UNIVERSITÀ DEGLI STUDI FIRENZE

Supersymmetric Wilson loops and the Bremsstrahlung function in ABJ(M) Theories

The String Theory Universe, Mainz September 22-26 2014

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Florence University and INFN

Based on:

(a) M. Bianchi, L. Griguolo, M. Leoni, S. Penati and D. S., JHEP 1406, 123 (2014) [arXiv:1402.4128 [hep-th]].

(b) L. Griguolo, D. Marmiroli, G. Martelloni and D. S,, JHEP 1305, 113 (2013) [arXiv:1208.5766 [hep-th]].

(c) V. Cardinali, L. Griguolo, G. Martelloni and D. S., Phys. Lett. B 718, 615 (2012) [arXiv:1209.4032 [hep-th]].

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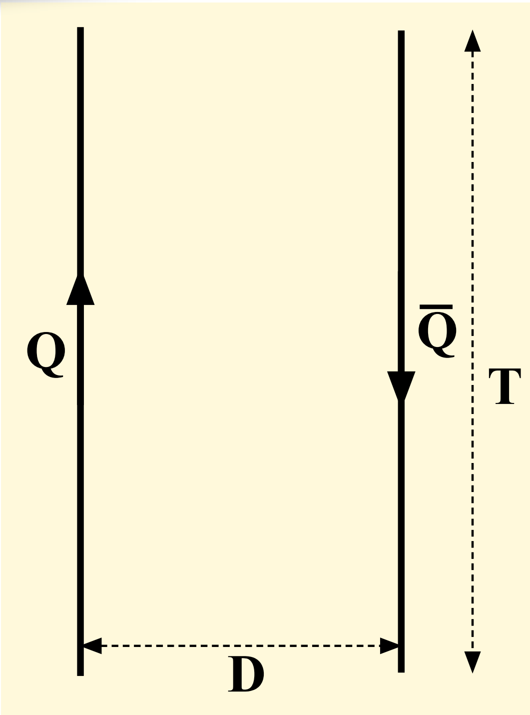
Motivations and Introduction



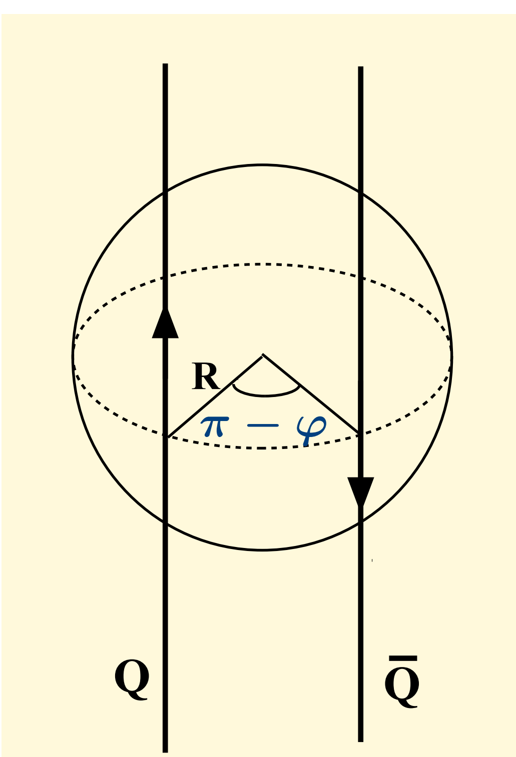
▶ $Q\bar{Q}$potential in gauge theories is captured by an holonomy of suitable gauge connection along anti-parallel lines: **T>>D**

$$\mathcal{W} \simeq \mathrm{e}^{-T\,V(\lambda,D)} \qquad [\lambda = g^2 N]$$

▶ In a conformally invariant field theory the dependence on D is trivial (fixed by symmetry)

$$V(\lambda, D) = \frac{V(\lambda)}{D}$$

the function V(λ) carries the only non-trivial information.



▶ Theory on the cylinder: Two probe charges on the big circle of the sphere separated by $\delta=\pi-\varphi$

$$\mathcal{W} \simeq \mathrm{e}^{-\frac{T}{R}\,V(\lambda,\varphi)}$$

flat space $\delta \to 0$ (Rδ finite)

$$\frac{1}{R} V(\lambda, \varphi) \sim \frac{V(\lambda)}{R\delta}$$

Relation with cusp physics:

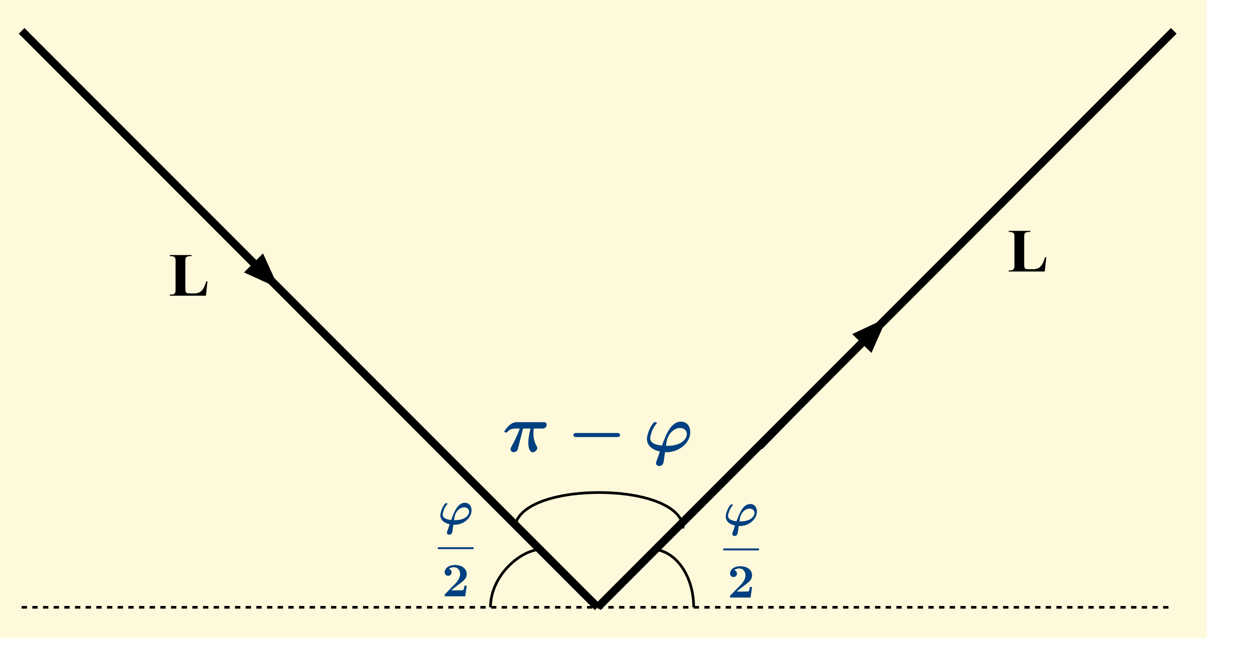
By means of the exponential map the parallel lines configuration is mapped to a cusp-like one:

$$\mathcal{W} \simeq e^{-\log\left(\frac{L}{\epsilon}\right)\Gamma_{\rm cusp}(\lambda,\varphi)}$$

$$\frac{T}{R} = \log\left(\frac{L}{\epsilon}\right)$$

L=Length of the edges (IR cut-off)

ε=UV cut-off



$$V(\lambda,\varphi) = \Gamma_{\rm cusp}(\lambda,\varphi)$$

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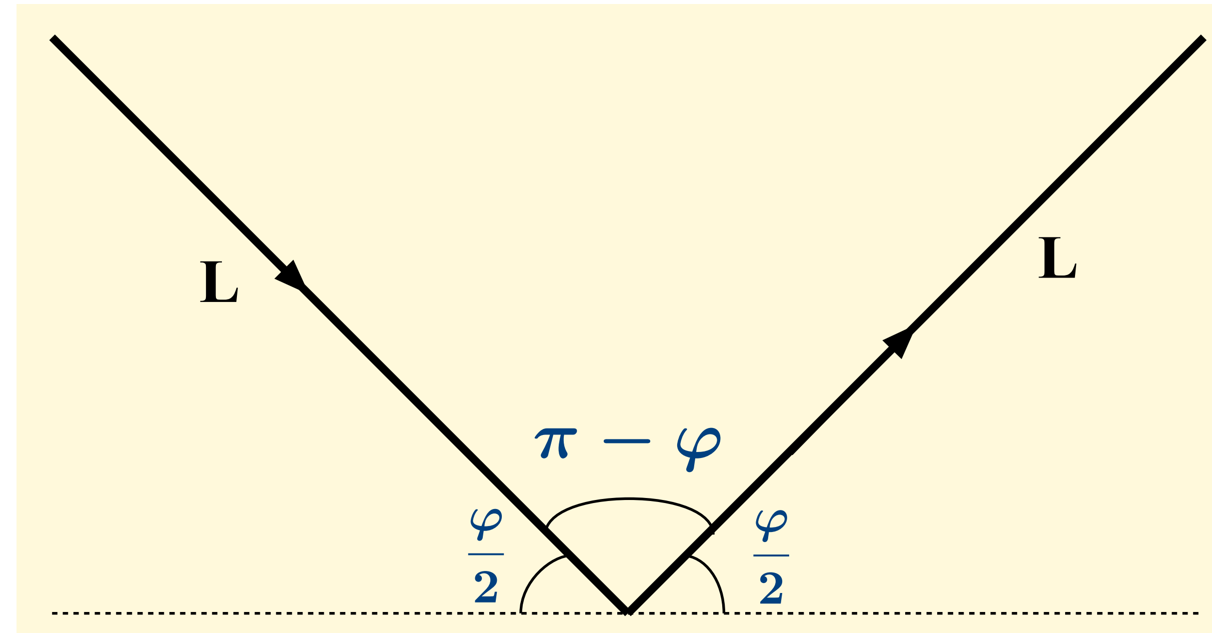

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$$V(\lambda,\varphi) = \Gamma_{\rm cusp}(\lambda,\varphi)$$

$\varphi \to i\,\varphi \quad \varphi \to \infty$

$$\Gamma_{\rm cusp}(\lambda, i\varphi) \sim \varphi\Gamma^{\infty}_{\rm cusp}(\lambda)$$

Universal cusp anomaly

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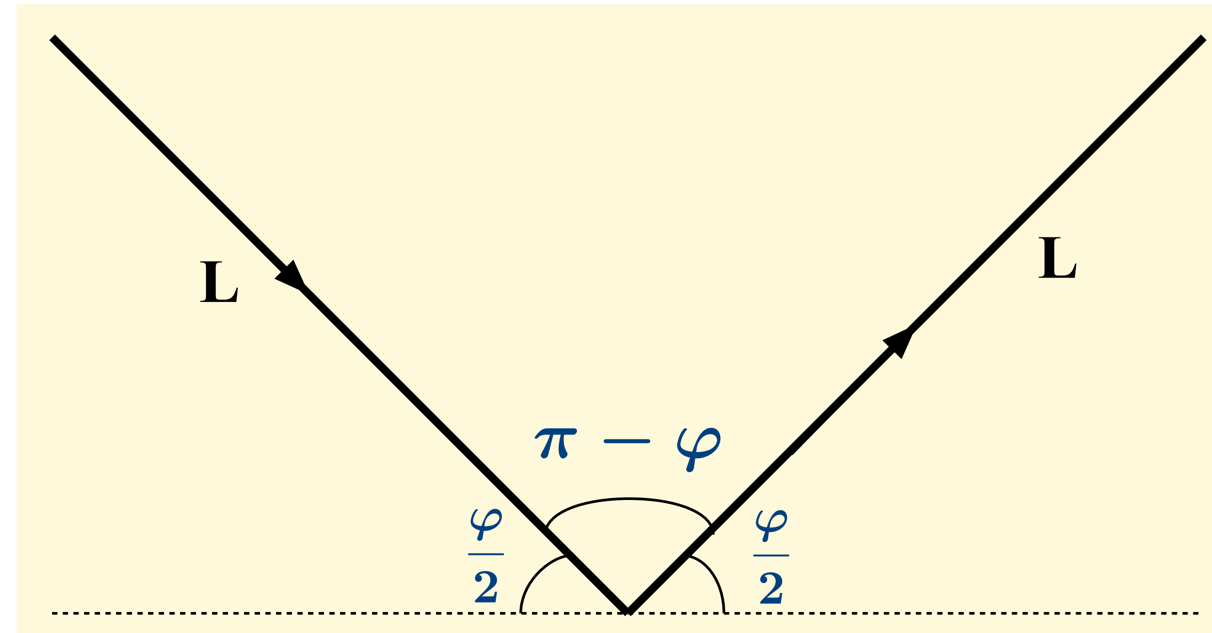

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φ* → i *φ ***φ* → ∞**

$$\Gamma_{\text{cusp}}(\lambda, i\varphi) \sim \varphi\Gamma^{\infty}_{\text{cusp}}(\lambda)$$

<u>Universal cusp anomaly</u>

δ = *π*−*φ* → 0

$$\Gamma_{\text{cusp}}(\lambda,\varphi) \sim \frac{V(\lambda)}{\delta}$$

<u>Q$\bar{\text{Q}}$ potential</u>

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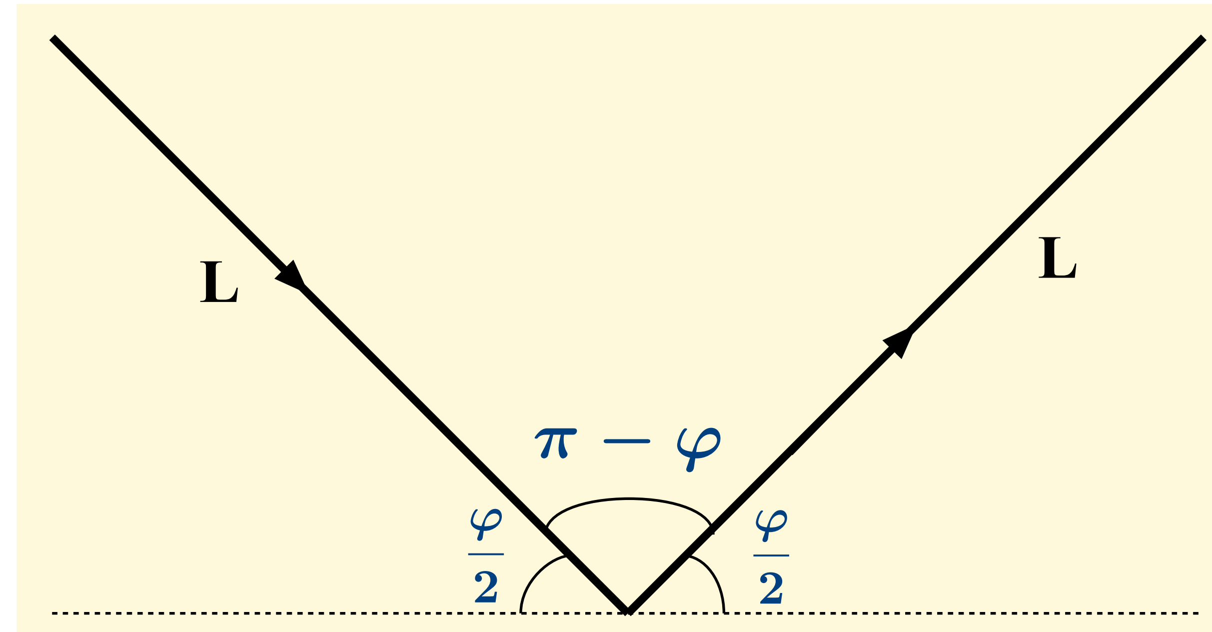

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<u>Universal cusp anomaly</u>

$\delta = \pi - \varphi \to 0$

$$\Gamma_{\rm cusp}(\lambda,\varphi) \sim \frac{V(\lambda)}{\delta}$$

<u>$Q\bar{Q}$ potential</u>

$\varphi \to 0$

$$\Gamma_{\rm cusp}(\lambda,\varphi) \sim -\varphi^2 B(\lambda)$$

<u>Bremsstrahlung function</u>

Bridge between Integrability and Wilson loops:

In N=4 SYM these connections among potential, cusp and B has been used to connect Integrability and Wilson loops:

- A set of TBA equations for $\Gamma_{cusp}(\lambda,\varphi)$ (and its generalisations) were written by Correa, Maldacena, Sever (2012).
- Analytic results were obtained in different limits: in particular $B(\lambda)$ was obtain in closed form

Bridge between Integrability and localization:

In N=4 SYM:

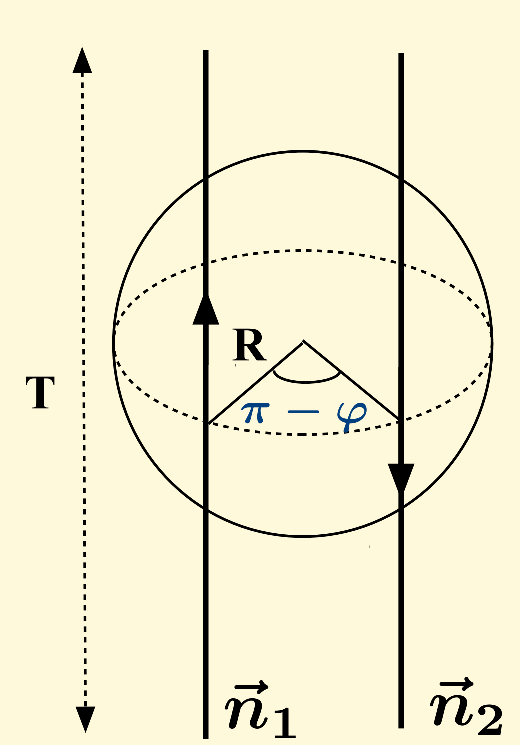
- The Bremsstrahlung function $\mathbf{B(\lambda)}$ was computed by means of Localization results [Correa, Henn, Maldacena, Sever, 2012].
- Surprisingly one is able to extract some Non-BPS observables starting from BPS results: first Deviation from BPS condition

Remark: Deforming the observable

[Drukker, Forini 2011]
[Correa, Henn, Maldacena, Sever 2012]

$$\vec{n}_1 = \left(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}, 0\right)$$

$$\vec{n}_2 = \left(\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}, 0\right)$$



R-symmetry deformation:

$$\mathcal{W} = \mathrm{Tr}\left[\mathrm{P}\exp\left(\oint A_\mu dx^\mu + \vec{n}\cdot\vec{\phi}ds\right)\right] \simeq$$

$$\simeq \exp\left[-\frac{T}{R}V(\lambda,\varphi,\theta)\right]$$

$V(\lambda,\varphi,\theta)$ is called Generalized Potential

Via the usual exponential map t=log r, we can also define a Generalised cusp and show that $\Gamma_{cusp}(\lambda,\varphi,\theta)=V(\lambda,\varphi,\theta)$

For $\theta^2=\varphi^2$ this configuration turn out to be BPS and

$$V(\lambda,\varphi,\theta)=0 \quad \text{for} \quad \theta^2=\varphi^2$$

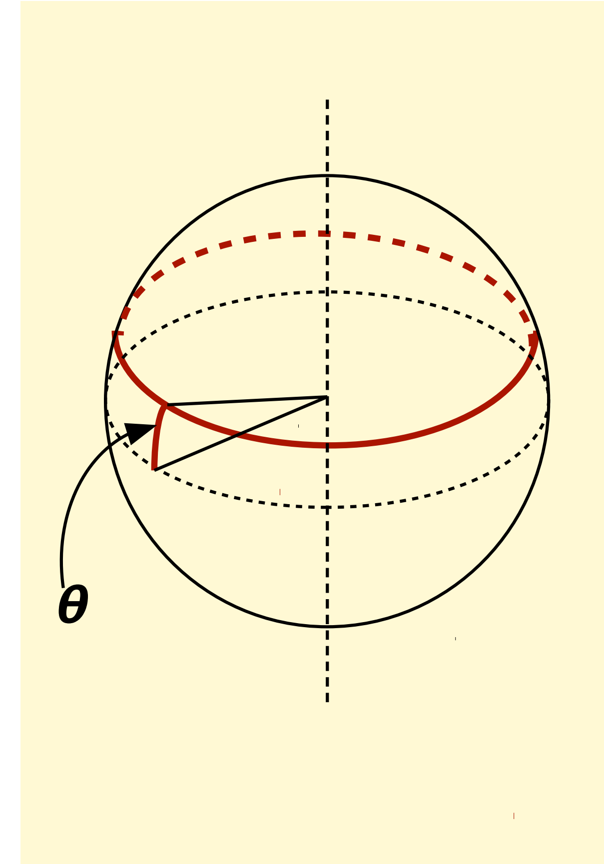
namely

$$V(\lambda,\varphi,\theta)=(\theta^2-\varphi^2)\, B(\lambda)+\ldots$$

The Bremsstrahlung function $B(\lambda)$ can be also extracted by considering the small θ behaviour

$B(\lambda)$ from the latitude on S^2

[Correa, Henn, Maldacena, Sever 2012]



$$\mathcal{W} = \mathrm{Tr}\left[\mathrm{P}\exp\left(\oint A_\mu dx^\mu + \vec{n}\cdot\vec{\phi}ds\right)\right]$$

where scalar coupling are given by

$$n_1 + in_2 = \sin\theta e^{i\tau} \qquad n_3 = \cos\theta$$

Exploiting the fact that the straight line and the circle are related by a conformal transformation, one can show that

$$B(\lambda) = -\frac{1}{2\pi^2}\frac{d^2}{d\theta^2}\log W_{\mathrm{lat.}}\Big|_{\theta=0}$$

$W_{\mathrm{lat.}}$ can be computed in closed form by exploiting localisation techniques: [Drukker 2006; Drukker, Giombi, Ricci, Trancanelli, 2007, 2008; Pestun 2009]

$$W_{\mathrm{lat.}} = \frac{2}{\sqrt{\lambda}\cos\theta} I_1(\sqrt{\lambda}\cos\theta) \quad \longrightarrow \quad B(\lambda) = \frac{1}{4\pi^2}\frac{\sqrt{\lambda}I_2(\sqrt{\lambda})}{I_1(\sqrt{\lambda})}$$

The goal is to extend some of these results and relations to ABJ(M) theories:

- In ABJM theories, as in N=4 SYM, the anomalous dimensions of composite operators are computed by an integrable auxiliary spin chain [Gromov, Vieira 2008]

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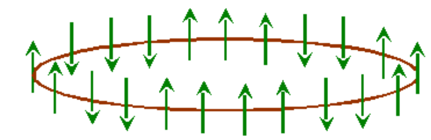


- Many results can be extracted through localisation [Kapustin, Willett , Yaakov, 2009; Marino, Putrov 2009; Marino, Putrov, Drukker 2010.....]

An additional interesting reason: In the integrability approach the key-ingredient is dispersion relation of the magnon moving on the chain:

$$\epsilon(p) = \sqrt{1 + 4h^2(\lambda)\sin^2\frac{p}{2}}$$

The function h(λ),introduced by [Nishioka, Takayanagi 2008; Gaiotto,Giombi, Yin 2008; Grignani, Harmark, Orselli 2008], is not (completely) fixed:

$$h^2(\lambda) \simeq \lambda^2 - \frac{2\pi}{3}\lambda^4 + O(\lambda^6) \quad \lambda \ll 1, \qquad h(\lambda) \simeq \sqrt{\frac{\lambda}{2}} - \frac{\log 2}{2\pi} + O(\frac{1}{\sqrt{\lambda}}). \quad \lambda \ll 1$$

[Minahan, Ohlsson Sax, Sieg, 2009; Leoni, Mauri, Minahan, Ohlsson Sax, Santambrogio, Sieg, Tartaglino Mazzucchelli 2010]

CHMS' suggestion:

1. Compute exactly **B(λ)** through localisation
2. Compute exactly **B(λ)** through integrability
3. Compare and extract **h(λ)**

Recently a conjecture on the exact form of **h(λ)** was put forward by Gromov and Sizov (2014). It is implicitly given by

$$\lambda = \frac{\sinh 2\pi h(\lambda)}{2\pi}\, {}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};1,\frac{3}{2};-\sinh^2 2\pi h(\lambda)\right)$$

The conjecture is obtained two ``unrelated" calculation:

(a) "slope-function" as exact solution of the ABJM spectral curve (integrability) [Cavaglià, Fioravanti, Gromov, Tateo 2014]

(b) 1/6 BPS circular Wilson loop (localization) [Marino, Putrov, 2010; Drukker, Marino, Putrov, 2010]

This conjecture was verified at strong coupling up to two loops [Bianchi, Bianchi, Forini, Bres,Vescovi 2014] [see Forini's talk on Friday]

Step 1:

(A) Construct the cusped Wilson loop for ABJM theories

(B) Check its behaviour and consistency (Exponentiation)

(C) Extract perturbatively $\Gamma_{cusp}(\lambda,\varphi,\theta)$, $V(\lambda,\varphi,\theta)$ and $B(\lambda)$

Step 2:

Construct more general family of supersymmetric Wilson loops (only the line and the maximal circle was known):

General classes of BPS loops on R^3 and S^2

Step 3:

(A) Evaluation of latitude on S^2

(B) Use the results on the latitude to conjecture the form of $B(\lambda)$ to all orders.

Short review of Wilson loops in ABJ(M)

ABJ(M) Theory dictionary:

1. Gauge symmetry U(N) X U(M) : 2 Chern-Simons of levels (k,- k)
2. Matter: 4 Complex scalar Z_I and 4 Dirac spinors Ψ_I in the bifundamental
3. Non gauge couplings: Yukawa $Z^2\Psi^2$; sextic scalar potential
4. 12 Poincaré +12 Superconformal supercharges; String dual: IIA on AdS_4XCP^3

ABJ(M) two types of Wilson loops:

1. Locally 1/6 BPS Wilson loops: [Drukker, Plefka, Young 2009; Chen, Wu 2009]

$$\mathcal{W}_{N,M} = \mathrm{Tr}_{N,M}\left[\mathrm{P}\exp\left(\oint \mathcal{A}_{N,M}\right)\right]$$

$$\mathcal{A}_N = A_\mu \dot{x}^\mu - \frac{2\pi}{\kappa} M_J{}^I Z_I \bar{Z}^J$$

$$\mathcal{A}_M = \hat{A}_\mu \dot{x}^\mu - \frac{2\pi}{\kappa} \bar{M}^I{}_J \bar{Z}^J Z_I$$

$M_J{}^I = \bar{M}^I{}_J$ = diag(1,1,-1,-1) for the circle or the line [or locally in general]. They possess SU(2)XSU(2) R-symmetry.

This dual is not dual to the fundamental string in AdS_4XCP^3

2. Locally 1/2 BPS Wilson loops: [Drukker Trancanelli 2009]

$$\mathcal{W} = \mathrm{P}\,\exp\left(-i\int_{-T}^{T}\mathcal{L}(\lambda)d\lambda\right)$$

$$\mathcal{L} = \begin{pmatrix} \mathcal{A}_N & -\sqrt{\frac{2\pi}{\kappa}}\eta_I\bar{\Psi}^I \\ \sqrt{\frac{2\pi}{\kappa}}\Psi_I\bar{\eta}^I & \mathcal{A}_M \end{pmatrix}$$

(A) But $M_J^{\ I} = \overline{M}^I_{\ J} = \mathrm{diag}(-1,1,1,1)$ for the circle or the line [or locally in general].

(B) $\mathcal{L}$ is an auxiliary super-connection living the Lie algebra of U(N|M)

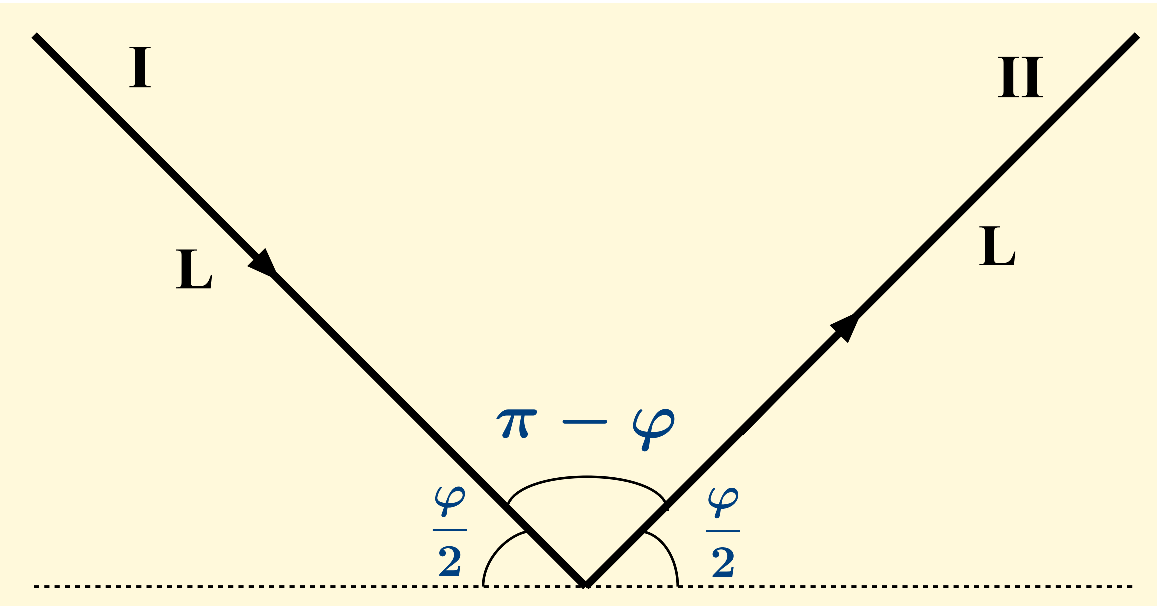
(C) They possess U(1)X SU(3) symmetry: **DUAL TO THE FUNDAMENTAL STRING IN ADS$_4$XCP^3**

(D) Bosonic spinor coupling $\boldsymbol{\eta}_I$, $\bar{\boldsymbol{\eta}}^I$

(E) SUSY invariance ⟷ $\boldsymbol{\delta\mathcal{L}=D_\lambda G}$ (super-gauge transformation of U(N|M))

The generalized cusp in ABJ(M) theory

[Griguolo, Martelloni, Marmiroli, S. 2012]



Two local 1/2 BPS lines

$$M_I(\theta) = M_{II}(-\theta) = \begin{pmatrix} -\cos\frac{\theta}{2} & \sin\frac{\theta}{2} & 0 & 0 \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(\eta_J^\alpha)_I = (\eta_J^\alpha)_{II}(-\theta, -\varphi) = \begin{pmatrix} \cos\frac{\theta}{4} \\ \sin\frac{\theta}{4} \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} e^{i\frac{\varphi}{4}} & e^{-i\frac{\varphi}{4}} \end{pmatrix}$$

- **BPS condition:** $\theta^2 = \varphi^2$ [No BPS configuration for a bosonic cusp]

$$\mathcal{W}(\lambda, \varphi, \theta) = \frac{M \exp(\Gamma_N(\lambda, \varphi, \theta)) + N \exp(\Gamma_M(\lambda, \varphi, \theta))}{N + M},$$

- **Two loop results:** [Γ_M is obtained by exchanging M↔N]

$$\Gamma_N = \left(\frac{2\pi}{\kappa}\right) N \left(\frac{\Gamma(\frac{1}{2} - \epsilon)}{4\pi^{3/2-\epsilon}}\right) (\mu L)^{2\epsilon} \left[\frac{1}{\epsilon}\left(\frac{\cos\frac{\theta}{2}}{\cos\frac{\varphi}{2}} - 1\right) - 2\frac{\cos\frac{\theta}{2}}{\cos\frac{\varphi}{2}} \log\left(\sec\left(\frac{\varphi}{2}\right) + 1\right) + \log 4\right] +$$
$$+ \left(\frac{2\pi}{\kappa}\right)^2 N^2 \left(\frac{\Gamma(\frac{1}{2} - \epsilon)}{4\pi^{3/2-\epsilon}}\right)^2 (\mu L)^{4\epsilon} \left[\frac{1}{\epsilon} \log\left(\cos\frac{\varphi}{2}\right)^2 \left(\frac{\cos\frac{\theta}{2}}{\cos\frac{\varphi}{2}} - 1\right) + O(1)\right]$$

Remarks on the perturbative analysis:

- The perturbative result is consistent with the BPS condition $\theta^2=\varphi^2$ up to two loops:

$$\Gamma(\lambda,\pm\theta,\theta)=0$$

- The result is consistent with a double exponentiation

$$\frac{M\exp(\Gamma_N(\lambda,\varphi,\theta))+N\exp(\Gamma_M(\lambda,\varphi,\theta))}{N+M},$$

- The light-like limit yields the correct universal cusp anomaly at two loops:

$$\gamma= N^2/K^2+\ldots$$

- The $Q\bar{Q}$ potential can be extract by taking $\theta=0$, $\delta=\pi-\varphi\to 0$

$$V_N(L)=\frac{N}{\kappa}\frac{1}{L}-\left(\frac{N}{\kappa}\right)^2\frac{1}{L}\log\frac{T}{L}$$

- The Bremsstrahlung function at two loops:

$$B(\lambda)=\frac{\lambda}{8}+O(\lambda^3)$$

New Supersymmetric Wilson loops in ABJ(M) theory

[Cardinali, Grigolo, Martelloni, S. 2012]

We start from the super-connection:

$$\mathcal{L} = \begin{pmatrix} \mathcal{A}_N & -\sqrt{\frac{2\pi}{\kappa}}\eta_I \bar{\Psi}^I \\ \sqrt{\frac{2\pi}{\kappa}}\Psi_I \bar{\eta}^I & \mathcal{A}_M \end{pmatrix}$$

The local U(1)XSU(3) is realised by choosing a direction

$n_I(\tau)$

and by selecting the following ansatz for the couplings:

$$\eta_I^\alpha(\tau) = n_I(\tau)\eta^\alpha(\tau), \qquad M_J{}^I = \delta_J^I - 2n_J(\tau)\bar{n}^I(\tau),$$

If we impose that the supersymmetry variation of $\mathcal{L}$ can be recast as a super-gauge transformation:

$$\delta_{\text{susy}}\mathcal{L}(\tau) = \mathfrak{D}_\tau G \equiv \partial_\tau G + i\{\mathcal{L}, G],$$

we find two set of constraints for the superconformal spinor $\bar{\Theta}^{IJ} = \bar{\theta}^{IJ} - (x\cdot\gamma)\bar{\epsilon}^{IJ}$,

Algebraic Constraints
$$\begin{cases} \epsilon_{IJKL}(\eta\bar{\Theta}^{IJ})\bar{n}^K = 0 \\ n_I(\bar{\eta}\bar{\Theta}^{IJ}) = 0, \end{cases}$$

Differential Constraints
$$\begin{cases} \bar{\Theta}^{IJ}\partial_\tau\bar{\eta}^K\epsilon_{IJKL} = 0 \\ \bar{\Theta}^{IJ}\partial_\tau\eta_I = 0 \end{cases}$$

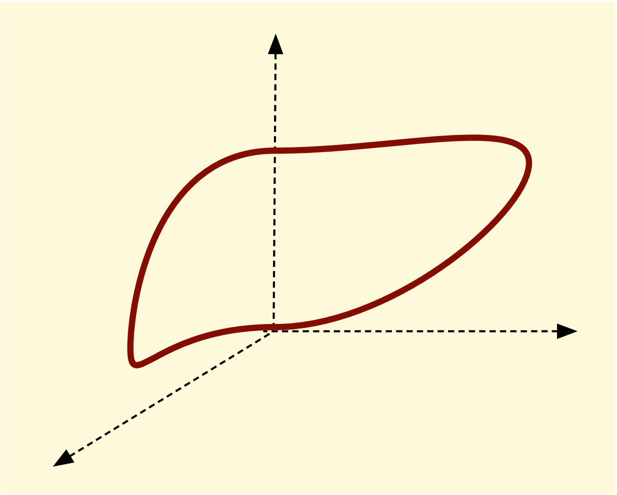
Two sets of significant solutions:

- **Zarembo-like Wilson loops:** Contour of arbitrary shape in R³. They posses only Poincaré supercharges (ϵ^{IJ}=0). They are generically 1/12 BPS:

$$\eta_I^{\alpha} = i s_I^{\beta}\left(\mathbb{1} + \ell\frac{\dot{x}\cdot\gamma}{|\dot{x}|}\right)_{\beta}^{\ \alpha}$$

$$M_K{}^J = \ell\left(\delta_K^J - 2 i s_K \bar{s}^J - 2 i \ell \frac{\dot{x}^{\mu}}{|\dot{x}|} s_K \gamma_{\mu} \bar{s}^J\right).$$

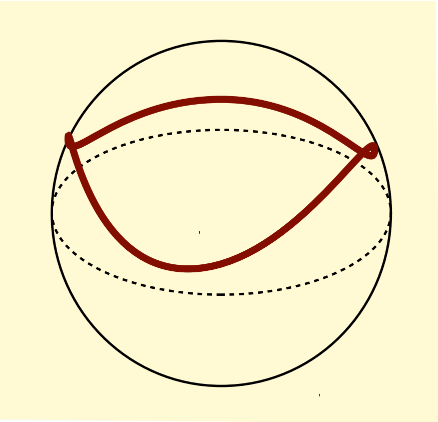
$$\boxed{\bar{s}^I_{\beta} s_I^{\alpha} = \frac{1}{2i}\delta^{\alpha}_{\beta}.}$$



- **Wilson loops on S2 (DGRT-like):** Contour of arbitrary shape on S^2 embedded in R³. They are generically 1/12 BPS: [$U = \cos\alpha\ \mathbb{1} + i\sin\alpha\ (x^{\mu}\gamma_{\mu})$]

$$\eta_I^{\beta} = \frac{i}{r_0} e^{\frac{i}{2}\ell(\sin 2\alpha)s}\left[s_I U^{\dagger}\left(\mathbb{1} + \ell\frac{\dot{x}\cdot\gamma}{|\dot{x}|}\right)\right]^{\beta},$$

$$M_K{}^J = 2 i s_K U^{\dagger}\left(\mathbb{1} + \ell\frac{\dot{x}\cdot\gamma}{|\dot{x}|}\right) U \bar{s}^J$$

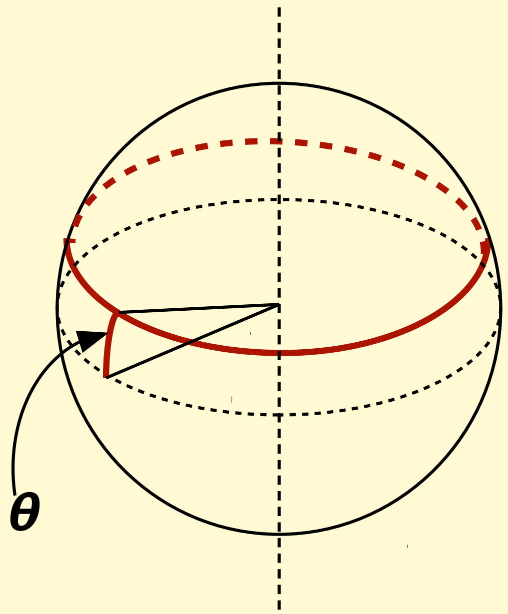


Tricky point: Fermionic couplings and thus the super-gauge transformation obey non-trivial boundary conditions on the closed loop $\Longrightarrow$ Twist matrix T

$$\mathcal{W} = \mathrm{P}\exp\left(\oint \mathcal{L}\right) \xrightarrow[\text{quantity}]{\text{invariant}} \mathrm{STr}(\mathcal{W}\mathcal{T}) \quad \text{with} \quad \mathcal{T} = \begin{pmatrix} e^{\frac{i}{4}(\sin 2\alpha)L} & 0 \\ 0 & e^{-\frac{i}{4}(\sin 2\alpha)L} \end{pmatrix}$$

Supersymmetric Latitude in ABJM

[Bianchi, Griguolo, Leoni, Penati, S. 2014]



$\nu = \sin 2\alpha \cos\theta$

- **Perturbative computation:** very technical but still possible analytically. It requires a careful use of the Dimensional Reduction
- **Susy:** the loop is 1/6 BPS and it possesses a 1/12 BPS bosonic avatar.
- **Exploring the relation with the Bremsstrahlung function**

Couplings:

$$\mathcal{M}_I^{\ J} = \begin{pmatrix} -\nu & e^{-i\tau}\sqrt{1-\nu^2} & 0 & 0 \\ e^{i\tau}\sqrt{1-\nu^2} & \nu & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \eta_I^\alpha \equiv \frac{e^{\frac{i\nu\tau}{2}}}{\sqrt{2}} \begin{pmatrix} \sqrt{1+\nu} \\ -\sqrt{1-\nu}\,e^{i\tau} \\ 0 \\ 0 \end{pmatrix}_I (1, -ie^{-i\tau})^\alpha$$

Quantity to compute:

$$W[\Gamma] = \frac{\mathrm{STr}(\mathcal{W}\mathcal{T})}{\mathrm{STr}(\mathcal{T})} \qquad \text{with} \qquad \mathcal{T} = \begin{pmatrix} e^{-\frac{i\pi\nu}{2}}\mathbb{1}_N & 0 \\ 0 & e^{\frac{i\nu\pi}{2}}\mathbb{1}_M \end{pmatrix}$$

ν=1: we must recover the 1/2 BPS circle

ν=0: Zarembo-Latitude

Relation of the fermionic latitude with its bosonic counterpart:

We can also define a merely bosonic counterpart of the fermionic latitude. It is given in terms of the U(N) connection:

$$\mathcal{L}_b \equiv A_\mu \dot{x}^\mu - \frac{2\pi i}{k}|\dot{x}|\widehat{\mathcal{M}}_J{}^I C_I \bar{C}^J \quad \text{with} \quad \widehat{\mathcal{M}}_J^I = \begin{pmatrix} -\nu & e^{-i\tau}\sqrt{1-\nu^2} & 0 & 0 \\ e^{i\tau}\sqrt{1-\nu^2} & \nu & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

analogously we can introduce a U(M) connection. Both connections define supersymmetric Wilson loops (W_B, $\hat{W}_B$) which are 1/12 BPS and they share common super-symmetries with the fermionic loop.

If one defines:

$$\mathbf{W}_B(\nu) \equiv \frac{\left[N e^{-\frac{\pi i \nu}{2}} W_B(\nu) - M e^{\frac{\pi i \nu}{2}} \hat{W}_B(\nu)\right]}{\mathrm{STr}(\mathcal{T})}$$

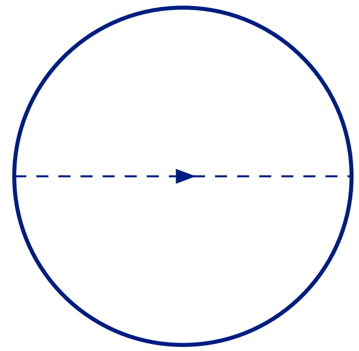
one can show the following cohomological relation between these loops:

$$\mathcal{W}_F - \mathbf{W}_B = Q(V)$$

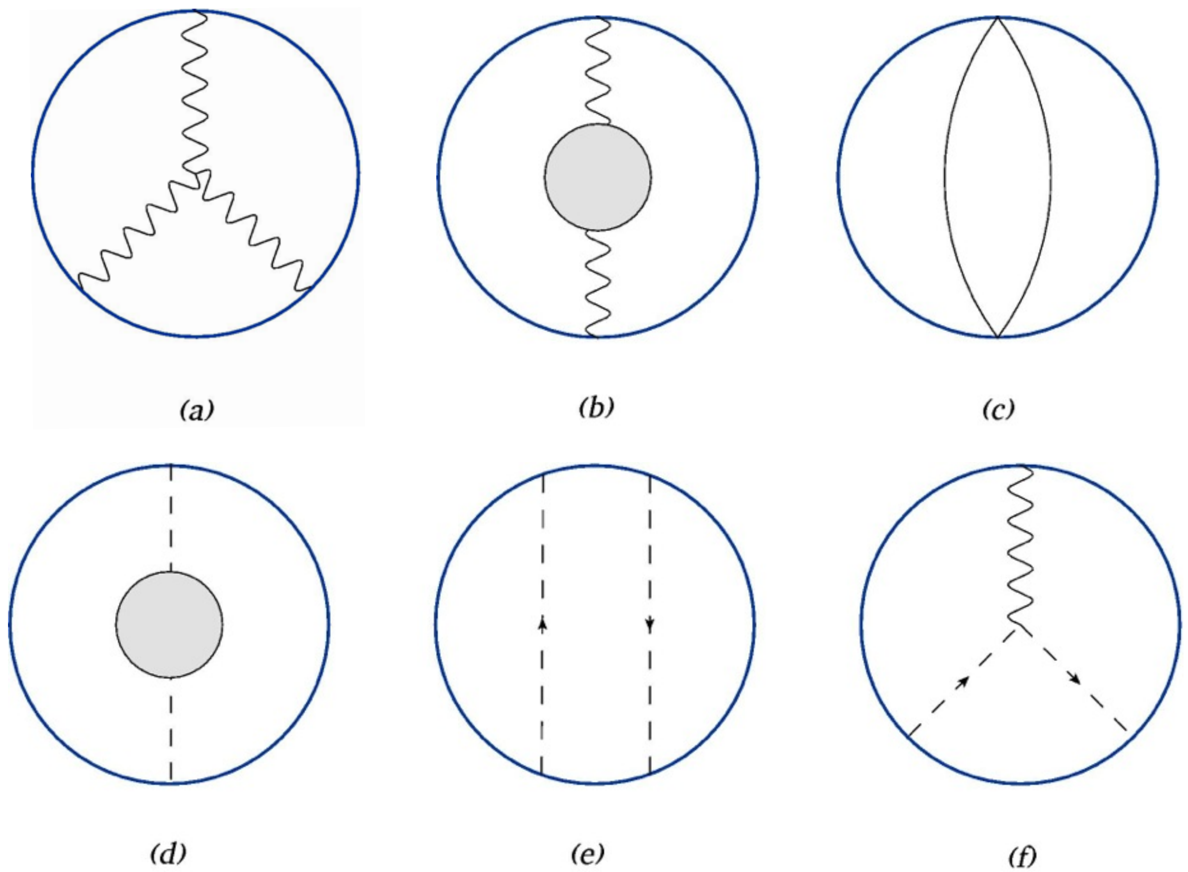
Similar to what happens in the case of the 1/2 BPS circle [Drukker,Trancanelli 2009].

Perturbative analysis of the latitude:

[Bianchi, Griguolo, Leoni, Penati, S. 2014]



One-loop Diagram



Bosonic Diagram

Two-loop Diagrams

Fermionic Diagrams

$$\langle W_F(\nu)\rangle_0 = e^{-\frac{i\pi\nu(N-M)}{k}} \frac{\left[N\ e^{-\frac{i\pi\nu}{2}+\frac{i\pi\nu N}{k}}\ \langle W_B(\nu)\rangle_0 - M\ e^{\frac{i\pi\nu}{2}-\frac{i\pi\nu M}{k}}\ \langle \hat{W}_B(\nu)\rangle_0\right]}{\mathrm{STr}(\mathcal{T})}$$

$$\langle W_B(\nu)\rangle = 1 + \frac{\pi^2}{k^2}\left[\frac{1}{2}(1+\nu^2)MN - \frac{N^2-1}{6}\right] + \mathcal{O}\left(k^{-3}\right)$$

$$\langle \hat{W}_B(\nu)\rangle = 1 + \frac{\pi^2}{k^2}\left[\frac{1}{2}(1+\nu^2)MN - \frac{M^2-1}{6}\right] + \mathcal{O}\left(k^{-3}\right)$$

The additional phases are expected and they a simple interpretation for ν=1

Framing

ABJM Bremsstrahlung function

[Bianchi, Griguolo, Leoni, Penati, S. 2014]

- Can we relate this latitude to the Bremsstrahlung function ? Let us apply (blindly) the same receipt of N=4 SYM

$$B_{1/2}(\lambda) = \frac{1}{4\pi^2}\,\partial_\nu\,\log\langle W_F(\nu)\rangle_0\,\Big|_{\nu=1}$$

exploiting the two-loop perturbative results we can immediately find

$$B_{1/2}(\lambda) = \frac{\lambda}{8} + \mathcal{O}\left(\lambda^3\right)$$

in perfect agreement with the two-loop computation of the cusp!

- If we express the $B_{1/2}(\lambda)$ in terms of the bosonic loops **W_B** and **$\hat{W}_B$**

$$B_{1/2}(\lambda) = \frac{1}{4\pi^2}\left[\underbrace{\partial_\nu\,\log\left(\langle W_B(\nu)\rangle_\nu + \langle\hat{W}_B(\nu)\rangle_\nu\right)\Big|_{\nu=1}}_{\text{[even powers in }\lambda\text{]}} - \underbrace{\frac{i\pi}{2}\,\frac{\langle W_B(1)\rangle_1 - \langle\hat{W}_B(1)\rangle_1}{\langle W_B(1)\rangle_1 + \langle\hat{W}_B(1)\rangle_1}}_{\text{[odd powers in }\lambda\text{]}}\right]$$

we can predict the subsequent term in the expansion:

$$B_{1/2}(\lambda) = \frac{\lambda}{8} - \frac{\pi^2}{48}\lambda^3 + \mathcal{O}\left(\lambda^5\right)$$

Checking in progress [Bianchi, Griguolo, Mauri, Penati, S]

An all order conjecture for $B_{1/2}(\lambda)$:

Maldacena and Lewkowycz (2013) were able to propose an exact expression for a "putative" $B_{1/6}(\lambda)$ based on n-winding 1/6 BPS circular Wilson loop.

In their analysis they argue that it is possible to trade the derivative with respect to their relevant geometric parameter (the squashing of the sphere **b**) with a derivative with respect to the winding **n** of the loop.

Let us assume that this can be done also in our case:

$$\partial_\nu \log\left(\langle W_B(\nu)\rangle_\nu + \langle \hat{W}_B(\nu)\rangle_\nu\right)\Big|_{\nu=1} = \partial_n \log\left(\langle W_n^{1/6}\rangle + \langle \hat{W}_n^{1/6}\rangle\right)\Big|_{n=1}$$

We discover that

$$\partial_n \log\left(\langle W_n^{1/6}\rangle + \langle \hat{W}_n^{1/6}\rangle\right)\Big|_{n=1} = 0$$

and we obtain the following simple all-order formula in terms of the 1/2 BPS circle:

$$B_{1/2}(\lambda) = -\frac{i}{8\pi}\frac{\langle W^{1/6}\rangle_1 - \langle \hat{W}^{1/6}\rangle_1}{\langle W^{1/6}\rangle_1 + \langle \hat{W}^{1/6}\rangle_1} = \begin{cases} \frac{\lambda}{8} - \frac{\pi^2}{48}\lambda^3 + \frac{\pi^4}{60}\lambda^5 - \frac{841\pi^6}{40320}\lambda^7 + \mathcal{O}\left(\lambda^9\right) & \text{weak coupling} \\ \frac{\sqrt{2\lambda}}{4\pi} - \frac{1}{4\pi^2} - \frac{1}{96\pi}\frac{1}{\sqrt{2\lambda}} + \mathcal{O}\left(\lambda^{-1}\right) & \text{strong coupling} \end{cases}$$

The leading term at strong coupling is in agreement with [Forini, Giangreco Marotta Puletti, Sax 2012] and [Aguilera-Damia, Correa, Silva 2014].

Conclusions and Outlook:

- Reviewed the the connection between localisation and integrality
- Shown how to define the generalised cusp in ABJ(M) theories and computed up to 2-loops
- Introduced two new families of susy Wilson loop in ABJ(M)
- Discussed the latitude in ABJ(M) and evaluated at 2-loops.
- Using the result on the latitude, we propose an all order expression for $B_{1/2}(\lambda)$ in ABJM theories.
- Further check: Three loop computation [in progress]

  Resumming ladder diagrams [in progress]
- Compute the exact expression of the ABJM latitude through localisation
- Compute $B_{1/2}(\lambda)$ through TBA, compare with the localisation result, determine $h(\lambda)$