Thermodynamics of the BMN matrix model at strong coupling

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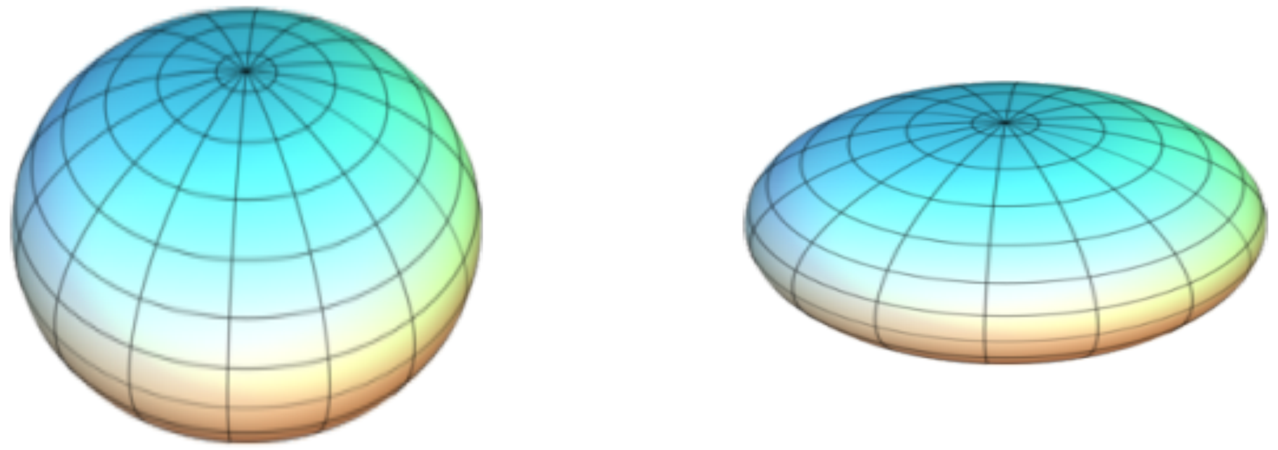
Work with L. Greenspan, J. Penedones and J. Santos



The String Theory Universe, Mainz - September 2014

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- Gauge/gravity duality as definition of quantum gravity in AdS

  Dual CFT is renormalizable and unitary. Problem: how to decode the hologram?

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  Test and understand the gauge/gravity duality with observables that are not protected by SUSY and can not be computed using integrability.

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Idea: Study thermodynamics of black holes dual to Matrix Quantum Mechanics that can be simulated on a computer using Monte-Carlo methods.

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- Can put theory on a computer using Monte Carlo simulations

[Catterall, Wiseman ´07,´08,´09; Anagnostopoulos et al ´07; Hanada et al ´08,´13]

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- 11D SUGRA solution (near horizon geometry of non-extremal D0-brane)

$$ds^2 = \frac{dr^2}{f(r)} + r^2 d\Omega_8^2 + \left(\frac{R}{r}\right)^7 dz^2 + f(r)dt\left(2dz - \left(\frac{r_0}{R}\right)^7 dt\right)$$

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^7, \qquad \left(\frac{R}{\ell_s}\right)^7 = 60\pi^3 g_s N, \qquad \left(\frac{r_0}{\ell_s}\right)^5 = \frac{120\pi^2}{49}\left(2\pi g_s N\right)^{\frac{5}{3}} \tau^2$$

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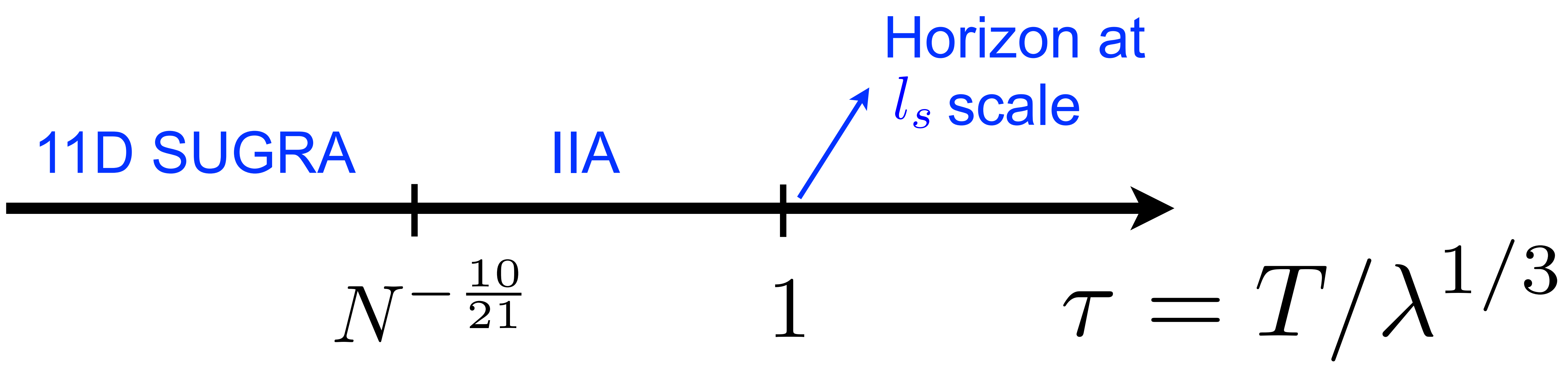
- Classical gravity domain (at horizon)



$$l_s^2 \mathcal{R}(r_0) \ll 1 \;\Rightarrow\; \tau \ll 1$$

$$g_s e^{\phi(r_0)} \ll 1 \;\Rightarrow\; \tau \gg N^{-\frac{10}{21}}$$

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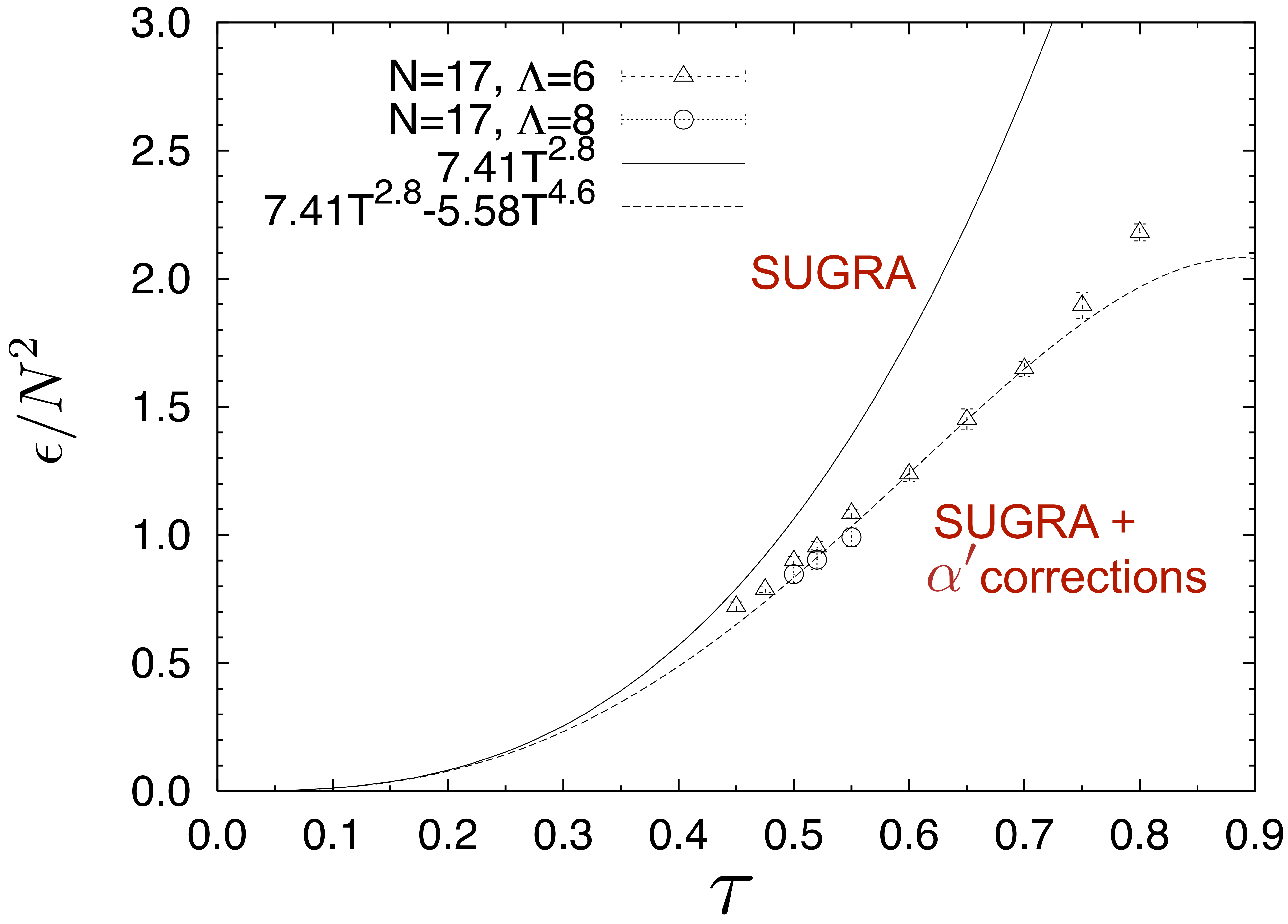

Monte-Carlo simulation of MQM

✕ negligible

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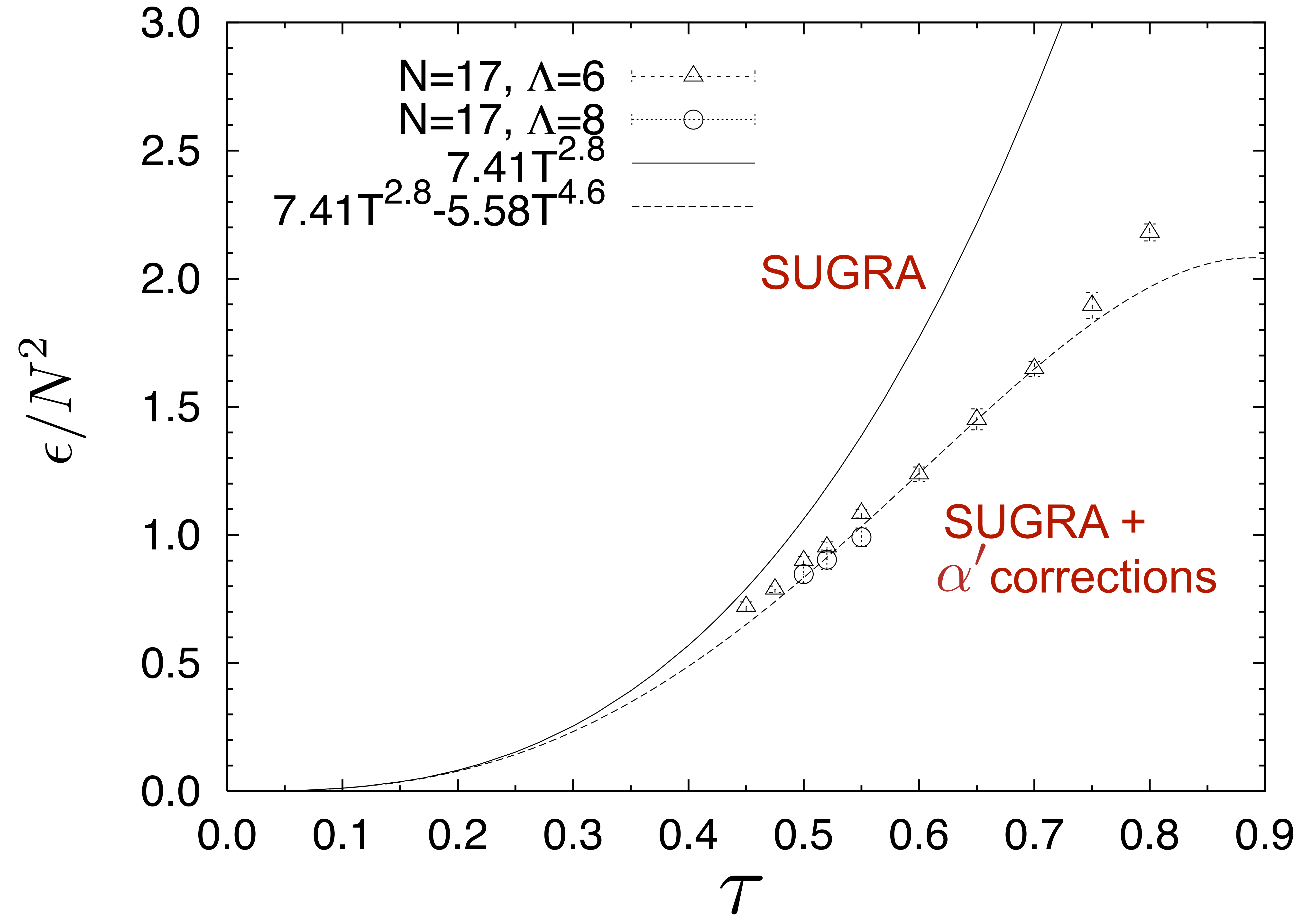

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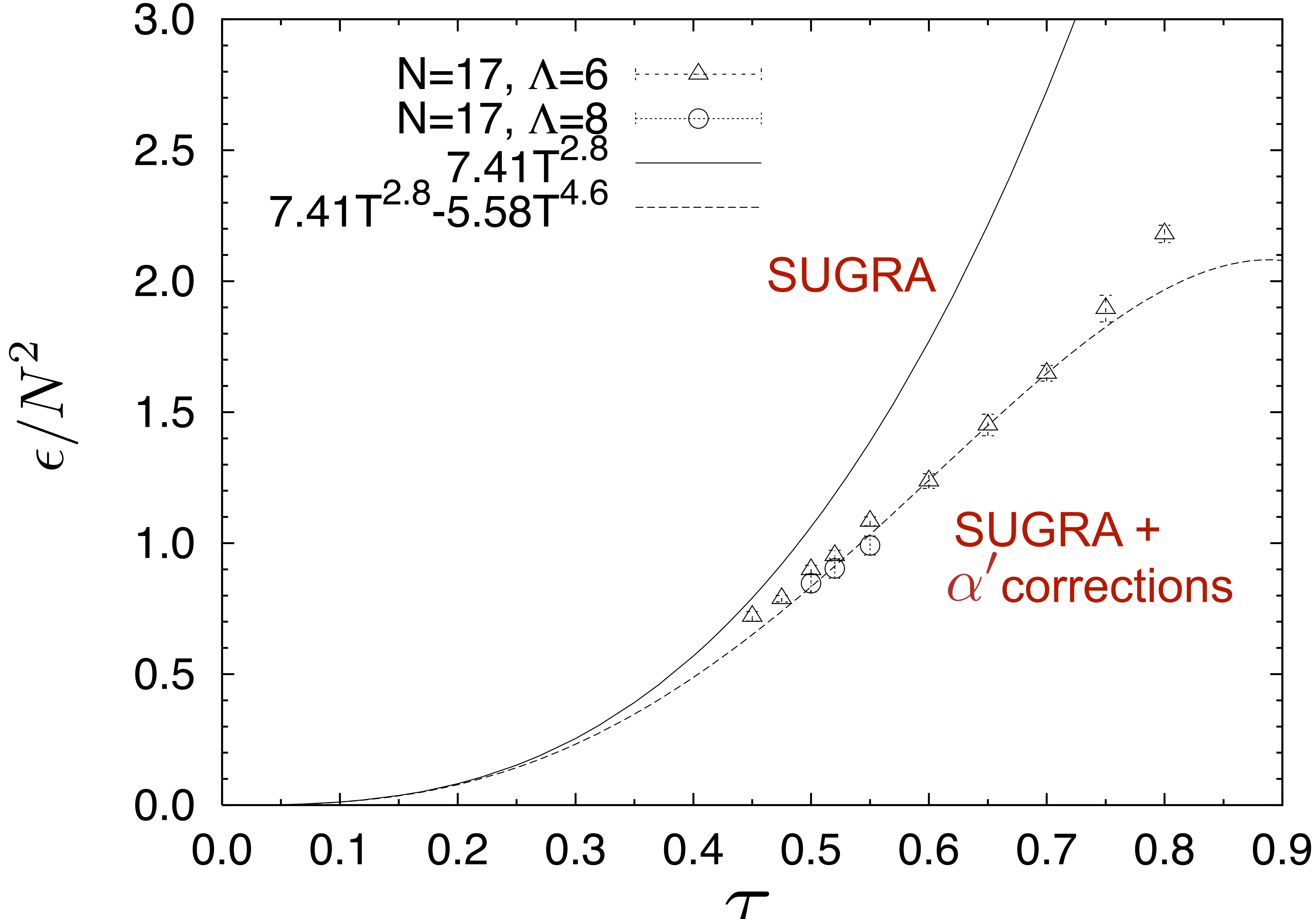

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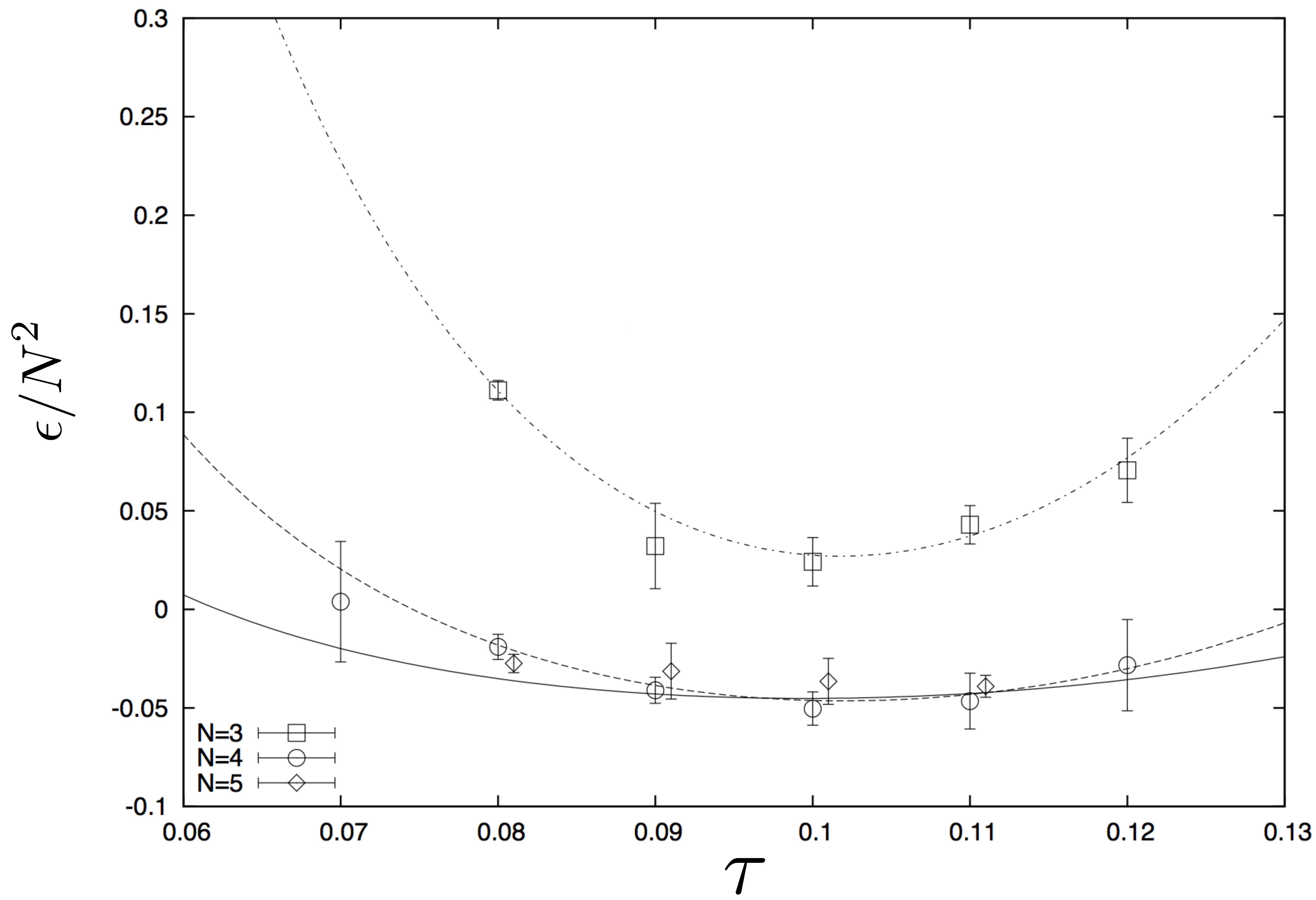

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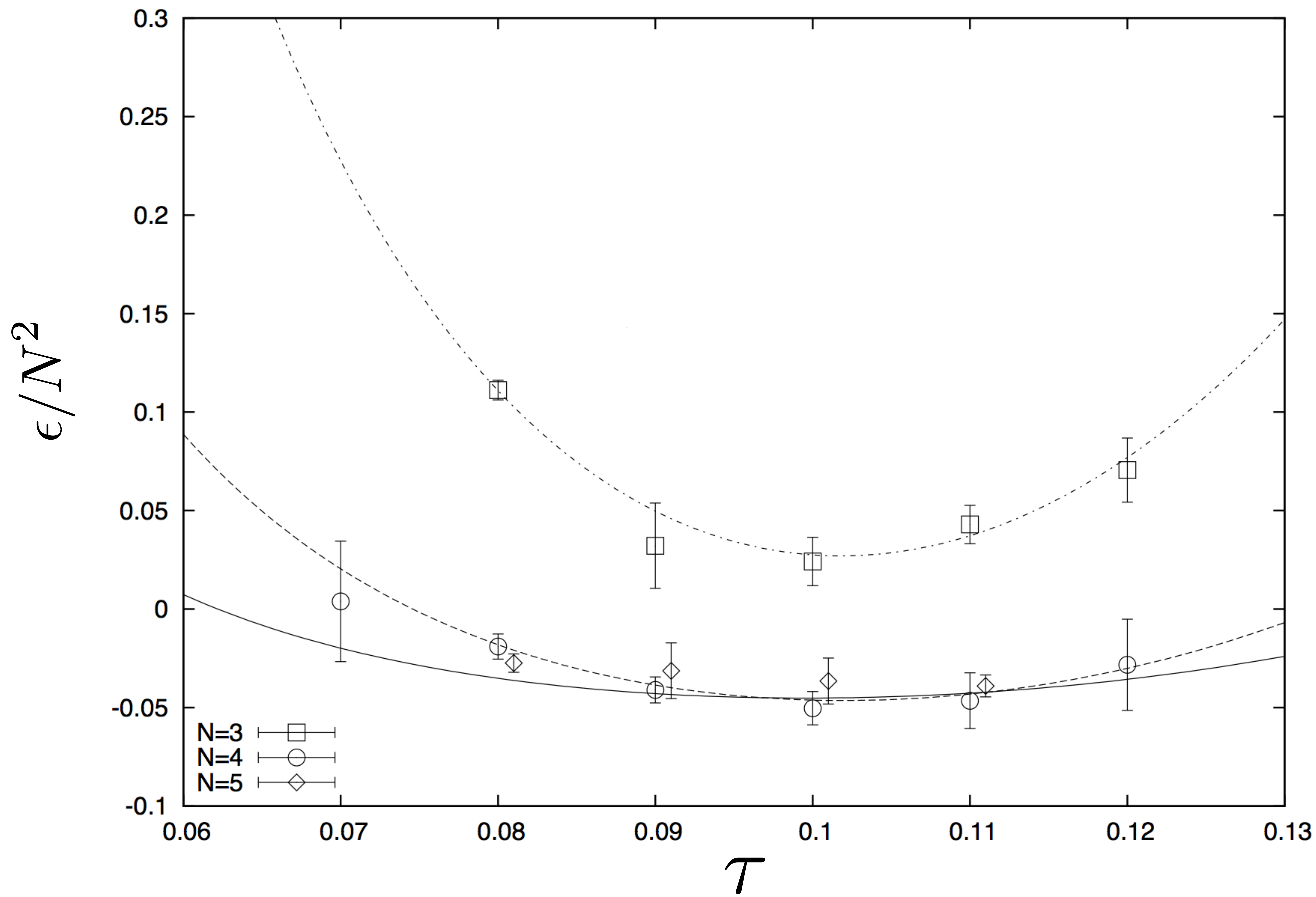

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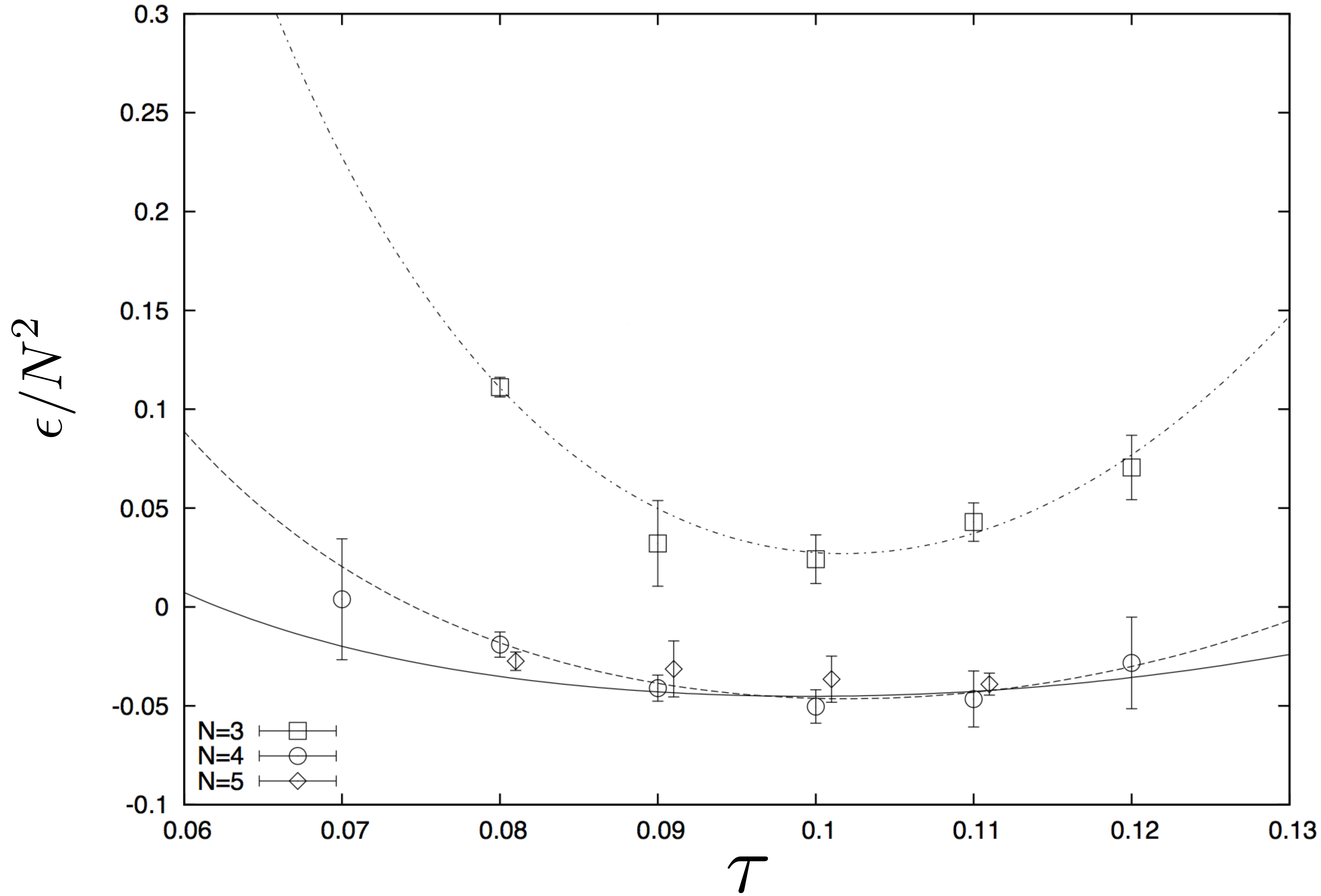

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$$\frac{F(T,r)}{N^2} \sim \mathcal{F}_{finite}(T) + \frac{9}{N} \ln r$$

Instability corresponds to Hawking radiation of D0-branes. At large N this is suppressed and black hole is stable (positive specific heat).

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• **Today's talk is about BMN matrix model** [Berenstein, Maldacena, Nastase ´02]

Mass deformation resolves IR divergence - canonical ensemble well defined.

Much richer thermodynamics with a 1st order phase transition
(at large N there are two dimensionless parameters).

BMN matrix model

$$S = S_{D0} - \frac{N}{2\lambda}\int dt\,\mathrm{Tr}\left[\frac{\mu^2}{3^2}(X^i)^2 + \frac{\mu^2}{6^2}(X^a)^2 + \frac{\mu}{4}\Psi^\alpha\left(\gamma^{123}\right)_{\alpha\beta}\Psi^\beta + i\frac{2\mu}{3}\epsilon_{ijk}X^iX^jX^k\right]$$

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Massive deformation of D0-brane MQM. Preserves SUSY but **breaks** $SO(9) \to SO(6) \times SO(3)$

$a = 4, \ldots, 9 \quad i = 1, 2, 3$

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Canonical ensemble is well defined and may still be simulated on a computer.

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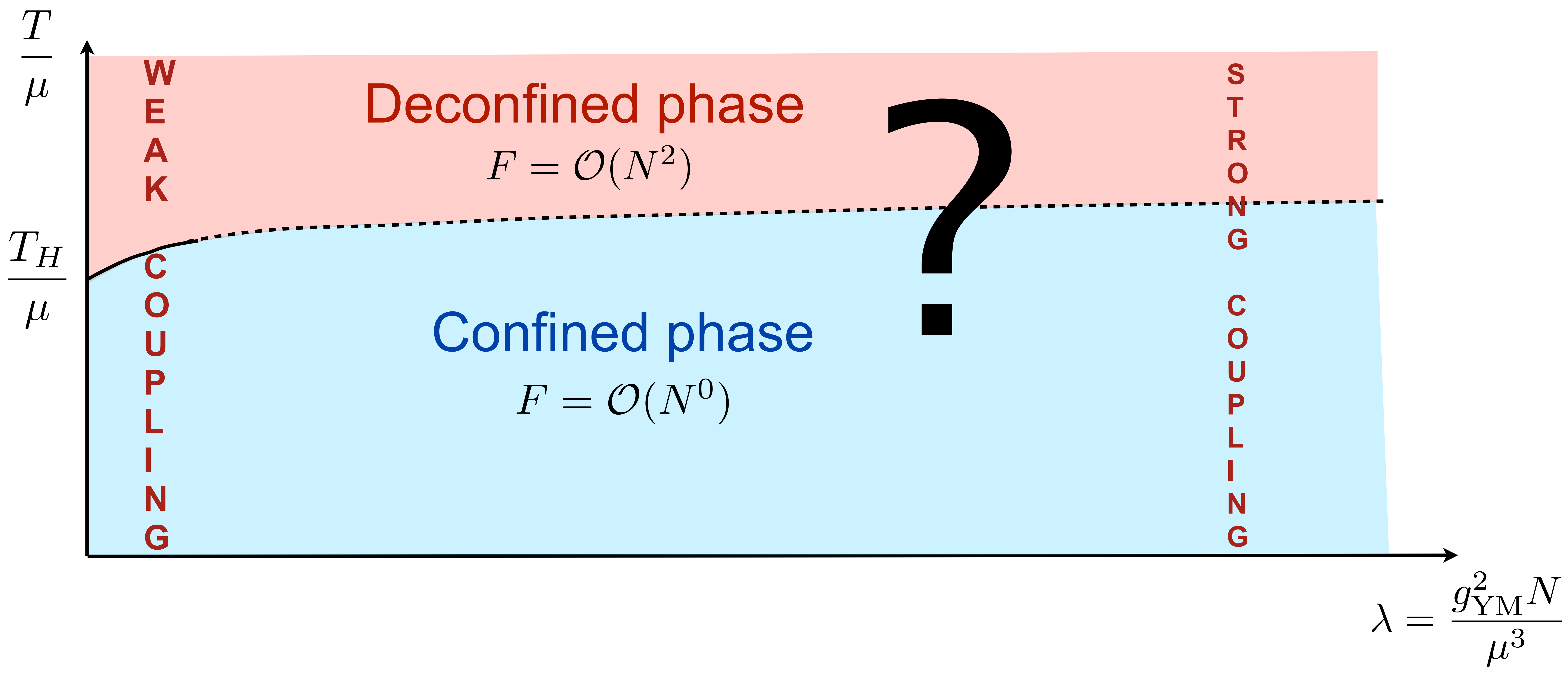

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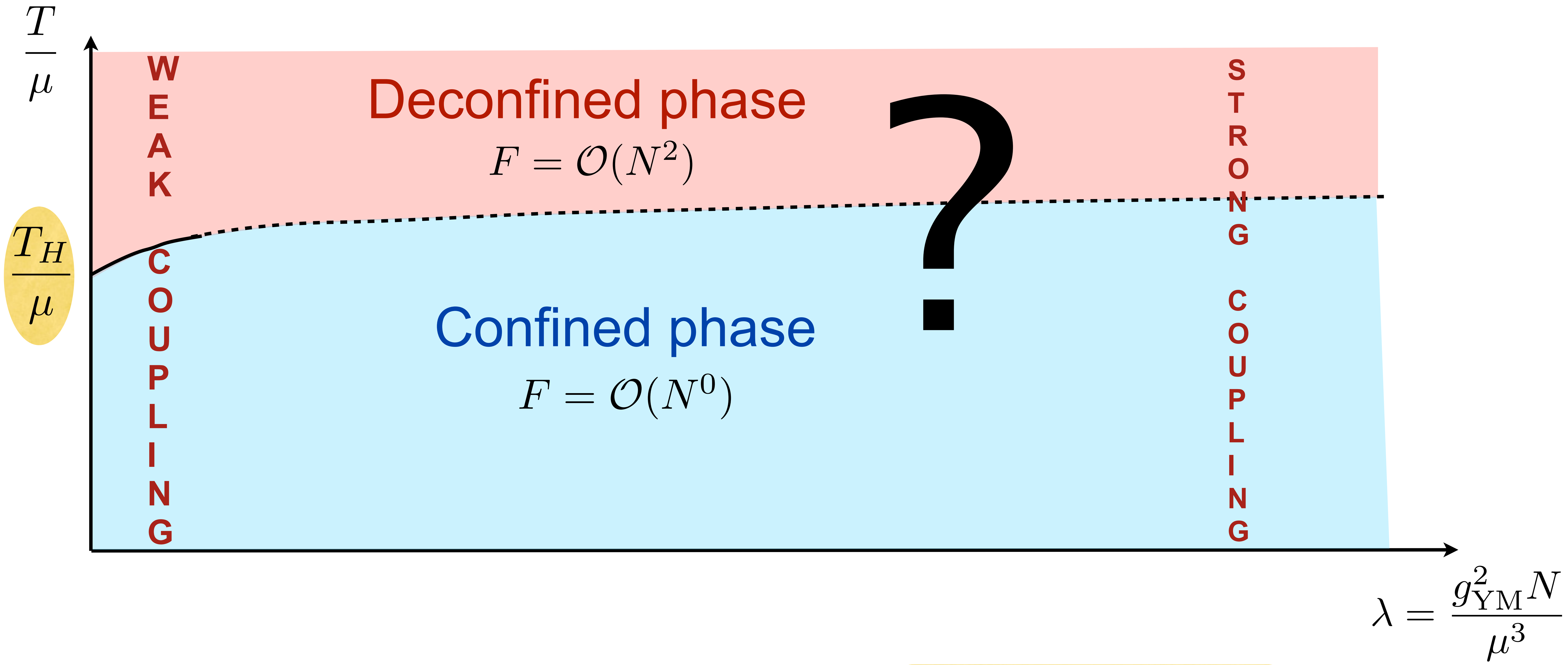

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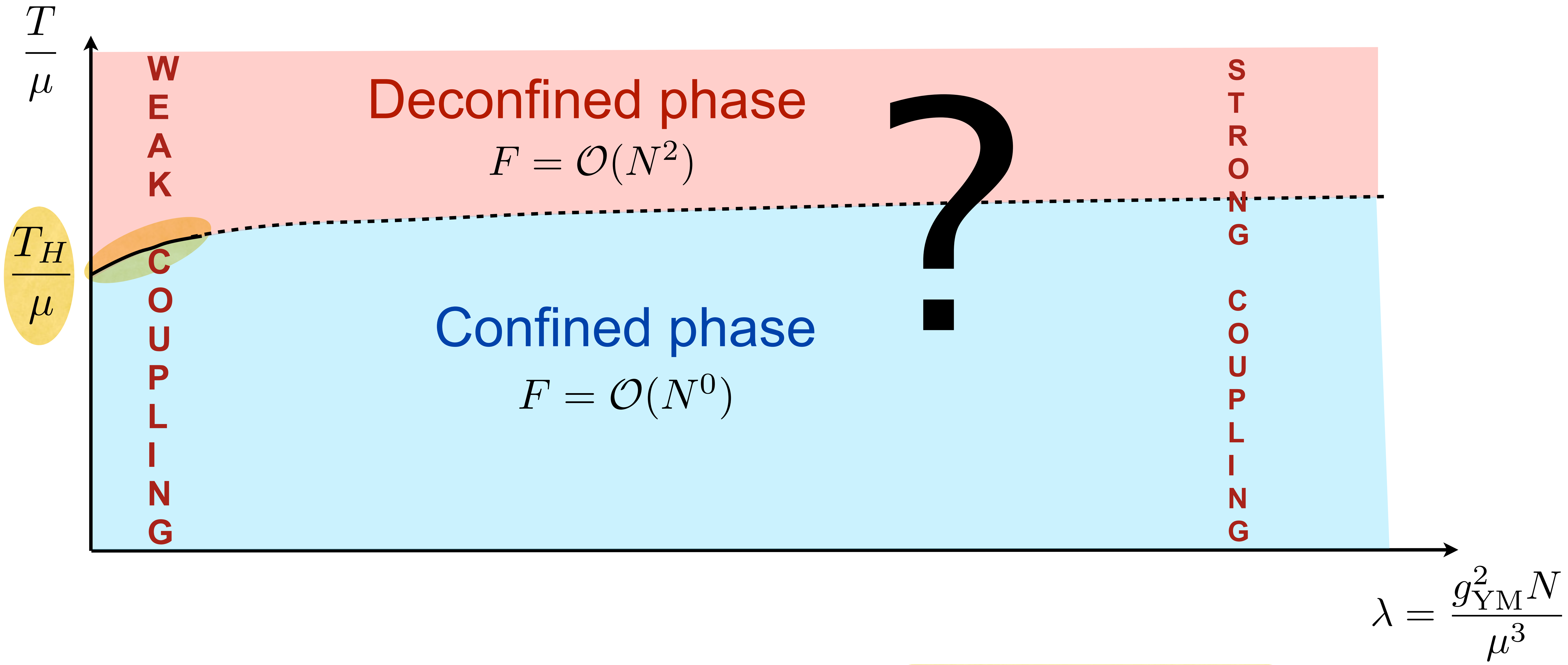

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First-order phase transition at
$$\frac{T_c}{\mu} = \frac{1}{12 \log 3}\left[1 + \frac{2^6 5}{3^4}\lambda - c\,\lambda^2 + O(\lambda^3)\right] \approx 0.076 + \mathcal{O}(\lambda)$$

[Hadizadeh, Ramadanovic, Semenoff, Young '04]

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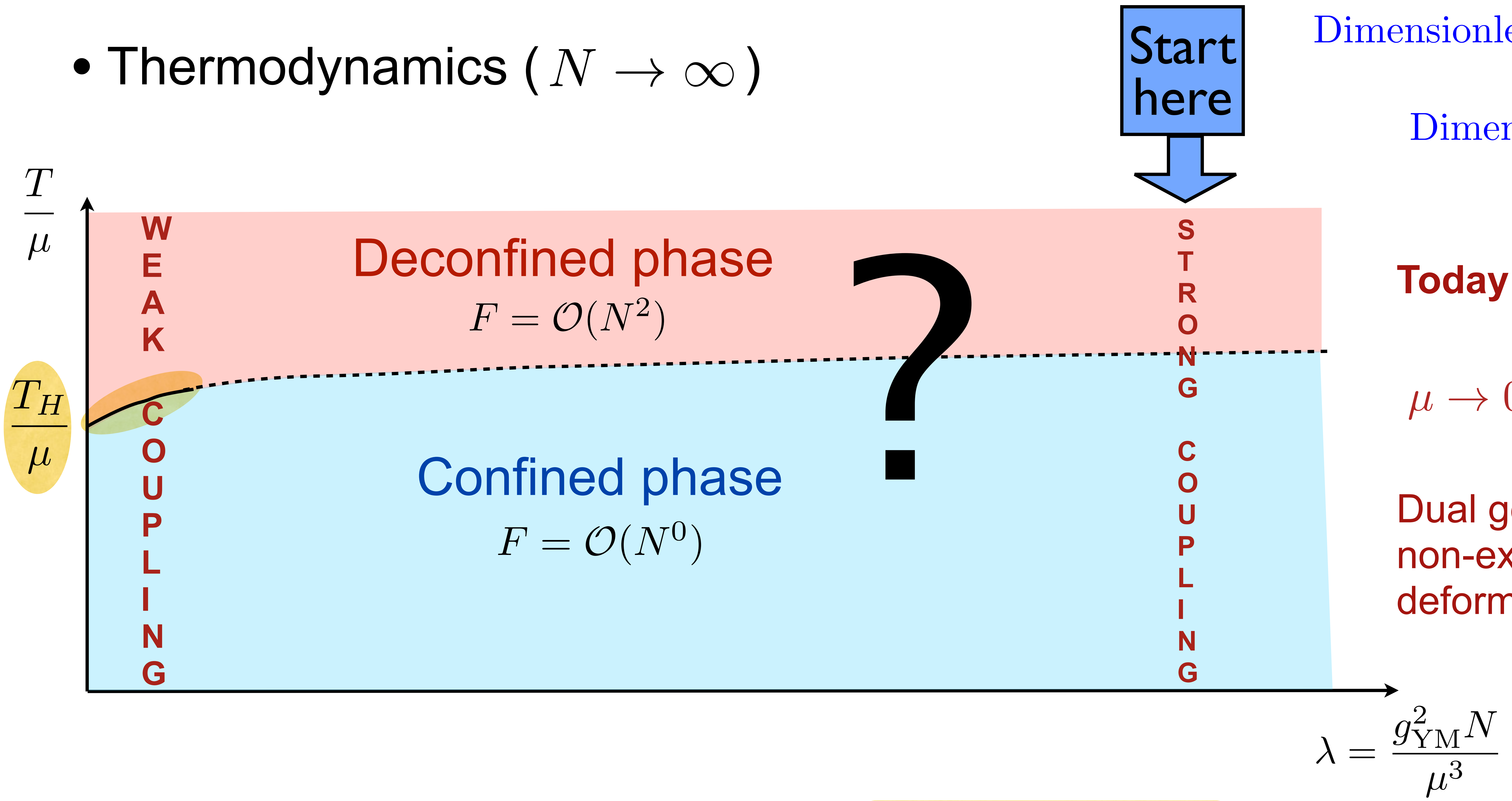

Today: strongly coupled limit

$$\mu \to 0\,, \quad \frac{T}{\mu} \text{ fixed and large}$$

Dual geometry is SO(9) invariant non-extremal D0-brane with deformation turned on

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Need back-reaction to decrease temperature and study phase transition at strong coupling. In particular,

$$SO(9) \to SO(6) \times SO(3)$$

- Ansatz for 11D SUGRA

$$ds^2 = -A\,\frac{(1-y^7)}{y^7}d\eta^2 + T_4\,y^7\left[d\zeta + \Omega\,\frac{(1-y^7)d\eta}{y^7}\right]^2$$
$$+\frac{1}{y^2}\left[B\,\frac{(dy+F dx)^2}{(1-y^7)y^2} + T_1\,\frac{4dx^2}{2-x^2} + T_2\,x^2(2-x^2)d\Omega_2^2 + T_3\,(1-x^2)^2 d\Omega_5^2\right]$$

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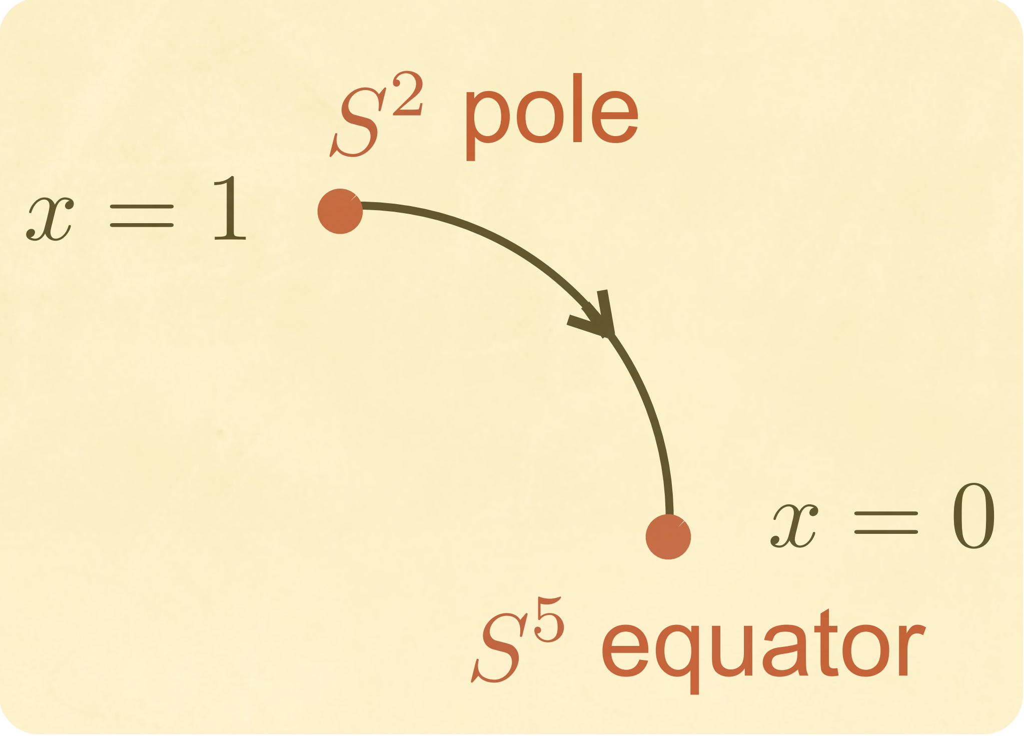

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x is a angular coordinate on compact 8-dimensional space with S^8 topology

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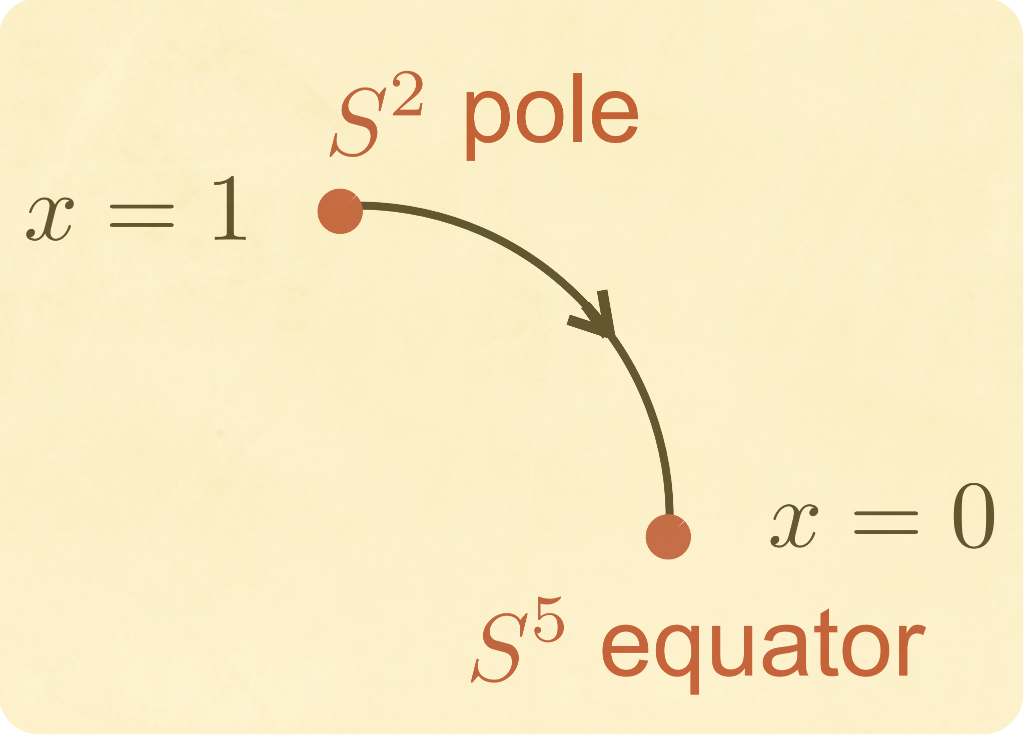

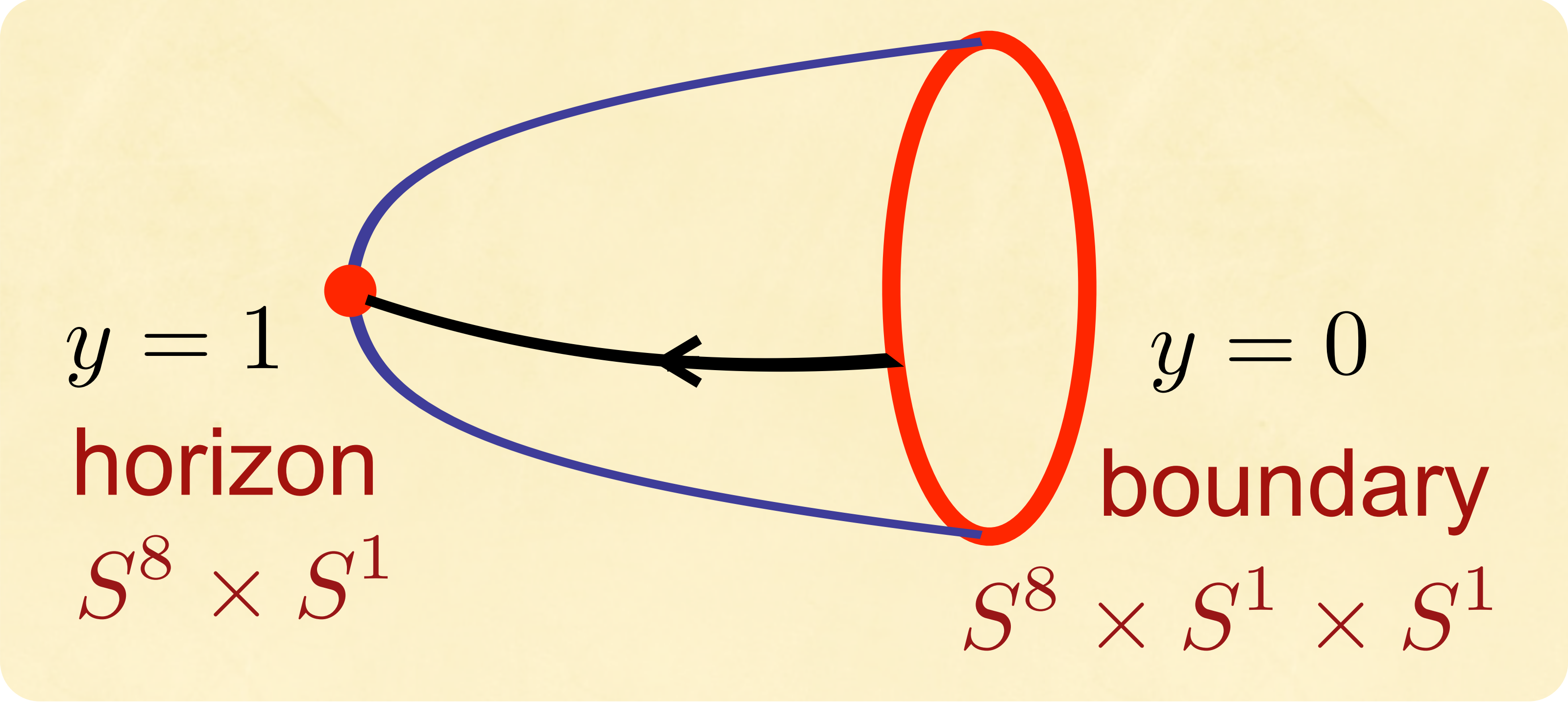

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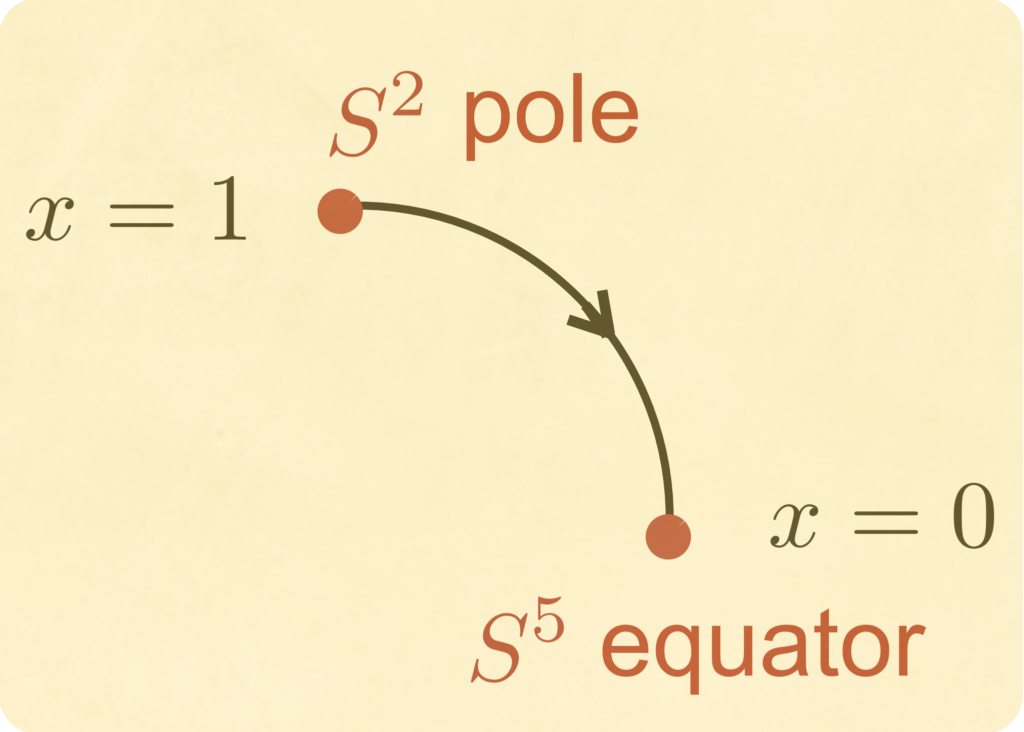

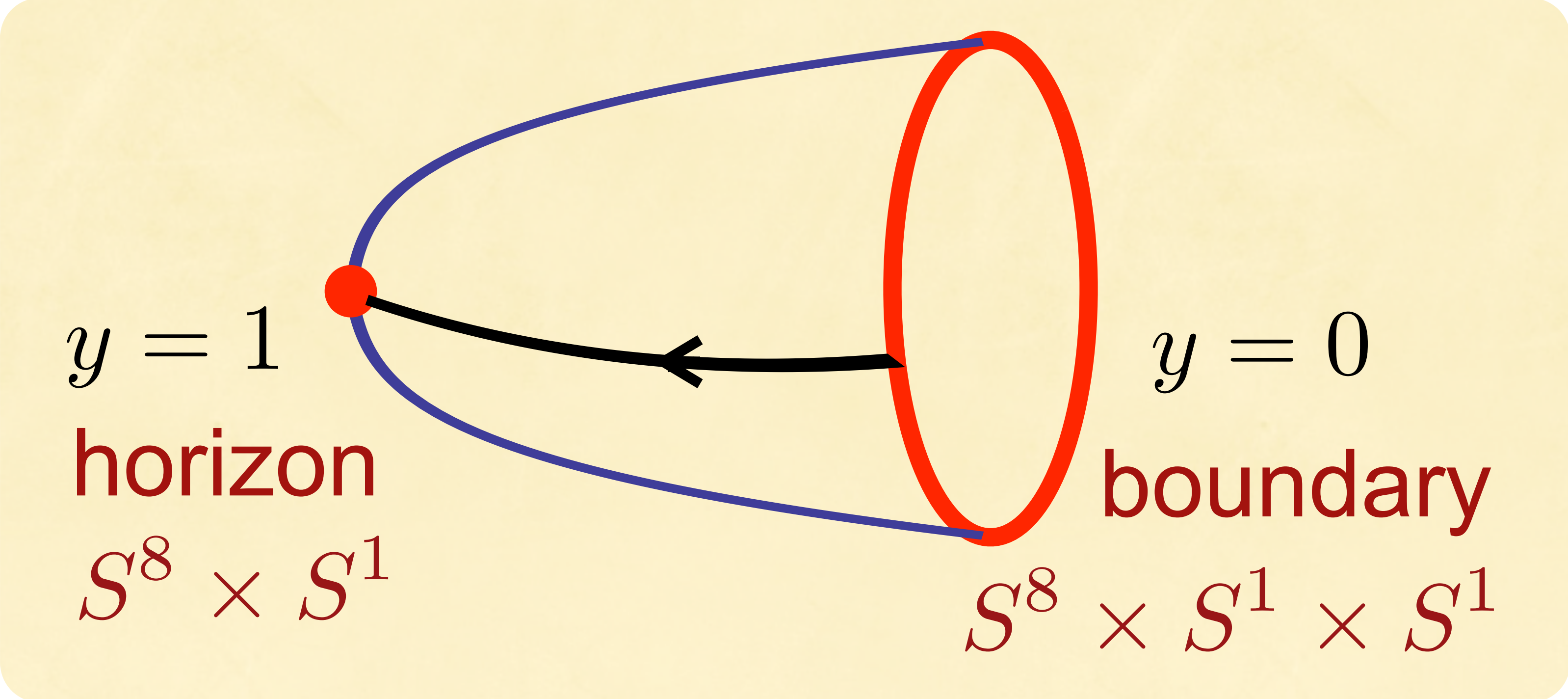

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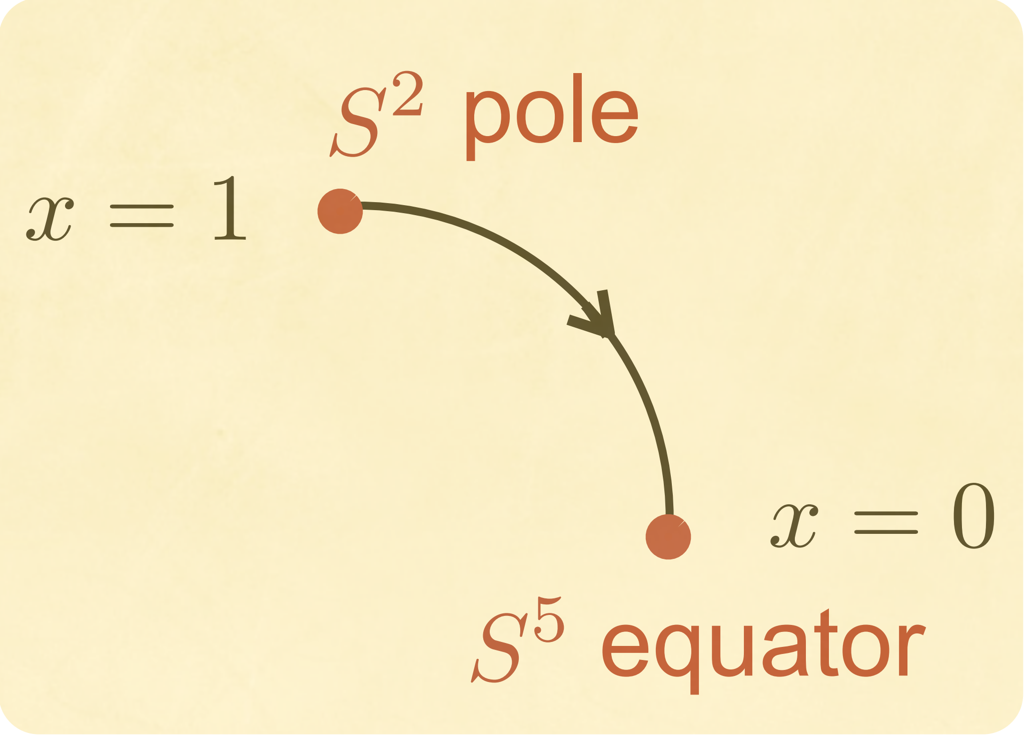

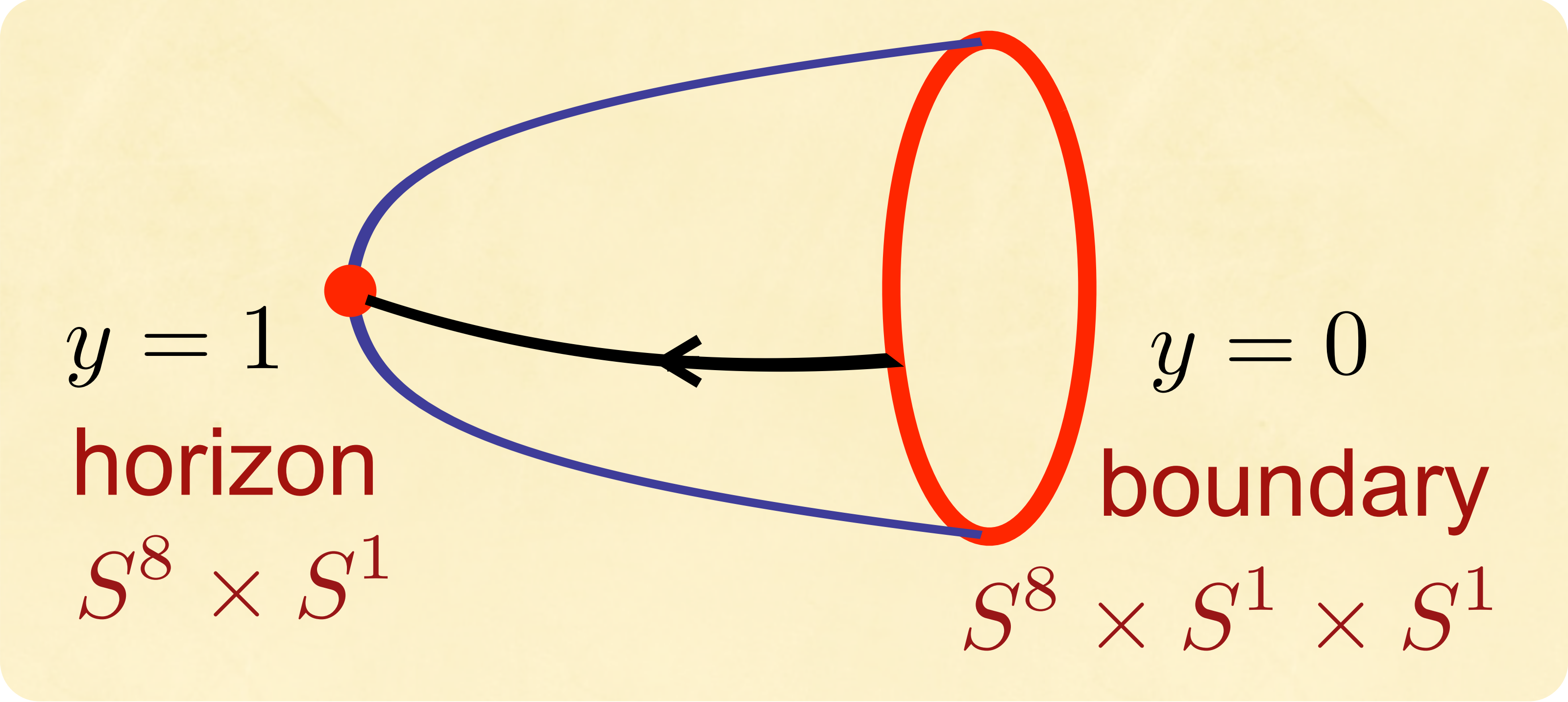

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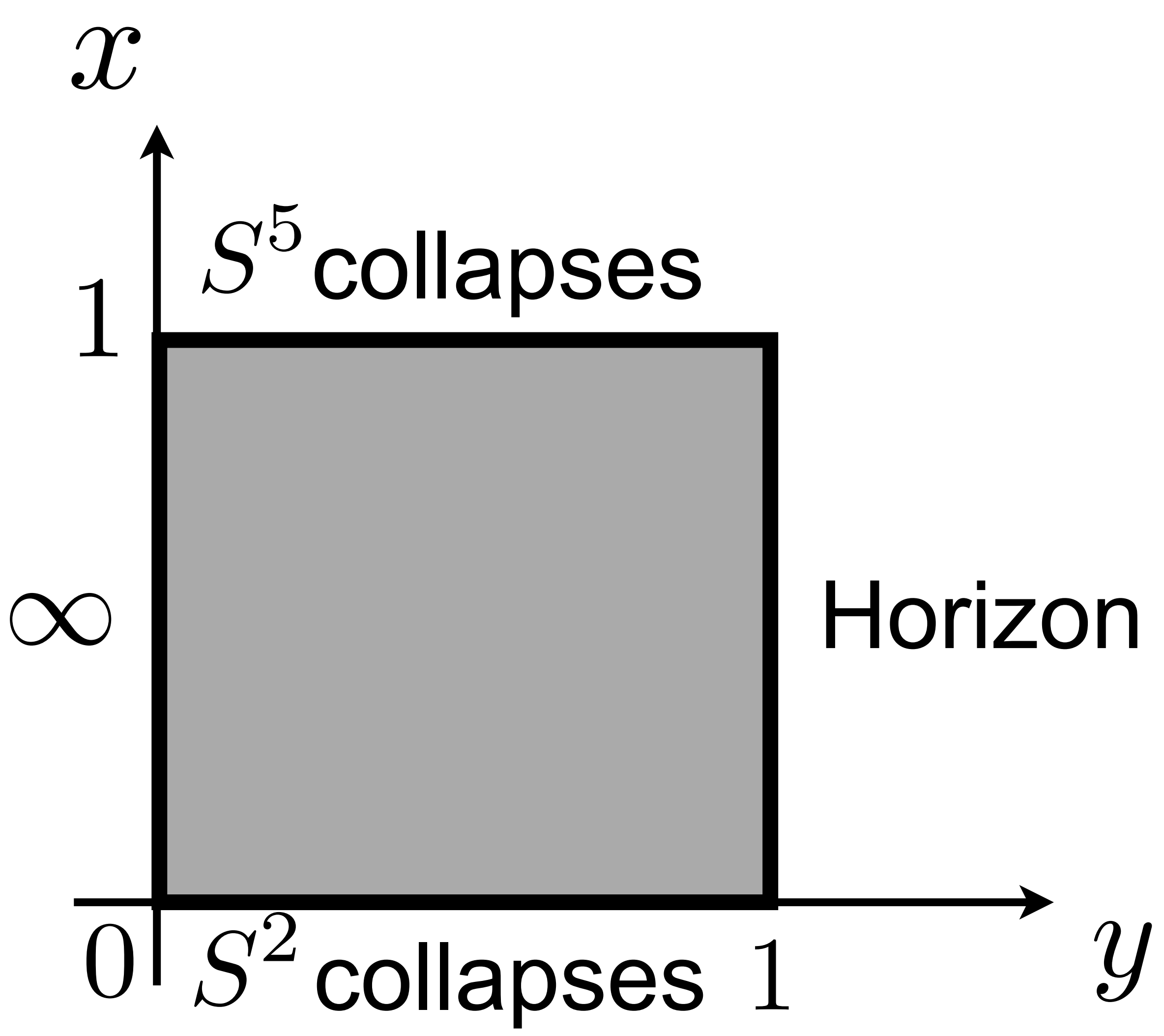
Tailored to numerical implementation
(domain of unknown is the unit square; everything dimensionless)



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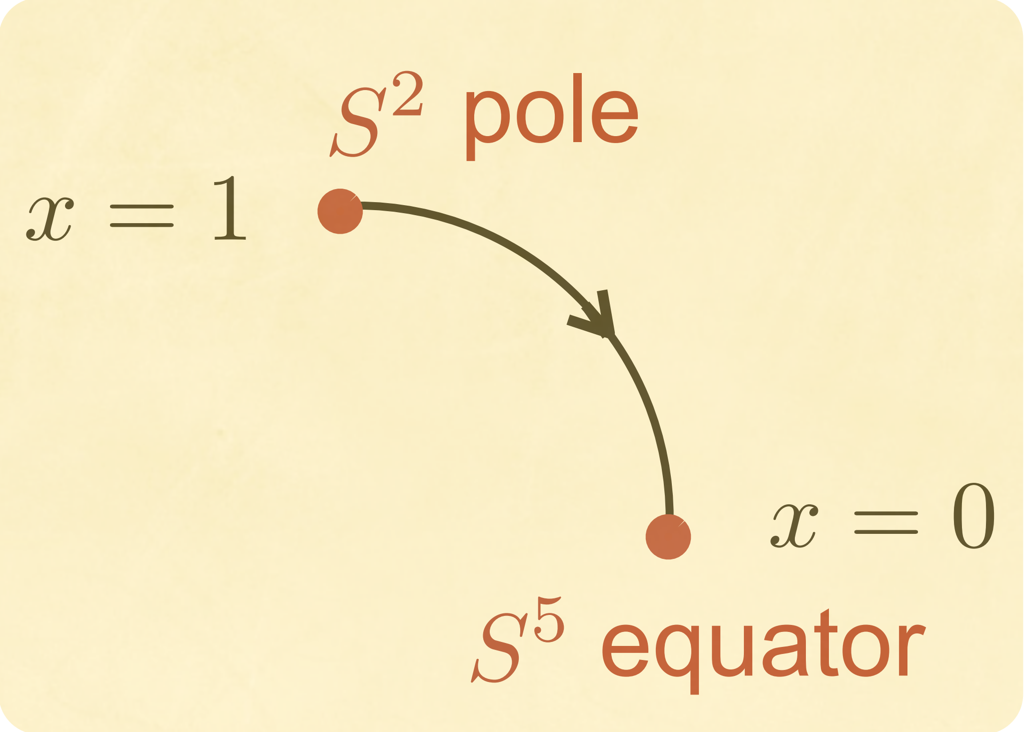

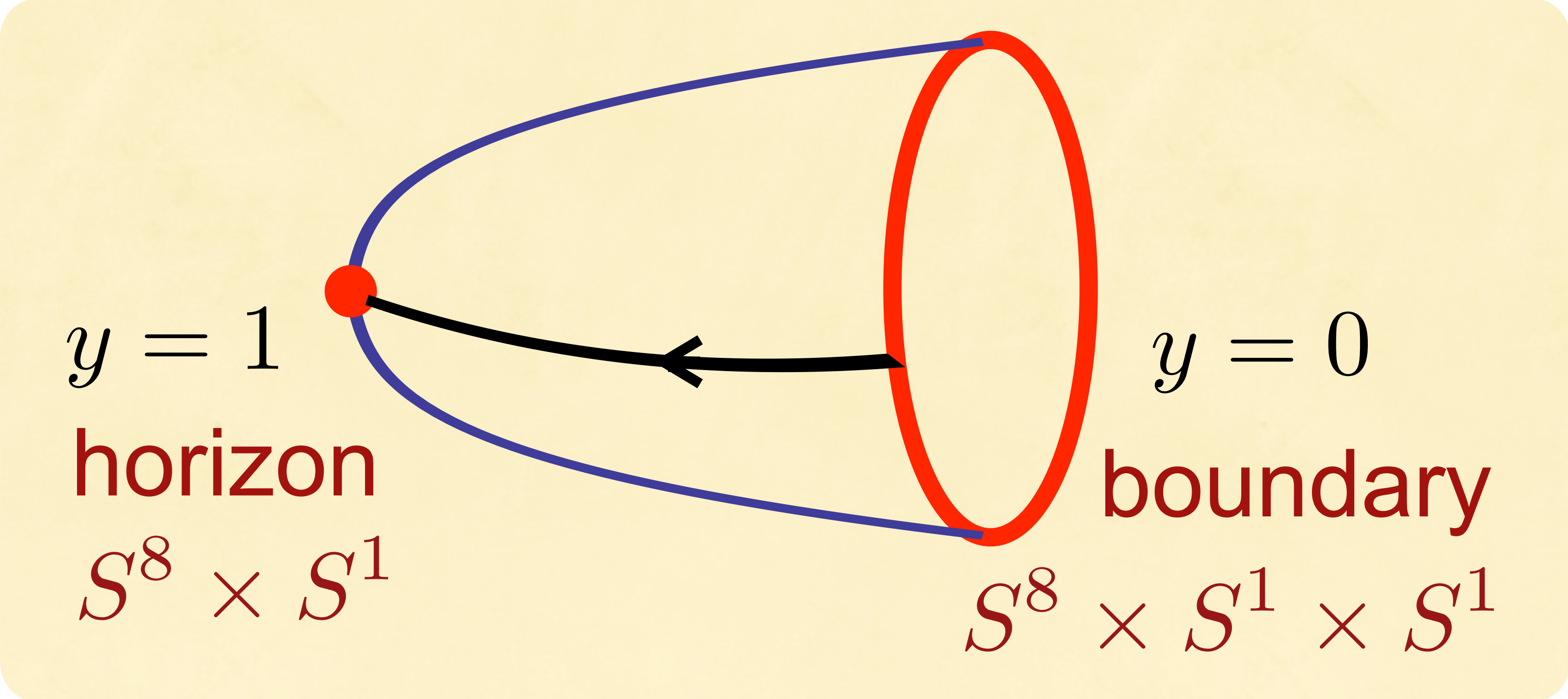

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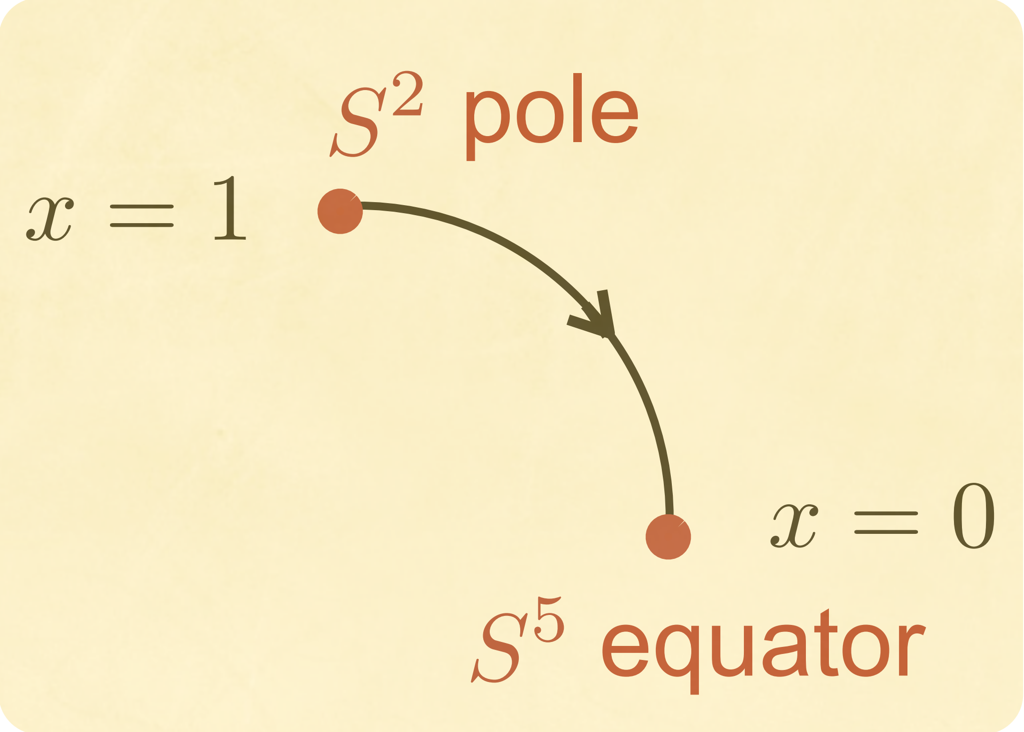

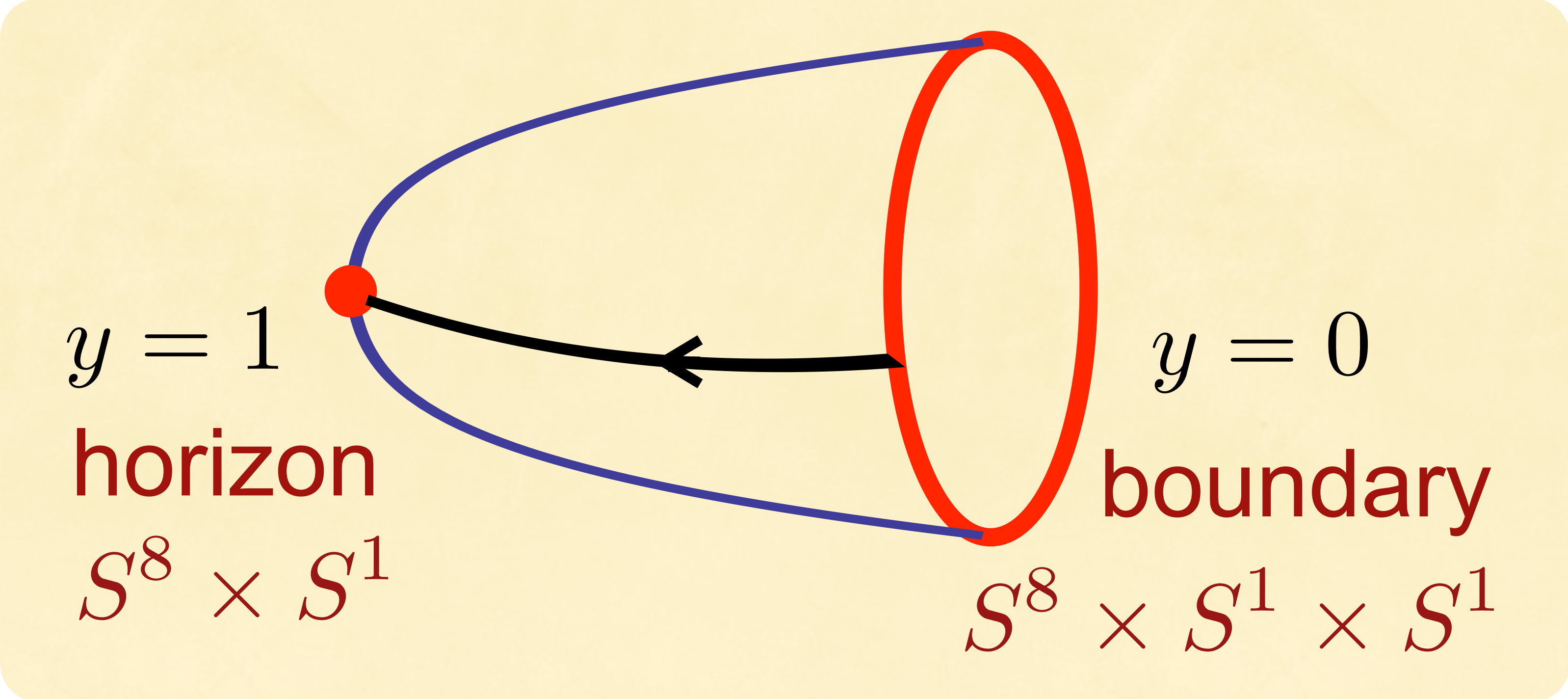

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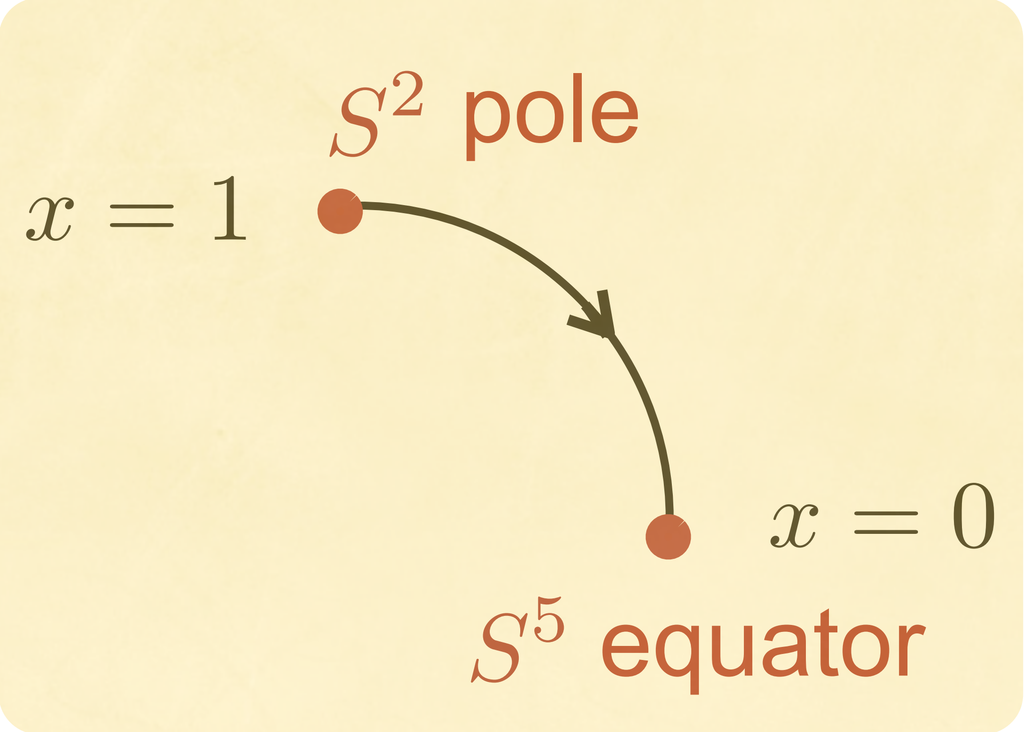

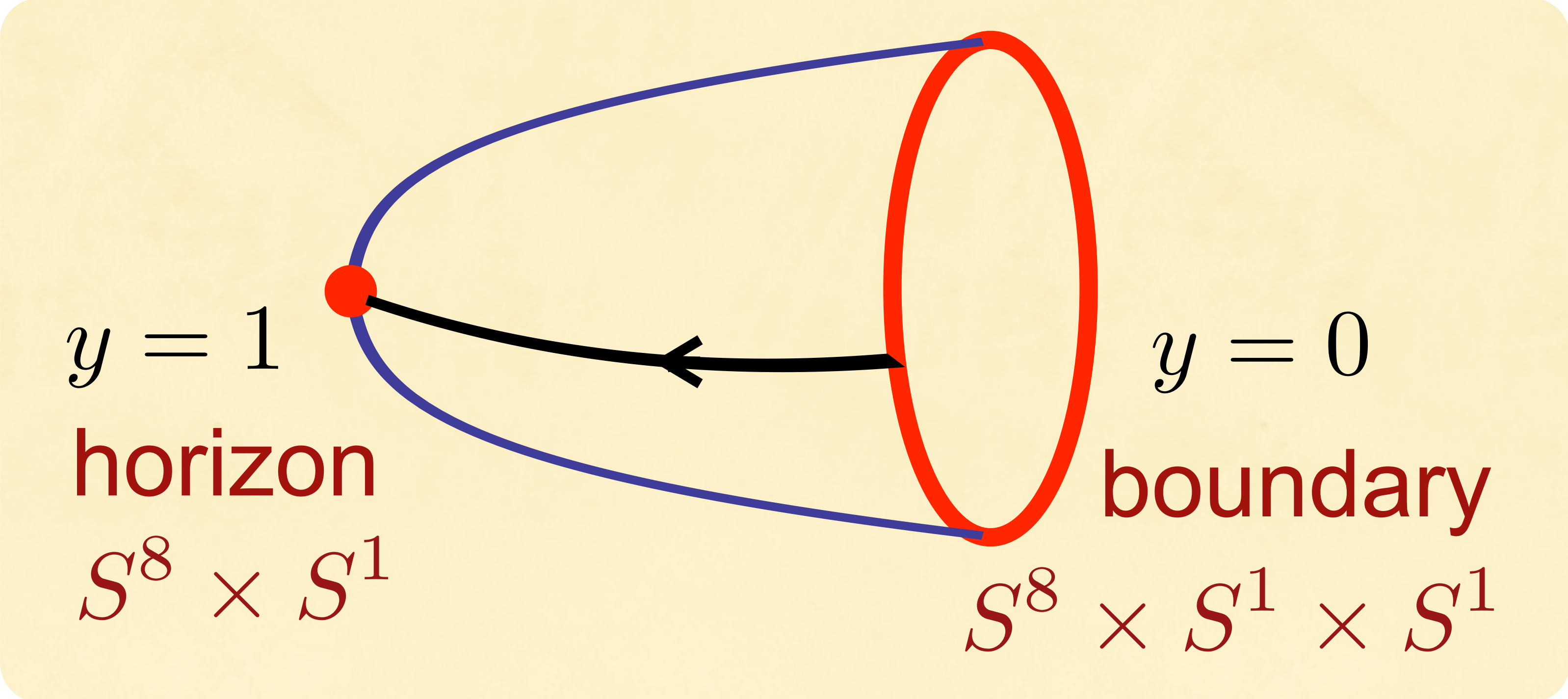

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and need to use scaling symmetry of 11D SUGRA action

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$$\zeta\sim\zeta+2\pi \ \to\ \zeta\sim\zeta+2\pi s' \qquad \Rightarrow \qquad I\to s' I$$

with

$$s=r_0$$

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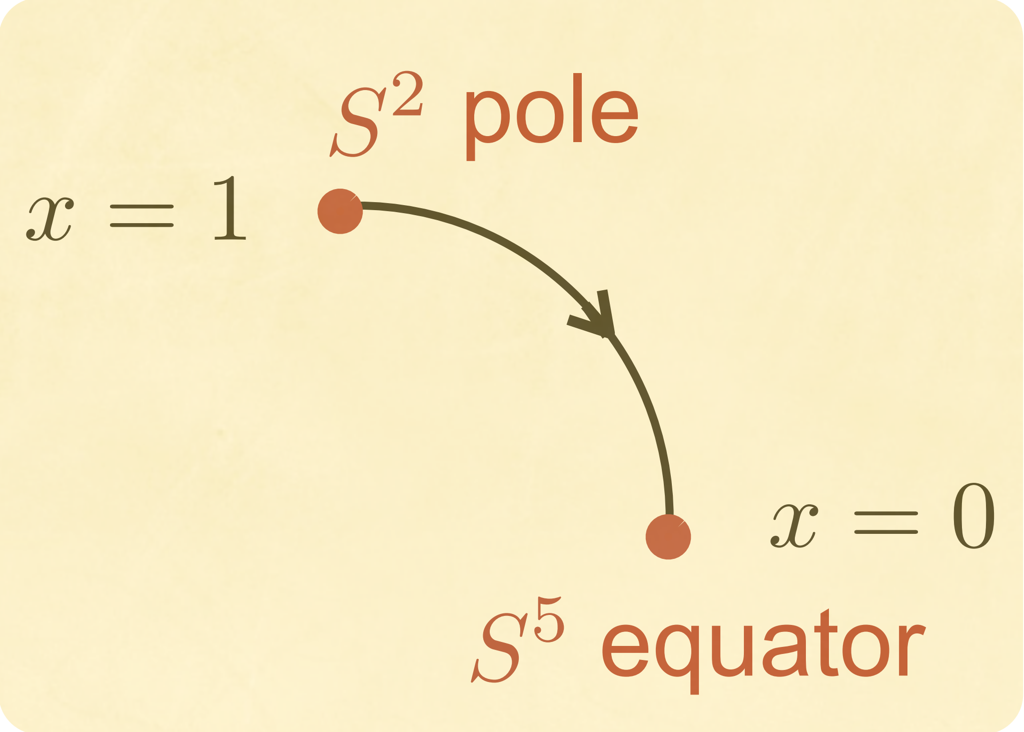

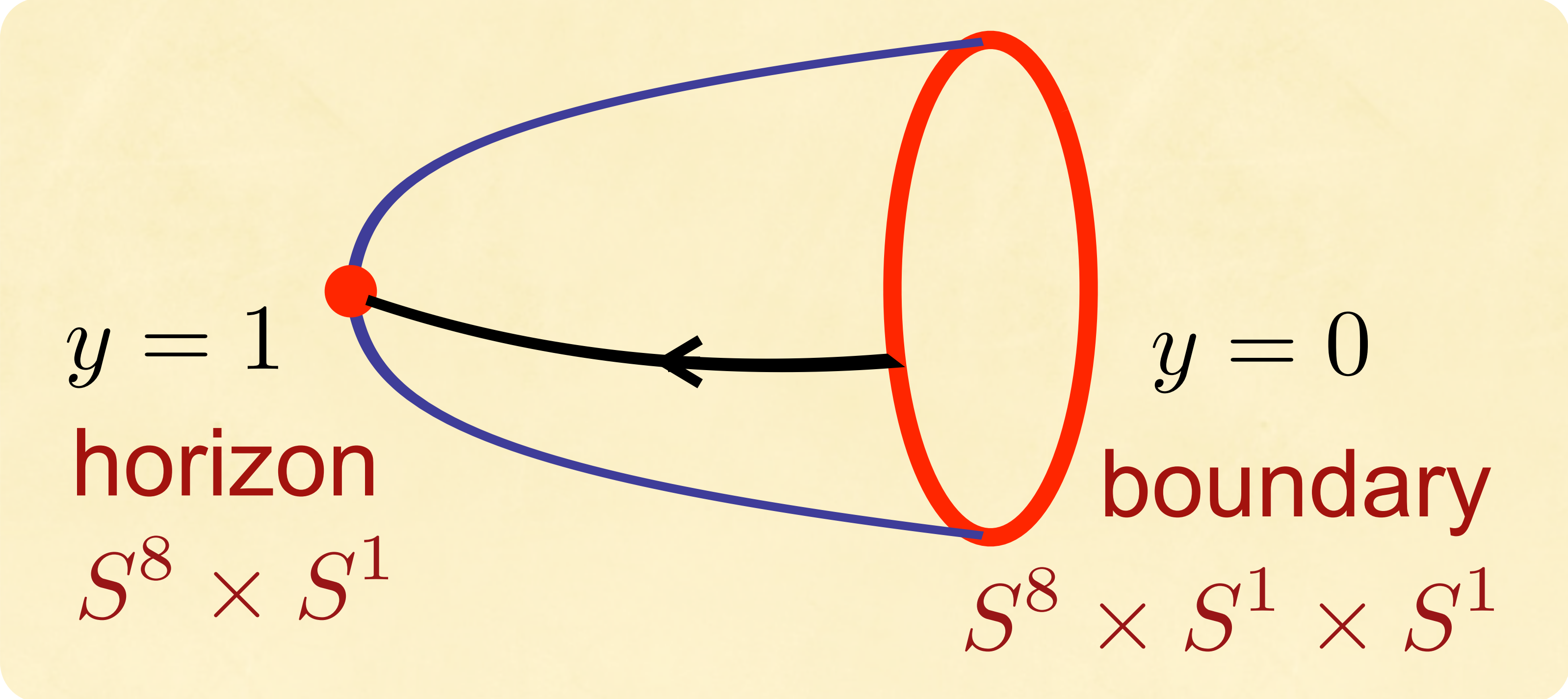

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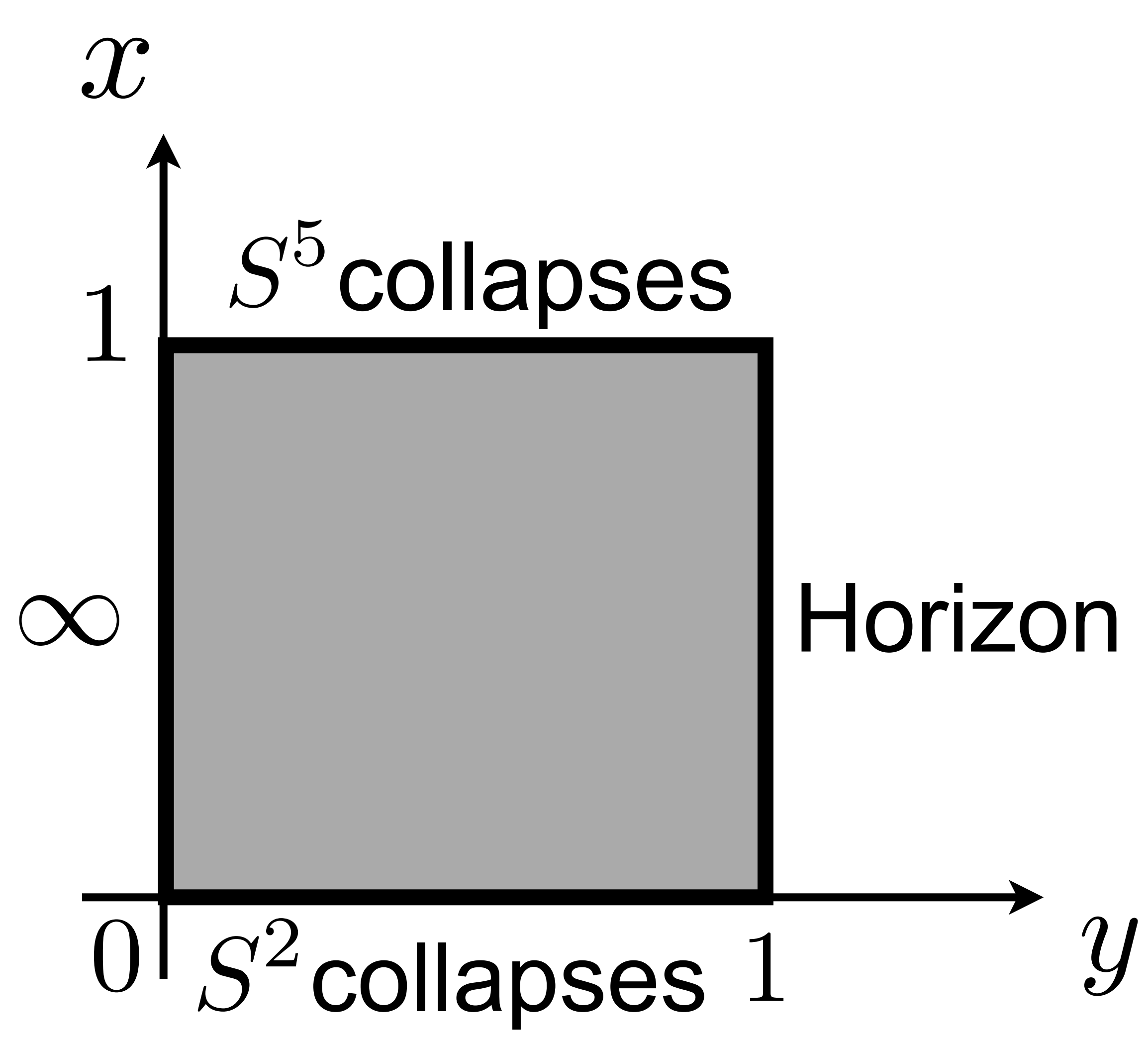
This scaling symmetry will be important later...

- Boundary conditions



- Boundary conditions

At infinity ($y=0$): $A, B, T_1, T_2, T_3, T_4, \Omega \to 1\,, \quad F \to 0$

$$M \to \hat{\mu}\, \frac{x^3(2-x^2)^{\frac{3}{2}}}{y^3}\,, \quad L \to \frac{3}{2}\,\hat{\mu}\, y^4 x^3 (2-x^2)^{\frac{3}{2}}$$

Recall that

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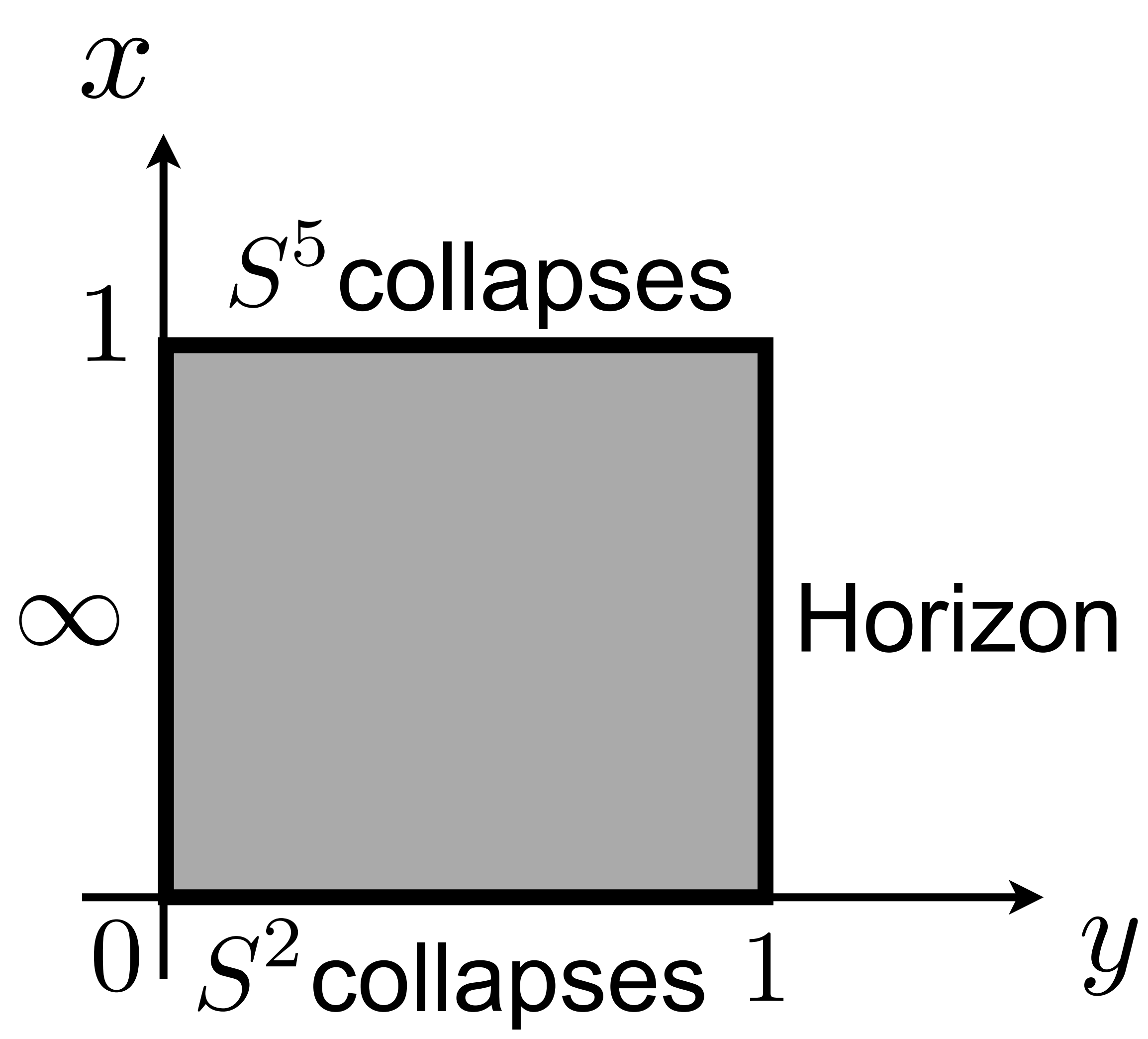

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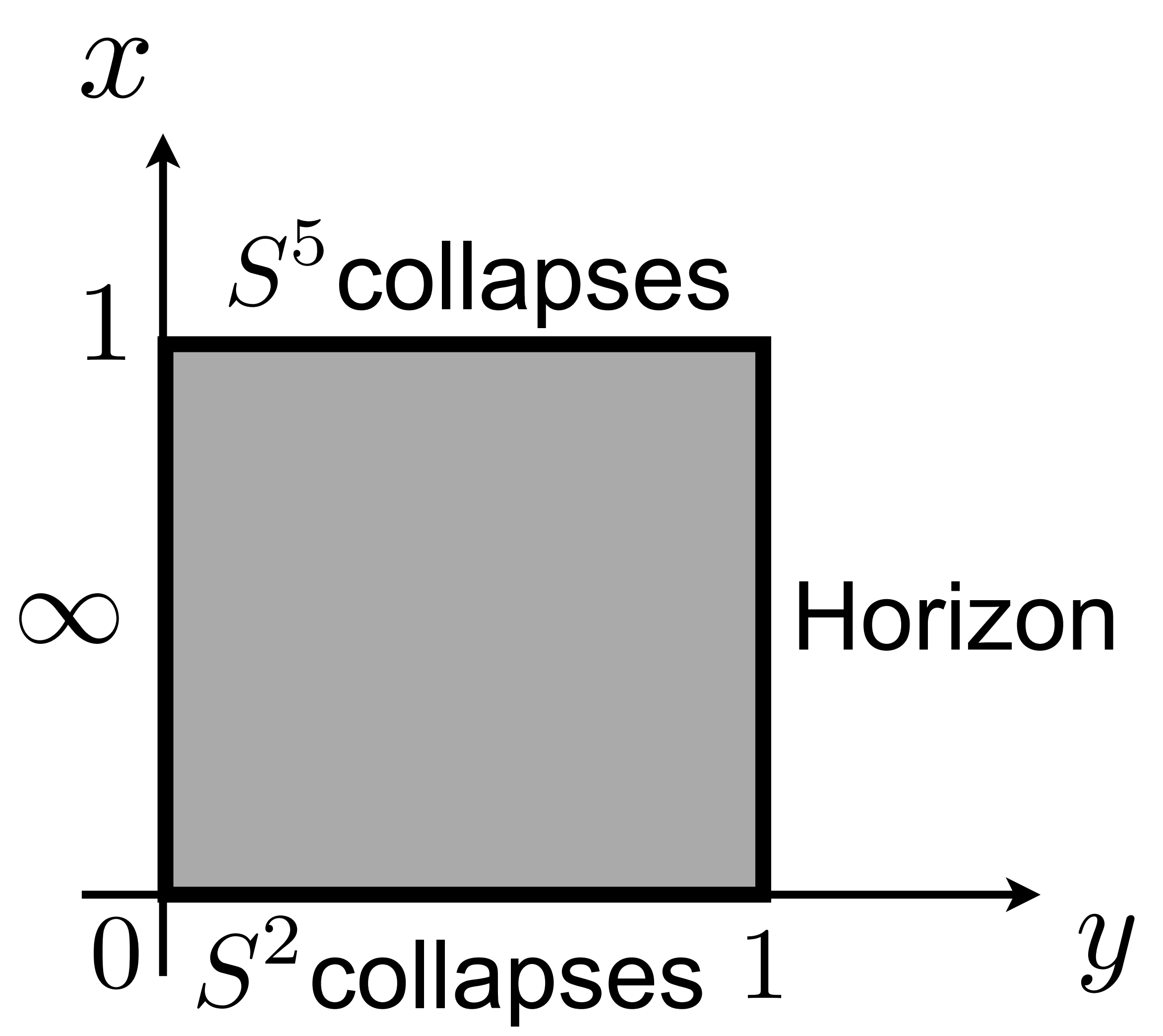

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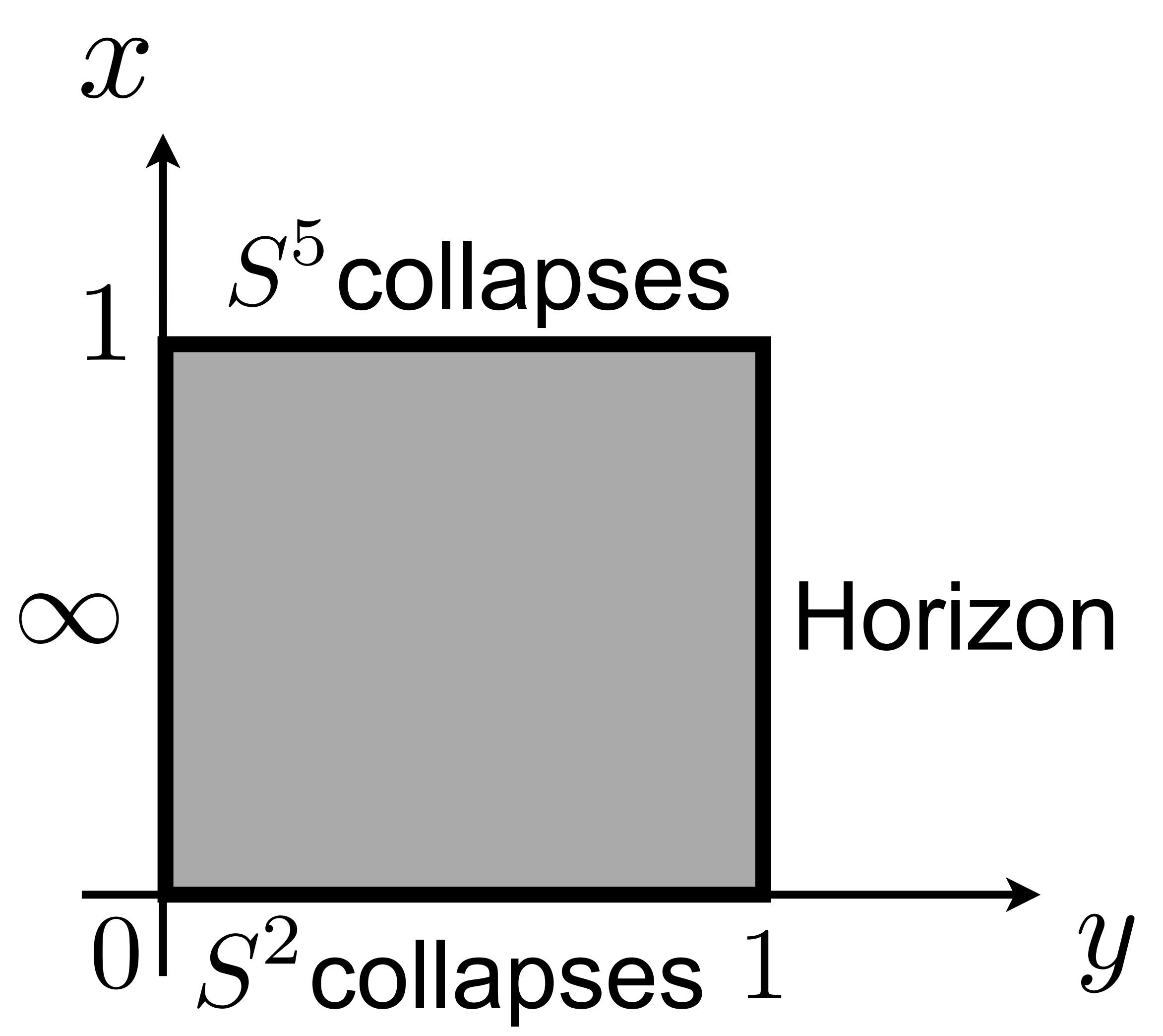

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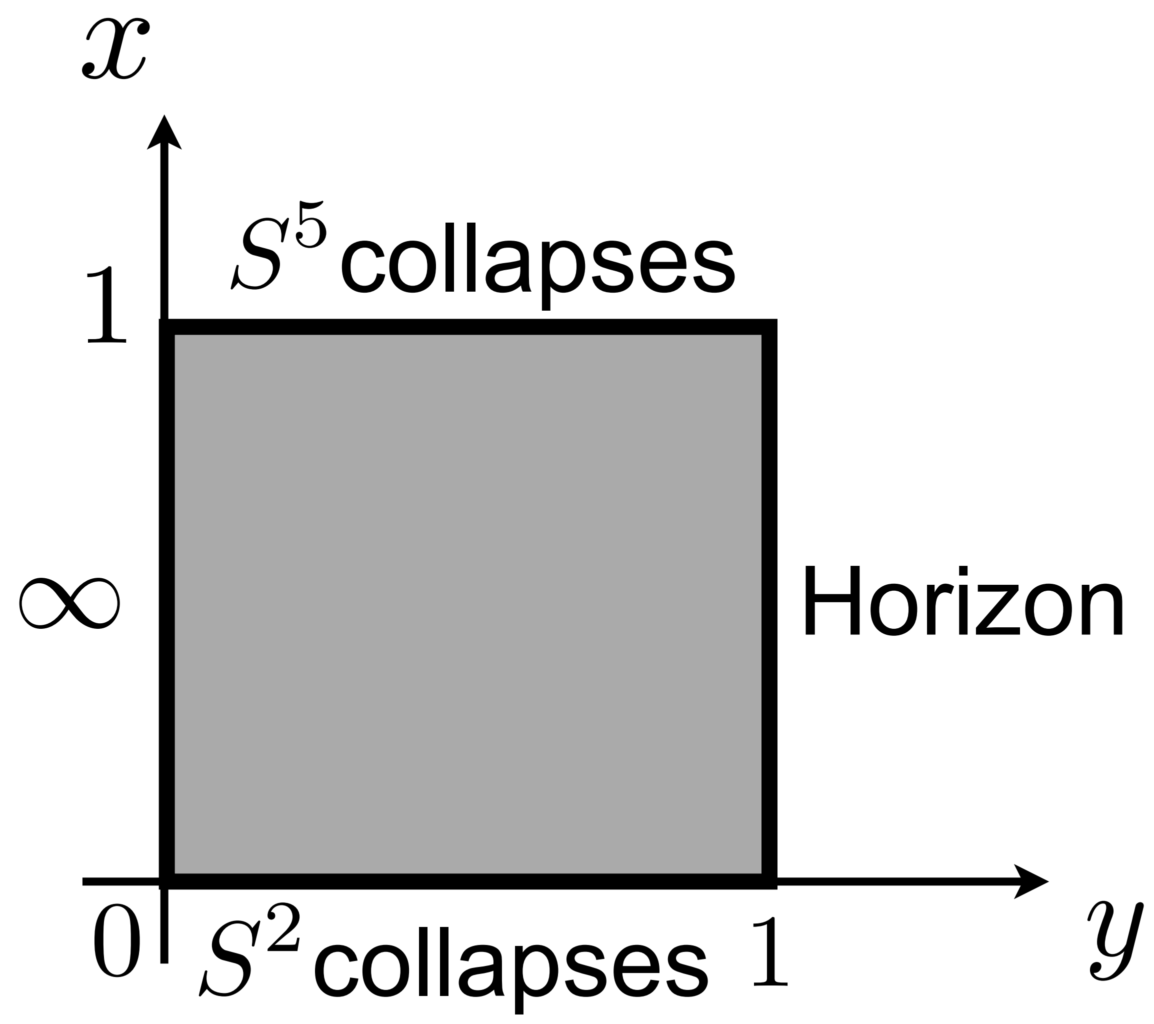

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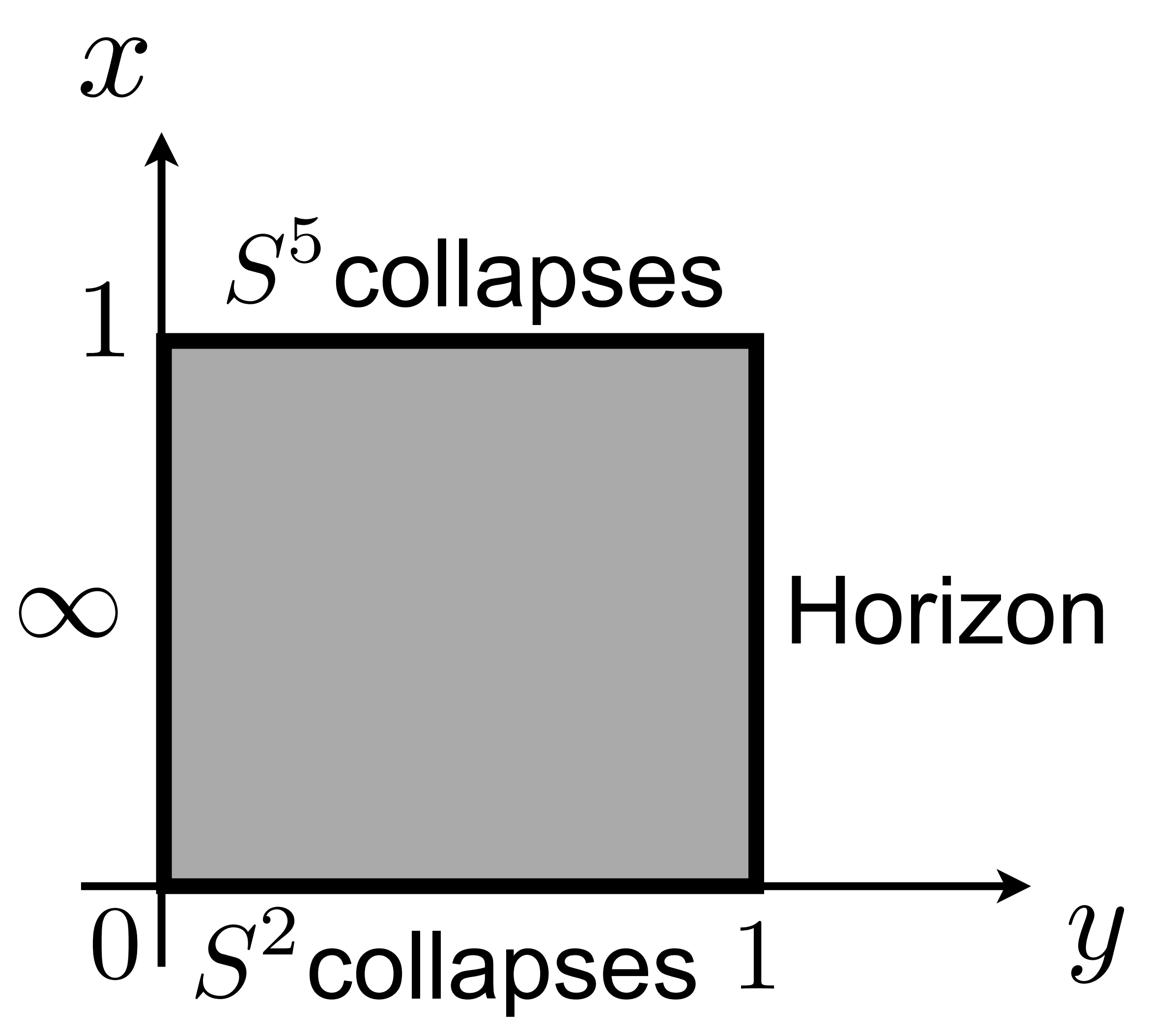

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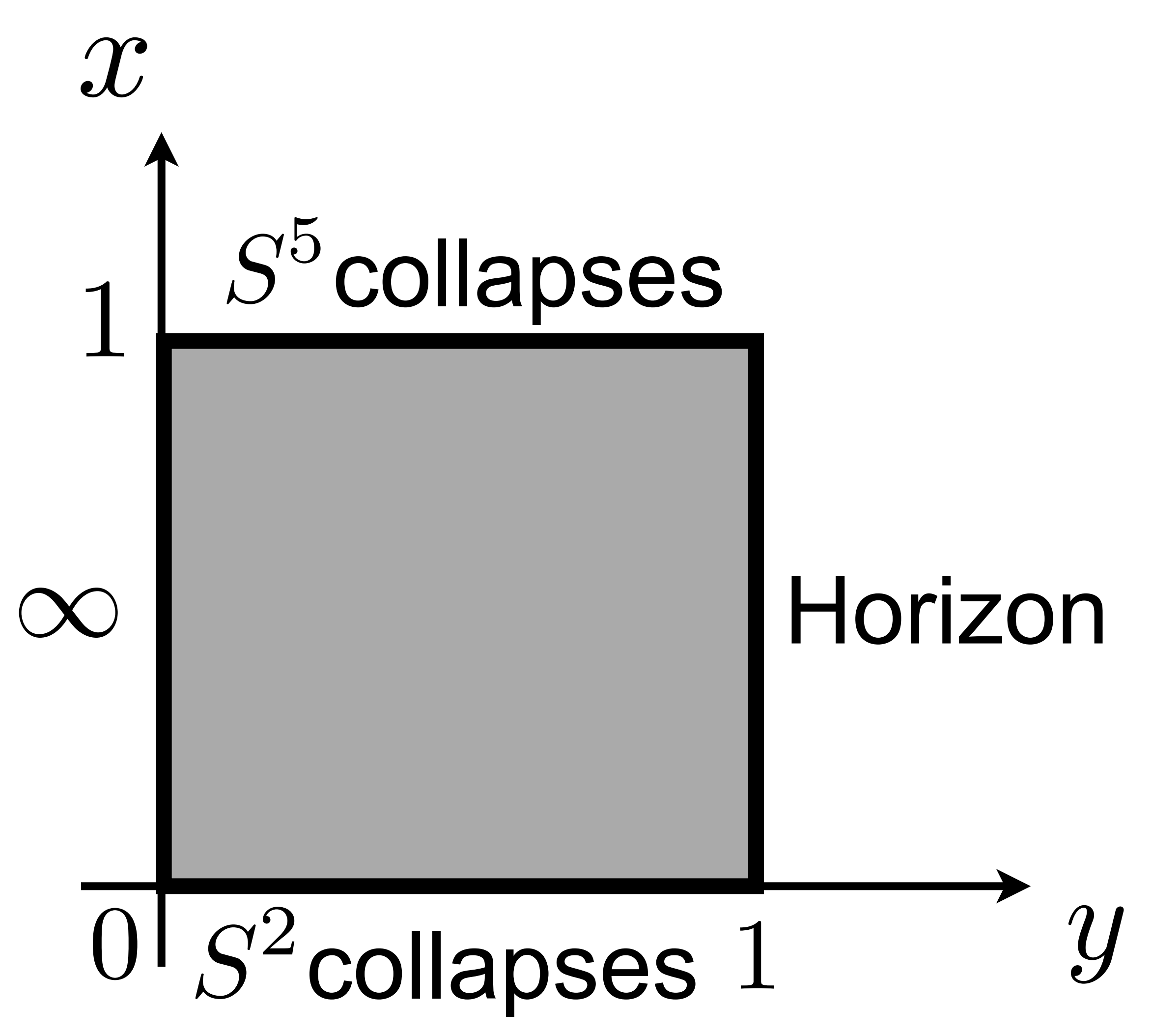

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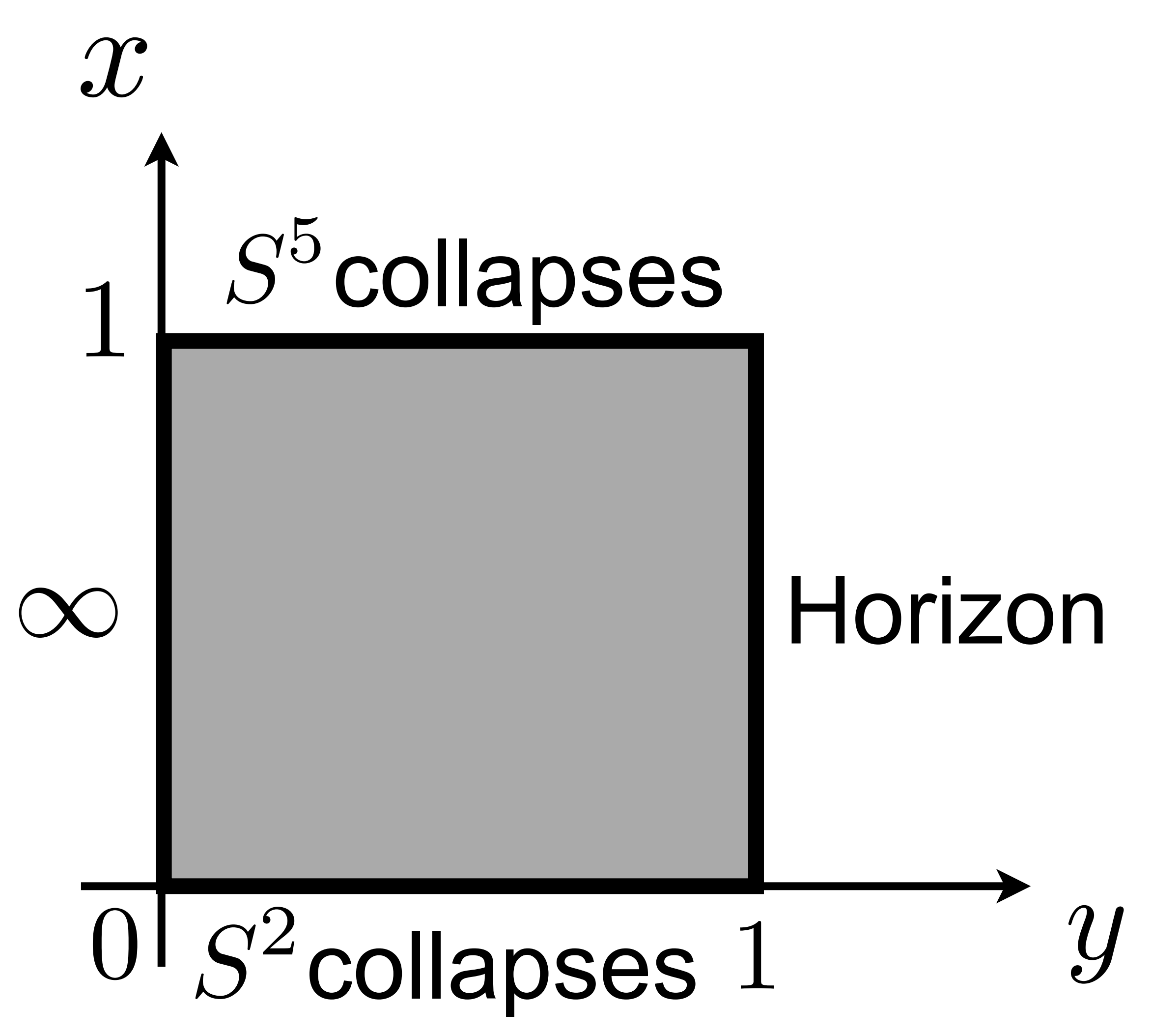

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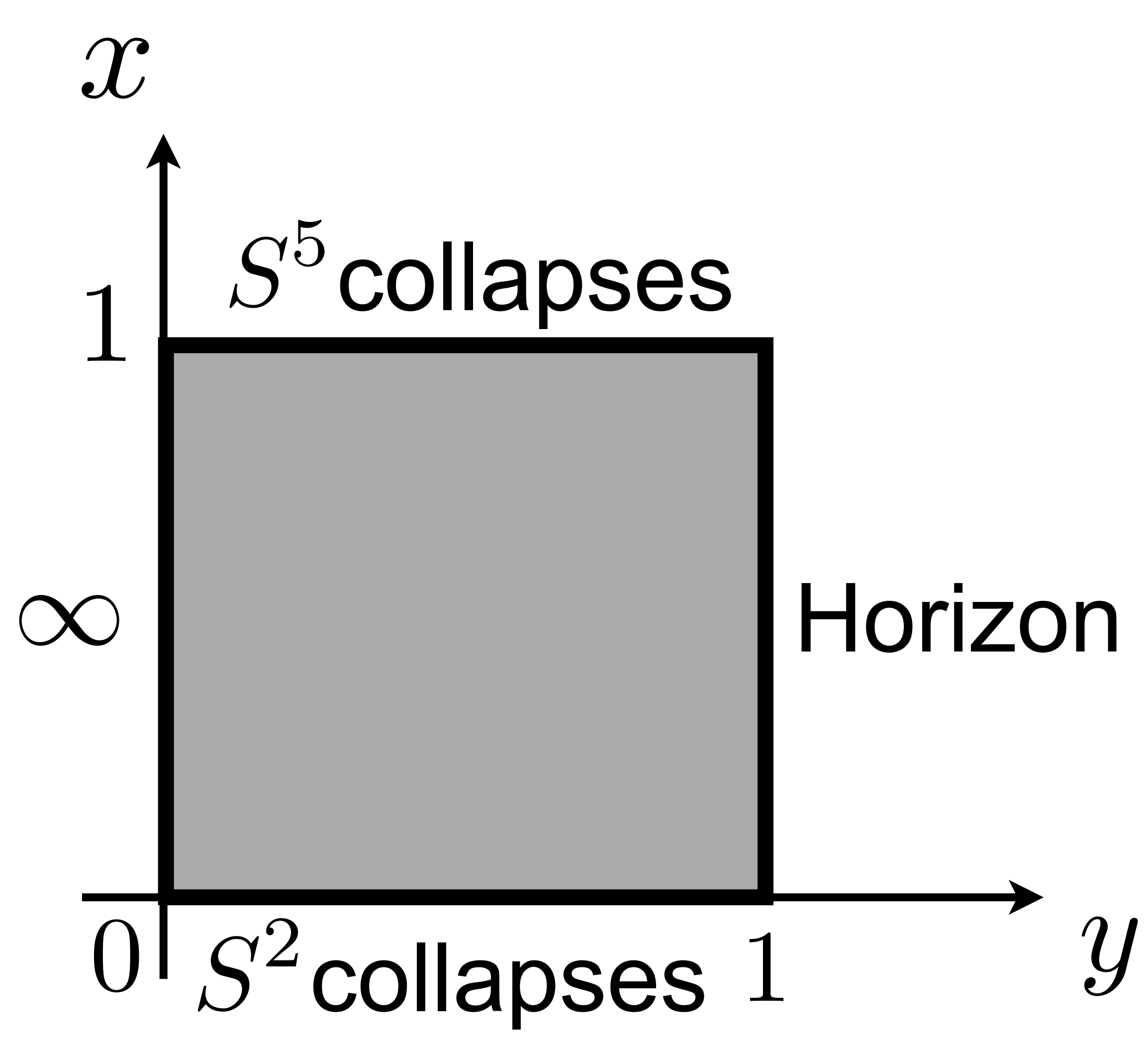

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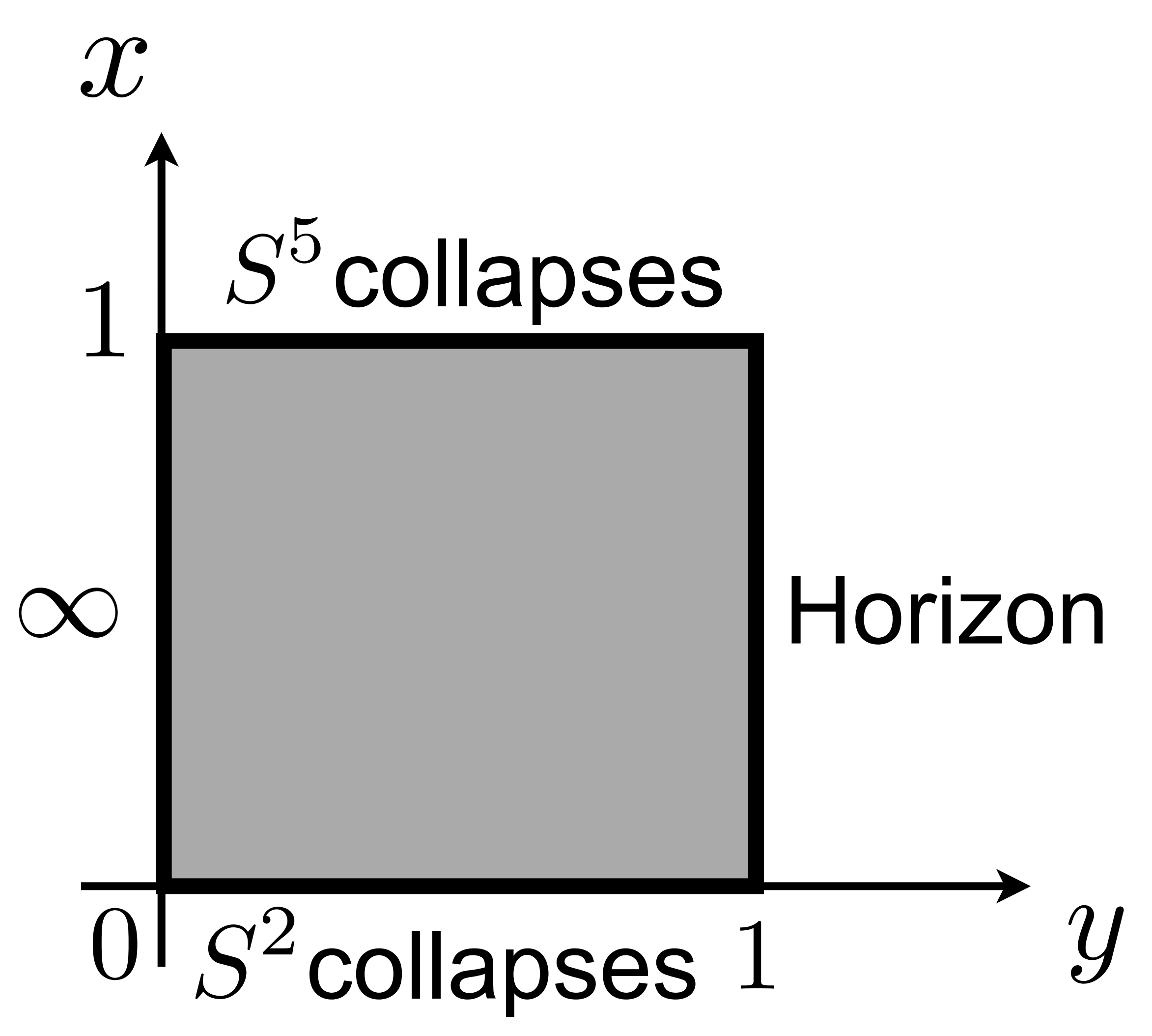

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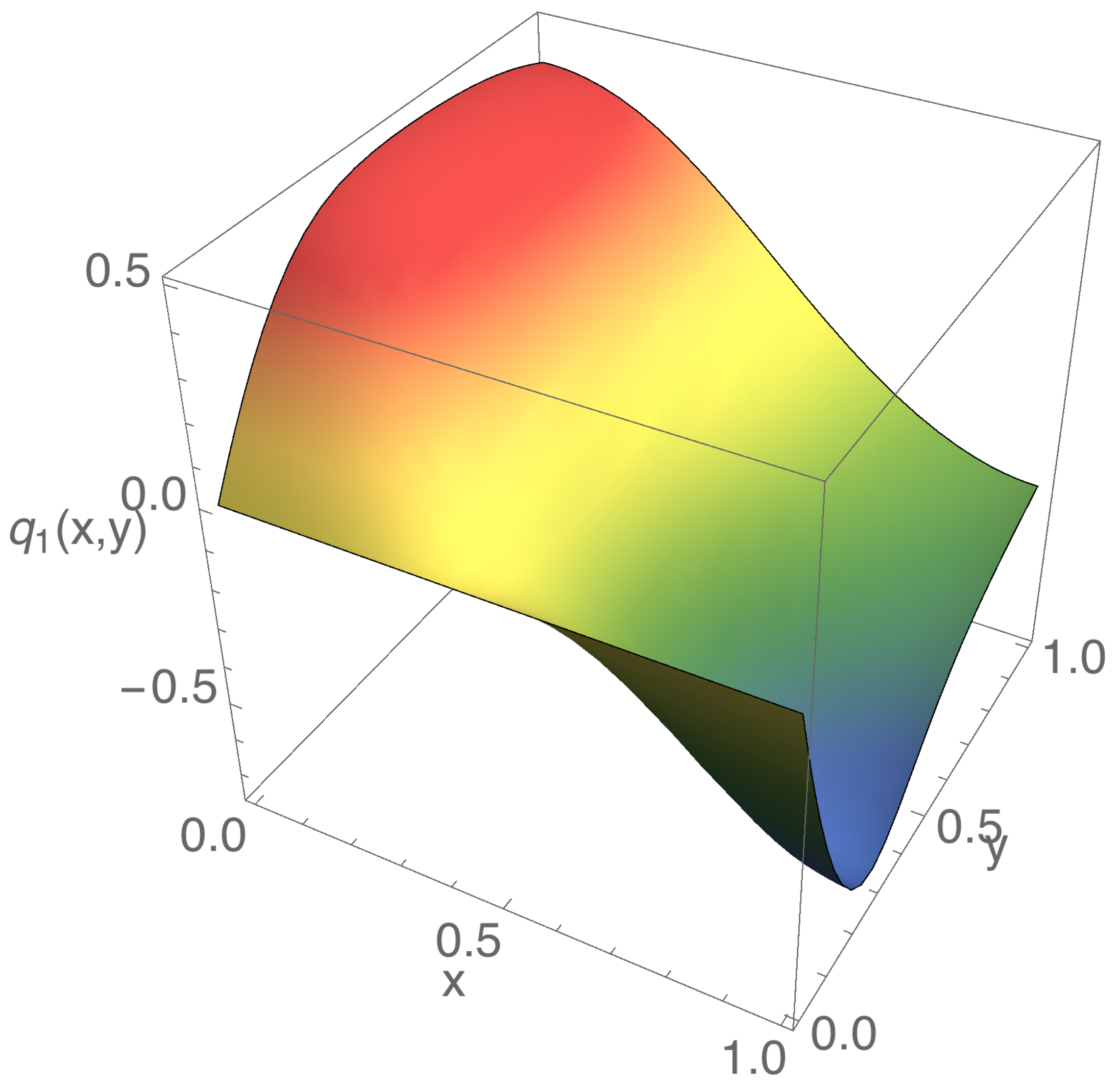

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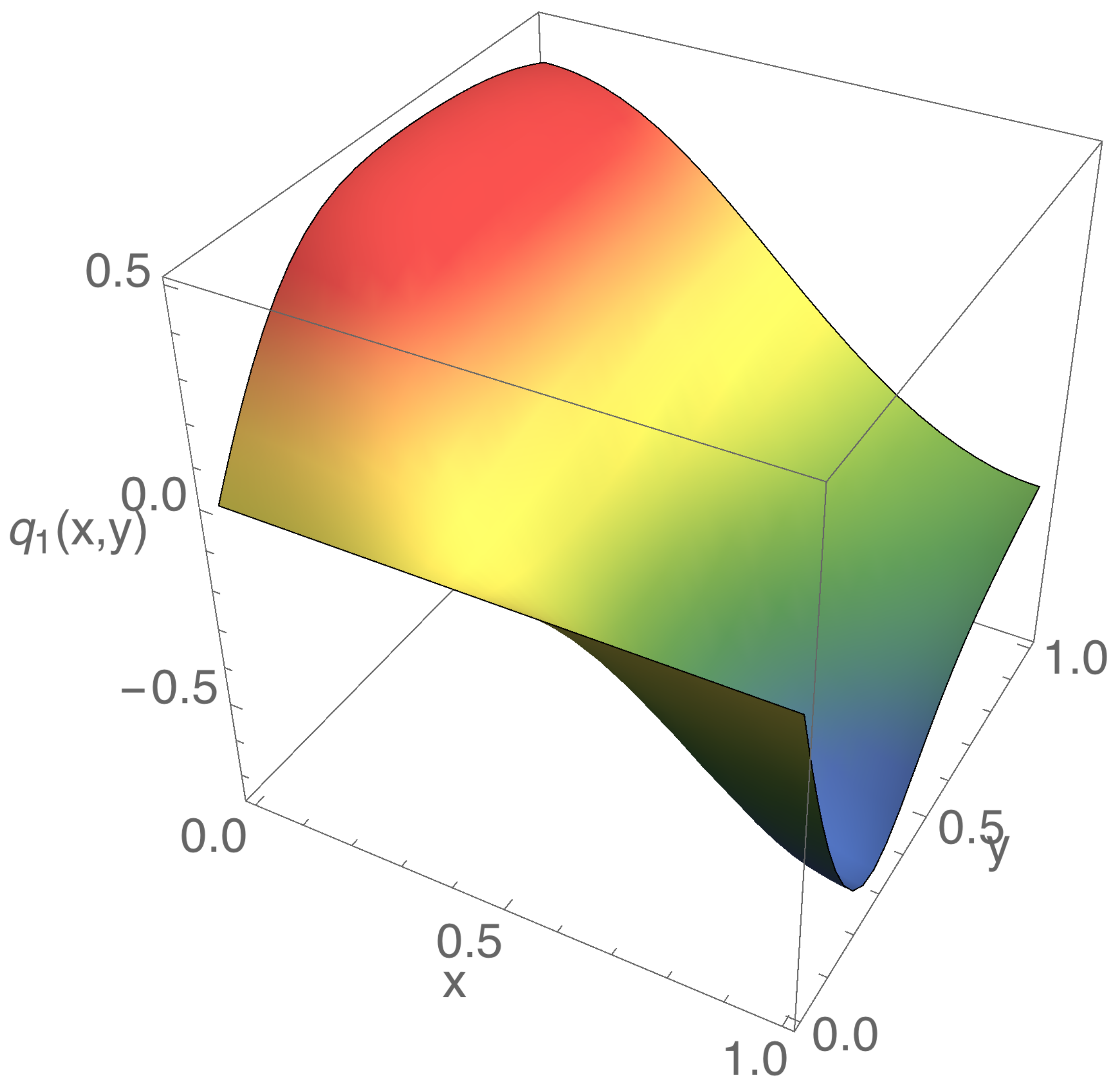


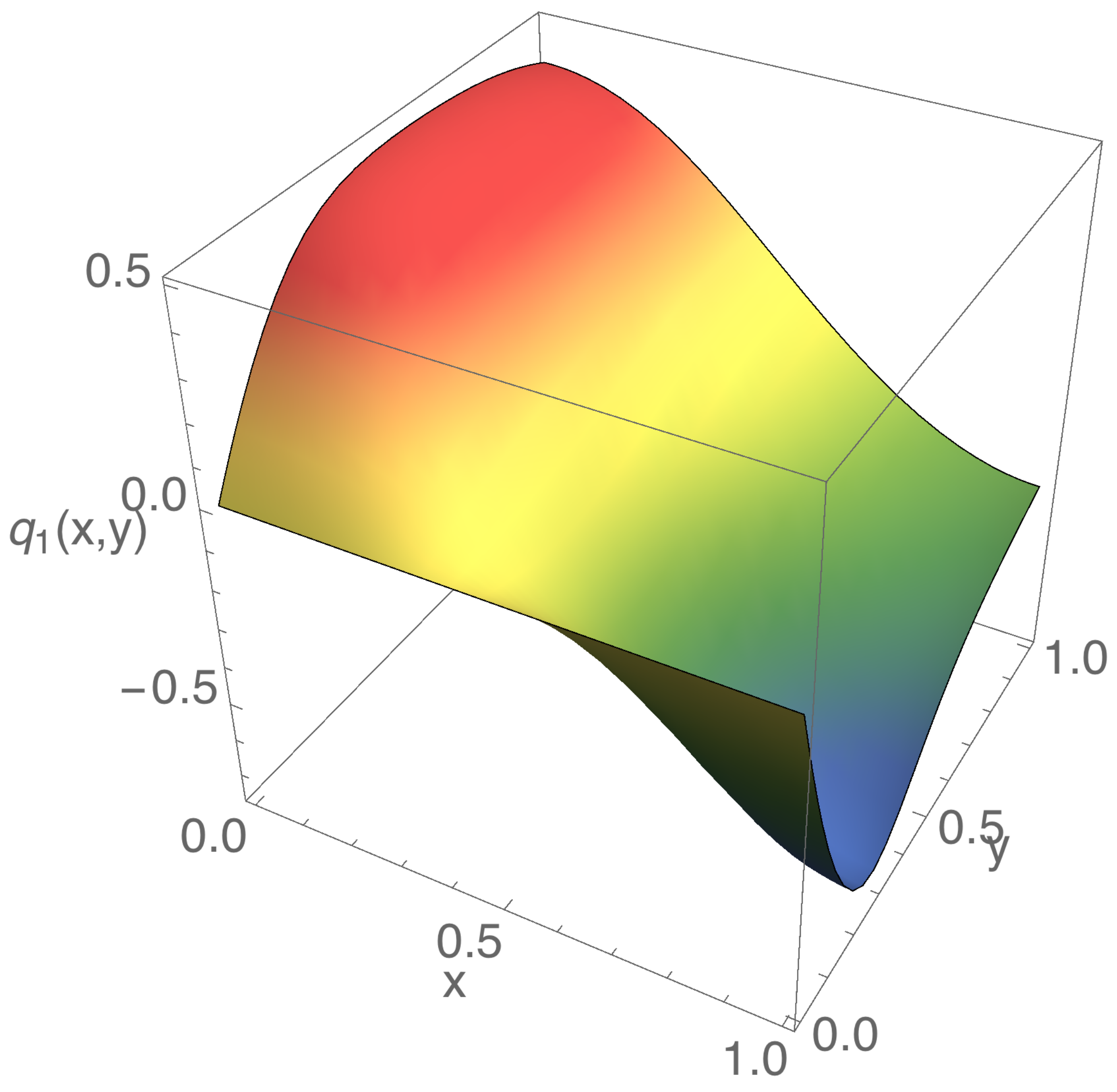
- Einstein-deTurck equations [Headrick, Kitchen, Wiseman '09]

$$R_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} = \frac{1}{12}\left(F_{\mu\alpha\beta\gamma}F_{\mu}{}^{\alpha\beta\gamma} - \frac{1}{12}g_{\mu\nu}F^2\right)$$

DeTurck term that makes Einstein equations elliptic

$$\xi^\mu = g^{\alpha\beta}\left(\Gamma^\mu_{\alpha\beta} - \tilde{\Gamma}^\mu_{\alpha\beta}\right)$$

The solution



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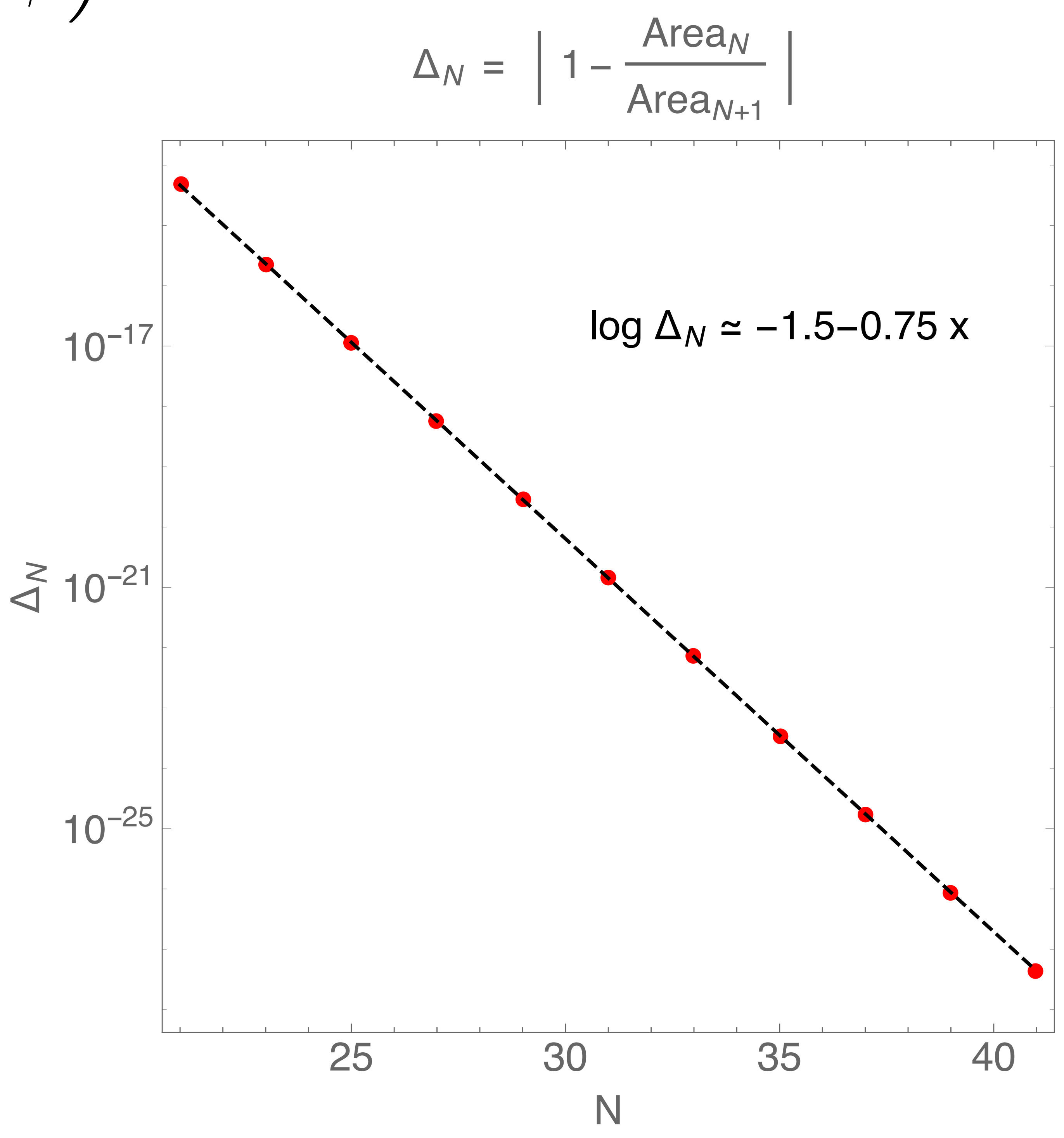
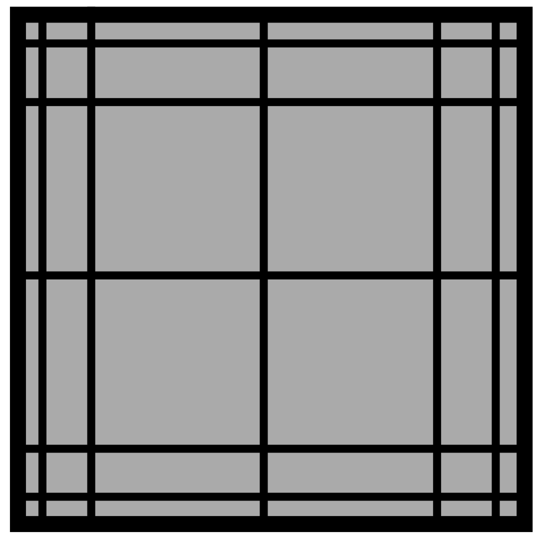


- Descretize PDEs with $N \times N$ Chebyshev grid

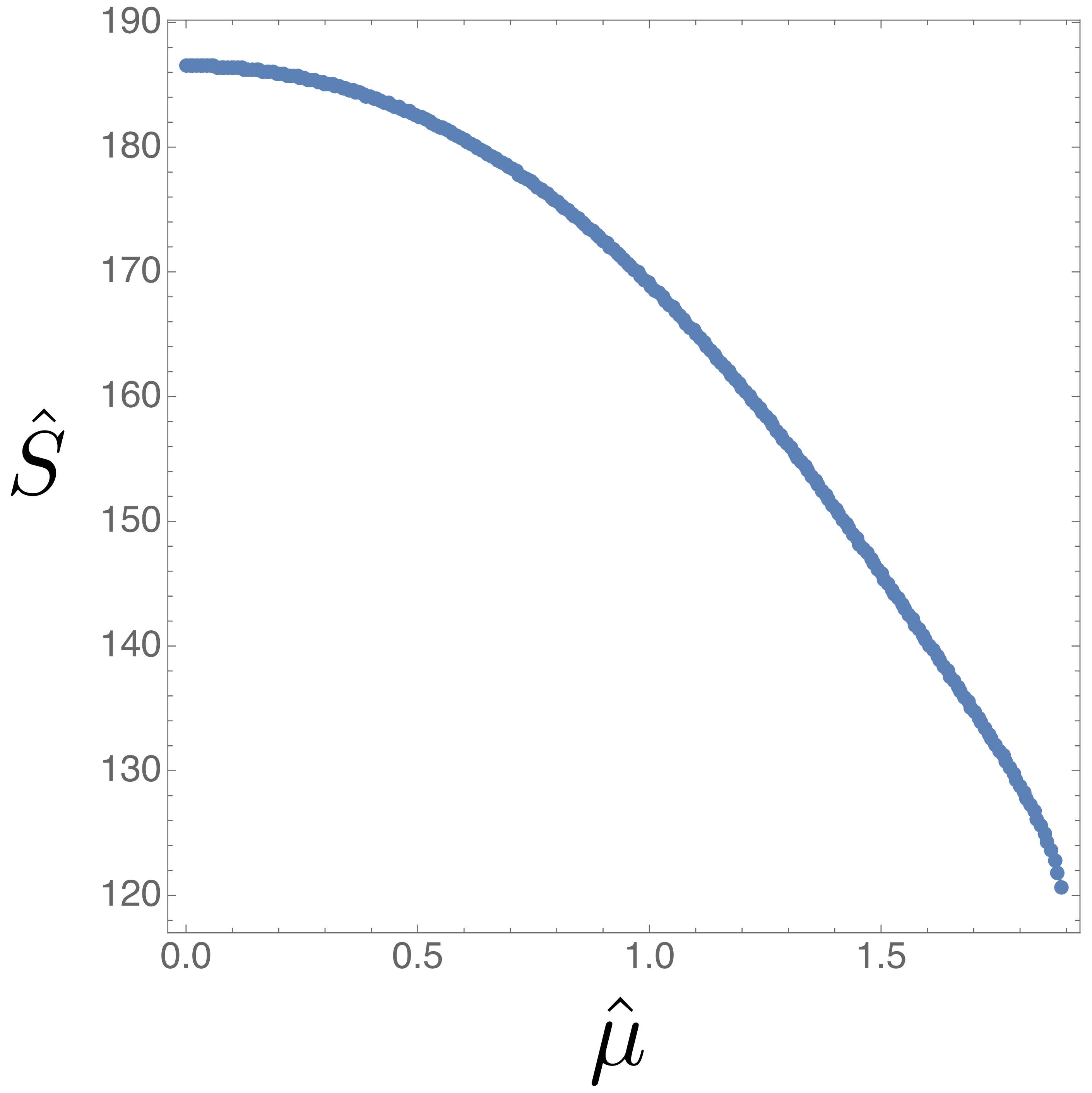


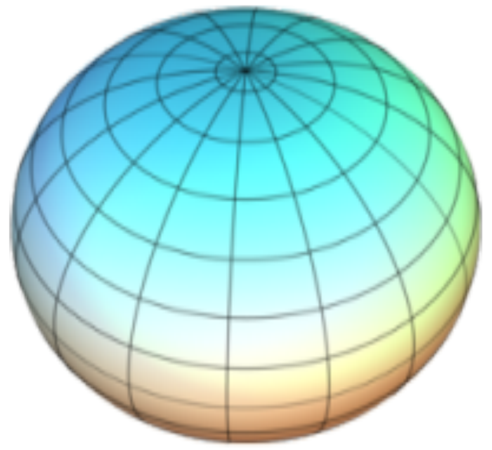
Derivatives are estimated using polynomial approximation that involves all points in the grid

spectral methods - exponential convergence

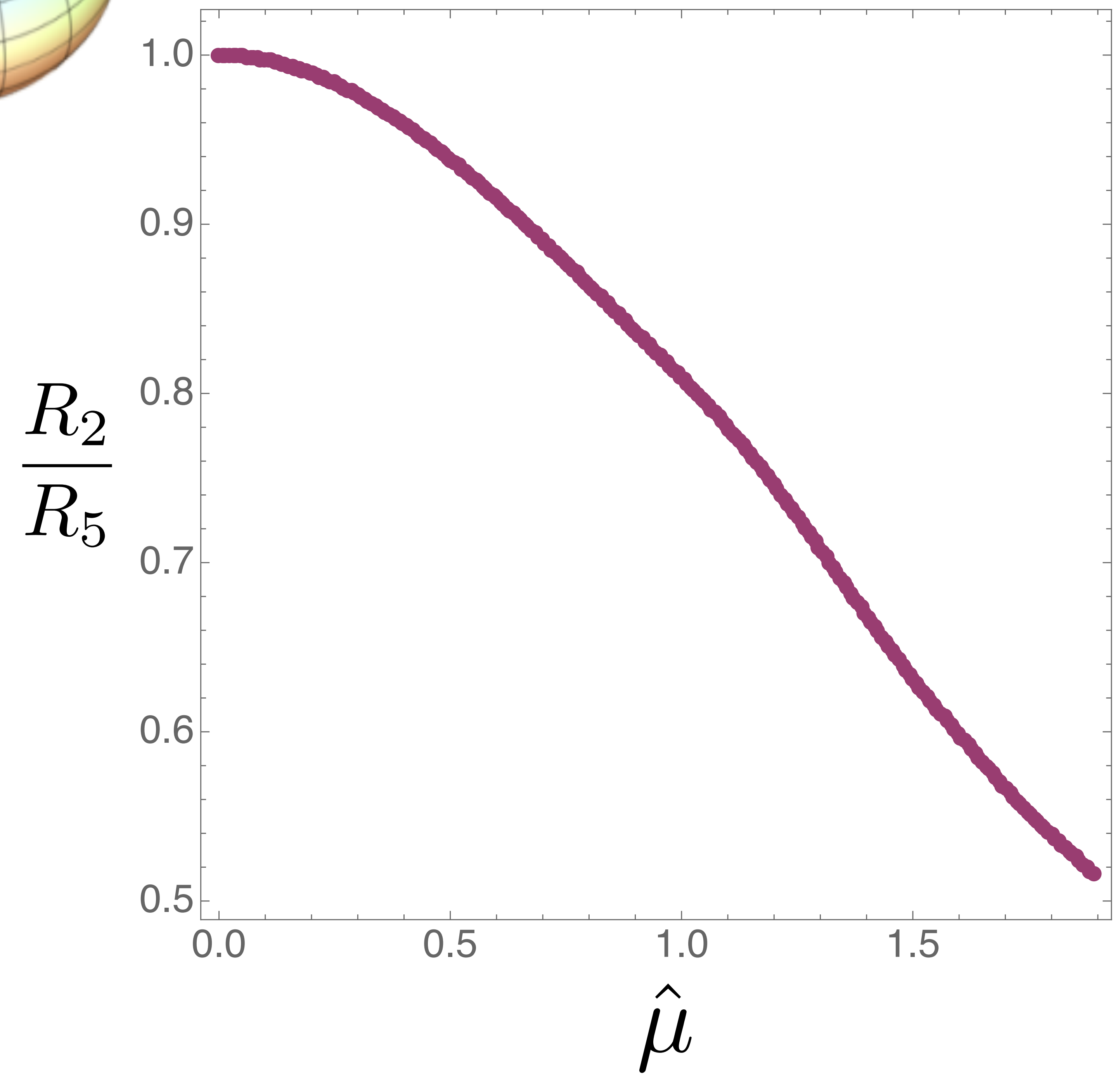
- Horizon area and shape

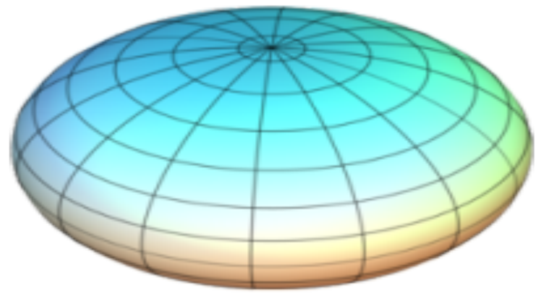
Horizon area





Ratio of maximal radius of S^2 to S^5





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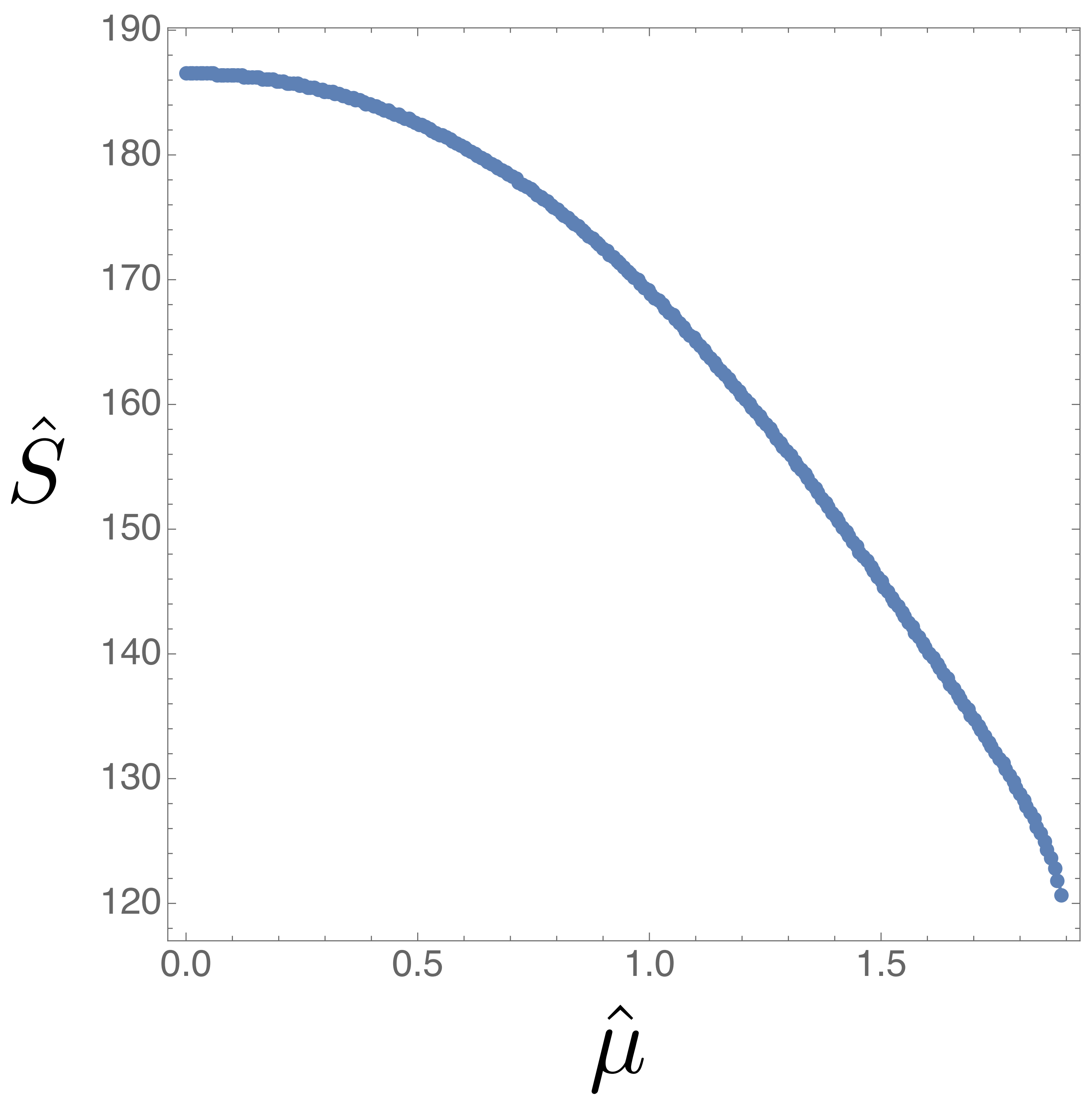

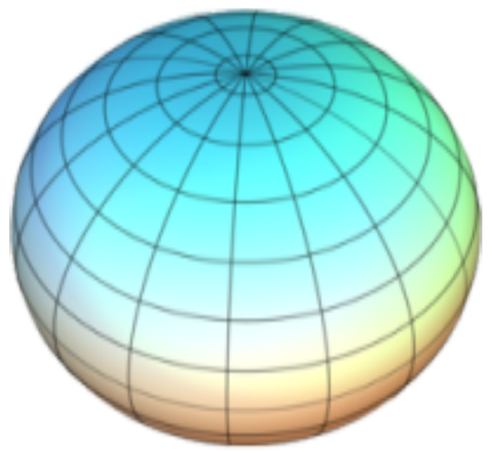

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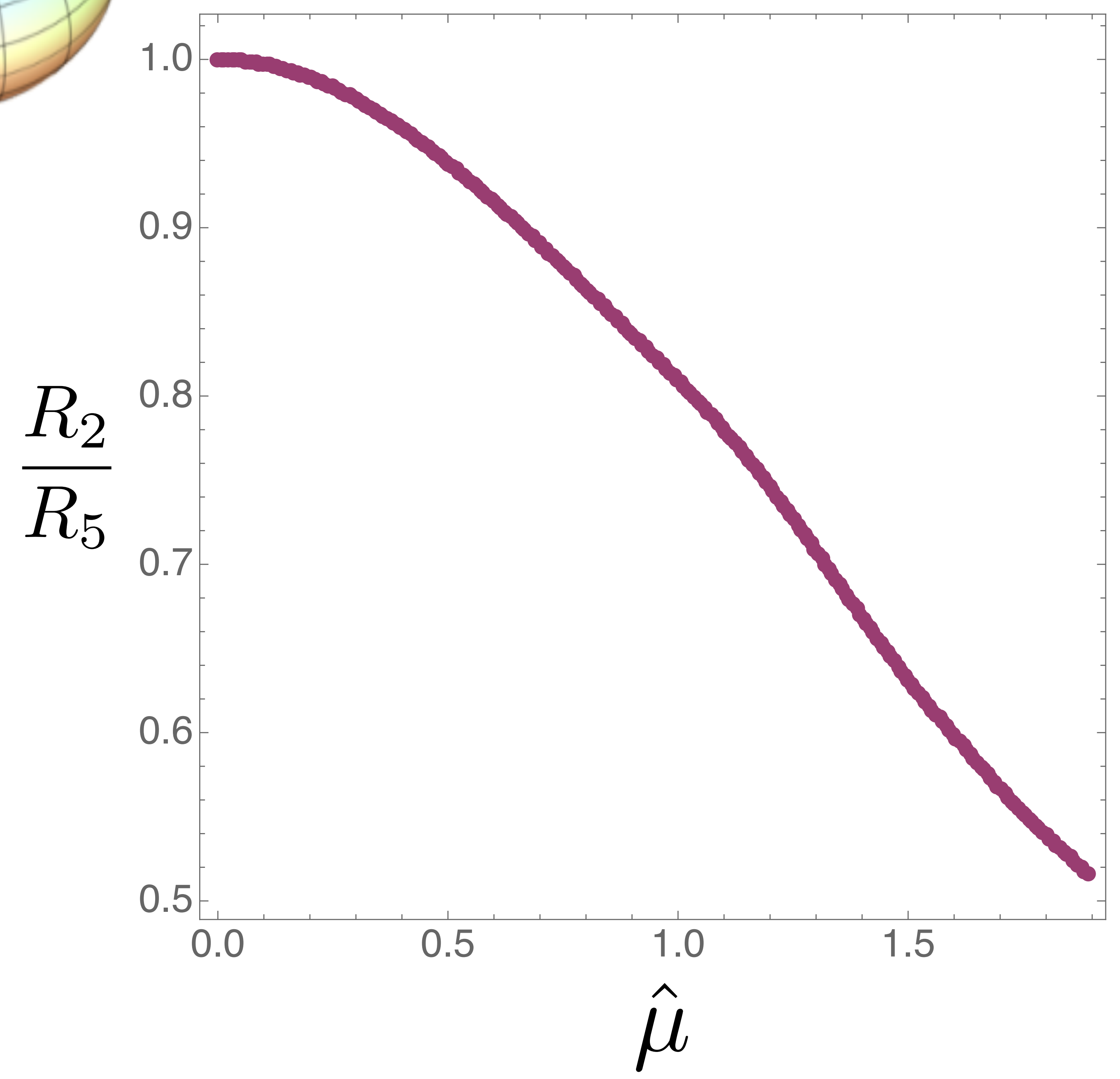

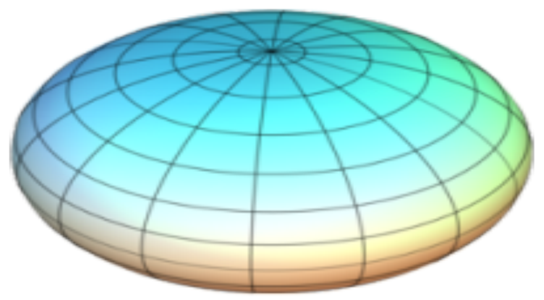

After scaling symmetry to obtain physical metric:

$$S = \frac{15\pi}{7}\left(\frac{15}{14^2\pi^8}\right)^{\frac{2}{5}} N^2 \left(\frac{T}{\lambda^{\frac{1}{3}}}\right)^{\frac{9}{5}} \hat{S}\left(\frac{\mu}{T}\right)$$

$$R_i = a_i \left(\frac{T}{\lambda^{\frac{1}{3}}}\right)^{\frac{2}{5}} \hat{R}_i\left(\frac{\mu}{T}\right)$$

Reproduces scalings predicted from strongly coupled low energy moduli estimate [Wiseman '13]

Black hole thermodynamics - critical temperature

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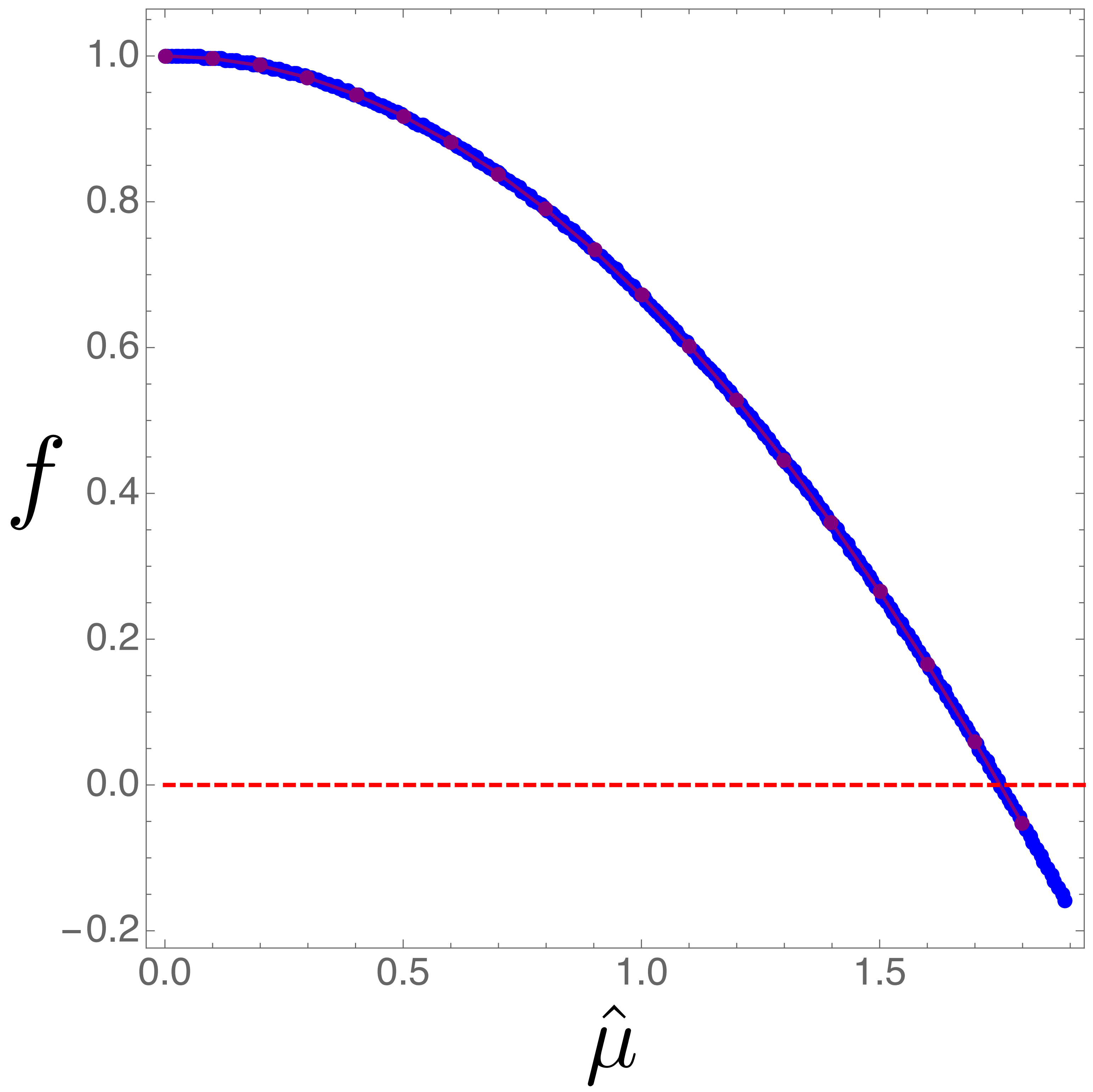

$$F(T,\mu) = F(T,0)\, f(\hat{\mu})$$
$$= -c_1 T^{\frac{14}{5}} f(\hat{\mu})$$

both using 1st law or holographic renormalization

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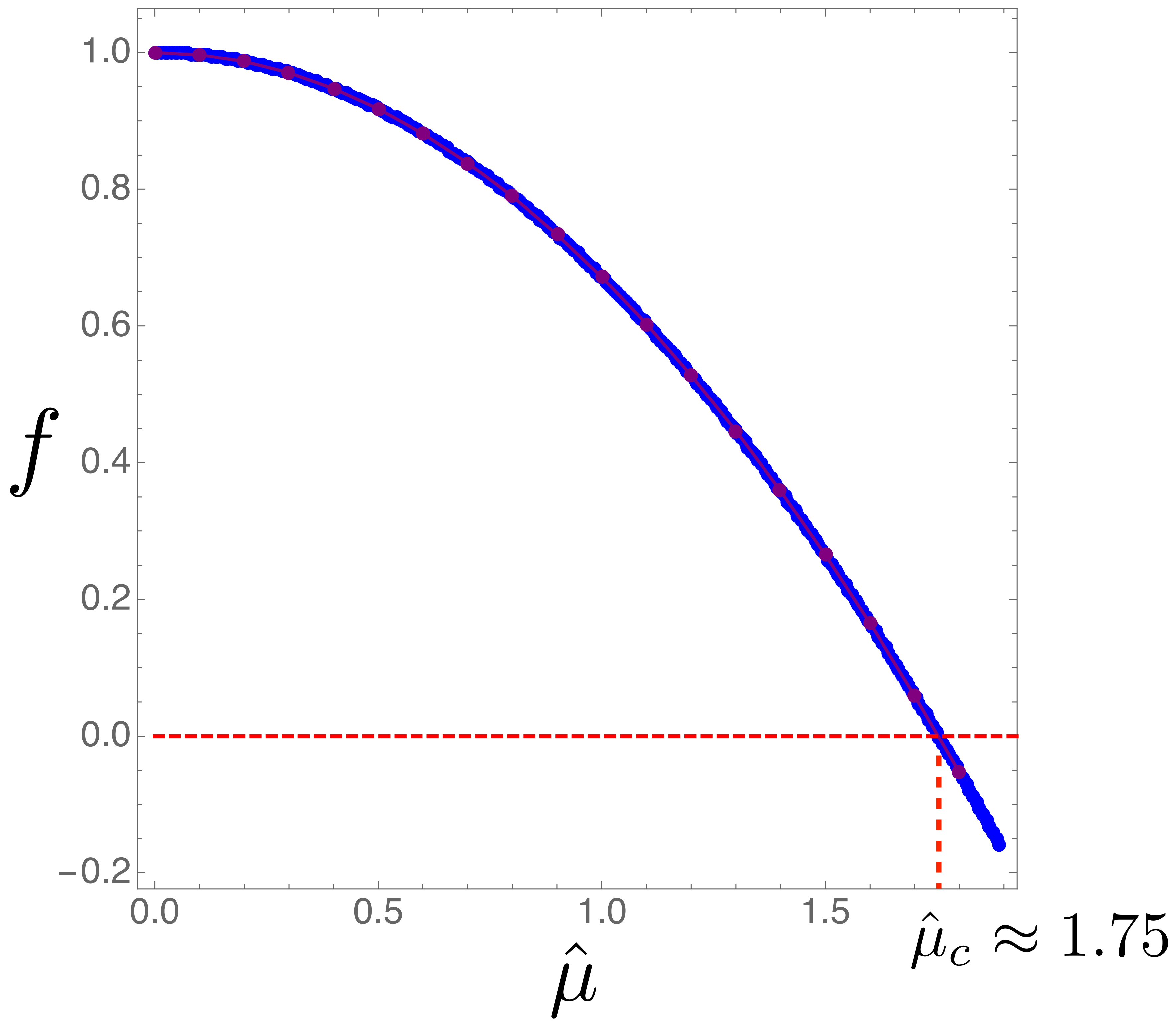

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both using 1st law or holographic renormalization

- Phase transition occurs when free energy changes sign, since for $T < T_c$ geometry without horizon is favoured $F \sim \mathcal{O}(N^0)$ [Lin, Maldacena '05]

$$\frac{T_c}{\mu} = \frac{7}{12\pi\hat{\mu}_c} \approx 0.106$$

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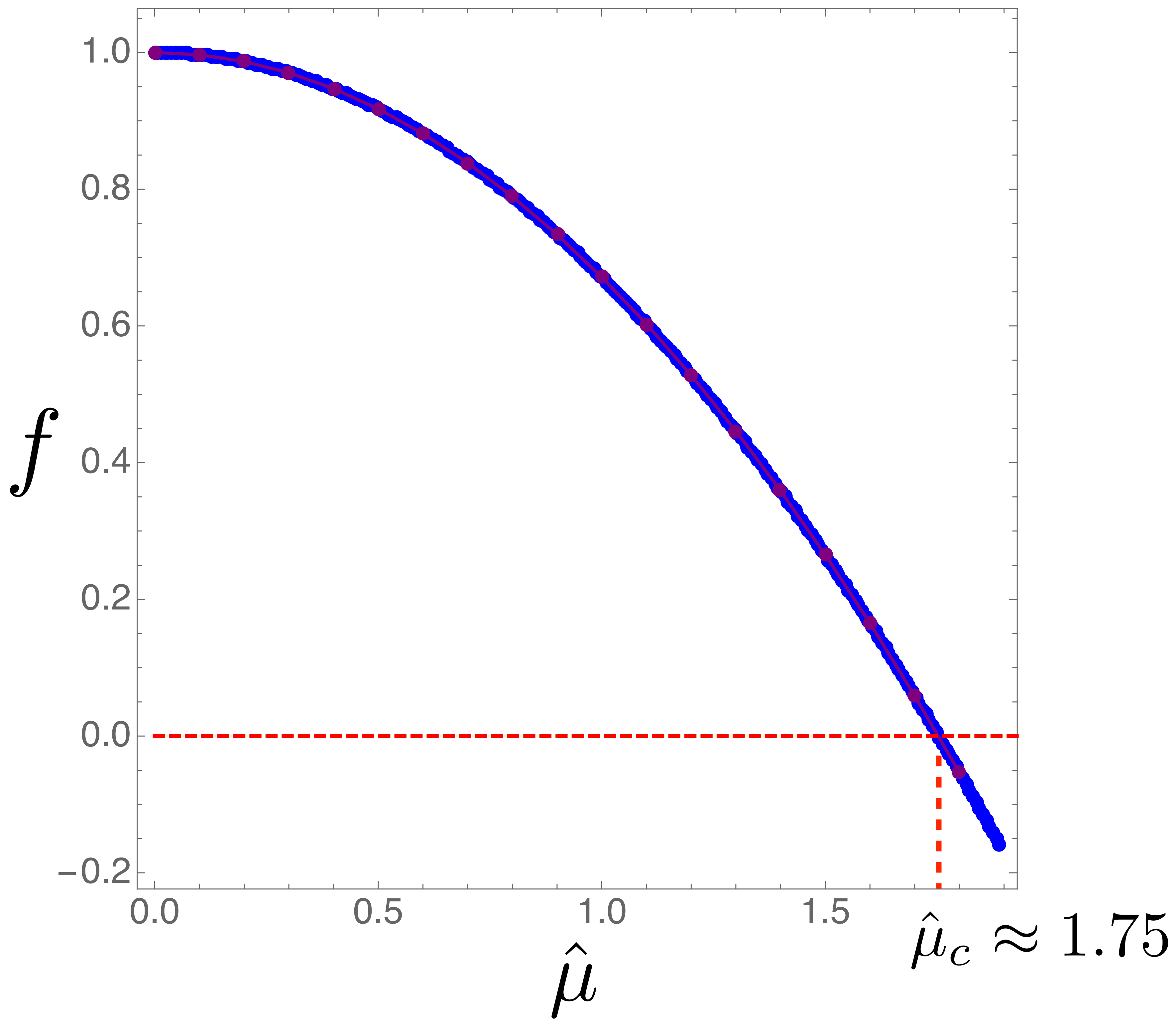

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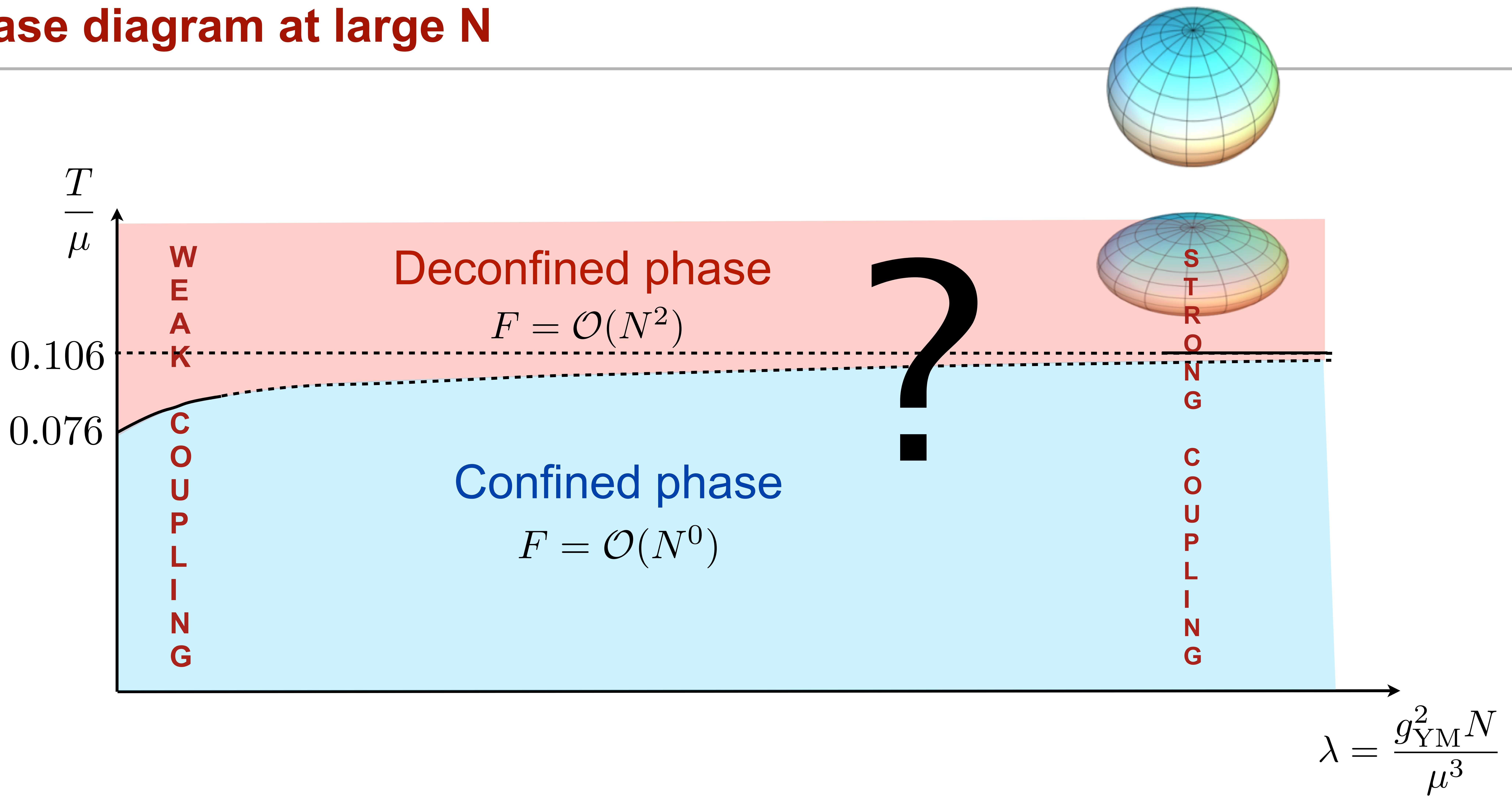
- BH is thermodynamically stable for $\hat{\mu} < \hat{\mu}_c$

$$c = T\left(\frac{\partial S}{\partial T}\right)_\mu \Rightarrow \frac{c}{S} = \frac{9}{5} - \hat{\mu}\frac{\partial}{\partial \hat{\mu}} \log s(\hat{\mu}) > 0$$

Phase diagram at large N

Phase diagram at large N



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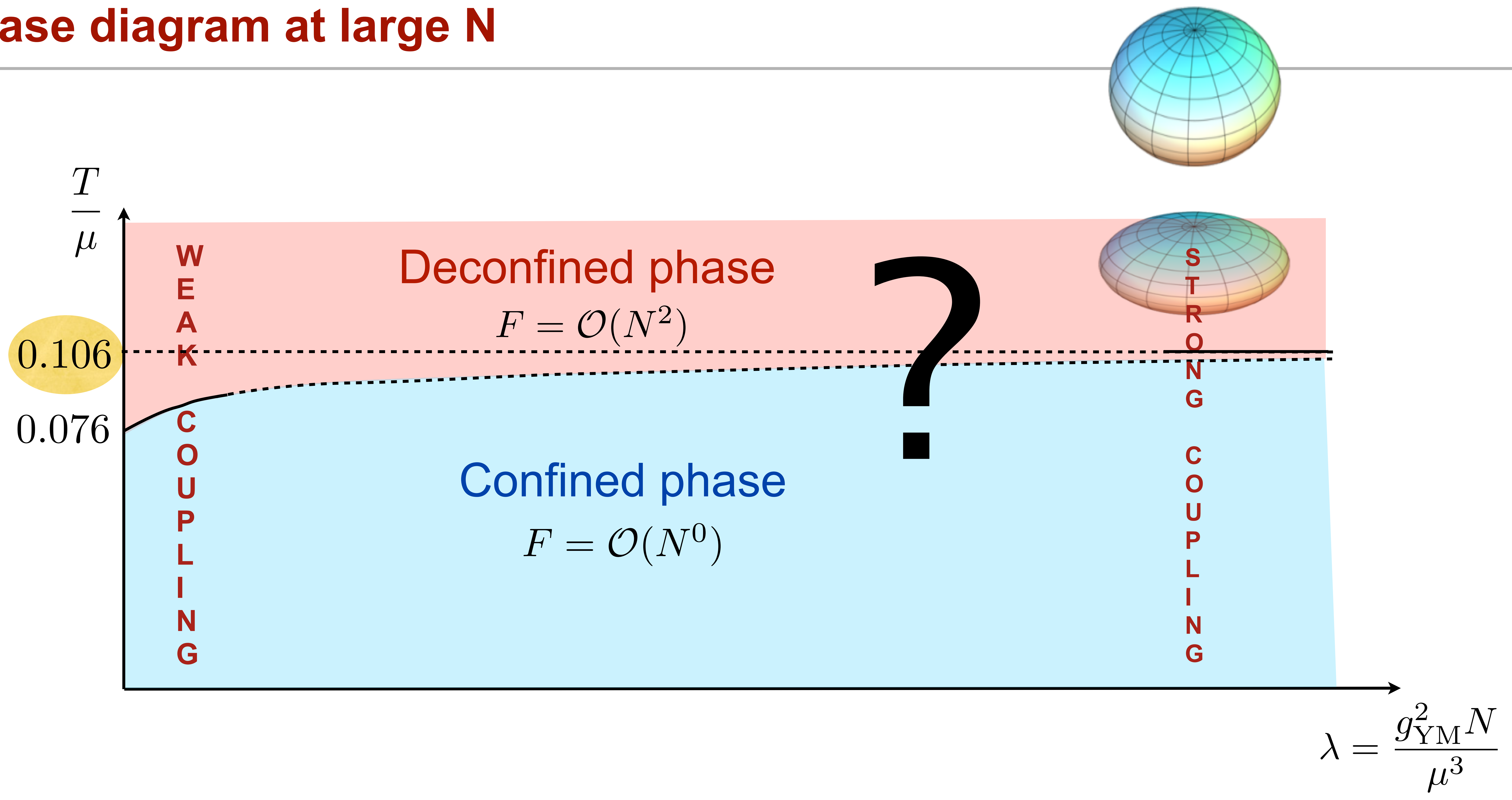

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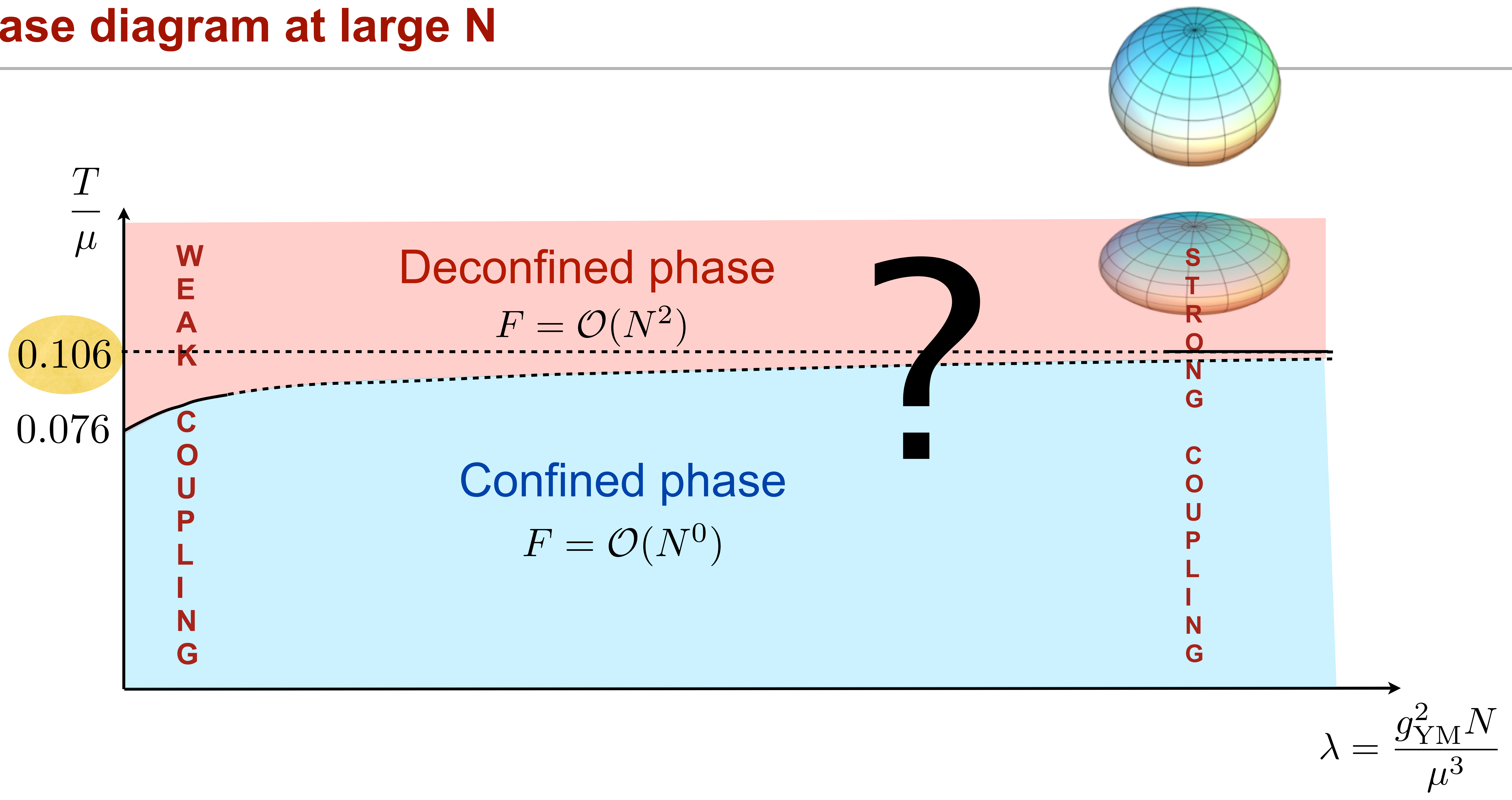

Very similar to SYM on a 3-sphere $(\mu \equiv 1/R)$

[Aharony, Marsano, Minwalla, Papadodimas, van Raamsdonk '03]

Boundary data

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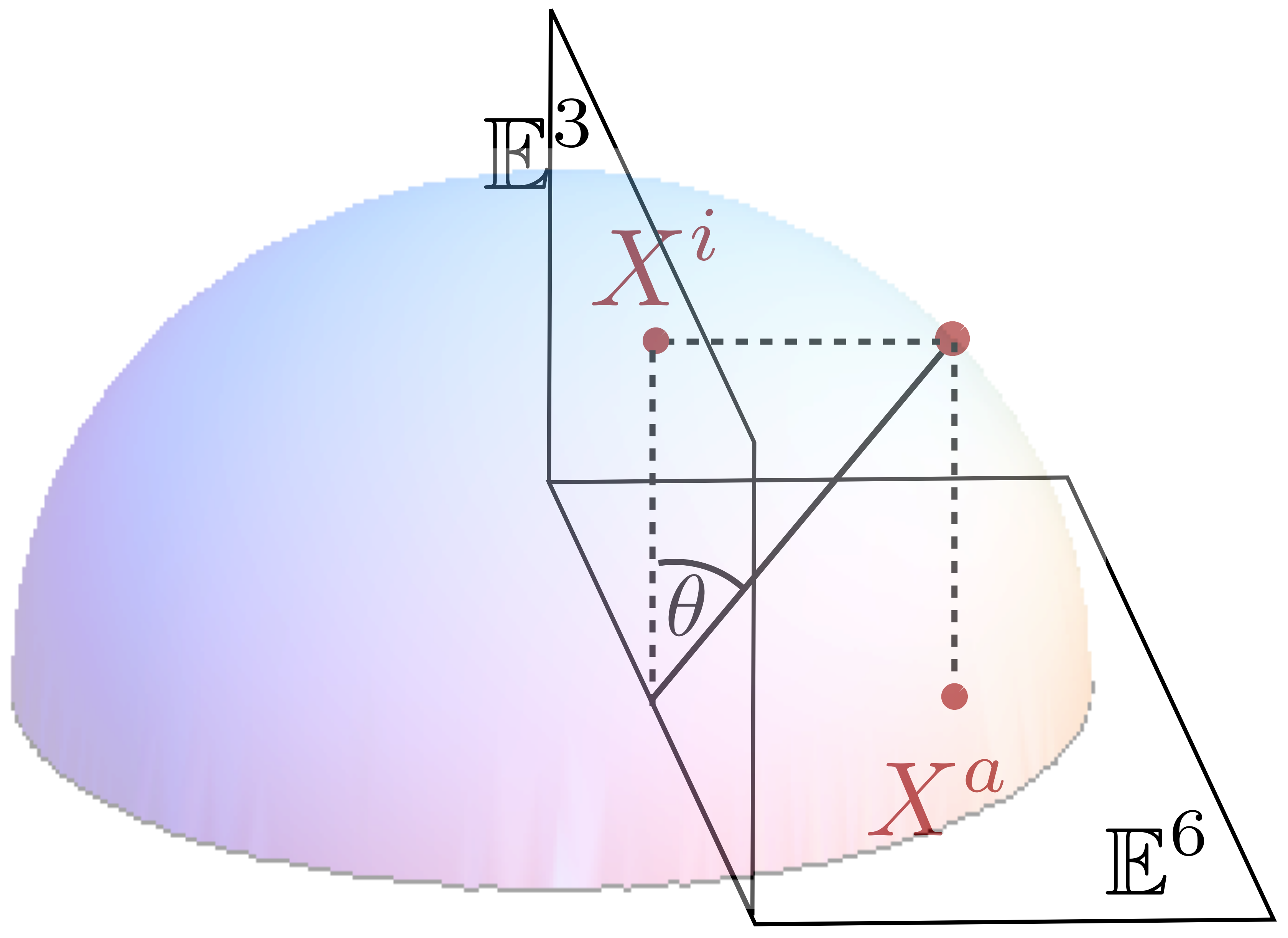

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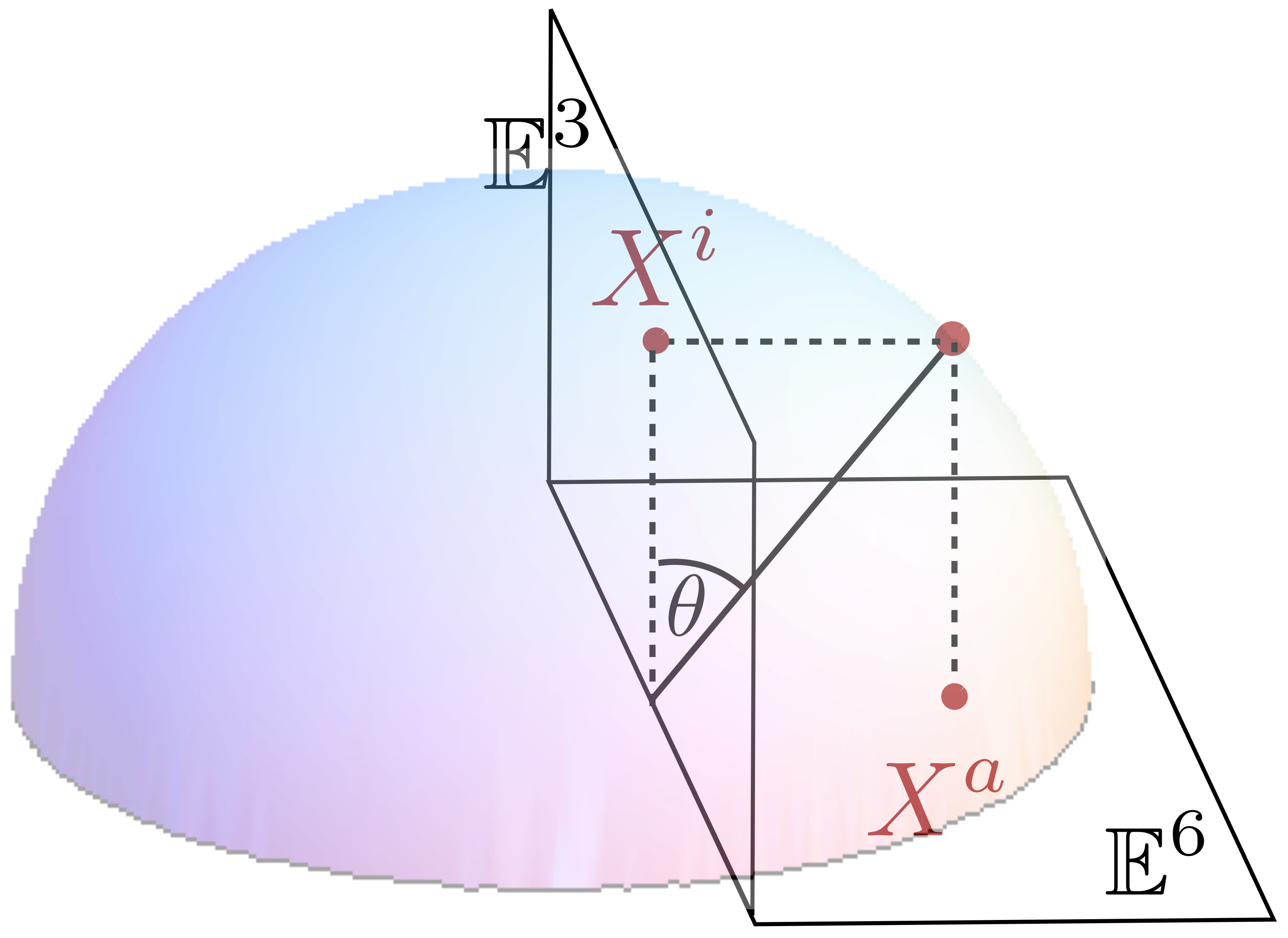

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• Boundary metric has $SO(9)$ symmetry, so $\tilde{Q}_i^j(x)$ are harmonic functions on S^8 . Thus we can classify the $SO(6)\times SO(3)$ invariant perturbations according to $SO(9)$ spin. This helps to establish bulk field / operator correspondence.

- 2- form modes in the asymptotic expansion $C = (M\, d\eta + L\, d\zeta) \wedge d^2\Omega_2$

$$v(x,y) = \sum_{l\, odd} \Big(\alpha_l\, f_l(y) + \tilde{\alpha}_l\, \tilde{f}_l(y) \Big)\, \mathbb{H}_l(x) \;+\; \text{back reaction}$$

$$f_l(y) \sim y^{1+l}$$

$$\tilde{f}_l(y) \sim y^{1-l}$$

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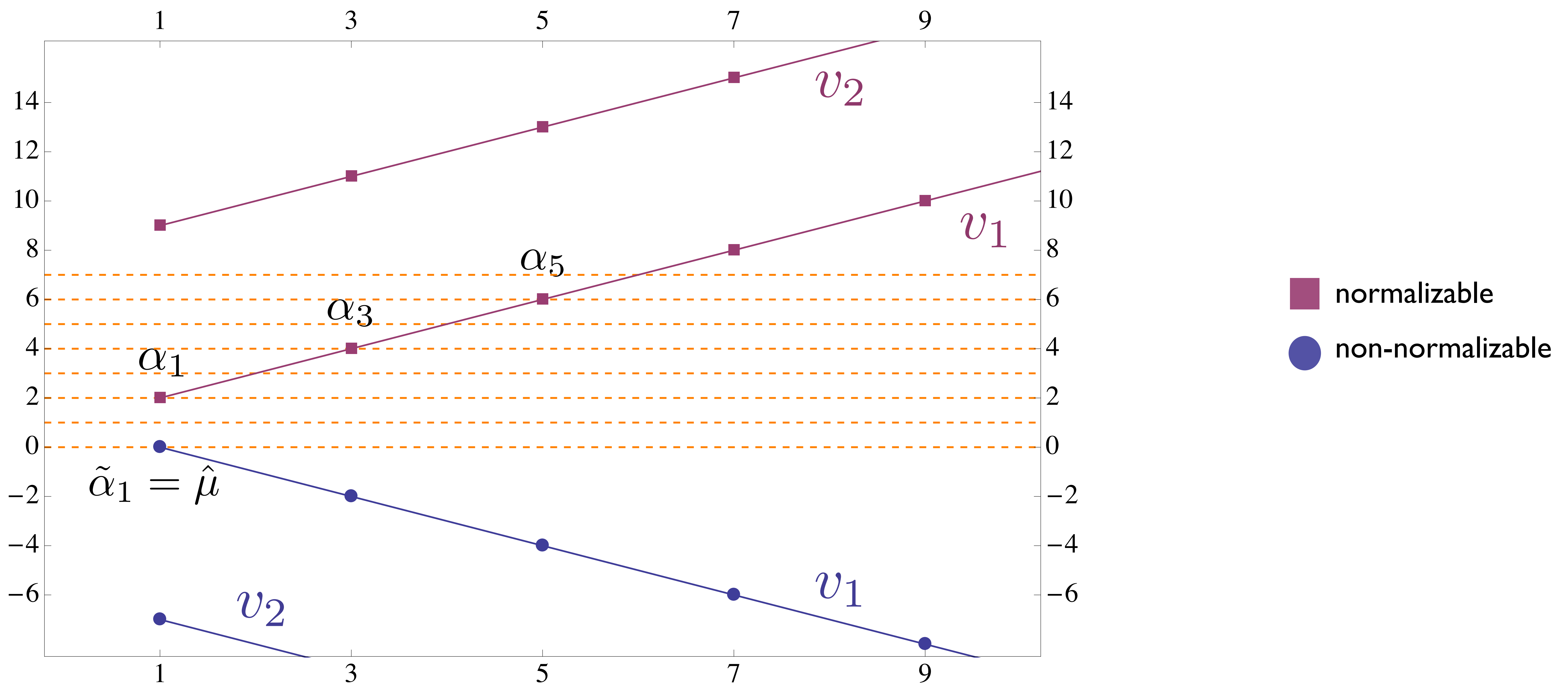
$n(l)$



$$\mathcal{O} \sim \epsilon^{ijk}\mathrm{Tr}\left(X_i X_j X_k\, X_{A_1} \dots X_{A_{l-1}}\right), \qquad l \geq 1 \text{ odd}$$

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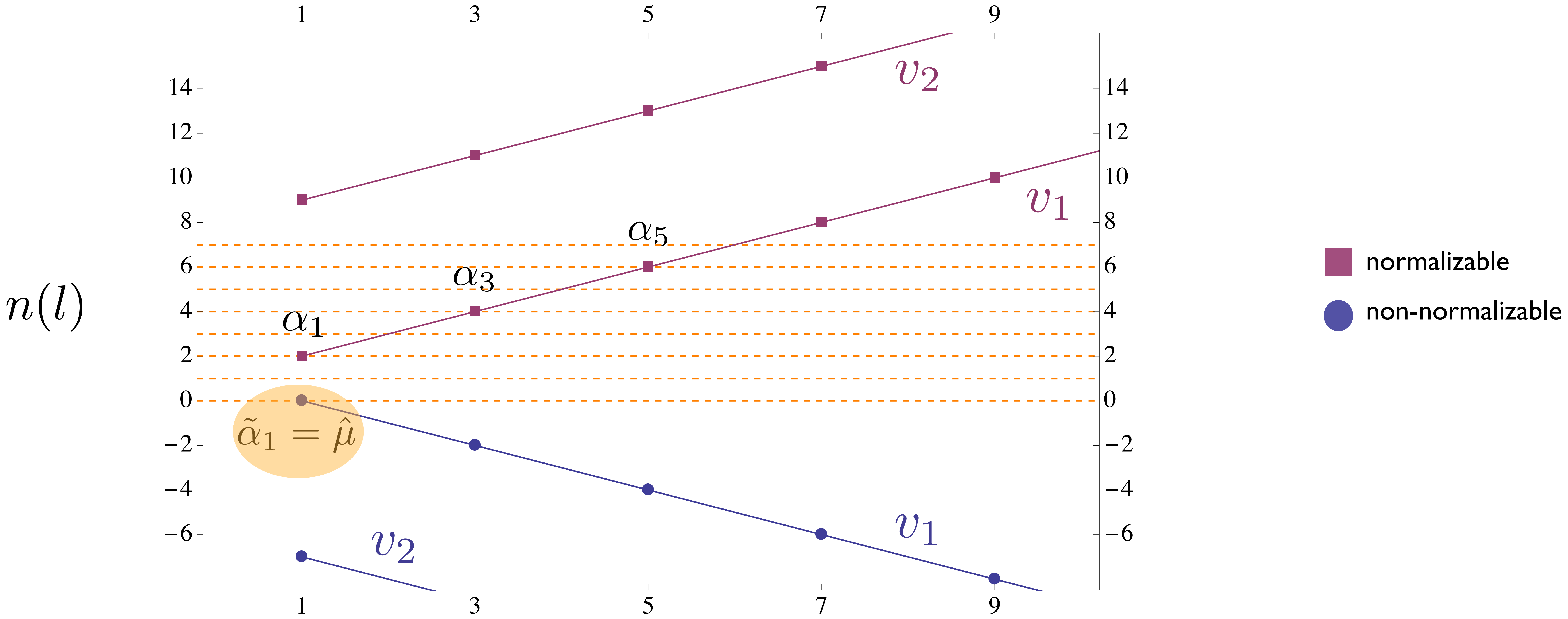

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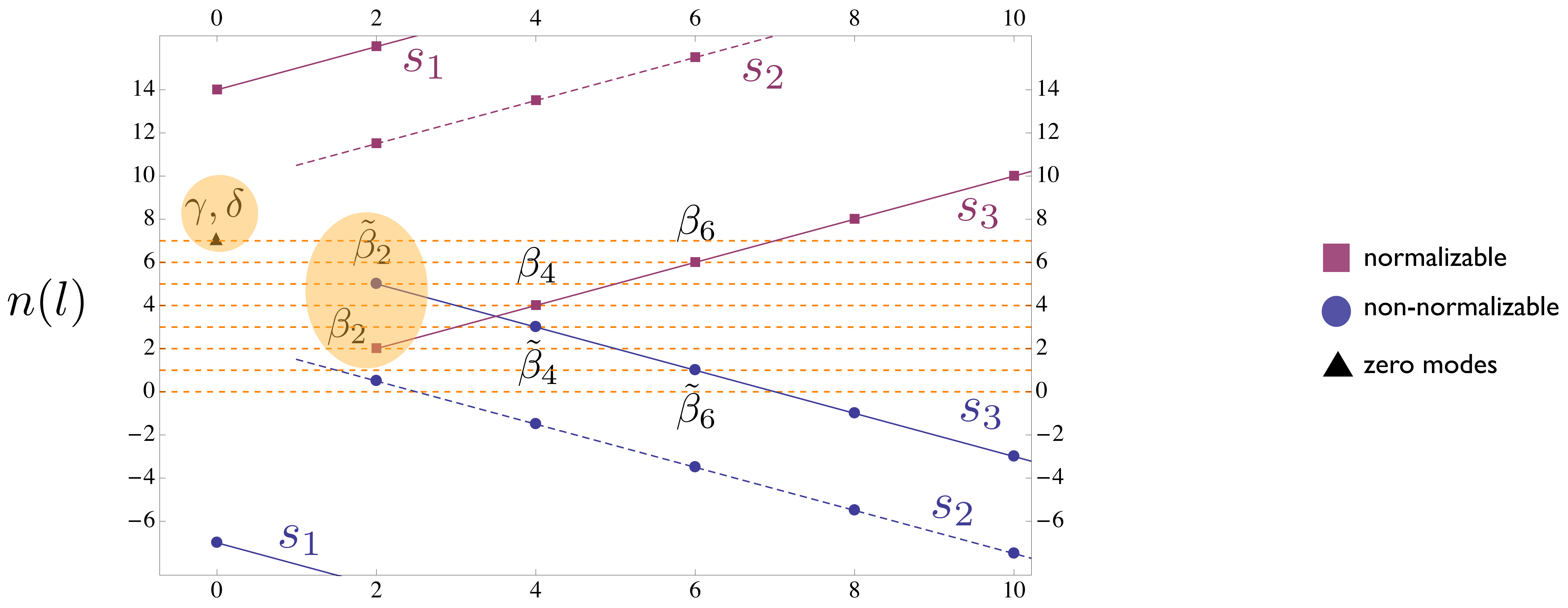
- scalar modes in the asymptotic expansion

$$s(x,y) = \sum_{l\,even} \left(\beta_l\, g_l(y) + \tilde{\beta}_l\, \tilde{g}_l(y)\right) \mathbb{S}_l(x) \;+\; \text{back reaction}$$

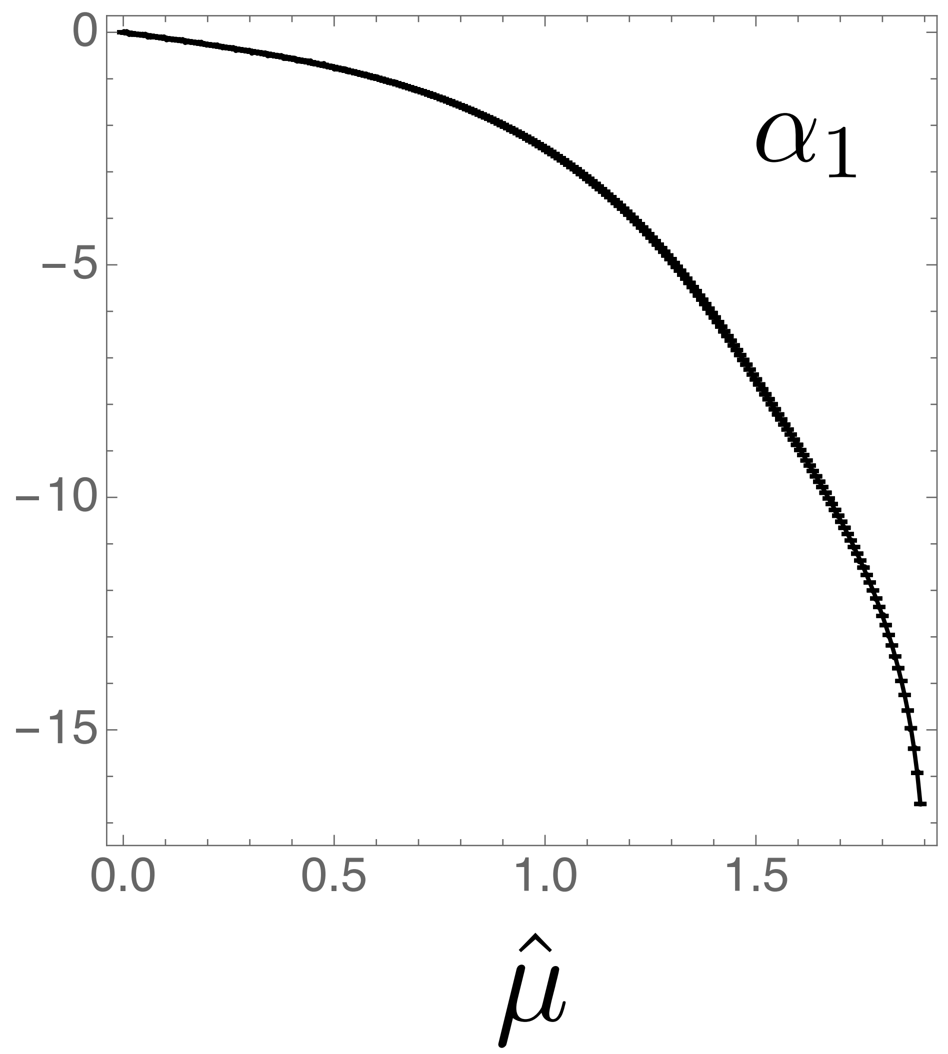
$SO(6)\times SO(3)$ invariant harmonic scalar



$$\mathcal{O} \sim \mathrm{Tr}\left(X_{A_1}\dots X_{A_l}\right)\,, \qquad l \geq 2 \;\; \text{even}$$

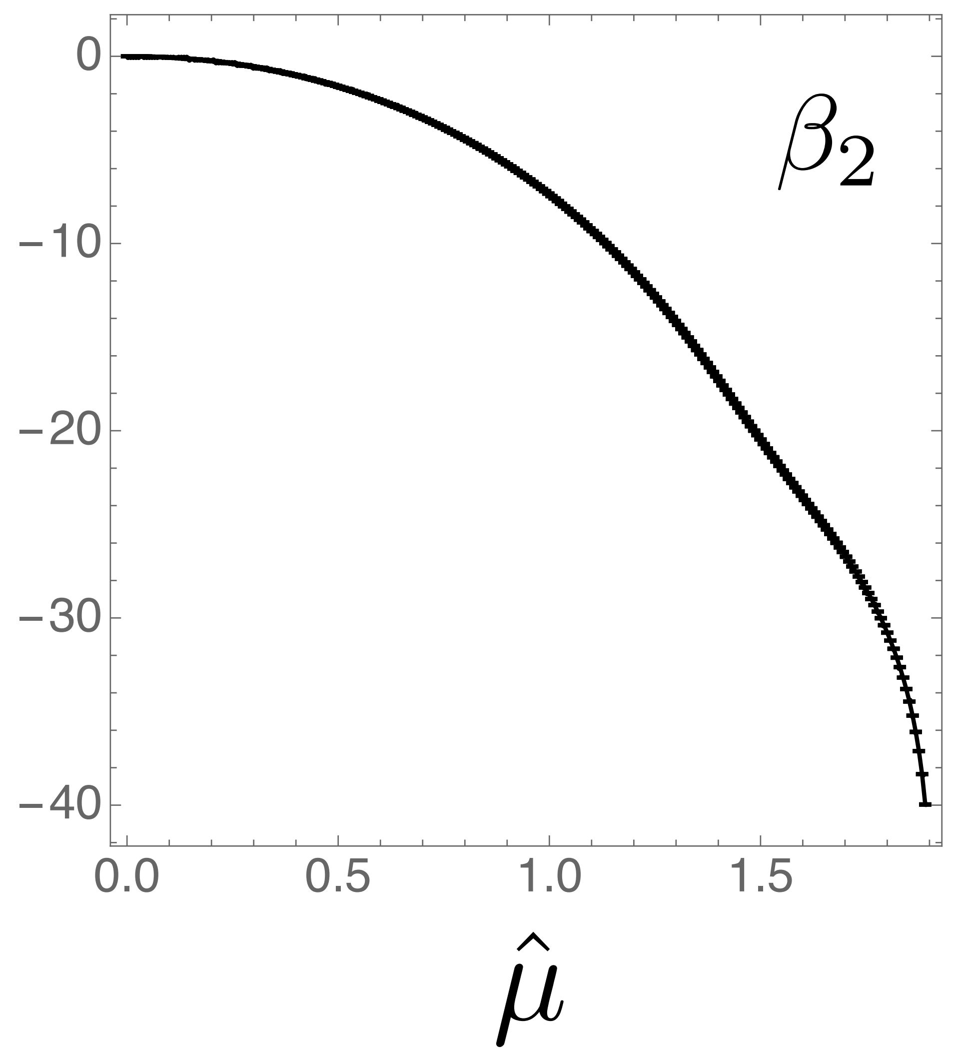
- Vevs read from normalizable modes appear first at order y^2

2-form $(l = 1)$



$\epsilon^{ijk}\langle\mathrm{Tr}\,(X_i X_j X_k)\rangle$

Scalar $(l = 2)$



$\langle\mathrm{Tr}\left(2X_i X^i - X_a X^a\right)\rangle$

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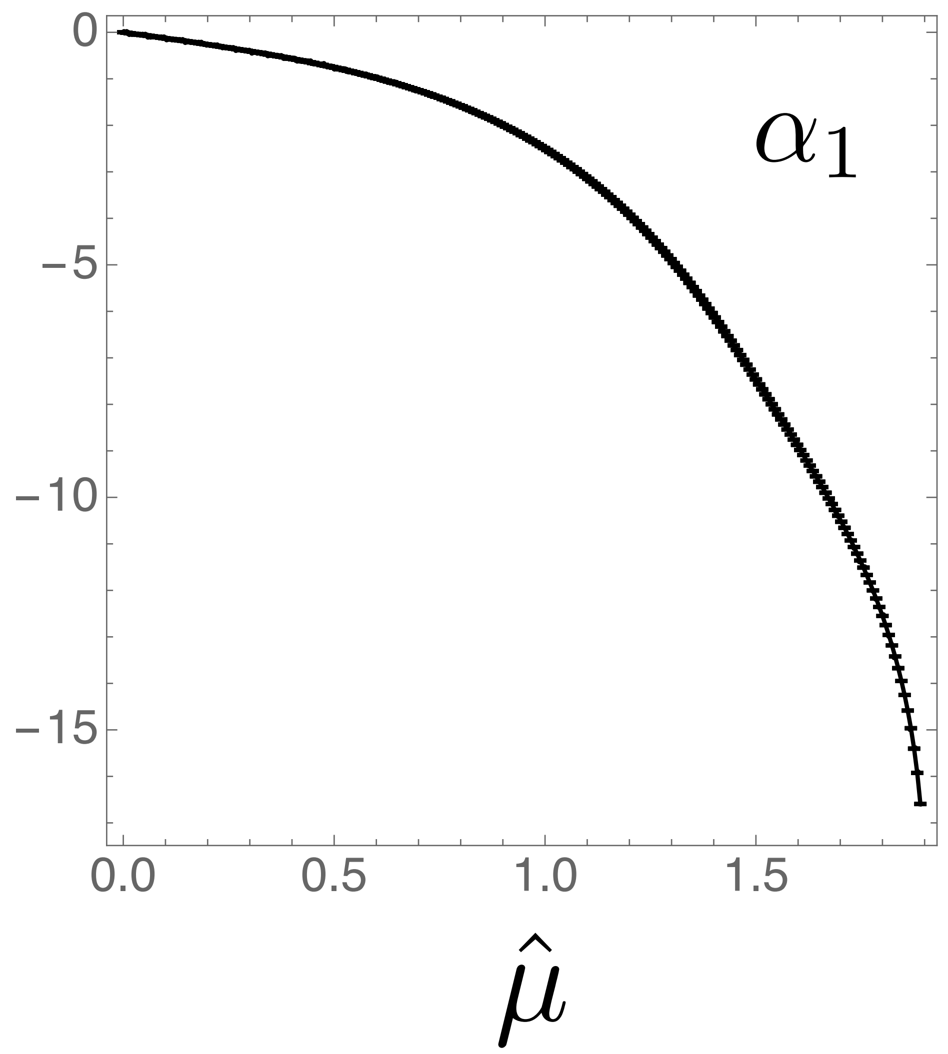
2-form $(l=1)$



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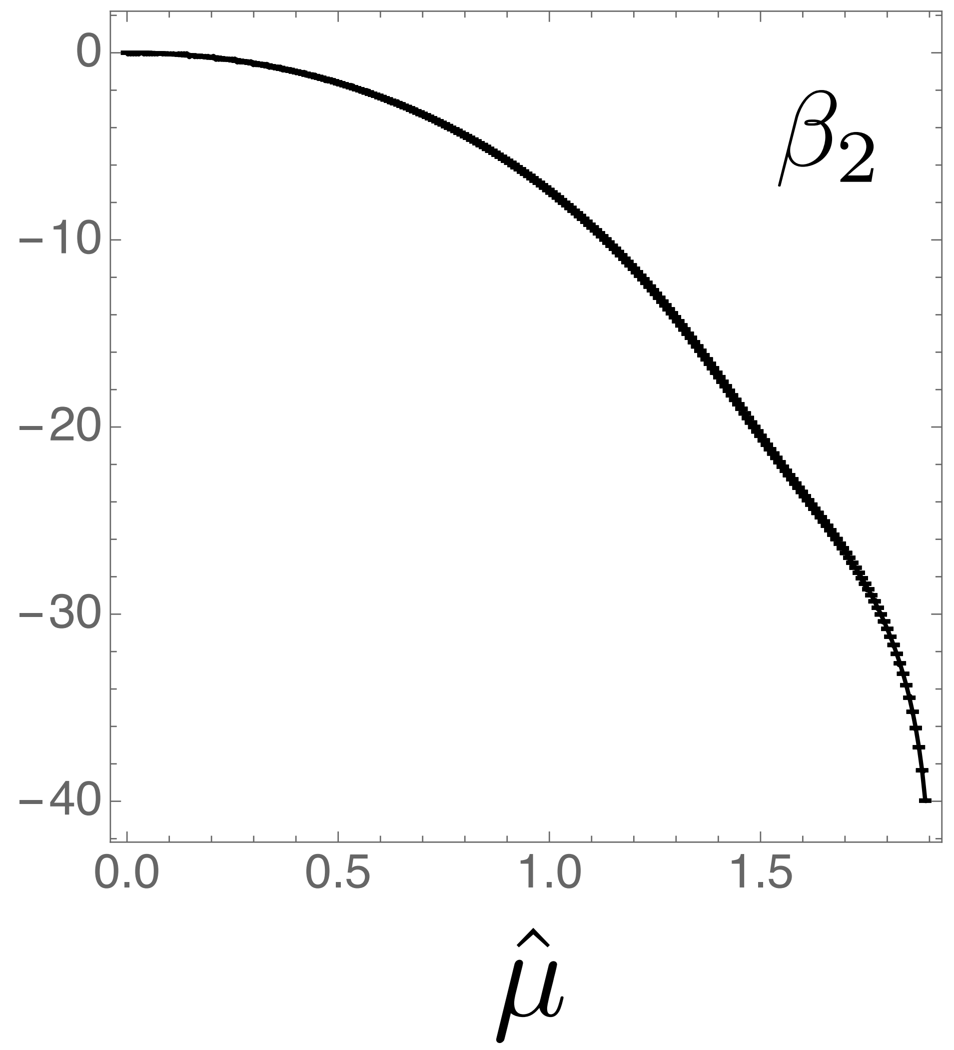
Scalar $(l=2)$

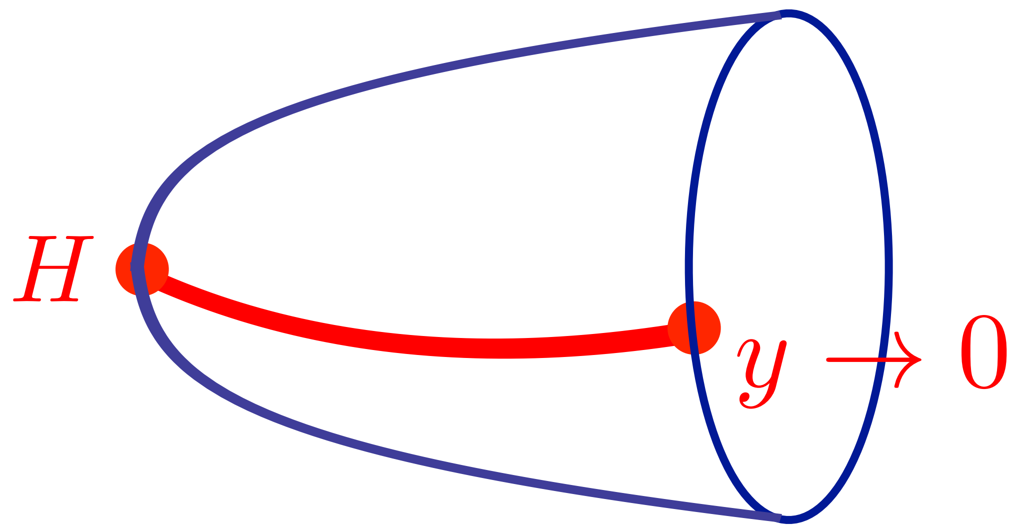


$\langle \mathrm{Tr}\left(2X_i X^i - X_a X^a\right)\rangle$

- Smarr formulae involve coefficients in asymptotic expansion up to order y^7

Numerics pass this highly non-trival check with 0.05% accuracy

$$d\left(\star K_v\right) = 0$$



Future work

- Confirm phase diagram with Monte-Carlo simulations of PWMM; confirm predictions for expectation values of operators dual to normalizable modes that are turned on

- Study dynamical stability of our BH

- Construct BH duals of other vacua (different horizon topology)
(caveat: we really only determined upper limit on critical temperature)

- **Deeper question:** What makes the PWMM special?
What are the minimal ingredients of a quantum mechanical system such that it gives rise to classical gravity in the limit of many degrees of freedom?

THANK YOU