The geometry of five dimensional gauge theories with rigid supersymmetry

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1401.3266 and work in progress with Yiwen Pan

7 years in one slide

1. Supergravity in the rigid limit

- Pick a supergravity theory (off-shell).
- Solve the killing spinor equations.

Festuccia, Seiberg; Dumitrescu, Closset, Komargodski; Klare, Tomasiello, Zaffaroni; …

2. Localisation

$Q^2 = \pounds_V + G_\Lambda$

- Exact calculation of supersymmetric observables.

Pestun; Kapustin, Willett, Yaakov; Gomis, Okuda; Cassani, Martelli; Alday, Richmond, Sparks; Kim, Kim, Lee, Park; Hosomichi, Seong, Terashima; Källen; Qiu, Zabzine; …

Outline

1. Super Yang-Mills theories on Sasaki-Einstein manifolds

The perturbative partition function counts holomorphic functions on the Calabi-Yau cone.

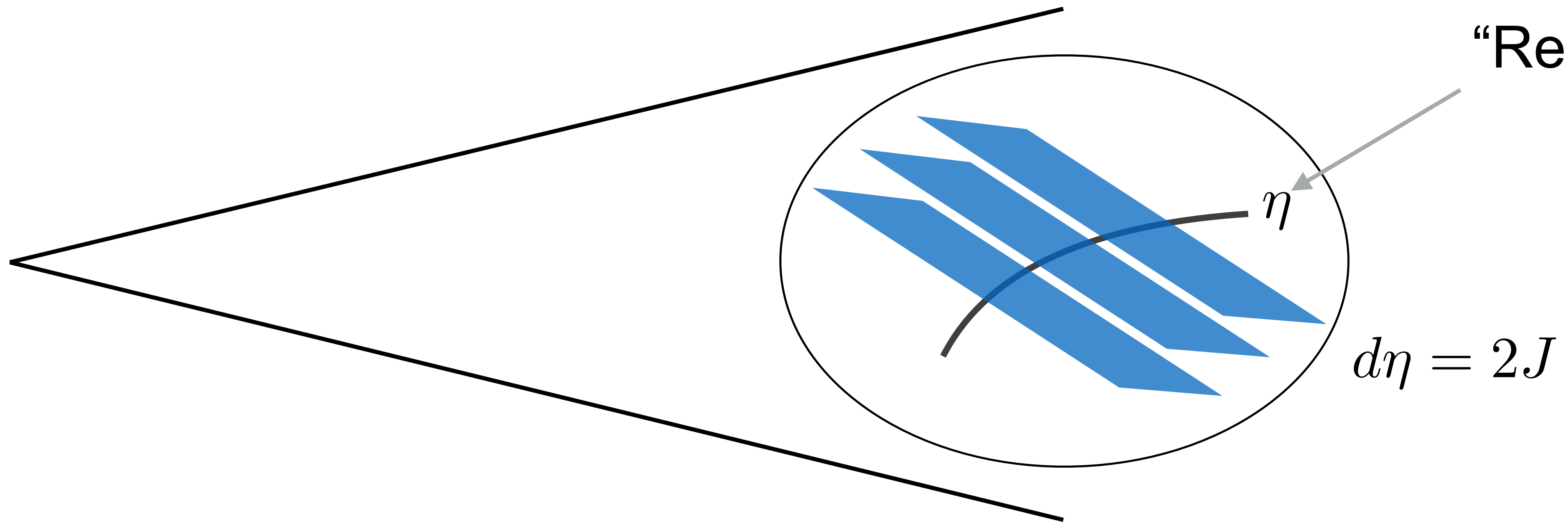
2. The geometry of five dimensional theories

d=5 theories yield examples of geometric structures that might be typical for supersymmetry in odd dimensions.

1. Super Yang-Mills theories on Sasaki-Einstein manifolds

Källen; Qiu, Zabzine; J.S.

Sasaki-Einstein geometry



Decomposition of the tangent bundle

$$J^2 = -1 + \eta \otimes \eta,$$
$$T_{\mathbb{C}}Y = T^{1,0}Y \oplus T^{0,1}Y \oplus \mathbb{C}\eta$$

Gauntlett, Martelli, Sparks, Waldram; Boyer, Galicki

Kohn-Rossi cohomology

Integrability

$$[T^{1,0}Y, T^{1,0}Y] \subseteq T^{1,0}Y \qquad \Rightarrow \qquad d = \partial_b + \bar{\partial}_b + \eta \wedge \pounds_\eta$$

Kohn-Rossi cohomology

$$\dots \xrightarrow{\bar{\partial}_b} \Omega^{p,q-1} \xrightarrow{\bar{\partial}_b} \Omega^{p,q} \xrightarrow{\bar{\partial}_b} \Omega^{p,q+1} \xrightarrow{\bar{\partial}_b} \dots \qquad H^{p,q}_{\bar{\partial}_b}(Y)$$

- Cohomology groups can often be calculated on the CY cone.

$$H^{0,0}_{\bar{\partial}_b}(Y) \cong H^0(\mathcal{O}_{C(Y)})$$

super Yang-Mills on Sasaki-Einstein manifolds

S^5-theory of Hosomichi, Seong, Terashima

- SUSY requires only Killing spinors. This defines the theory on SE-mfds.

Localisation locus

Contact instantons $$(1 + \imath_\eta \star)F = 0, \qquad \imath_\eta F = 0.$$

Källen, Qiu, Zabzine

Perturbative partition function

Evaluated in terms of holomorphic functions on C(Y).

$$\mathrm{sdet}'\, L = \left(\det{}'_{H^{0,0}_{\bar\partial_b}}(L) \det{}_{H^{0,0}_{\partial_b}}(L+3) \det{}'_{H^{0,0}_{\partial_b}}(L) \det{}_{H^{0,0}_{\bar\partial_b}}(L-3) \right)^{\frac{1}{2}}$$

J.S.

2. The geometry of five dimensional theories

Closset, Dumistrescu, Festuccia, Komargodski; Imamura, Matsuno; Pan; J.S.

d=5, $\mathcal{N}$=1 Supergravity

Weyl multiplet (- a scalar)

$SU(2)_R$ triplet $\quad$ $SU(2)_R$ R-symmetry $\quad$ U(1) $\quad$ two-form

$$D_m \xi_I = t_I{}^J \gamma_m \xi_J + A_{mI}{}^J \xi_J + \mathcal{F}_{mn} \gamma^n \xi_I + \frac{1}{2} \gamma_{mpq} \mathcal{V}^{pq} \xi_I$$

Kugo, Ohashi; Zucker

Spinor bilinears

$$\eta_m = \xi_I \gamma_m \xi^I, \qquad \Theta_{IJmn} = \xi_I \gamma_{mn} \xi_J .$$

Algebraic properties

G-structure & SU(2) isomorphism

$$SO(5) \to SO(4) \cong SU(2)_+ \times SU(2)_- \xleftrightarrow{\Theta_{mnI}{}^J} SU(2)_{\mathcal{R}}$$

- Non-vanishing global $SU(2)_R$ section $t_I{}^J$ defines SU(2) structure.

"Canonical" almost contact metric structure

$$\imath_\eta \Theta_{IJ} = 0, \quad \left(\frac{1}{\sqrt{\det t}} t^{IJ} \Theta_{IJ}\right)^2 = -1 + \eta \otimes \eta, \quad T_{\mathbb{C}}M = T^{1,0}M \oplus T^{0,1}M \oplus \mathbb{C}\eta$$

- Foliation with leaves along "Reeb" vector.

Candidate geometries

Transversally holomorphic foliation (THF)

- Appears generically in d=3.

$$[T^{1,0}M \oplus \mathbb{R}\eta, T^{1,0}M \oplus \mathbb{R}\eta] \subseteq T^{1,0}M \oplus \mathbb{R}\eta$$

Closset, Dumistrescu, Festuccia, Komargodski

Integrable Cauchy-Riemann structure (CR)

- Trivially integrable in d=3.

$$[T^{1,0}M, T^{1,0}M] \subseteq T^{1,0}M$$

Contact geometry

- Almost generic in d=5.

Qiu, Zabzine

Holomorphy conditions

Niejenhuis tensor

- tedious, yet useful

Spinorial conditions

$X \in T^{1,0}$ iff

- CR $$X^a H_I{}^J \gamma_a \xi_J = 0$$
- THF $$X^a \Pi_a{}^b H_I{}^J \gamma_b \xi_J = 0$$
- where $$H_I{}^J = t_I{}^J + \imath\sqrt{\det t}\,\delta_I{}^J$$

- Note: $H_I{}^J$ has zero determinant - i.e. picks a single spinor.

Integrability conditions

The $SU(2)_R$ section $t_I{}^J$ has to be covariantly constant on the transverse space.

Transversally holomorphic foliated if and only if

$$\Pi_m{}^n(D_n t_I{}^J) = 0$$

Integrable CR-structure if and only if

$$\Pi_m{}^n(D_n t_I{}^J) = 0 \quad \wedge \quad (\mathcal{F} + \mathcal{V})^{2,0} = 0$$

Tentative results.

Yiwen Pan, J.S.

Comments

Localisation on SE_5

- Interpretation of the d=6 counting - holography?
- Kohn-Rossi cohomology also appears in AdS/CFT duality.
- Contact Instantons.

Rigid supersymmetry in d=5

- Examples!
- Interpretation for $t_I{}^J$ not covariantly constant?
- Partition functions - cohomology.
- Do THF or CR imply supersymmetry?

Additional Material

The partition function

The partition function

$$Z = \int_{\imath\mathfrak{t}} dx Z_{\text{class}}(x)\,\text{sdet}'(-\imath\pounds_{\eta} + G_x) Z_{\text{inst.}}(x)$$

We will focus on the perturbative terms.

bosonic	$\Omega^1(Y,\mathfrak{g}) \oplus H^0(Y,\mathfrak{g}) \oplus H^0(Y,\mathfrak{g})$
fermionic	$\Omega^{2,0}(Y,\mathfrak{g}) \oplus \Omega^{0,2}(Y,\mathfrak{g}) \oplus \Omega^0(Y,\mathfrak{g}) \oplus \Omega^0(Y,\mathfrak{g}) \oplus \Omega^0(Y,\mathfrak{g})$

One-loop contribution

$$\operatorname{sdet}' L = \left(\frac{\det_{\Omega^0} L \, \det_{\Omega^{2,0}} L}{\det_{\Omega^{1,0}} L} \, \frac{\det_{\Omega^0} L \, \det_{\Omega^{0,2}} L}{\det_{\Omega^{0,1}} L} \right)^{\frac{1}{2}} \frac{1}{\det_{H^0} L}, \qquad L = -\imath \pounds_\eta$$

$$\frac{f}{\bar\partial_b f} \, \frac{f \bar\Omega}{\partial_b f \lrcorner \bar\Omega} \, \frac{\bar\partial_b \alpha}{\alpha}$$

$$\operatorname{sdet}' L = \left({\det}'_{H^{0,0}_{\bar\partial_b}}(L) \det_{H^{0,0}_{\partial_b}}(L+3) {\det}'_{H^{0,0}_{\partial_b}}(L) \det_{H^{0,0}_{\bar\partial_b}}(L-3) \right)^{\frac{1}{2}}$$