Mathieu Moonshine: Facts and Fantasies

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Plan:

1. Mathieu Moonshine: The problem
2. The solution: Facts and Fantasies
3. The evidence: Massless representation of M_{24}

[Taormina/W11] *The overarching finite symmetry group of Kummer surfaces in the Mathieu group M_{24}*, JHEP **1308**:152 (2013); arXiv:1107.3834 [hep-th]

[Taormina/W12] *A twist in the M_{24} moonshine story*; arXiv:1303.3221 [hep-th]

[Taormina/W13] *Symmetry-surfing the moduli space of Kummer K3s*; arXiv:1303.2931 [hep-th]

[W14] *Snapshots of conformal field theory*; to appear in "Mathematical Aspects of Quantum Field Theories", Mathematical Physics Studies, Springer; arXiv:1404.3108 [hep-th]

1. Mathieu Moonshine on K3: The observation

M: a K3 surface, that is, a cpct. Calabi-Yau 2-fold, $h^{1,0}(M) = 0$

ELLIPTIC GENUS of M for $\tau, z \in \mathbb{C}$, $\mathrm{Im}(\tau) > 0$:

$$\mathcal{E}_{\mathrm{K3}}(\tau, z) = 8\left(\frac{\vartheta_2(\tau,z)}{\vartheta_2(\tau,0)}\right)^2 + 8\left(\frac{\vartheta_3(\tau,z)}{\vartheta_3(\tau,0)}\right)^2 + 8\left(\frac{\vartheta_4(\tau,z)}{\vartheta_4(\tau,0)}\right)^2.$$

For every $N = (2,2)$ SCFT at central charges $c = \overline{c} = 6$ with space-time SUSY and integral $U(1)$ charges:

Its CFT elliptic genus

$$\mathcal{E}_{CFT}(\tau, z) = \mathrm{sTr}_{\mathcal{H}_R}\left(y^{J_0} q^{L_0 - c/24} \overline{q}^{\overline{L}_0 - \overline{c}/24}\right), \quad (q = e^{e\pi i\tau},\ y = e^{2\pi i z})$$

$\mathcal{H}_R$: Ramond sector,

$J_0, L_0, \overline{L}_0$: zero modes of the $U(1)$-current and Virasoro fields in the SCA

either vanishes, or it agrees with $\mathcal{E}_{\mathrm{K3}}(\tau, z)$.

The theory has $N = (4,4)$ SUSY.

Examples: orbifolds of toroidal SCFTs, Gepner models at $c = \overline{c} = 6$

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Definition (K3 THEORY)
An $N = (4,4)$ SCFT at $c = \overline{c} = 6$ with space-time SUSY, integral $U(1)$ charges and CFT elliptic genus $\mathcal{E}_{K3}(\tau, z)$.

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1. Decomposition into irreducible $N = 4$ characters

3 types of $N = 4$ irreps $\mathcal{H}_\bullet$ with $\chi_\bullet(\tau, z) = \mathrm{sTr}_{\mathcal{H}_\bullet}\left(y^{J_0} q^{L_0 - 1/4}\right)$:

- vacuum $\mathcal{H}_0$ with $\chi_0(\tau, 0) = -2$
- massless matter $\mathcal{H}_{\mathrm{m.m.}}$ with $\chi_{\mathrm{m.m.}}(\tau, 0) = 1$
- massive matter $\mathcal{H}_h$ $(h \in \mathbb{R}_{>0})$, $\chi_h(\tau, z) = q^h \widetilde{\chi}(\tau, z)$, $\chi_h(\tau, 0) = 0$

Ansatz:

$$\begin{aligned}\mathcal{H}_R \;=\; & \mathcal{H}_0 \otimes \overline{\mathcal{H}}_0 \;\oplus\; 20\, \mathcal{H}_{\mathrm{m.m.}} \otimes \overline{\mathcal{H}}_{\mathrm{m.m.}} \\ & \oplus \left(\bigoplus_{0<n\in\mathbb{N}} \left[f_n \mathcal{H}_n \otimes \overline{\mathcal{H}}_0 \oplus \overline{f}_n \mathcal{H}_0 \otimes \overline{\mathcal{H}}_n \right] \right) \\ & \oplus \left(\bigoplus_{0<m\in\mathbb{N}} \left[g_m \mathcal{H}_m \otimes \overline{\mathcal{H}}_{\mathrm{m.m.}} \oplus \overline{g}_m \mathcal{H}_{\mathrm{m.m.}} \otimes \overline{\mathcal{H}}_m \right] \right) \\ & \oplus \bigoplus_{0<h,\overline{h}\in\mathbb{R}} k_{h,\overline{h}} \mathcal{H}_h \otimes \overline{\mathcal{H}}_{\overline{h}}\end{aligned}$$

where all f_n, $\overline{f}_n$, g_m, $\overline{g}_m$, $k_{h,\overline{h}}$ are non-negative integers.

$$\Longrightarrow \quad \begin{aligned}\mathcal{E}_{K3}(\tau, z) &= -2\chi_0(\tau, z) + 20\chi_{\mathrm{m.m.}}(\tau, z) + 2e(\tau)\widetilde{\chi}(\tau, z), \\ 2e(\tau) &= \sum_{n=1}^{\infty} (g_n - 2f_n) q^n\end{aligned}$$

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Conjecture [Eguchi/Ooguri/Tachikawa10] For all n, g_n-2f_n gives the dimension of a non-trivial representation of the Mathieu group M_{24}.

2. Solving Mathieu Moonshine? – Facts

Theorem **[Gannon12]** using results of **Cheng, Duncan, Gaberdiel, Hohenegger, Persson, Ronellenfitsch, Volpato**

There exists a representation $\mathcal{R}_n$ of M_{24} for every $n \in \mathbb{N}$, s.th.

$$\mathcal{R}_{\text{Gannon}} := (-2)\mathcal{H}_0 \oplus 20\,\mathcal{H}_{\text{m.m.}} \oplus \bigoplus_{n=1}^{\infty} \mathcal{R}_n \otimes \mathcal{H}_n$$

yields $\mathcal{E}_{K3}^{(g)} = \mathrm{sTr}_{\mathcal{R}_{\text{Gannon}}}(g\, y^{J_0} q^{L_0 - 1/4}), g \in M_{24}$, the twisted genera.

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WHY?

HOW?

Is there an underlying structure of a vertex algebra?

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Theorem [Mukai88]

If G is a symmetry group of a K3 surface M,

> that is, G fixes the two-forms that define the hyperkähler structure of M,

then G is isomorphic to a subgroup of the Mathieu group M_{24},

and $|G| \leq 960 \ll 244.823.040 = |M_{24}|$.

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[Gaberdiel/Hohenegger/Volpato11]

M_{24} cannot act as symmetry group of a K3 theory.

2. Solving Mathieu Moonshine? – Facts and Fantasies

Observation **[Taormina/W10-13]**
The map $\mathcal{H}_R \twoheadrightarrow \mathcal{H}_{R,gen} \stackrel{!}{=} \mathcal{R}_{\text{Gannon}}$ depends on the choice of a geometric interpretation; SO: restrict to geom. symmetry groups.

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Conjecture **[Taormina/W10-13]**
In every geometric interpretation,

$$\mathcal{H}_{R,gen} \cong (-2)\mathcal{H}_0 \;\oplus\; \mathcal{R}_{\text{m.m.}} \otimes \mathcal{H}_{\text{m.m.}} \;\oplus\; \bigoplus_{n=1}^{\infty} \mathcal{R}_n \otimes \mathcal{H}_n$$

as a representation of the geometric symmetry group $G \subset M_{24}$; the rhs collects the symmetries from distinct points of the moduli space.

We call this procedure SYMMETRY SURFING.

3. Symmetries of $\mathbb{Z}_2$-orbifold CFTs on K3

G: geometric symmetry group of a $\mathbb{Z}_2$-orbifold CFT
with geometric interpretation on $X = T_\Lambda/\mathbb{Z}_2$, $T_\Lambda = \mathbb{C}^2/\Lambda$

using [Fujiki88] $\Longrightarrow$

$$G = (\mathbb{Z}_2)^4 \rtimes G_\Lambda \subset (\mathbb{Z}_2)^4 \rtimes \mathrm{GL}_4(\mathbb{F}_2) = \mathrm{Aff}(\mathbb{F}_2^4),$$
$$\mathbb{F}_2^4 \cong \tfrac{1}{2}\Lambda/\Lambda, \quad G_\Lambda \subset \mathrm{SO}(3)$$

- $G_\Lambda \subset G_{\Lambda_k}$, one of three maximal finite groups
$$G_{\Lambda_1} = A_4, \; G_{\Lambda_2} = S_3, \; G_{\Lambda_0} = \mathbb{Z}_2^2$$
- our choice of lattice generators $\Lambda = \mathrm{span}_\mathbb{C}\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ for the corresponding maximally symmetric lattices Λ_k is

$$\begin{aligned}
\Lambda_1 &= \mathrm{span}_\mathbb{Z}\left\{(1,0,0,0), (0,1,0,0), (0,0,1,0), \tfrac{1}{2}(1,1,1,1)\right\},\\
\Lambda_2 &= \mathrm{span}_\mathbb{Z}\left\{(1,0,0,0), \tfrac{1}{2}(-1,\sqrt{3},0,0), (0,0,1,0), \tfrac{1}{2}(0,0,-1,\sqrt{3})\right\},\\
\Lambda_0 &= \mathrm{span}_\mathbb{Z}\left\{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\right\}.
\end{aligned}$$

3. Symmetry surfing the moduli space of Kummer K3s

Result **[Taormina/W11&12]**
For the $\mathbb{Z}_2$-orbifold CFTs on K3 with geometric interpretation on some $X = \widetilde{T_\Lambda/\mathbb{Z}_2}$, the joint action of all symmetry groups yields the maximal subgroup $\mathrm{Aff}(\mathbb{F}_2^4) \overset{\text{[Jordan1870]}}{\cong} (\mathbb{Z}_2)^4 \rtimes A_8 \subset M_{24}$.

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Note: $\mathbb{Z}_2^4 \rtimes A_8$ is not a subgroup of M_{23}.

Recall:

$$\mathcal{H}_{R,gen} \cong (-2)\mathcal{H}_0 \oplus \mathcal{R}_{\mathrm{m.m.}} \otimes \mathcal{H}_{\mathrm{m.m.}} \oplus \bigoplus_{n=1}^{\infty} \mathcal{R}_n \otimes \mathcal{H}_n$$

Result **[Taormina/W13]**
$\mathcal{R}_1$ can be constructed as a 90-dim. space of states common to all K3-theories that are $\mathbb{Z}_2$-orbifolds of toroidal theories.
As common representation space of all geometric symmetry groups of Kummer K3s, $\mathcal{R}_1$ carries an action of $\mathbb{Z}_2^4 \rtimes A_8$ induced from $\mathcal{R}_1 \cong \mathbf{45} \oplus \overline{\mathbf{45}}$ with irreps $\mathbf{45}$, $\overline{\mathbf{45}}$ of M_{24}.

3. A simpler mathematical observation

Geometric elliptic genus of M: $\quad (T := T^{1,0}M)$

$$\mathbb{E}_{q,-y} := y^{-D/2} \otimes \bigotimes_{n=1}^{\infty} \left[\Lambda_{-yq^{n-1}} T^* \otimes \Lambda_{-y^{-1}q^n} T \otimes S_{q^n} T^* \otimes S_{q^n} T \right]$$

$$\begin{aligned} \mathcal{E}_{\mathrm{K3}}(\tau, z) &= \int_{\mathrm{K3}} \mathrm{Td}(\mathrm{K3}) \mathrm{ch}(\mathbb{E}_{q,-y}) \\ &= -2\chi_0(\tau, z) + 20\chi_{\mathrm{m.m.}}(\tau, z) + \sum_{n=1}^{\infty} (g_n - 2f_n)\chi_n(\tau, z) \end{aligned}$$

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Conjecture [W13]

There are polynomials p_n for every $n \in \mathbb{N}$, such that

$$\mathbb{E}_{q,-y} = -\mathcal{O}_{\mathrm{K3}}\chi_0(\tau, z) - T\chi_{\mathrm{m.m.}}(\tau, z) + \sum_{n=1}^{\infty} p_n(T)\chi_n(\tau, z),$$

where $\dim(\mathcal{R}_n) = g_n - 2f_n = \int_{\mathrm{K3}} \mathrm{Td}(\mathrm{K3}) p_n(T) = \chi(p_n(T))$ for all $n \in \mathbb{N}$. Hence $p_n(T) \twoheadrightarrow \mathcal{R}_n$ carries a natural action of every geometric symmetry group $G \subset M_{24}$ of K3.

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proof: [Creutzig/Hoehn/W14]

The End

Thank you for your attention!