Anomalous magneto-response and the Stückelberg axion in holography

A. Jimenez, L. Melgar, K.L. [arXiv:1407.8162]

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Outline

- Review of anomalous transport
- Stückelberg axion and anomaly
- Axionic magneto-response
- Summary and Outlook

Anomalous transport

- Chiral magnetic effect (CME)

[Vilenkin],
[Giovannini,Shaposhnikov]
[Alekseev, Chaianov, Fröhlich]
[Fukushima, Kharzeev, McLarren]
[Fukushima, Kharzeev,Warringa]

$$\vec{J} = \sigma_{\mathrm{CME}} \vec{B}$$

Realized (possibly) in:

- *Heavy ion collisions*
- *Early universe*
- *Weyl semi-metals*

$$\sigma^{ab}_{\mathrm{CME}} = \frac{d^{abc}}{4\pi^2} \mu_c$$

Anomaly coefficient

Anomalous transport

- Chiral vortical effect (CVE) $\vec{J} = \sigma_{\rm CVE}\vec{\omega}$

[Vilenkin]
[Batthacharya et al.]
[Erdmenerg et al.]

- CME and CVE in energy currents

$$\vec{J}_\epsilon = \sigma^\epsilon_{\rm CVE}\vec{B}$$

$$\vec{J}_\epsilon = \sigma^\epsilon_{\rm CVE}\vec{\omega}$$

$$\sigma^a_{\rm CVE} = \frac{d^{abc}}{8\pi^2}\mu_b\mu_c + \frac{b_a}{24}T^2$$

$$\sigma^\epsilon_{\rm CVE} = \frac{d^{abc}}{12\pi^2}\mu_a\mu_b\mu_c + \frac{b_a}{12}\mu_a T^2$$

$$\sigma^{\epsilon a}_{\rm CME} = \sigma^a_{CVE}$$

[Vilenkin]
[Megias, K.L.., Pena-Benitez]

Anomalous transport

- Non-renormalization:
  free fermions = holographic theories

  [Yee] [Schmitt,Stricker,Rebhan]
  [Gynther, K.L., Rebhan, Pena-Benitez]
  [Megias,, K.L., Pena-Benitez]

- *proof: e.g. hydrodynamics, effective action approach*

  [Son, Surowka], [Neiman, Oz] [Jensen, Loganayagam, Yarom] [DiPietro, Komargodski]

- *HOWEVER: 2-loop correction found in T^2 term and later in all anomalous transport coefficients*

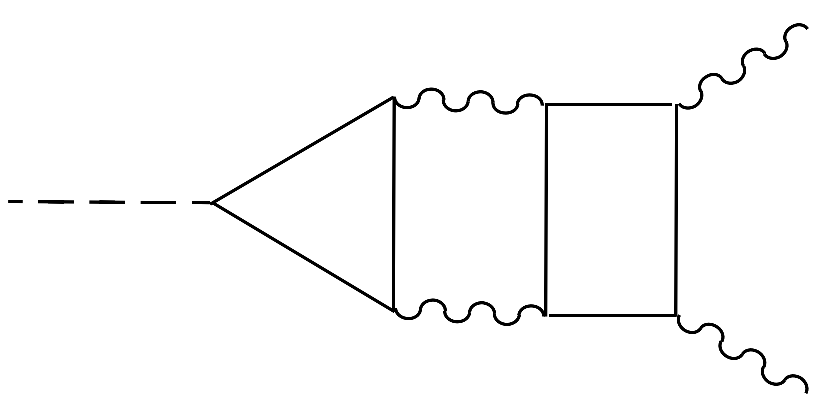
  [Hou, Lui, Ren] [Golkar, Son], [Jensen, Kovtun, Ritz] [Gorbar, Miransky, Shovkovy, Wang]

- *Ingredient: dynamical gauge fields in anomaly*

Anomalous transport

$$(D_\mu J^\mu)^a = \epsilon^{\mu\nu\rho\lambda} \left(\frac{d^{abc}}{32\pi^2} F^b_{\mu\nu} F^c_{\rho\lambda} + \frac{b_a}{768\pi^2} R^\alpha{}_{\beta\mu\nu} R^\beta{}_{\alpha\rho\lambda} \right)$$

- Non-renormalization holds if gauge fields and metric are classical fields
- If gauge fields (or metric) are quantum fields: current is not a dimension three operator
- Axial anomaly in QED: anomalous dimension for axial current J_5^μ



Anomalous transport

A subtlety: ***covariant*** anomaly vs. ***consistent*** anomaly

$$\delta J_\mu = i[\lambda, J_\mu] \qquad \textbf{\textit{or}} \qquad J_\mu = \frac{\delta W_{\text{eff}}}{\delta A_\mu}$$

In V-A theory (Dirac fermion):

$$J^\mu_{\text{cons}} = J^\mu_{\text{cov}} + \frac{1}{4\pi^2}\epsilon^{\mu\nu\rho\lambda}A^5_\nu F_{\rho\lambda}$$

- Only consistent vector current $\partial_\mu J^\mu = 0$
- Chiral magnetic effect: $\vec{J} = \left(\frac{\mu_5}{2\pi^2} - \frac{A^5_0}{2\pi^2}\right)\vec{B}$
- Chiral separation effect: $\vec{J}_5 = \frac{\mu}{2\pi^2}\vec{B}$

Stückelberg Axion

U(1)$_A$ anomaly QCD:

$$\partial_\mu J^\mu_A = \epsilon^{\alpha\beta\gamma\delta}\left(C_1 F_{\alpha\beta}F_{\gamma\delta} + C_2 \mathrm{tr}(G_{\alpha\beta}G_{\gamma\delta}) + C_2 F^5_{\alpha\beta}F^5_{\gamma\delta}\right)$$

"QED" "QCD" external

strongly coupled

Path integral:

$$e^{iW_{\text{eff}}} = \int D\Psi D\bar{\Psi} D\mathcal{A}_q \exp\left[i\int d^4x\left(-\frac{1}{2}\mathrm{tr}(G.G) + \bar{\Psi}D\Psi + \theta\mathcal{O}_A\right)\right]$$

$W_{\text{eff}}[A, A_5, \theta]$ invariant under: $\delta A_5 = d\lambda\ ,\quad \delta\theta = -\lambda$

Stückelberg Axion

Bottom up approach: substitute path integral by dynamics in AdS-space keeping the symmetries

$$\mathcal{L} = \Big[-\frac{1}{4}F^2 - \frac{1}{4}H^2 - \frac{m^2}{2}(A_\mu - \partial_\mu \theta)(A^\mu - \partial^\mu \theta) + \frac{\kappa}{2}\epsilon^{\mu\alpha\beta\gamma\delta}(A_\mu - \partial_\mu \theta)\left(F_{\alpha\beta}F_{\gamma\delta} + 3H_{\alpha\beta}H_{\gamma\delta}\right)\Big]$$

- Two gauge symmetries
- One conserved current
- One non-conserved current = massive vector
- CS term reflects anomalies in weakly coupled sector
- Stückelberg mechanism and anomaly more general discussion and model

[Klebanov, Ouyang, Witten] [Gursoy, Jansen]

Stückelberg Axion

- Asymptotic expansion: $A_{i(N.N.)} \sim A_{i(0)} r^{\Delta}\,; \quad A_{i(N.)} \sim \tilde{A}_{i(0)} r^{-2-\Delta}\,; \quad \Delta = -1 + \sqrt{1+m^2}\,.$
- Dimension of current: $\dim(\tilde{A}_{i(0)}) = [J_i] = 3 + \Delta\,.$
- Holographic counterterms ($\Delta < 1/3$):

$$S_{CT} = \int_{\partial} d^4x \sqrt{-\gamma} \left(\frac{\Delta}{2} B_i B^i - \frac{1}{4(\Delta+2)} \partial_i B^i \partial_j B^j + \frac{1}{8\Delta} F_{ij} F^{ij} \right)$$

number CTs depends on Δ , for $\Delta = 1$ infinite number of CTs

- Renormalized WI: $\langle \partial_i J^i \rangle = \lim_{r\to\infty} \sqrt{-g} r^{\Delta} \left(m^2 \partial^r \theta + r\Delta \partial_i A^i - \frac{\kappa}{3} \epsilon^{ijkl} F_{ij} F_{kl} + \tilde{X} \right)$

$$\stackrel{\text{Ren.}}{=} 2(1+\Delta) \partial_i \tilde{A}^i_{(0)}$$

- No covariant or consistent current, gauge fields do not allow to construct the CS currents of correct dimension

Stückelberg Axion

Response to magnetic field in AdS-Schwarzschild black brane (high T plasma phase), decoupling limit

- Chemical potential becomes source for operator $A = \Phi(r) dt$

$$\Phi'' + \frac{3}{r}\Phi' - \frac{m^2}{f}\Phi = 0$$

$$\phi(r_H) = 0\,, \qquad \phi(r \to \infty) \sim \mu_5 r^{\Delta}\,,$$

- ”Work” needed to bring unit of charge behind the horizon

$$"\delta\mu" = \lim_{r\to\infty} \int_{r_H}^{r} \partial_r A_t dr \to \infty$$

- Although charge is not conserved, stationary solution (it is sourced)

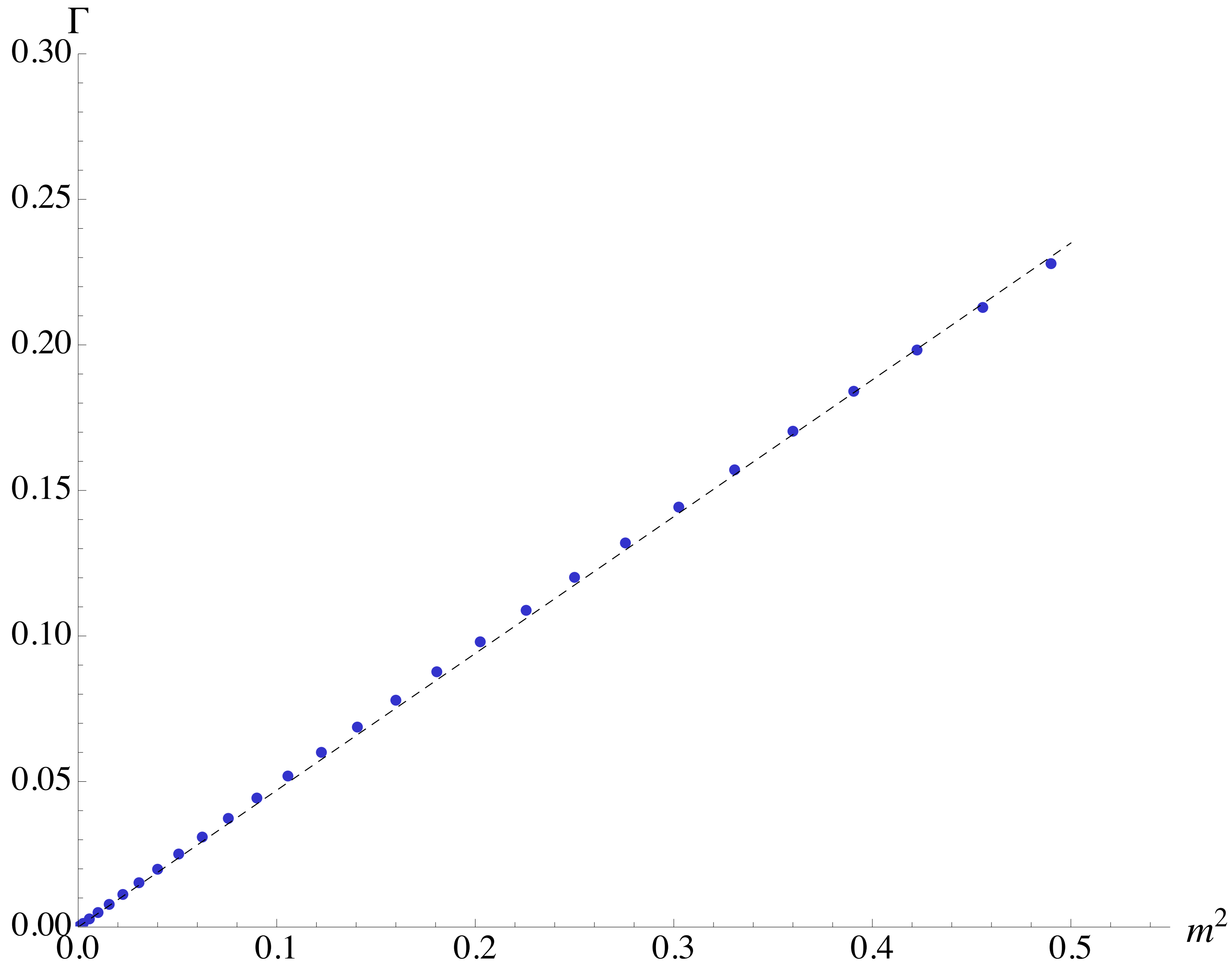
$$\frac{d}{dt} J_5^0 = 0$$

Stückelberg Axion

what happens to charge diffusion?

$$\omega = -i\Gamma - iDk^2$$

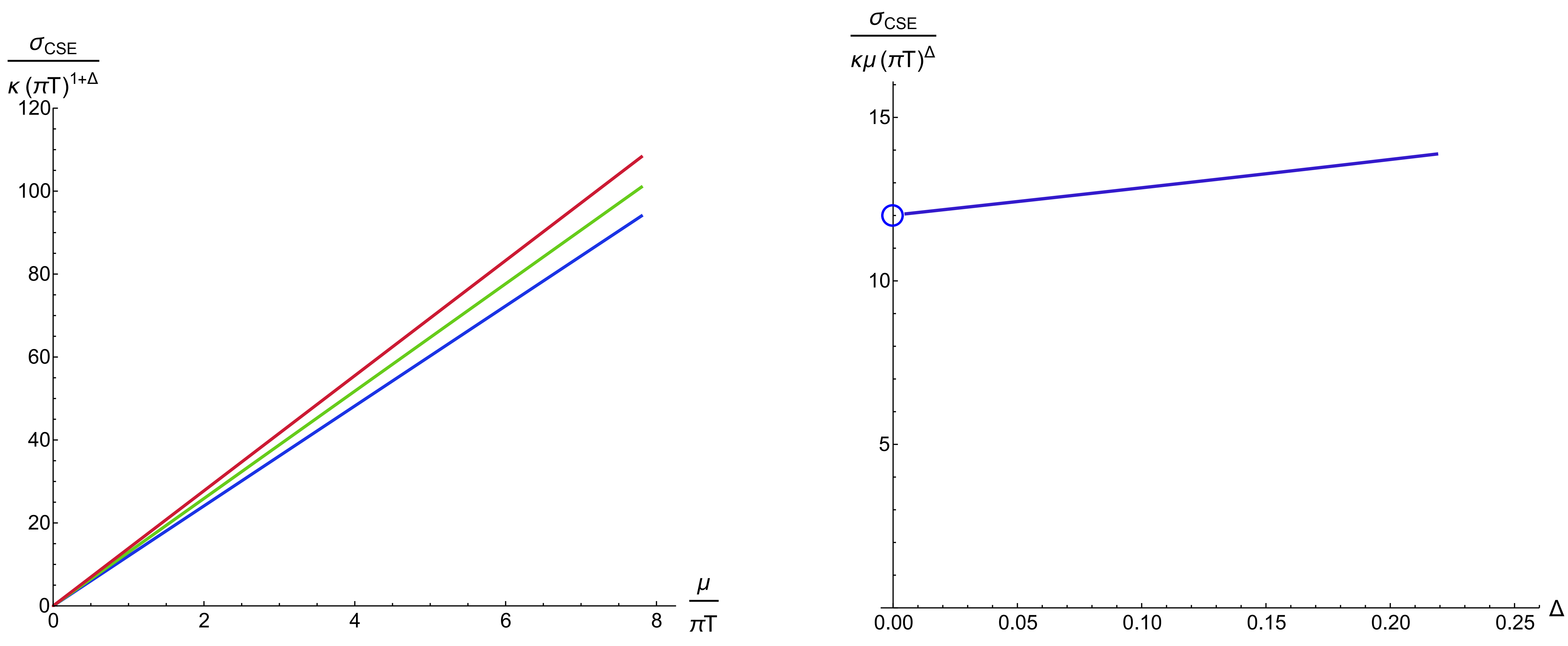
gap opens up in diffusive mode



Axionic magnetoresponse

what happens to Chiral Separation Effect?

$$\vec{J}_5 = \sigma_{CSE}\vec{B}$$



Axionic magnetoresponse

what happens to Chiral Magnetic Effect?

$$\vec{J} = \sigma_{CME} \vec{B}$$

we find: $\sigma_{CME} = 0$

- Natural generalization of result for $A_5^0 = \mu_5$

- Natural generalization of consistent current

$$J^\mu = \frac{\delta S_{\rm ren}^{\rm on-shell}}{\delta V_\mu}$$

- Does this mean CME is absent? NO!

Axionic magnetoresponse

Another manifestation of CME is the Chiral Magnetic Wave:

[Kharzeev, Yee]

- In constant magnetic field, propagating wave of charge to axial charge fluctuation induced by interaction between CME and CSE
- "Hydrodynamic" model:

$$j_V^x = \frac{\kappa \rho_A B}{\chi_A} - D\partial_x \rho_V \qquad \partial_\mu J_V^\mu = 0$$

$$j_A^x = \frac{\kappa \rho_V B}{\chi_V} - D\partial_x \rho_A \qquad \partial_\mu J_A^\mu = -\Gamma \rho_A$$

- Prediction for QNMs in magnetic field

$$\omega_\pm = -\frac{i\Gamma}{2} - iDk^2 \pm \sqrt{\frac{B^2 k^2 \kappa^2}{\chi_A \chi_V} - \frac{\Gamma^2}{4}}$$

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$$j_V^x = \underbrace{\frac{\kappa \rho_A B}{\chi_A}}_{\text{CME and CSE}} - \underbrace{D\partial_x \rho_V}_{\text{Fick's law}} \qquad \partial_\mu J_V^\mu = 0$$

$$j_A^x = \frac{\kappa \rho_V B}{\chi_V} - D\partial_x \rho_A \qquad \partial_\mu J_A^\mu = \underbrace{-\Gamma \rho_A}_{\text{non-conservation}}$$

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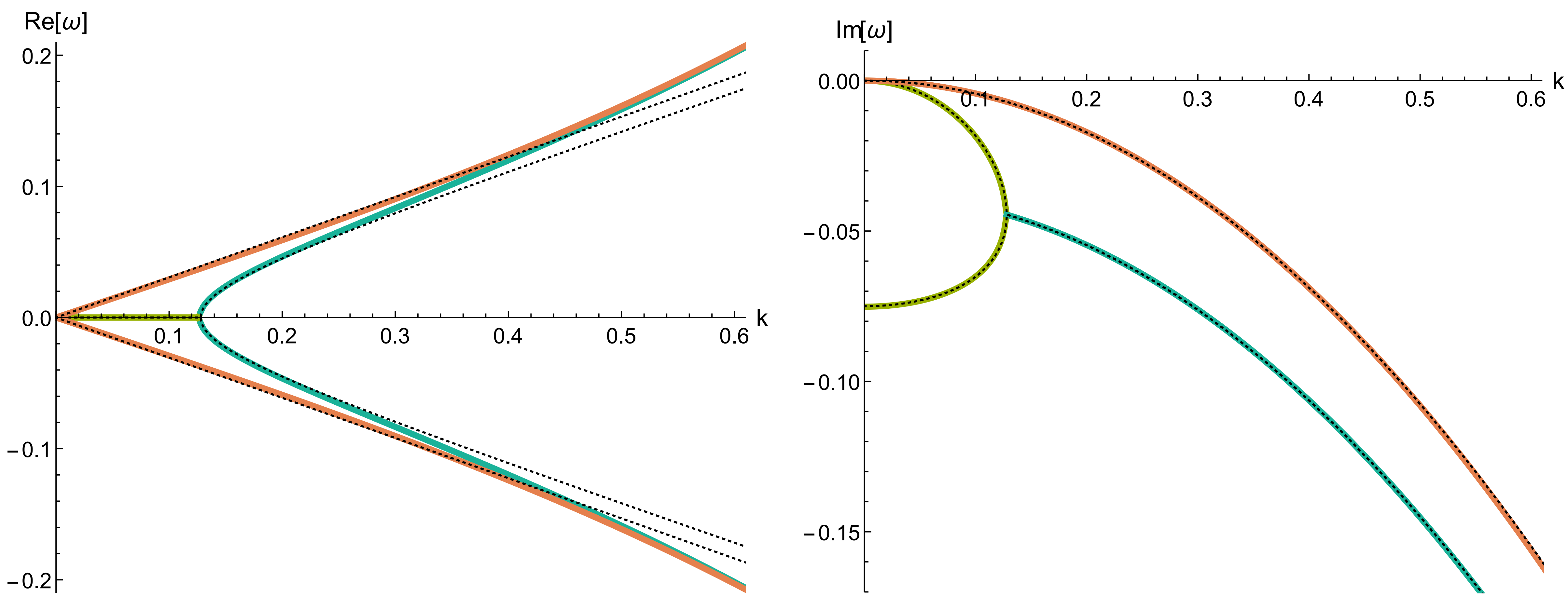

CME present for freely fluctuating charges !

Summary and Outlook

- Non-conserved axial current is consequence of QCD
- At strong coupling: holographic (bottom up) model
- Magnetoresponse still present
    - Subtleties consistent vs. covariant need better understanding
    - CMV is not a wave anymore (gapped) for small momenta
    - Also present: finite negative magneto-resistivity
- More work needed:
    - Include backreaction, study energy current
    - Include gravitational contribution to anomaly
    - Better understanding of dual field theory interpetation


HELAU!
MNK