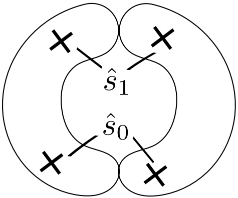

- One two-section becomes rational
  $\rightarrow$ U(1)
- second two-section **not rational**
  $\rightarrow \mathbb{Z}_2$ discrete symmetry

F_2 State:	$\mathbf{1}_{(0,-)}$
multiplicity	Matches Higgsing ✓

Conclusion

We have constructed

Global 6D F-theory fibrations with fibers as hypersurfaces in all 16, 2D toric varieties and computed

- Gauge group, matter, moduli and their multiplicities
- full 6D gauge and gravity anomaly cancellation
- All possible extremal transitions and matched the field theories

Our Highlights

We found

- A mirror-symmetric Network of toric-fibrations
- Fibrations with discrete symmetries and their charges
- $q = 3$ U(1) charged matter
- Interesting fibrations for particle physics (Trinification, Pati-Salam and Standard model)

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Thank You !

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