Holographic graphene bilayers

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Overview

- Graphene $\longrightarrow$ conformal system of massless fermions in 2+1-dim interacting through electromagnetic forces
  - $\alpha_{\text{graphene}} = \frac{U}{T} = \frac{e^2}{4\pi\hbar c}\frac{c}{v_F} \sim \frac{300}{137} = 2.2$
- AdS/CFT $\longrightarrow$ D3/probe D5 $\longrightarrow$ top-down approach
- Dual theory $\longrightarrow$ $\mathcal{N} = 4$ SYM at large 't Hooft coupling λ coupled to fundamental hypermultiplets along a 2+1-dim defect

  [DeWolfe, Freedman, Ooguri,hep-th/0111135]
- We study the D3/probe D5-$\overline{\text{D5}}$ system as an holographic model of a graphene bilayer
- The effects of both an external magnetic field and of a charge density are examined
- Two channels for chiral symmetry breaking
  - intra-layer condensate $\langle\bar{\psi}_1\psi_1\rangle$
  - inter-layer condensate $\langle\bar{\psi}_1\psi_2\rangle$

D3/probe D5-$\overline{\text{D5}}$

- Stack of N D3-branes $\longrightarrow$ $\mathrm{AdS}_5 \times S^5$ background

$$ds^2 = r^2\left(-dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{1}{r^2}\left(dr^2 + r^2 d\psi^2 + r^2 \sin^2\psi d^2\Omega_2 + r^2\cos^2\psi d^2\tilde{\Omega}_2\right)$$

where $d^2\Omega_2 = \sin\theta d\theta d\phi$ and $d^2\tilde{\Omega}_2 = \sin\tilde{\theta} d\tilde{\theta} d\tilde{\phi}$

- It is useful to introduce other coordinates

$$\rho = r\sin\psi\ , \qquad l = r\cos\psi$$

$$ds^2 = (\rho^2 + l^2)\left(-dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{1}{\rho^2 + l^2}\left(d\rho^2 + \rho^2 d^2\Omega_2 + dl^2 + l^2 d^2\tilde{\Omega}_2\right)$$

Poincaré horizon at $r = 0 \longrightarrow \rho = l = 0$.

- l asymptotically gives the distance between the D3- and the D5-brane $\longrightarrow$ the bare fermion mass.

- Embed N_5 D5 and $\overline{\text{D5}}$ probes in this background ($N_5 \ll N$)

- DBI + WZ actions

$$S = T_5 N_5 \left[- \int d^6\sigma \sqrt{-\det(g + 2\pi\alpha' F)} + 2\pi\alpha' \int C^{(4)} \wedge F \right]$$

- Worldvolume coordinates and ansatz for the embedding of the D5-$\overline{\text{D5}}$

	t	x	y	z	ρ	l	θ	ϕ	$\tilde{\theta}$	$\tilde{\phi}$
D3	$\times$	$\times$	$\times$	$\times$						
D5/$\overline{\text{D5}}$	$\times$	$\times$	$\times$	$z(\rho)$	$\times$	$l(\rho)$	$\times$	$\times$		

Embed the D5 on (t, x, y, ρ and Ω_2).

- $z(\rho)$ and $l(\rho)$ give the brane a non trivial profile in ρ.

D5-$\overline{\text{D5}}$ embedding

- Induced metric on the D-branes worldvolume

$$ds^2 = (\rho^2 + l^2)\left(-dt^2 + dx^2 + dy^2\right) + \frac{\rho^2}{\rho^2 + l^2} d^2\Omega_2 + \frac{d\rho^2}{\rho^2 + l^2}\left(1 + ((\rho^2 + l^2)z')^2 + l'^2\right)$$

- Charge density and external magnetic field $\longrightarrow$ D5 world-volume gauge fields (in the $a_\rho = 0$ gauge)

$$\frac{2\pi}{\sqrt{\lambda}} F = a_0'(\rho) d\rho \wedge dt + b dx \wedge dy$$

$$b = \frac{2\pi}{\sqrt{\lambda}} B \qquad a_0 = \frac{2\pi}{\sqrt{\lambda}} A_0$$

DBI action

- DBI action for N_5 D5 ($\overline{\text{D5}}$)

$$S = \mathcal{N}_5 \int d\rho \frac{\rho^2}{\rho^2 + l^2} \sqrt{(\rho^2 + l^2)^2 + b^2} \sqrt{1 + l'^2 + ((\rho^2 + l^2) z')^2 - {a_0'}^2}$$

where $\mathcal{N}_5 = \dfrac{\sqrt{\lambda} N N_5}{2\pi^3} V_{2+1}$

- $a_0(\rho)$ and $z(\rho)$ are cyclic variables $\longrightarrow$ their canonical momenta are constants

$$Q = -\frac{\delta \mathscr{L}}{\delta a_0'} \equiv \frac{2\pi \mathcal{N}_5}{\sqrt{\lambda}} q \qquad q = \frac{\rho^2 a_0' \sqrt{(\rho^2 + l^2)^2 + b^2}}{(\rho^2 + l^2)\sqrt{1 + l'^2 + ((\rho^2 + l^2) z')^2 - (a_0')^2}}$$

$$\Pi_z = \frac{\delta \mathscr{L}}{\delta z'} \equiv \mathcal{N}_5 f \qquad f = \frac{(\rho^2 + l^2)\rho^2 z' \sqrt{(\rho^2 + l^2)^2 + b^2}}{\sqrt{1 + l'^2 + ((\rho^2 + l^2) z')^2 - {a_0'}^2}}$$

  - $q =$ charge density on the D5 ($\overline{\text{D5}}$)

Equations of motion

Solving for $a_0'(\rho)$ and $z'(\rho)$ in terms of q and f we get

$$a_0' = \frac{q(\rho^2+l^2)\sqrt{1+l'^2}}{\sqrt{\rho^4\left(b^2+(\rho^2+l^2)^2\right)+q^2(\rho^2+l^2)^2-f^2}}$$

$$z' = \frac{f\sqrt{1+l'^2}}{(\rho^2+l^2)\sqrt{\rho^4\left(b^2+(\rho^2+l^2)^2\right)+q^2(\rho^2+l^2)^2-f^2}}$$

By Legendre transforming the action one gets the Routhian

$$R_{fq} = \int d\rho \frac{\sqrt{(l'^2+1)\left(-f^2+l^2\left(l^2+2\rho^2\right)\left(\rho^4+q^2\right)+\rho^4\left(\rho^4+q^2+b^2\right)\right)}}{l^2+\rho^2}$$

From which the EoM for $l(\rho)$ can be derived as

$$\begin{aligned} &-\left(l^2+\rho^2\right) l'' \left(-f^2+l^2\left(l^2+2\rho^2\right)\left(\rho^4+q^2\right)+\rho^4\left(\rho^4+q^2+b^2\right)\right) \\ &-2\left(l'^2+1\right)\left(\rho\left(f^2+\rho^2 l^2\left(3\rho^2 l^2+l^4+3\rho^4+b^2\right)+\rho^8\right) l'+\left(\rho^4-f^2\right) l\right)=0 \end{aligned}$$

Note: the magnetic field b can be rescaled to 1 by rescaling $\rho \to \sqrt{b}\rho$, $f \to b^2 f$, $q \to b\, q$

Asymptotic behaviour

Asymptotic behaviour at $\rho \to \infty$ for the embedding functions $z(\rho)$, $l(\rho)$ and the gauge field $a_0(\rho)$

- $$z(\rho) \underset{\rho\to\infty}{\simeq} \pm\frac{L}{2} \mp \frac{f}{5\rho^5} + \dots \qquad \text{(for D5/}\overline{\text{D5}}\text{)}$$
  - $L =$ separation between the D5 and the $\overline{\text{D5}}$
  - $f \propto$ expectation value for the inter-layer chiral condensate

- $$l(\rho) \underset{\rho\to\infty}{\simeq} m + \frac{c}{\rho} + \dots$$
  - $m \propto$ mass term for the fermions $\longrightarrow$ we consider solution with $m = 0$
  - $c \propto$ expectation value for the intra-layer chiral condensate

- $$a_0(\rho) \underset{\rho\to\infty}{\simeq} \mu - \frac{q}{\rho} + \dots$$
  - $\mu =$ chemical potential

Classification of the solutions

Scheme of the possible types of solutions

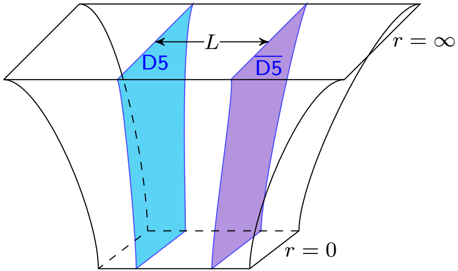
	$f = 0$	$f \neq 0$
$c = 0$	unconnected, $l = 0$ BH, chiral symm.	connected, $l = 0$ Mink, inter
$c \neq 0$	unconnected, $l(\rho)$ not constant BH/Mink, intra	connected, $l(\rho)$ not constant Mink, intra/inter

Unconnected solutions

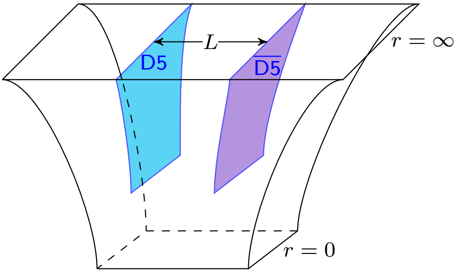
Eq. for $z(\rho) \longrightarrow z' = \dfrac{f\sqrt{1+l'^2}}{(\rho^2+l^2)\sqrt{\rho^4\left(b^2+(\rho^2+l^2)^2\right)+q^2(\rho^2+l^2)^2-f^2}}$

If $f=0 \longrightarrow$ the solution is trivial $\longrightarrow z = \pm L/2$ (for D5/$\overline{\text{D5}}$)

Unconnected solution



"Black hole" embedding



Minkowski embedding

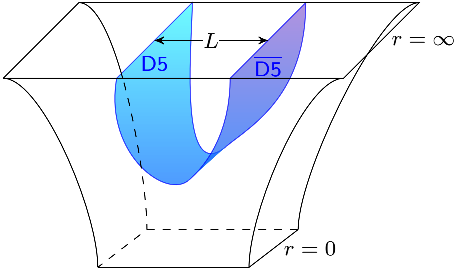
$r=0 \longrightarrow l=\rho=0$

Connected solutions

If $f \neq 0$ the solution for $z(\rho)$ is

$$z(\rho) = f \int_{\rho_0}^{\rho} d\tilde{\rho} \frac{\sqrt{1 + l'^2}}{(\rho^2 + l^2)\sqrt{\rho^4 \left(b^2 + (\rho^2 + l^2)^2\right) + q^2(\rho^2 + l^2)^2 - f^2}}$$

ρ_0 such that $\rho_0^4 \left(b^2 + (\rho_0^2 + l^2(\rho_0))^2\right) + q^2(\rho_0^2 + l(\rho_0)^2) - f^2 = 0$

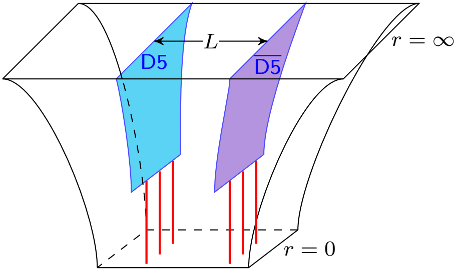


Minkowski embedding

- D-brane worldvolume confined in the region $\rho \geq \rho_0$
- in order to have a sensible solution we have to glue smoothly the D5/$\overline{\text{D5}}$ solutions at $\rho = \rho_0$ $\longrightarrow$ connected solution
- $f_{\text{D5}} = -f_{\overline{\text{D5}}}$ and $q_{\text{D5}} = -q_{\overline{\text{D5}}}$ $\longleftrightarrow$ D5-$\overline{\text{D5}}$ system is neutral

Minkowski vs. BH embeddings

- $(f = 0, c \neq 0)$-solutions can in principle be either BH or Mink. embeddings
- In practice if $q \neq 0$ only BH embeddings are allowed
- Mink. embeddings $\longrightarrow$ D-brane pinches off at $\rho = 0$
- If $q \neq 0 \longrightarrow$ there must be charge sources $\longrightarrow$ F-strings suspended between the D5 and the Poincaré horizon $(r = 0)$
- $T_{\mathsf{F1}} > T_{\mathsf{D5}} \longrightarrow$ strings pull the D5 to $r = 0 \longrightarrow$ BH embed.
  [Kobayashi et al. hep-th/0611099]
- For unconnected solutions $(f = 0)$ Mink. embeddings are allowed only if $q = 0$



D-brane separation and chemical potential

- Separation between the D5 and the $\overline{\text{D5}}$ for the connected solution $(f \neq 0)$

$$L = 2\int_{\rho_0}^{\infty} d\rho\, z'(\rho) = \int_{\rho_0}^{\infty} d\rho\, \frac{2f\sqrt{1+l'^2}}{(\rho^2+l^2)\sqrt{\rho^4\left(b^2+(\rho^2+l^2)^2\right)+q^2(\rho^2+l^2)^2-f^2}}$$

- Chemical potential

$$\mu = \int_{\rho_0}^{\infty} a_0'(\rho)\, d\rho = \int_{\rho_0}^{\infty} d\rho\, \frac{q(\rho^2+l^2)\sqrt{1+l'^2}}{\sqrt{\rho^4\left(b^2+(\rho^2+l^2)^2\right)+q^2(\rho^2+l^2)^2-f^2}}$$

where, for $f \neq 0$, ρ_0 is the solution of

$$\rho_0^4\left(b^2+(\rho_0^2+l^2(\rho_0))^2\right)+q^2(\rho_0^2+l(\rho_0)^2)-f^2=0$$

if $f=0 \longrightarrow \rho_0 = l(\rho_0) = 0$ for $q \neq 0$ and $\rho_0 = 0$ for $q = 0$

D-brane separation and chemical potential

For the constant solution $l = 0$ the integrals can be done analitically

- The turning point ρ_0 of the connected solution is

$$\rho_0 = \frac{\sqrt[4]{\sqrt{\left(b^2+q^2\right)^2+4f^2}-b^2-\rho^2}}{\sqrt[4]{2}}$$

- The separation between the branes for the connected solution is

$$L = \frac{f\sqrt{\pi}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{7}{4};-\frac{f^2}{{\rho_0}^8}\right)}{2{\rho_0}^5\Gamma\left(\frac{7}{4}\right)}$$

- The chemical potential is

$$\mu = \frac{q\sqrt{\pi}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{f^2}{{\rho_0}^8}\right)}{\rho_0\Gamma\left(\frac{3}{4}\right)}$$

Solutions

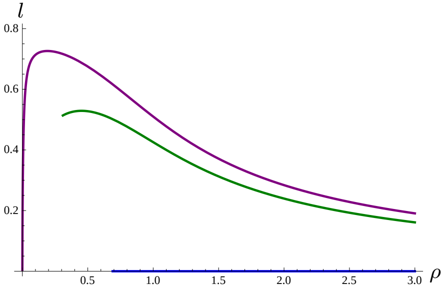
- We must look for non-trivial (*i.e.* non-constant) solutions for $l(\rho)$
- EoM for l is a non-linear ODE
- Numerical method to find solutions imposing the suitable asymptotic condition

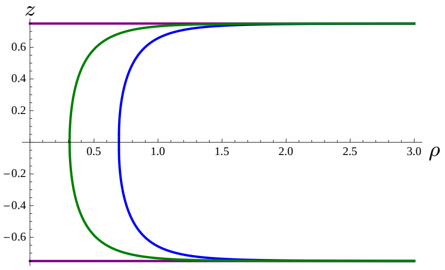
$$l(\rho) \underset{\rho\to\infty}{\simeq} \frac{c}{\rho} + \dots \quad \text{massless fermions!}$$

- We used a shooting technique
- Four types of solutions are allowed
  - $f = 0$, $c = 0$ $(z = \pm L/2,\ l = 0)$ $\longrightarrow$ chiral symm.
  - $f = 0$, $c \neq 0$ $\longrightarrow$ intra
  - $f \neq 0$, $c = 0$ $\longrightarrow$ inter
  - $f \neq 0$, $c \neq 0$ $\longrightarrow$ intra and inter

Plot of solutions

- Example of plots of non-trivial solutions with $\sqrt{b}L \simeq 1.5$ and $\mu/\sqrt{b} \simeq .77$
  - $f = 0$, $c \neq 0 \longrightarrow$ intra
  - $f \neq 0$, $c = 0 \longrightarrow$ inter
  - $f \neq 0$, $c \neq 0 \longrightarrow$ inter and intra

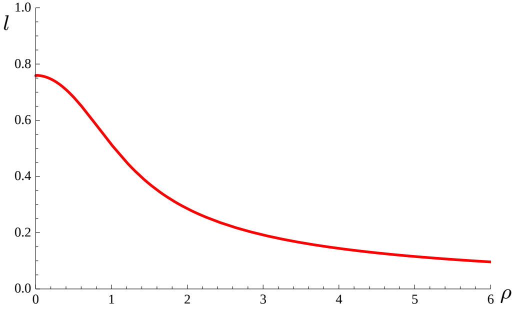




Solutions with zero charge density

- We are interested in solutions at fixed L and μ
- Eq. for a_0 is $\longrightarrow$ $a_0' = \dfrac{q(\rho^2+l^2)\sqrt{1+l'^2}}{\sqrt{\rho^4\left(b^2+(\rho^2+l^2)^2\right)+q^2(\rho^2+l^2)^2-f^2}}$
- It has a trivial solution $\longrightarrow$ $a_0 = \text{const}$ when $q=0$
- Other solutions with $q=0$ and $a_0=\mu$
- Among these the only relevant one $\longrightarrow$ Minkowski embedding with $f=0$ and $c\neq 0$

[Evans,Kim 1311.0149]



Free energy

Which configuration is favored?

- Compare the free energies of the different solutions at the same L and μ

- The right quantity to define the free energy is the action evaluated on solutions $\longrightarrow \mathcal{F}[L,\mu] = S[l,z,a_0]$

$$\delta\mathcal{F} = \int_0^\infty d\rho \left(\delta l \frac{\partial \mathcal{L}}{\partial l'} + \delta a_0 \frac{\partial \mathcal{L}}{\partial a_0'} + \delta z \frac{\partial \mathcal{L}}{\partial z'}\right)' = -q\delta\mu + f\delta L$$

$$\mathcal{F}[L,\mu] = \mathcal{N}_5 \int_{\rho_0}^\infty d\rho \, \frac{\rho^4 \left(1+\left(l^2+\rho^2\right)^2\right) \sqrt{\frac{1+l'^2}{f^2-q^2\left(l^2+\rho^2\right)^2-\rho^4\left(1+\left(l^2+\rho^2\right)^2\right)}}}{l^2+\rho^2}$$

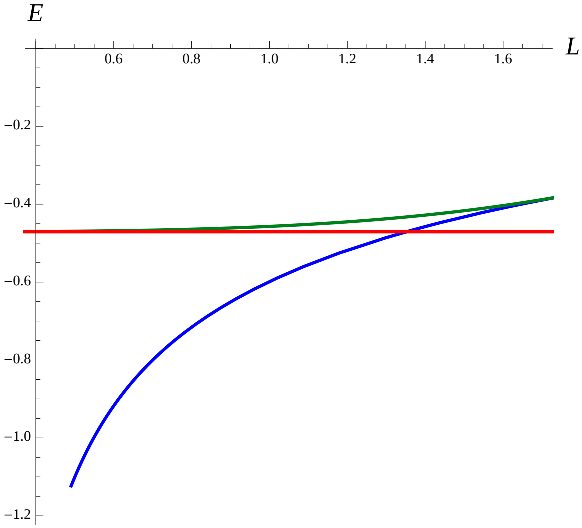
- $\mathcal{F} \longleftrightarrow$ implicit function of L and μ

Free Energy

- The free energy of each solution is UV divergent
- Regularization $\longrightarrow$ subtracting to the free energy of each solution that of the trivial $(f = 0, c = 0; \rho \neq 0)$-solution (with the same μ)
- We use the regularized free energy to study the dominant configuration at fixed values of L and μ
- We construct the phase diagram working on a series of constant L slices

Free Energy as a function of the separation: no charge

[Evans,Kim 1311.0149]

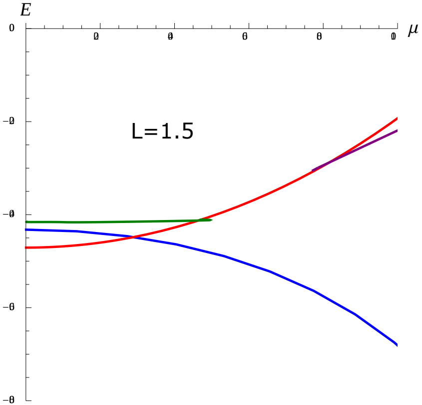


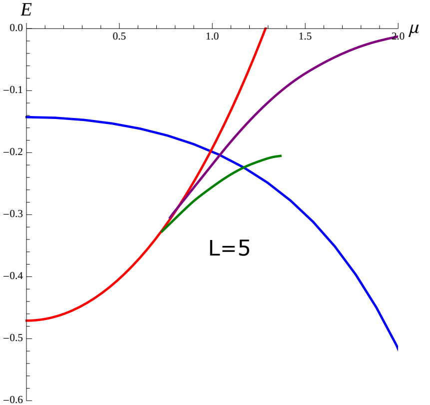
red line: Minkowski embedding unconnected, only intra

blue line: connected ρ-independent, only inter

green line: connected ρ-dependent, both inter and intra

Free Energy as a function of the chemical potential





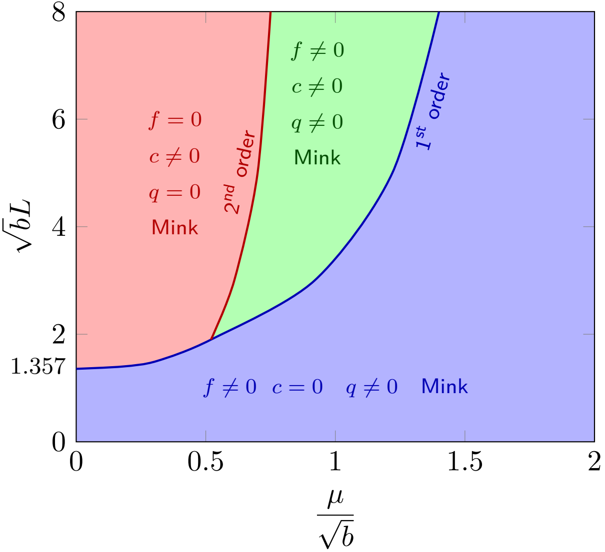
red line: Minkowski embedding unconnected, only intra

purple line: Black-hole embedding unconnected, only intra

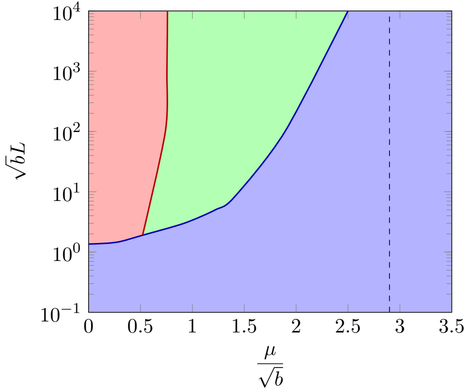
blue line: connected ρ-independent, only inter

green line: connected ρ-dependent, both inter and intra

D3/D5-$\overline{\text{D5}}$ Phase diagram



Large L limit



Conclusions

D3/probe D5-$\overline{\text{D5}}$ system as an holographic model of a graphene bilayer

- Two channels for chiral symmetry breaking $\longrightarrow$ intra/inter-layer condensates
- Inter-layer condensate is possible only for overall neutral system
- There is a pahse with both inter- and intra-layer condensates
- Study of the phase diagram $\left(\mu/\sqrt{b}, \sqrt{b}L\right)$
- For two layers at a finite distance with an external magnetic field and a chemical potential $\longrightarrow$ chiral symmetry is always broken
- Three relevant phases $\longrightarrow$ intra $q = 0$, intra $q \neq 0$, inter

Outlook

This work can be extended in several directions:

- The temperature can be taken into account
- Study of non-neutral system ($\rho_{\mathsf{D5}} + \rho_{\overline{\mathsf{D5}}} \neq 0$)
- We can use a different holographic model for bilayer semi-metal $\longrightarrow$ D3/probe D7-$\overline{\text{D7}}$

[Davis, Kraus, Shah. arXiv:0809.1876]