Sigma-model perturbation theory and AdS/CFT spectrum

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Emmy Noether Research Group
Gauge Fields from Strings

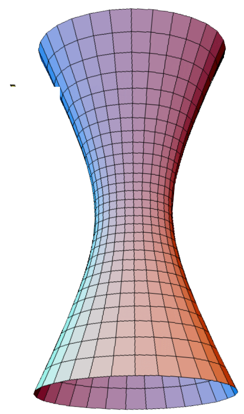
with L. Bianchi, B. Hoare
with L. Bianchi, M.S. Bianchi, A. Bres and E. Vescovi

Here, earlier: talks of D. Seminara, P. Sundin, poster of E. Vescovi

The String Theory Universe
COST, Mainz, September 26 2014

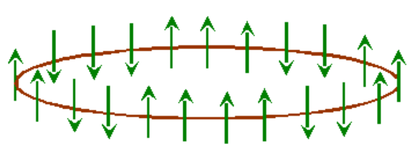
Sigma-model perturbation theory, AdS/CFT and integrability

Unescapable tool to understand string theory in **nontrivial backgrounds**
(es. quantum consistency of proposed actions, UV finiteness)



Here: test AdS/CFT and check **exact** methods/results

> based on **integrability**



[Minahan Zarembo 02 ..]
[Beisert Staudacher 03 ..]
[....]

>> solid fact *classically*
(quantum: pure spinor language [Giangreco M. Puletti 08])

[Bena, Polchinski, Roiban 03]
[Sorokin Wulff 09]

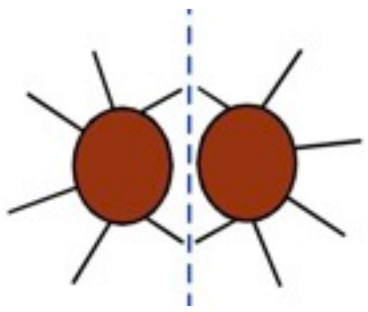
> based on **supersymmetric localization** [Pestun 07] [Drukker Marino Putrov 10]

> based on **integrability <u>and</u> localization** [Correa Henn Maldacena Sever 12] [Gromov Sizov 14]

Outline

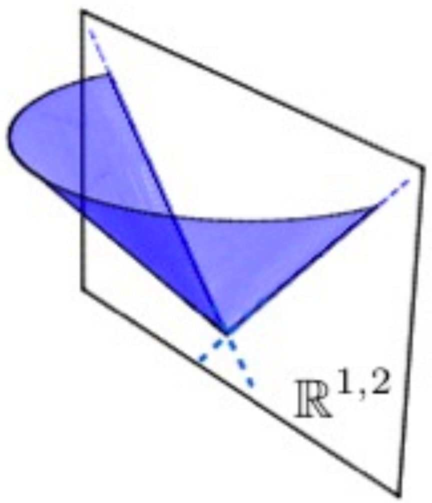
Sigma-model perturbation theory I

Unitarity methods for scattering in 2d



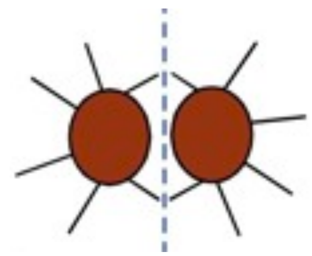
Sigma-model perturbation theory II

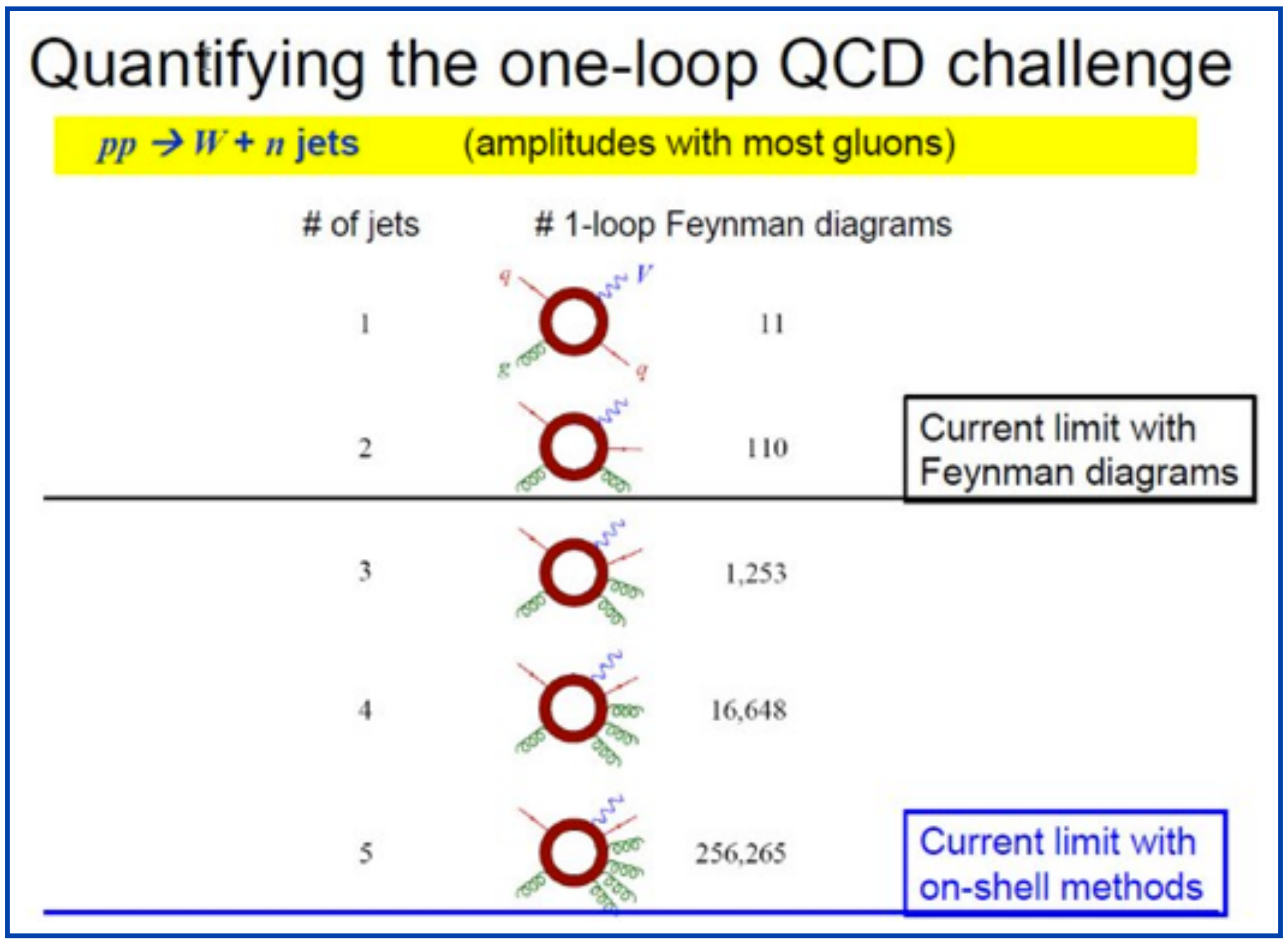
ABJM cusp anomaly at two loops and the interpolating function h(λ)



Calculating scattering amplitudes efficiently

Remarkable **efficiency** of unitarity-based methods [Bern, Dixon, Dunbar, Kosower, 1994] for calculation of amplitudes in various qft's and various dimensions (non-abelian gauge theories, Chern-Simons theories, supergravity).





[from a L. Dixon talk]

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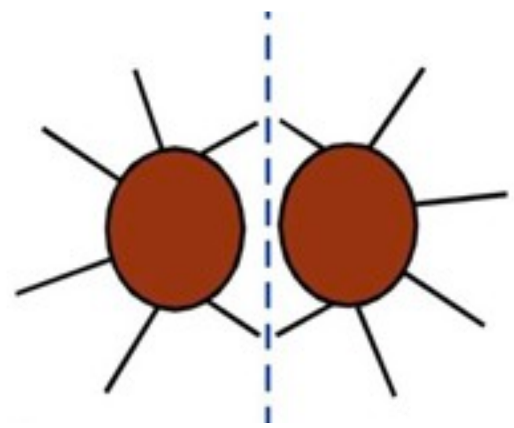


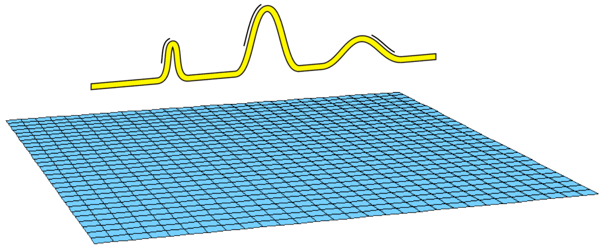
Goal: apply to evaluation of amplitudes of two-dimensional cases of interest.

- **Methodological**: techniques never really applied in two dimensions.
- Provide tests of **quantum integrability** for certain string backgrounds.
- Provide 2d scattering perturbation theory with **efficient tools**.
  Extract information on **integrable worldsheet** S-matrices

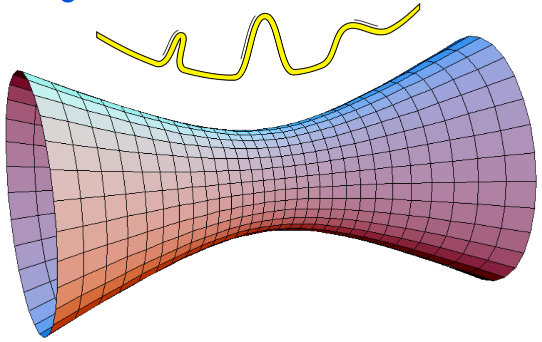
String worldsheet scattering

- Worldsheet amplitudes ($N \to \infty$, free strings), scattering of the (2d) lagrangean excitations. Non-trivial interactions due to highly non trivial background.

flat space



AdS_5xS^5 with RR fluxes



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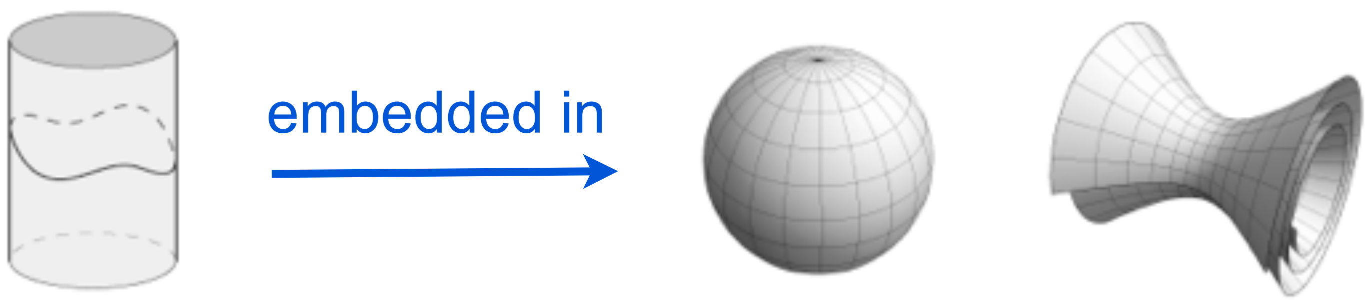

$$\frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$$

- Because of RR-background need a GS formulation

$$S = \frac{\sqrt{\lambda}}{4\pi}\int d^2\sigma \sqrt{-h}\, h^{ab} G_{MN}(X)\partial_a X^M \partial_b X^N + \text{fermions}$$

loop counting parameter $\hat{g} = \frac{2\pi}{\sqrt{\lambda}}$

- Work on a gauge-fixed sigma model (uniform light-cone gauge)

[Arutyunov, Frolov, Plefka, Zamaklar 2006]

$$H_{ws} = \int d\sigma\, \mathcal{H}_{ws} = -\int d\sigma\, p_- \equiv E - J$$

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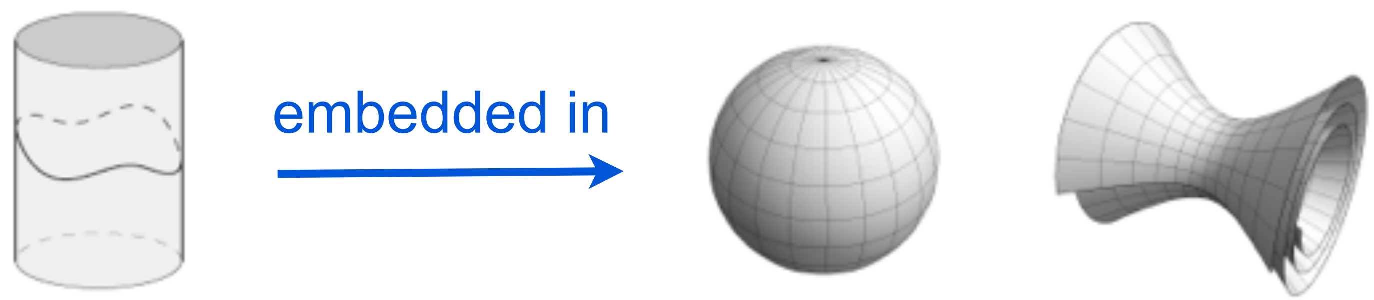

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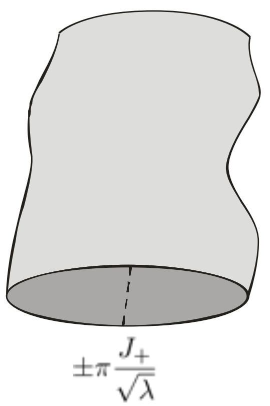


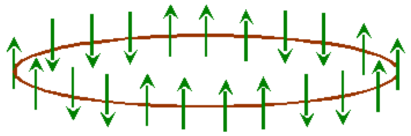
- Decompactification limit and large tension expansion $\hat{g} \to \infty$

→ sensible definition of a **perturbative worldsheet S-matrix**

AdS/CFT (internal) S-matrix I

[Klose McLoughlin Roiban Zarembo 2007]

- This S-matrix is the strong coupling perturbative expansion of the exact AdS_5/CFT_4 S-matrix *aka* "spin chain S-matrix"



[Staudacher 2004]
[Beisert Staudacher 2005]
[Beisert 2005]

$$
\begin{aligned}
1 &= \prod_{j}^{K_4} \frac{x^+_{4j}}{x^-_{4j}} \\
1 &= \prod_{\substack{j=1\\ j\neq k}}^{K_2} \frac{u_{2k}-u_{2j}-i}{u_{2k}-u_{2j}+i} \prod_{j=1}^{K_3} \frac{u_{2k}-u_{3j}+\frac{i}{2}}{u_{2k}-u_{3j}-\frac{i}{2}} \\
1 &= \prod_{j=1}^{K_2} \frac{u_{3k}-u_{2j}+\frac{i}{2}}{u_{3k}-u_{2j}-\frac{i}{2}} \prod_{j=1}^{K_4} \frac{x_{3k}-x^+_{4j}}{x_{3k}-x^-_{4j}} \\
\left(\frac{x^+_{4k}}{x^-_{4k}}\right)^L &= \prod_{j=1}^{K_4} \left(\frac{u_{4k}-u_{4j}+i}{u_{4k}-u_{4j}-i}\, e^{2i\theta(x_{4k},x_{4j})}\right) \prod_{j=1}^{K_3} \frac{x^-_{4k}-x_{3j}}{x^+_{4k}-x_{3j}} \prod_{j=1}^{K_5} \frac{x^-_{4k}-x_{5j}}{x^+_{4k}-x_{5j}} \\
1 &= \prod_{j=1}^{K_6} \frac{u_{5k}-u_{6j}+\frac{i}{2}}{u_{5k}-u_{6j}-\frac{i}{2}} \prod_{j=1}^{K_4} \frac{x_{5k}-x^+_{4j}}{x_{5k}-x^-_{4j}} \\
1 &= \prod_{\substack{j=1\\ j\neq k}}^{K_6} \frac{u_{6k}-u_{6j}-i}{u_{6k}-u_{6j}+i} \prod_{j=1}^{K_5} \frac{u_{6k}-u_{5j}+\frac{i}{2}}{u_{6k}-u_{5j}-\frac{i}{2}}
\end{aligned}
$$

$$x(u) = \tfrac{1}{2}u + \tfrac{1}{2}u\sqrt{1-2g^2/u^2}$$

$$x^\pm(u) = x(u \pm \tfrac{i}{2})$$

$$\delta D = \delta E = g^2 \sum_{j=1}^{K_4}\left(\frac{i}{x^+_{4j}} - \frac{i}{x^-_{4j}}\right)$$

$$g^2 = \frac{\lambda}{8\pi^2}$$

Describe the **exact asymptotic spectrum**

> anomalous dimensions of local composite operators
> energies of their dual string configurations.

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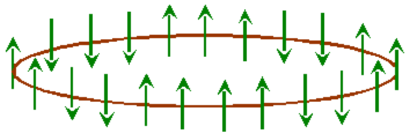
[Klose McLoughlin Roiban Zarembo 2007]

- This S-matrix is the strong coupling perturbative expansion of the exact AdS_5/CFT_4 S-matrix *aka* “spin chain S-matrix”



[Staudacher 2004]
[Beisert Staudacher 2005]
[Beisert 2005]

- Structure of two-particle S-matrix determined by supergroup PSU(2,2|4)

$$S_{12} = S^0 \, \mathbf{S_{12}}$$

up to one (/more) scalar factor(/s)

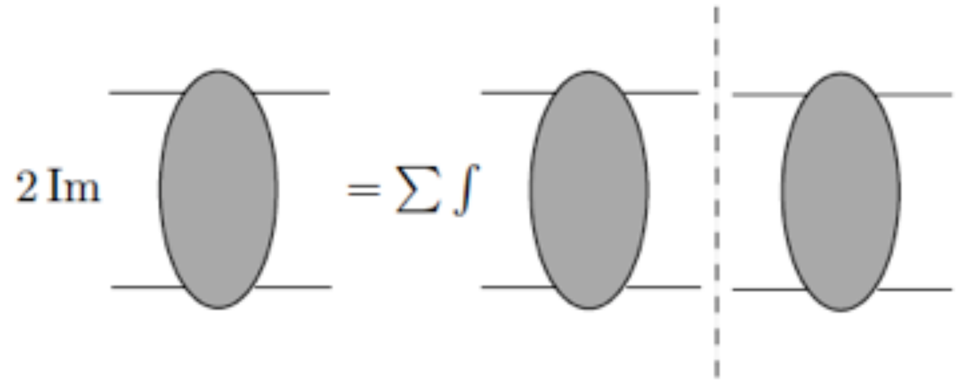
fixed with additional constraints like “crossing symmetry”

Hardest thing to compute, particularly in some models relevant in AdS_3/CFT_2 where solutions to crossing-like equations are difficult to determine.

Unitarity cuts method

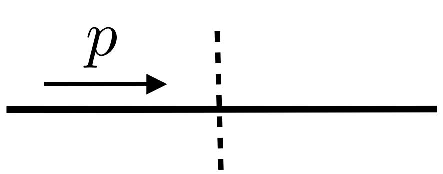
Consequence of the optical theorem

$$S = 1 + i\,T \xrightarrow[S^\dagger = S^{-1}]{\text{unitarity}} 2\,\mathrm{Im}(T) = T\,T^\dagger$$



Relates a certain loop amplitude to a lower order one.

Imaginary part of the amplitude contains the branch-cut information.



$$\xrightarrow[\text{rules}]{\text{Cutting (Cutkosky)}} 2\pi i\,\delta(p^2 - m^2)$$

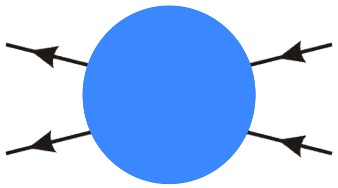
Unitarity cuts method: revert the order,
find n-loop amplitude fusing lower order ones

Only singular part can be reconstructed (logs or polilogs.)

Cut-constructibility of a theory always to be verified.

Two-dimensional scattering

Two-body scattering process of a theory invariant under space and time translations



described via the four-point amplitude

$$\langle \Phi^P(p_3)\Phi^Q(p_4)\,|\mathbb{S}|\,\Phi_M(p_1)\Phi_N(p_2)\rangle = (2\pi)^2\delta^{(d)}(p_1+p_2-p_3-p_4)\,\mathcal{A}_{MN}^{PQ}(p_1,p_2,p_3,p_4)$$

For d=2 and in the single mass case, scattering 2 → 2 is simple.

Particles either preserve or exchange their momenta

$$\delta^{(2)}(p_1+p_2-p_3-p_4) = J(p_1,p_2)\,\big(\delta(\mathrm{p}_1-\mathrm{p}_3)\delta(\mathrm{p}_2-\mathrm{p}_4)+\delta(\mathrm{p}_1-\mathrm{p}_4)\delta(\mathrm{p}_2-\mathrm{p}_3)\big)$$

The Jacobian $J(p_1,p_2)$ depends on dispersion relation.

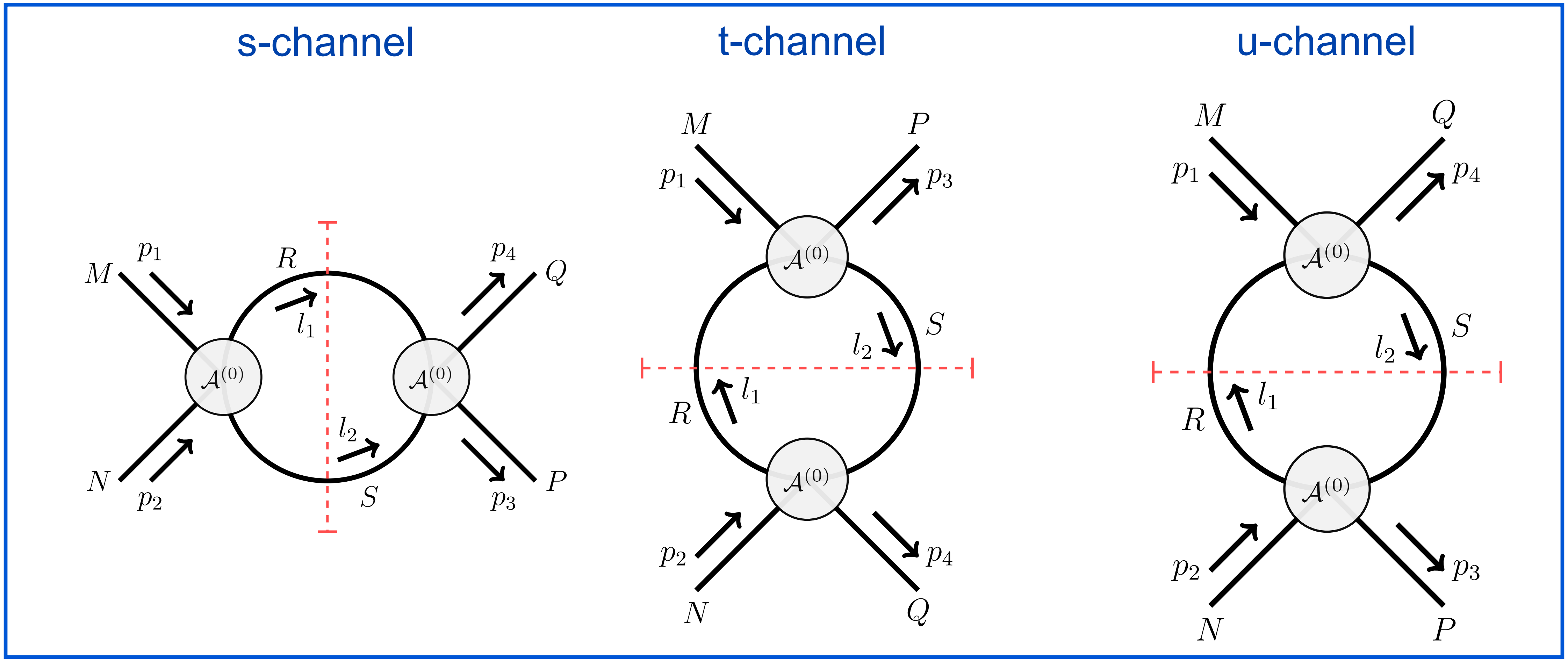
S-matrix element defined by

$$S_{MN}^{PQ}(p_1,p_2) \equiv \frac{J(p_1,p_2)}{4\epsilon_1\epsilon_2}\mathcal{A}_{MN}^{PQ}(p_1,p_2,p_1,p_2)$$

Dispersion relation for asymptotic states (equal masses =1): $\epsilon_i^2 = 1+\mathrm{p}_\mathrm{i}^2$

Scattering in d=2: unitarity cuts (1)

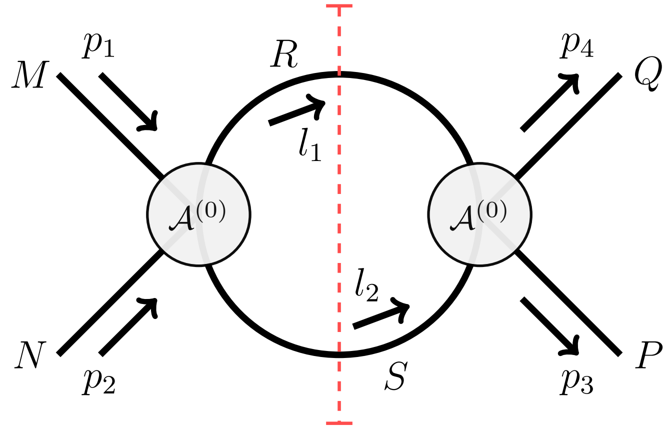
One-loop result from unitarity techniques: contributions from three cut-diagrams



Example: s-cut contribution. Glue tree-amplitudes.

$$\mathcal{A}^{(1)}{}_{MN}^{PQ}(p_1,p_2,p_3,p_4)|_{s-cut} = \frac{1}{2}\int \frac{d^2 l_1}{(2\pi)^2}\int \frac{d^2 l_2}{(2\pi)^2}\; i\pi\delta^+({l_1}^2-1)\; i\pi\delta^+(l_2^2-1)$$
$$\times\, \mathcal{A}^{(0)}{}_{MN}^{RS}(p_1,p_2,l_1,l_2)\mathcal{A}^{(0)}{}_{SR}^{PQ}(l_2,l_1,p_3,p_4)$$

Scattering in d=2: unitarity cuts (2)



- Use 2-momentum conservation at the first vertex

$$\widetilde{\mathcal{A}}^{(1)}{}^{PQ}_{MN}(p_1,p_2,p_3,p_4)|_{s-cut} = \frac{1}{2}\int \frac{d^2 l_1}{(2\pi)^2}\, i\pi\delta^+({l_1}^2-1)\, i\pi\delta^+((l_1-p_1-p_2)^2-1)$$
$$\times \widetilde{\mathcal{A}}^{(0)}{}^{RS}_{MN}(p_1,p_2,l_1,-l_1+p_1+p_2)\, \widetilde{\mathcal{A}}^{(0)}{}^{PQ}_{SR}(-l_1+p_1+p_2,l_1,p_3,p_4)$$

- Use the zeroes of δ - functions in the $\widetilde{\mathcal{A}}^{(0)}$: loop momenta are completely frozen.

  Can **pull** tree-level amplitudes **out** of the integral (like $f(x)\,\delta(x) = f(0)\,\delta(x)$)

- Restore loop momentum off-shell $i\pi\delta^+(l_1^2-1) \longrightarrow \dfrac{1}{l_1^2-1}$ and **uplift**

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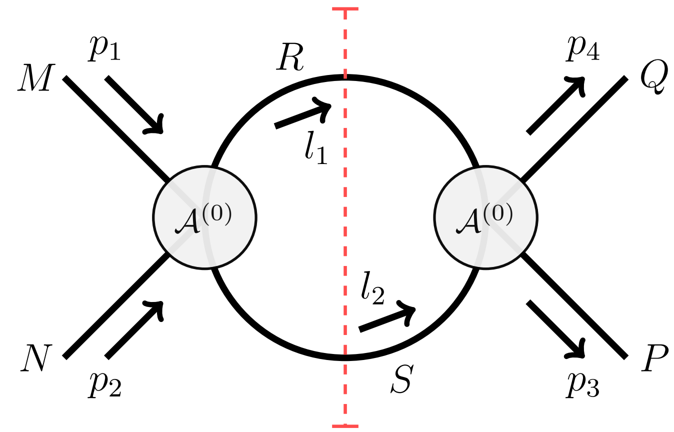

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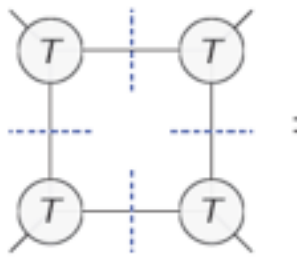
Two-particle cuts in d=2 at one loop are **maximal cuts.**

Expect same as quadrupole cuts in d=4: $A_4^{1-loop} = \sum (A_4^{tree})^4\, I_{box}$



4-points amplitude at one-loop

Tree-level amplitudes can be pulled out of the integral, evaluated at those zeroes.

A simple **sum over discrete solutions of the on-shell conditions**

$$
\begin{aligned}
\widetilde{\mathcal{A}}^{(1)}{}^{PQ}_{MN}(p_1,p_2,p_3,p_4) = \; & \frac{I(p_1+p_2)}{2}\Big[\widetilde{\mathcal{A}}^{(0)}{}^{RS}_{MN}(p_1,p_2,p_1,p_2)\widetilde{\mathcal{A}}^{(0)}{}^{PQ}_{SR}(p_2,p_1,p_3,p_4) \\
& + \widetilde{\mathcal{A}}^{(0)}{}^{RS}_{MN}(p_1,p_2,p_2,p_1)\widetilde{\mathcal{A}}^{(0)}{}^{PQ}_{SR}(p_1,p_2,p_3,p_4)\Big] \\
+ \; & I(p_1-p_3)\widetilde{\mathcal{A}}^{(0)}{}^{SP}_{MR}(p_1,p_3,p_1,p_3)\widetilde{\mathcal{A}}^{(0)}{}^{RQ}_{SN}(p_1,p_2,p_3,p_4) \\
+ \; & I(p_1-p_4)\widetilde{\mathcal{A}}^{(0)}{}^{SQ}_{MR}(p_1,p_4,p_1,p_4)\widetilde{\mathcal{A}}^{(0)}{}^{RP}_{SN}(p_1,p_2,p_4,p_3)
\end{aligned}
$$

weighted by scalar “bubble” integrals

$$
I(p) = \int \frac{d^2q}{(2\pi)^2}\frac{1}{(q^2-1+i\epsilon)((q-p)^2-1+i\epsilon)}
$$

Inherently **finite** formula.

One of initial motivation of our work: ordinary Feynman diagrammatics was problematic (divergencies did not cancel). Recently clarified in [Roiban, Sundin, Tseytlin, Wulff 14]

Sundin talk on Monday

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$[M]=0$ bosons
$[M]=1$ fermions

$$S^{(1)}{}_{MN}^{PQ}(p_1,p_2)=\frac{1}{4(\epsilon_2\,\mathrm{p}_1-\epsilon_1\,\mathrm{p}_2)}\Big[\tilde{S}^{(0)}{}_{MN}^{RS}(p_1,p_2)\tilde{S}^{(0)}{}_{RS}^{PQ}(p_1,p_2)I(p_1+p_2)$$

$$+(-1)^{[P][S]+[R][S]}\,\tilde{S}^{(0)}{}_{MR}^{SP}(p_1,p_1)\tilde{S}^{(0)}{}_{SN}^{RQ}(p_1,p_2)I(0)$$

$$+(-1)^{[P][R]+[Q][S]+[R][S]+[P][Q]}\tilde{S}^{(0)}{}_{MR}^{SQ}(p_1,p_2)\tilde{S}^{(0)}{}_{SN}^{PR}(p_1,p_2)I(p_1-p_2)\Big]$$

weighted by scalar "bubble" integrals

$$I_s\equiv I(p_1+p_2)=\frac{1}{\epsilon_2\,\mathrm{p}_1-\epsilon_1\,\mathrm{p}_2}-\frac{\operatorname{arsinh}(\epsilon_2\,\mathrm{p}_1-\epsilon_1\,\mathrm{p}_2)}{4\pi i\,(\epsilon_2\,\mathrm{p}_1-\epsilon_1\,\mathrm{p}_2)}$$

$$I_t\equiv I(0)=\frac{1}{4\pi i}$$

$$I_u\equiv I(p_1-p_2)=\frac{\operatorname{arsinh}(\epsilon_2\,\mathrm{p}_1-\epsilon_1\,\mathrm{p}_2)}{4\pi i\,(\epsilon_2\,\mathrm{p}_1-\epsilon_1\,\mathrm{p}_2)}$$

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\begin{aligned}
S^{(1)}{}^{PQ}_{MN}(p_1,p_2) = \frac{1}{4(\epsilon_2\,\mathrm{p}_1-\epsilon_1\,\mathrm{p}_2)}\Big[& \tilde{S}^{(0)}{}^{RS}_{MN}(p_1,p_2)\tilde{S}^{(0)}{}^{PQ}_{RS}(p_1,p_2) I(p_1+p_2) \\
& +(-1)^{[P][S]+[R][S]}\,\tilde{S}^{(0)}{}^{SP}_{MR}(p_1,p_1)\tilde{S}^{(0)}{}^{RQ}_{SN}(p_1,p_2) I(0) \\
& +(-1)^{[P][R]+[Q][S]+[R][S]+[P][Q]}\tilde{S}^{(0)}{}^{SQ}_{MR}(p_1,p_2)\tilde{S}^{(0)}{}^{PR}_{SN}(p_1,p_2) I(p_1-p_2)\Big]
\end{aligned}
$$

weighted by scalar “bubble” integrals

$$I_s \equiv I(p_1+p_2) = \frac{1}{\epsilon_2\,\mathrm{p}_1-\epsilon_1\,\mathrm{p}_2} - \frac{\operatorname{arsinh}(\epsilon_2\,\mathrm{p}_1-\epsilon_1\,\mathrm{p}_2)}{4\pi i\,(\epsilon_2\,\mathrm{p}_1-\epsilon_1\,\mathrm{p}_2)}$$

$$I_t \equiv I(0) = \frac{1}{4\pi i}$$

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Logarithms

Rational

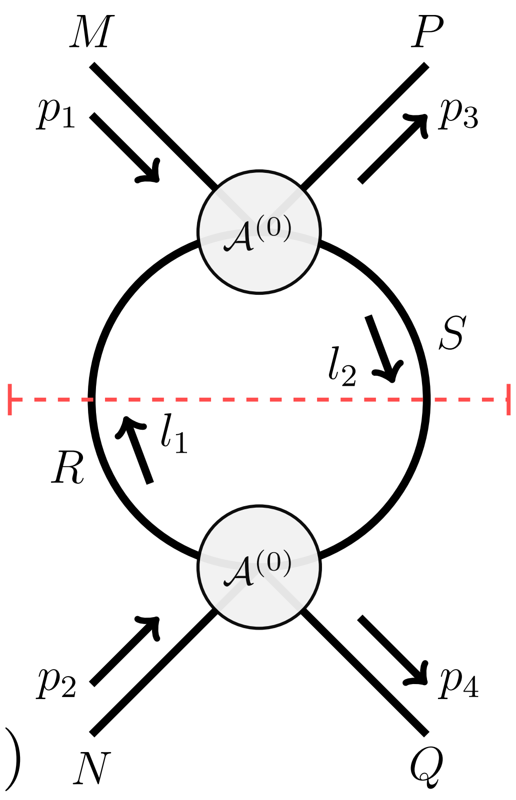
Logarithmic terms safe, rational could be not the whole story.

Subtleties

- The t-channel cut is special.

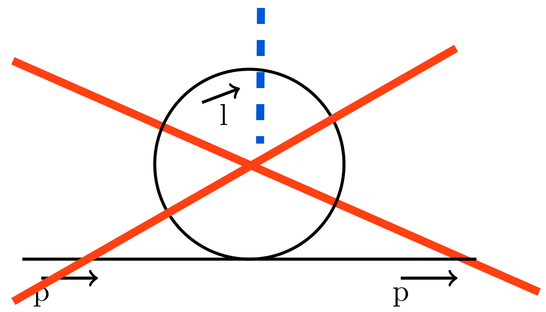
  - Using first $\delta(\mathrm{p}_1 - \mathrm{p}_3)\delta(\mathrm{p}_2 - \mathrm{p}_4)$ makes it ill-defined and requires a **prescription**: use delta-function only at the end of the calculation

  - Asymmetrical wrt choice of the vertex used to solve momenta: **consistency condition**



$$\tilde{S}^{(0)}{}^{SP}_{MR}(p_1, p_1)\,\tilde{S}^{(0)}{}^{RQ}_{SN}(p_1, p_2) = \tilde{S}^{(0)}{}^{PS}_{MR}(p_1, p_2)\,\tilde{S}^{(0)}{}^{QR}_{SN}(p_2, p_2)$$

- We are NOT including contributions from tadpoles (no physical cuts)



- A inherently finite result says **nothing** about UV-finiteness or renormalizability. Might be missing rational terms following from regularization procedure.

Cut-constructibility to be always checked

Relativistic models

- Bosonic: generalised sine-Gordon models

  gauged WZW model for a coset G/H = SO(n + 1)/SO(n)
  (n=1: sine-Gordon, n=2: complex sine-Gordon)

  The method works <u>up to a finite shift in the coupling</u>.

- Supersymmetric generalizations (``Pohlmeyer reductions'' of string theories):

  $\mathcal{N} = 1, 2$ supersymmetric sine-Gordon

  The method reproduces the <u>full</u> result.

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  The method reproduces the <u>full</u> result.

In two cases cut-constructibility is non trivial.

(complex sine-Gordon and Pohlmeyer-reduced AdS_3xS^3 theory)

Models *integrable only classically*, quantum counterterms restore e.g. Yang-Baxter eq,

The unitarity method gives the "<u>quantum integrable</u>" result.

AdS/CFT S-matrix: exact and perturbative structure

$AdS_5 x S^5$ worldsheet sigma-model: most complicated example.

Exact S-matrix based on a (centrally extended) $PSU(2|2)^2$ symmetry algebra.
From symmetries and integrability follows a **group factorization**

$$\mathbb{S} = e^{i\,\theta}\ \hat{S}^{PSU(2|2)} \otimes \hat{S}^{PSU(2|2)}$$

$$\hat{S}_{AB}^{CD} = \begin{cases} A\delta_a^c\delta_b^d + B\delta_a^d\delta_b^c \\ D\delta_\alpha^\gamma\delta_\beta^\delta + E\delta_\alpha^\delta\delta_\beta^\gamma \\ C\epsilon_{ab}\epsilon^{\gamma\delta} \qquad F\epsilon_{\alpha\beta}\epsilon^{cd} \\ G\delta_a^c\delta_\beta^\delta \qquad H\delta_a^d\delta_\beta^\gamma \\ L\delta_\alpha^\gamma\delta_b^d \qquad K\delta_\alpha^\delta\delta_b^c \end{cases}$$

Each factor has manifest $SU(2)\times SU(2)$ invariance

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Perturbatively

Light-cone gauge-fixing preserves $SO(4)\times SO(4)$ in the bosonic lagrangean

Worldsheet fields (embedding coords in AdS_5xS^5) $T,\ \Phi,\ Y^m,\ Z^m,\ \text{fermions}$

can be represented as bispinors $SO(4)\simeq(SU(2)\times SU(2))/\mathbb{Z}_2 \qquad Y_{a\dot{a}}=(\sigma_m)_{a\dot{a}}Y^m,$

LSZ reduction produces various tensor structures, translated in $SU(2)\times SU(2)$ language.

Tree-level S-matrix reproduces leading order of $\mathbb{S}$ [Klose McLoughlin Roiban Zarembo 2007]

AdS/CFT S-matrix: exact and perturbative structure

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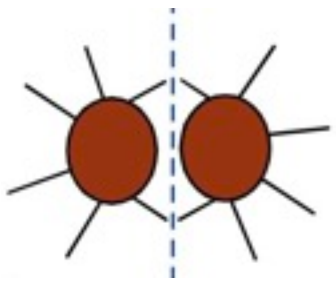
Each factor has manifest $SU(2)\times SU(2)$ invariance

Logarithms

Matrix structure,
rational dependence
on momenta



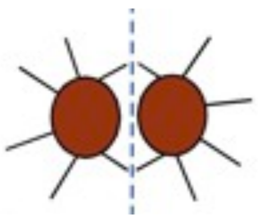
✓ "Bootstrapping" the tree-level S-matrix at one loop via unitarity cuts recover all the tensor structure, group factorization and exponentiation of the logarithms.
One-loop **non-trivial evidence of integrability and cut-constructibility**.

See also [Roiban, Sundin, Tseytlin, Wulff 14]

Remarks

- **For a large class of 2-d models (relativistic and not) four-points one-loop amplitudes are cut-constructible**

  > Standard unitarity (2-particle cuts) reproduces **all rational terms, up to shifts in the coupling**.



- Efficient way for

  > Proposing/checking matrix structure and overall phases for other models

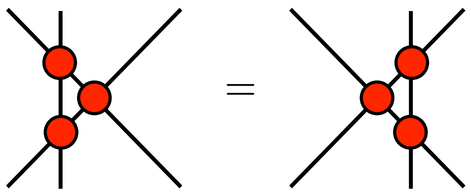
  - $AdS_3 \times S^3 \times T^4$ supported by pure RR flux
  - $AdS_3 \times S^3 \times S^3 \times S^1$ supported by pure RR flux
  - $AdS_3 \times S^3 \times T^4$ supported by a mix RR and NS NS fluxes

L. Bianchi, B. Hoare arXiv: 1405.7947

- Cut-constructibility "criterion"

  > Integrability is crucial asset

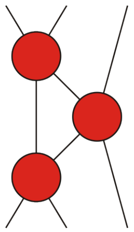
  > Structure of the one-loop S-matrix derived by unitarity cuts *automatically* satisfies the Yang-Baxter equation



Wish list

- **Two loops** rational terms (all logarithms reproduced in [Engelund McEwan Roiban 2013]

- **Higher points**: factorization should emerge $S_{3\to 3} = \left(S_{2\to 2}\right)^3$



- Extend to **off-shell objects**, including form factors and correlation functions.

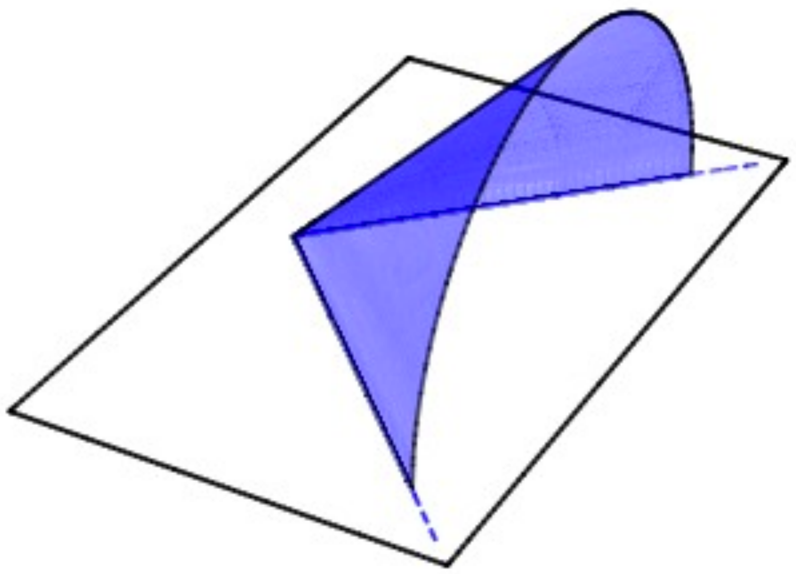
[Klose McLoughlin 2012/2013]
[Engelund McEwan Roiban 2013]

String sigma-model perturbation theory II

ABJM cusp anomaly at two loops and the interpolating function h(λ)

L. Bianchi, M.S. Bianchi, A. Bres, VF, E. Vescovi, arxiv:1407.4788

poster on Monday



AdS_4/CFT_3 and integrability

- Planar AdS_4/CFT_3 system $(\lambda = k/N,\ k, N \to \infty)$

$\mathcal{N}=6$ super Chern-Simons theory in 3d gauge group U(N)xU(N), CS levels k and −k.	**and**	Type IIA strings in $AdS_4 \times \mathbb{CP}^3$ with RR four- and two-form fluxes

believed to be **integrable**: formulation of Bethe equations (and TBA, and Pμ-system).

- Two peculiarities:

1. The relevant string background is **not** maximally supersymmetric.

Construction of the superstring action *complicated.*

2. All-integrability based calculations are given in terms of a function appearing in the **magnon dispersion relation**

$$\epsilon = \sqrt{1 + 4\,h^2(\lambda)\sin^2\frac{p}{2}}$$

which is **not** fixed by symmetries. It is **here** a **non-trivial**, interpolating function of λ .

Sundin talk on Monday

Integrable couplings

- In $\mathcal{N}=4$ SYM the function is "trivial": $h(\lambda_{YM}) = \frac{\sqrt{\lambda_{YM}}}{4\pi}$

  Checked **exactly** via comparison between integrability and localisation results for the ``Brehmstrahlung function'' of N=4 SYM. [Correa, Henn, Sever, Maldacena 2012]

  Seminara talk on Monday

- In ABJM non-trivial dependence on the t'Hooft coupling

$$
\begin{aligned}
h^2(\lambda) &= \lambda^2 - \frac{2\,\pi^3}{3}\,\lambda^4 + \mathcal{O}\left(\lambda^6\right) & \lambda \ll 1 \\
h(\lambda) &= \sqrt{\frac{\lambda}{2}} - \frac{\log 2}{2\pi} + \mathcal{O}(\sqrt{\lambda})^{-1} & \lambda \gg 1
\end{aligned}
$$

[Gaiotto Giombi Yin 08] [Grignani Harmark Orselli] [Nihsioka Takayanagi 08] [Minahan, Ohlsson Sax, Sieg 09] [Leoni, Mauri, Minahan, Ohlsson Sax, Santambrogio, Sieg, Tartaglino Mazzucchellu 10]

Finite coupling dependence unknown from first principles.
[Lewkowycz Maldacena 2013] [Bianchi, Griguolo, Leoni, Penati, Seminara 2014]

Knowledge of $h(\lambda)$ decisive to grant the conjecture integrability of ABJM theory a full predictive power.

A conjecture for the ABJM integrable coupling

A conjecture exist

$$\lambda = \frac{\sinh 2\pi h(\lambda)}{2\pi} \, {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; 1, \frac{3}{2}; -\sinh^2 2\pi h(\lambda)\right)$$

[Gromov Sizov 2014]

extrapolated by “similarities” between two all-order calculations:

> one based on **integrability**: “slope-function” as exact solution of the ABJM spectral curve

[Cavaglia’, Fioravanti, Gromov Tateo 2014]

> one based on **localization**: 1/6 BPS Wilson loop

[Marino, Putrov, 10] [Drukker, Marino, Putrov, 10]

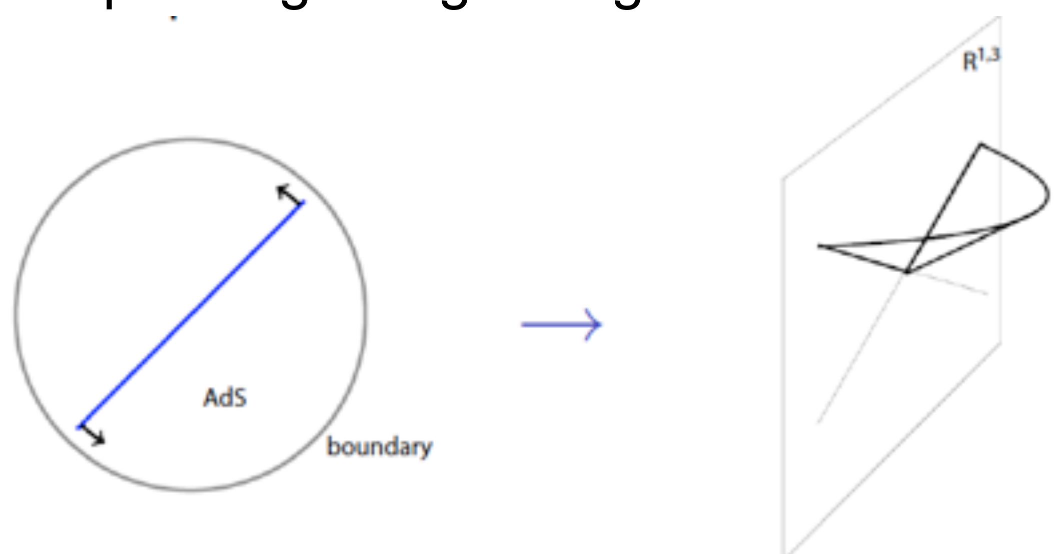
Its weak and strong coupling expansions are

$$h(\lambda) = \lambda - \frac{\pi^2}{3}\lambda^3 + \frac{5\pi^4}{12}\lambda^5 - \frac{893\pi^6}{1260}\lambda^7 + \mathcal{O}(\lambda^9) \qquad \lambda \ll 1$$

$$h(\lambda) = \sqrt{\frac{1}{2}\left(\lambda - \frac{1}{24}\right)} - \frac{\log 2}{2\pi} + \mathcal{O}\left(e^{-2\pi\sqrt{2\lambda}}\right) \qquad \lambda \gg 1$$

Cusp anomaly in AdS_5/CFT_4

- Weak coupling, appears in a variety of contexts:
  - > anomalous dimension of twist operators in large spin limit $\Delta_{\rm twist} \sim f(\lambda)\, \ln S\,, \qquad S \gg 1$
  - > renormalization of light-like cusped Wilson loops $\langle W_{\rm cusp}\rangle \sim e^{-f(\lambda)\phi \ln \frac{\Lambda}{\epsilon}}$
  - > leading IR behavior of log of scattering amplitudes $\log \mathcal{A} \sim \frac{f(\lambda)}{\epsilon^2} + \ldots$
- Strong coupling: corresponding string configurations are related



$$E_{\rm classical} \sim f(\lambda)\, \ln S\,, \qquad S \gg 1 \qquad\qquad \langle W_{\rm cusp}\rangle = Z_{\rm string} = \int [dX\, d\theta] e^{-S[X,\theta]}$$

[Gubser, Klebanov, Polyakov,02] [Kruczenski,02] [Kruczenski, Tirziu, Roiban, Tseytlin 07]

- Integrability gives an **all-order** equation for cusp anomaly $f(\lambda)$, BES equation matching all known independent perturbative results. [Beisert Eden Staduacher 2006]

ABJM cusp anomaly

- Despite nontrivial differences of the cusp physics in ABJM

  [MS Bianchi, Griguolo, Penati, Seminara 2013,14] [Marmiroli 2013] [Lewkowitz Maldacena 2013]

  integrability gives a BES equation only slightly modified, therefore the prediction

$$f_{\rm ABJM}(\lambda) = \frac{1}{2}\, f_{\mathcal{N}=4}(\lambda_{\rm YM})\Big|_{\frac{\sqrt{\lambda_{\rm YM}}}{4\pi}\to h(\lambda)}$$

[Gromov Vieira 2008]

  from which, knowing already the N=4 SYM case,

[Basso Korchemsky Kotanski 2007]
[Roiban Tseytlin 2007]

$$f_{\rm ABJM}(\lambda) = 2h(\lambda) - \frac{3\log 2}{2\pi} - \frac{K}{8\pi^2}\frac{1}{h(\lambda)} + \cdots$$

$$\lambda \gg 1$$

- **Direct** string sigma-model evaluation of the lhs

$$f_{\rm ABJM}(\lambda) = \sqrt{2\lambda} - \frac{5\log 2}{2\pi} + \mathcal{O}(\sqrt{\lambda})^{-1}$$

[ABJM]
[several papers]

  will give also an estimation of the rhs and thus of

$$h(\lambda) = \sqrt{\frac{\lambda}{2}} - \frac{\log 2}{2\pi} + \mathcal{O}(\sqrt{\lambda})^{-1}$$

Superstrings on $AdS_4 x CP^3$

- Solution of Type IIA sugra preserving 24 out of 32 supersymmetries. [Nilsson Pope 84]

- Supercoset approach à la flat space [Hennaux Mezincescu 85] and AdS5xS5 [Metsaev Tseytlin 98]

  Sigma-model action based on $$\frac{OSp(6|4)}{U(3)\times SO(1,3)}$$ [Arutyunov Frolov 08] [Stefanski 08]

  has 24 fermionic dof, and for strings only moving in AdS_4 kappa-symmetry has rank 12.

  Coset model misses 4 physical fermions corresponding to broken supersymmetries.

- Quantum studies of these configurations require starting from ***complete*** IIA string action in $AdS_4 x CP^3$ and make suitable kappa-symmetry gauge fixing.

  [Gomis Sorokin Wulff 08] [Grassi Sorokin Wulff 09]

String action and effective string tension

- Action obtained in [Uvarov, 09,10] from double dimensional reduction from D=11 action for membrane in $AdS_4 x S^7$ based on supercoset OSp(8/4)/(SO(1,3)x SO(7))

  [de Wit, Peeters, Plefka, Sevrin 98]

“AdS” light-cone gauge: light-cone coordinates entirely inside AdS_4

dramatically simplifies fermionic action: at most quartic in the remaining 16 fermions.

[Metsaev Tseytlin 00]
[Metsaev Thorn Tseytlin 00]

- Original ABJM **dictionary proposal** (R is the CP^3 radius) [ABJM 2008] for the effective string tension

$$T = \boxed{\frac{R^2}{2\pi\alpha'} = 2\sqrt{2\lambda}}$$

$$\lambda = \frac{N}{k}$$

is modified to (in planar limit) [Bergman Hirano 2009]

$$T = \boxed{\frac{R^2}{2\pi\alpha'} = 2\sqrt{2\left(\lambda - \frac{1}{24}\right)}} \sim 2\sqrt{2\lambda} - \frac{1}{12\sqrt{2\lambda}}$$

plays a role at 2-loops in perturbation theory

due to higher order (in the curvature) corrections to the background

Perturbative evaluation of path integral around the cusp

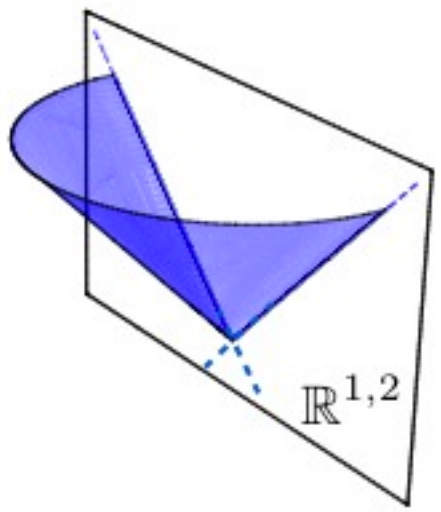
- Classical solution

$$w \equiv e^{2\varphi} = \sqrt{\frac{\tau}{\sigma}} \qquad x^+ = \tau \qquad x^- = -\frac{1}{2\sigma}$$

describe a surface bounded by a null cusp, as at the AdS4 boundary $0 = w^2 = -2x^+x^-$.

- To extract cusp anomaly, compute partition function around it.

$$\langle W_{cusp}\rangle = Z_{string} \equiv \int \mathcal{D}[x,w,z,\eta,\theta]\, e^{-S_E} \equiv e^{-W}$$



Expand around the solution $X = X_{\rm cl} + \tilde{X}$

and evaluate the path integral perturbatively $W = W_0 + W_1 + W_2 + \ldots$

$$Z_{string} \equiv e^{-\frac{1}{2}f(\lambda)V} \qquad V:\ \text{(infinite) 2d volume, } \sim \log S$$

As solution is "homogeneous", i.e. fluctuation lagrangean has constant coefficients, one can factor out V.

$$f(g) = g\left[1 + \frac{a_1}{g} + \frac{a_2}{g^2} + \ldots\right], \qquad g = \frac{T}{2}\ .$$

One loop

Very smooth calculation.

8 bosonic modes	8 fermionic modes
1 real scalar $\tilde{x}^1$ with mass $\frac{1}{\sqrt{2}}$,	2 massless modes,
1 real scalar $\tilde{\varphi}$ with mass 1,	6 massive excitations with mass $\frac{1}{2}$.
3 complex massless z^a, $a = 1, 2, 3$.	

Their determinant is easily evaluated

$$-\ln Z_1 = \frac{1}{2}\int \frac{d^2p}{(2\pi)^2}\left\{\ln(p^2+1)+\ln\left(p^2+\frac{1}{2}\right)+6\ln(p^2)-2\ln(p^2)-6\ln\left(p^2+\frac{1}{4}\right)\right\}$$

$$= -\frac{5\ln 2}{16\pi}\underbrace{\int dt ds}_{V}$$

One-loop finiteness, expected result: $a_1 = -\dfrac{5\log 2}{2\pi}$

[McLoughlin, Roiban, Tseytlin 08] [Alday Arutyunov Bykov 08]

Two loops

Expand the action up to quartic order in fluctuations
and compute all **connected vacuum** Feynman diagrams

$$W_2 = \langle S_{int} \rangle - \frac{1}{2} \langle S_{int}^2 \rangle_c$$

where S_{int} is the interacting part of the action at **cubic** and quartic order

$$\begin{aligned}
\mathcal{L}_{(3)} =& -8\varphi(\partial_s x^1)^2 - 2\varphi(x^1)^2 + 8\varphi x^1(\partial_s x) + 4\varphi^2(\partial_t\varphi - \partial_s\varphi) + 4\varphi[(\partial_t\varphi)^2 - (\partial_s\varphi)^2] \\
& + 4\varphi(\partial_t z^a \partial_t \bar{z}_a - \partial_s z^a \partial_s \bar{z}_a) + 2\varepsilon_{abc}\partial_t z^a \bar{\eta}^b \bar{\eta}^c - 2\varepsilon^{abc}\partial_t \bar{z}_a \eta_b \eta_c + 4\partial_t \bar{z}_a \bar{\eta}^a \bar{\eta}^4 - 4\partial_t z^a \eta_a \eta_4 \\
& i\Big\{\Big[2i\varepsilon_{acb} z^c \bar{\eta}^b \partial_s \bar{\theta}^a - i\varepsilon_{acb} z^c \bar{\eta}^b \bar{\theta}^a - 8\varphi \eta_a \partial_s \bar{\theta}^a + 4\varphi \eta_a \bar{\theta}^a - 2i\varepsilon^{adc}\eta_a \Big(\partial_s \bar{z}_d \theta_c + \bar{z}_d \partial_s \theta_c - \frac{1}{2}\bar{z}_d \theta_c\Big)\Big] + c.c.\Big\} \\
& - 4i\varphi(\partial_s \theta_4 \bar{\eta}^4 - \partial_s \eta_4 \bar{\theta}^4 + \eta_4 \partial_s \bar{\theta}^4 - \theta_4 \partial_s \bar{\eta}^4) + 8i\eta_a \bar{\eta}^a \partial_s x^1 - 4i\eta_a \bar{\eta}^a x^1 + 4i\theta_4 \bar{\theta}^4 \partial_s x^1 - 2i\theta_4 \bar{\theta}^4 x^1 \\
& + 4i\eta_4 \bar{\eta}^4 \partial_s x^1 - 2i\eta_4 \bar{\eta}^4 x^1 + 4\partial_s \bar{z}_a \bar{\eta}^a \bar{\theta}^4 + 4\partial_s z^a \eta_a \theta_4
\end{aligned}$$

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Expand the action up to quartic order in fluctuations
and compute all **connected vacuum** Feynman diagrams.

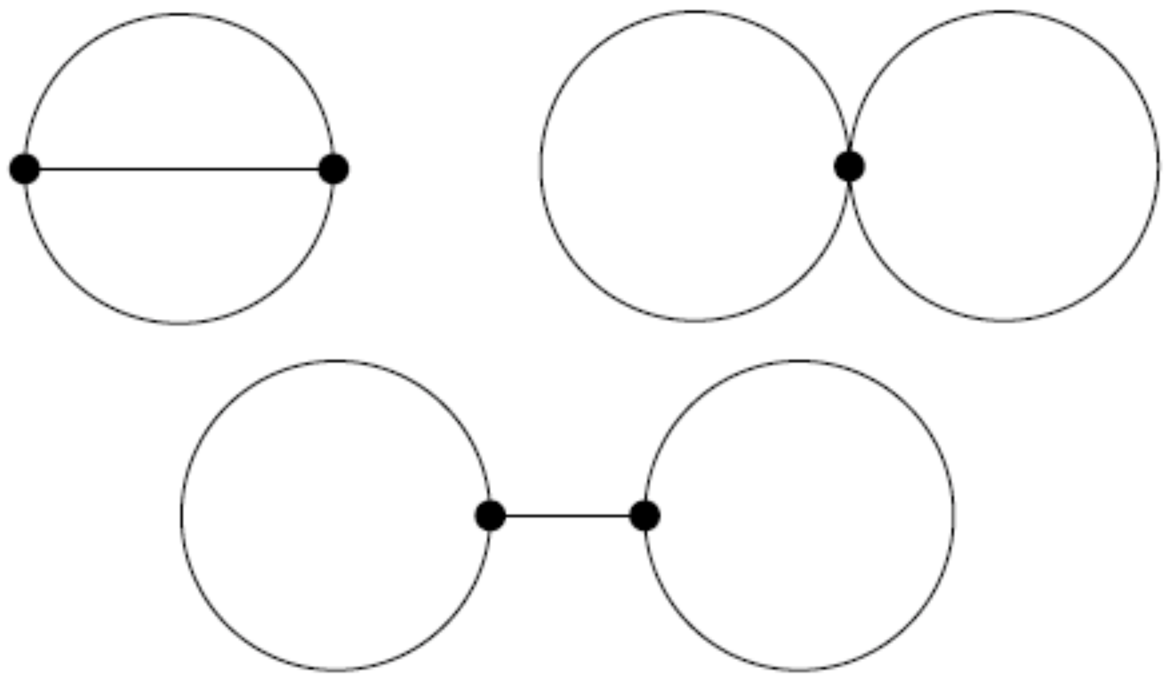
$$W_2 = \langle S_{int} \rangle - \frac{1}{2} \langle S_{int}^2 \rangle_c$$

where S_{int} is the interacting part of the action at cubic and **quartic** order

$$\begin{aligned}
\mathcal{L}_{(4)} = \; & 32\,\varphi^2(\partial_s x^1)^2 + 8\varphi^2(x^1)^2 - 32\varphi^2 x^1(\partial_s x^1) + \frac{4}{3}\varphi^4 + \frac{16}{3}\varphi^3(\partial_t\varphi) + 8\varphi^2(\partial_t\varphi)^2 \\
& + \frac{16}{3}\varphi^3(\partial_s\varphi) + 8\varphi^2(\partial_s\varphi)^2 + 8\varphi^2(\partial_t z^a\partial_t\bar{z}_a + \partial_s z^a\partial_s\bar{z}_a) + \frac{1}{3}\Big[\bar{z}_a\partial_t z^a\bar{z}_b\partial_t z^b + z^a\partial_t\bar{z}_a z^b\partial_t\bar{z}_b \\
& - z^b\bar{z}_b\partial_t z^a\partial_t\bar{z}_a - \bar{z}_a z^b\partial_t z^a\partial_t\bar{z}_b + \bar{z}_a\partial_s z^a\bar{z}_b\partial_s z^b + z^a\partial_s\bar{z}_a z^b\partial_s\bar{z}_b - z^b\bar{z}_b\partial_s z^a\partial_s\bar{z}_a - \bar{z}_a z^b\partial_s z^a\partial_s\bar{z}_b\Big] \\
& - 4i\partial_t\bar{z}_a(z^a\eta_b\bar{\eta}^b + \bar{\eta}^a z^b\eta_b) - 4i\varepsilon^{acb}\partial_t\bar{z}_a\bar{z}_c\eta_b\bar{\eta}^4 - 2i\varepsilon_{acb}\partial_t z^a z^c\bar{\eta}^b\eta_4 + 4i(\theta_4\bar{\theta}^4 + \eta_4\bar{\eta}^4)(\partial_t z^b\bar{z}_b - \partial_t\bar{z}_b z^b) \\
& + 8\Big[(\eta_a\bar{\eta}^a)^2 + \varepsilon_{abc}\bar{\eta}^a\bar{\eta}^b\bar{\eta}^c\bar{\eta}^4 + \varepsilon^{abc}\eta_a\eta_b\eta_c\eta_4 + 2\eta_4\bar{\eta}^4(\eta_a\bar{\eta}^a - \theta_4\bar{\theta}^4)\Big] + i\Big\{ + 2z^a\bar{z}_a\bar{\eta}_b\partial_s\theta_b - z^a\bar{z}_a\bar{\eta}^b\theta_b \\
& - 2\bar{\eta}^a\bar{z}_a z^b\partial_s\theta_b + \bar{\eta}^a\bar{z}_a z^b\theta_b - 8i\varepsilon_{acb}\varphi z^c\bar{\eta}^b\partial_s\bar{\theta}^a + 4i\varepsilon_{acb}\varphi z^c\bar{\eta}^b\bar{\theta}^a + 16\varphi^2\eta_a\partial_s\bar{\theta}^a - 8\varphi^2\eta_a\bar{\theta}^a \\
& - 2\eta_a\partial_s\bar{\theta}^a|z|^2 + \eta_a\bar{\theta}^a|z|^2 + 2\eta_a\partial_s\bar{\theta}^c\bar{z}_c z^a - \eta_a\bar{\theta}^c\bar{z}_c z^a + 8i\varphi\eta_a\varepsilon^{acb}\bar{z}_c\partial_s\theta_b - 4i\varphi\eta_a\varepsilon^{acb}\bar{z}_c\theta_b + c.c. \\
& + 8\varphi^2(\partial_s\theta_4\bar{\eta}^4 - \partial_s\eta_4\bar{\theta}^4 + \eta_4\partial_s\bar{\theta}^4 - \theta_4\partial_s\bar{\eta}^4) + 8i\varepsilon_{acb}z^c\bar{\eta}^b\bar{\eta}^a\partial_s x^1 - 4i\varepsilon_{acb}z^c\bar{\eta}^b\bar{\eta}^a x^1 \\
& - 8i\varepsilon^{adc}\eta_a\bar{z}_d\eta_c\partial_s x^1 + 4i\varepsilon^{adc}\eta_a\bar{z}_d\eta_c x^1 - 48\varphi\eta_a\bar{\eta}^a\partial_s x^1 + 24\varphi\eta_a\bar{\eta}^a x^1 - 24\varphi\theta_4\bar{\theta}^4\partial_s x^1 \\
& . + 12\varphi\theta_4\bar{\theta}^4 x^1 - 24\varphi\eta_4\bar{\eta}^4\partial_s x^1 + 12\varphi\eta_4\bar{\eta}^4 x^1 - 4\varepsilon^{acb}\partial_s\bar{z}_a\bar{z}_c\eta_b\bar{\theta}^4 - 4\varepsilon_{acb}\partial_s z^a z^c\bar{\eta}^b\theta^4 \\
& + 16i\varphi\partial_s\bar{z}_a\bar{\eta}^a\bar{\theta}^4 + 16i\varphi\partial_s z^a\eta_a\theta_4 + 4\Big[\theta_4\bar{\eta}^4\partial_s z^b\bar{z}_b - \theta_4\bar{\eta}^4\partial_s\bar{z}_b z^b - \eta_4\bar{\theta}^4\partial_s z^b\bar{z}_b + \eta_4\bar{\theta}^4\partial_s\bar{z}_b z^b\Big]\Big\}
\end{aligned}$$

Two loops

At two loops, possible topologies of connected vacuum diagrams are sunset, double bubble, double tadpole



where vertices carry up to two derivatives.
Finiteness is not obvious, each diagram is separately divergent.

Two loops

- Some simplification occurring from bosonic propagators being diagonal (a feature of this gauge).

- Standard reduction allows to rewrite every integral as linear combination of the two scalar integrals

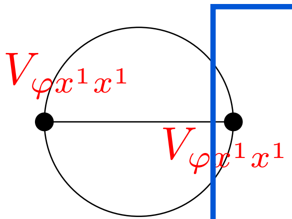
$$I\left(m^2\right) \equiv \int \frac{d^2p}{(2\pi)^2} \frac{1}{p^2+m^2}$$

$$I\left(m_1^2, m_2^2, m_3^2\right) \equiv \int \frac{d^2p\, d^2q\, d^2r}{(2\pi)^4} \frac{\delta^{(2)}(p+q+r)}{(p^2+m_1^2)(q^2+m_2^2)(r^2+m_3^2)}$$

- In fact, the sum of all (remaining) divergent integrals cancel out in the computation! **no need to pick up an explicit regularization scheme** to compute them.

Two loops

Only (two of the) cubic vertices give finite contributions



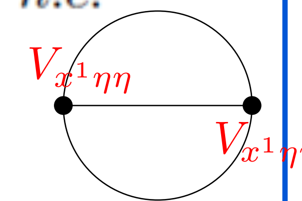
$$V_{\varphi x^1 x^1} = -4\,\varphi\,\left[(\partial_s - \tfrac{1}{2})\,x^1\right]^2 \qquad V_{\varphi^3} = 2\varphi\left[(\partial_t\varphi)^2 - (\partial_s\varphi)^2\right] \qquad V_{\varphi|z|^2} = 2\varphi\left[|\partial_t z|^2 - |\partial_s z|^2\right]$$

$$V_{z\eta\eta} = -\epsilon^{abc}\partial_t \bar{z}_a \eta_b \eta_c + h.c.$$

$$V_{z\eta\theta} = -2\,\epsilon^{abc}\bar{z}_a \eta_b(\partial_s - \tfrac{1}{2})\theta_c - h.c.$$

$$V_{\varphi\eta\theta} = -4\,i\,\varphi\,\eta_a(\partial_s - \tfrac{1}{2})\bar{\theta}^a - h.c.$$

$$V_{x^1\eta\eta} = -4\,i\,\bar{\eta}^a\eta_a(\partial_s - \tfrac{1}{2})x^1$$



$$V_{z\eta_a\eta_4} = -2\,\partial_t z^a \eta_a \eta_4 + h.c.$$

$$V_{z\eta_a\theta_4} = 2\,\partial_s z^a \eta_a \theta_4 - h.c.$$

$$V_{\varphi\eta_4\bar{\theta}^4} = -2\,i\,\varphi\,(\bar{\theta}^4\partial_s\eta_4 - \partial_s\bar{\theta}^4\eta_4) - h.c.$$

$$V_{x^1\bar{\psi}^4\psi_4} = -2\,i\,(\bar{\eta}^4\eta_4 + \bar{\theta}^4\theta_4)(\partial_s - \tfrac{1}{2})x^1$$

$$V_{\varphi^2 x^1 x^1} = 16\,\varphi^2\left[(\partial_s - \tfrac{1}{2})\,x^1\right]^2$$

$$V_{\varphi^2|z|^2} = 4\,\varphi^2\left[|\partial_t z|^2 + |\partial_s z|^2\right]$$

$$V_{\eta^2\eta_4\bar{\eta}_4} = 8\,\bar{\eta}^4\eta_4\bar{\eta}^a\eta_a$$

$$V_{\eta^4} = 4(\bar{\eta}^a\eta_a)^2$$

$$V_{\eta_4\bar{\eta}_4\theta_4\bar{\theta}_4} = -8\,\bar{\eta}^4\eta_4\bar{\theta}^4\theta_4$$

$$V_{\eta^3\eta_4} = 4\,\epsilon^{abc}\eta_a\eta_b\eta_c\eta_4 + h.c.$$

$$V_{\varphi z\eta_a\theta_4} = -8\,\varphi\,\partial_s z^a\eta_a\theta_4 - h.c.$$

$$V_{zz\bar{\eta}^a\theta_4} = 2\,i\,\epsilon_{abc}\partial_s z^a z^b\bar{\eta}^c\theta_4 - h.c.$$

$$V_{\varphi x^1\eta\eta} = 24\,i\,\varphi\,\bar{\eta}^a\eta_a(\partial_s - \tfrac{1}{2})x^1$$

$$V_{\varphi^2\eta\theta} = 8\,i\,\varphi^2\,\eta_a(\partial_s - \tfrac{1}{2})\bar{\theta}^a - h.c.$$

$$V_{\varphi^4} = 4\,\varphi^2\left[(\partial_t\varphi)^2 + (\partial_s\varphi)^2 + \frac{1}{6}\varphi^2\right]$$

$$V_{\dot{z}\bar{z}\bar{\psi}^4\psi_4} = -2\,i\,(\bar{\eta}^4\eta_4 + \bar{\theta}^4\theta_4)\bar{z}_b\partial_t z^b + h.c.$$

$$V_{z'\bar{z}\bar{\psi}^4\psi_4} = -2\,i\,(\bar{\eta}^4\theta_4 - \bar{\theta}^4\eta_4)\bar{z}_b\partial_s z^b - h.c.$$

$$V_{\varphi^2\eta_4\bar{\theta}^4} = 4\,i\,\varphi^2\,(\bar{\theta}^4\partial_s\eta_4 - \partial_s\bar{\theta}^4\eta_4) - h.c.$$

$$V_{\varphi x^1\bar{\psi}^4\psi_4} = 12\,i\,\varphi\,(\bar{\eta}^4\eta_4 + \bar{\theta}^4\theta_4)(\partial_s - \tfrac{1}{2})x^1$$

$$V_{zz\bar{\eta}^a\eta_4} = -2\,i\,\epsilon_{abc}\partial_t z^a z^b\bar{\eta}^c\eta_4 + h.c.$$

$$V_{\varphi z\eta\theta} = 8\,\varphi\epsilon^{abc}\bar{z}_a\eta_b(\partial_s - \tfrac{1}{2})\theta_c - h.c.$$

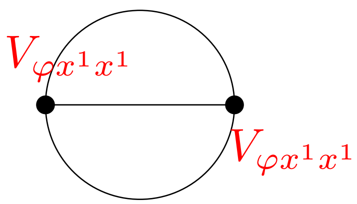
$$V_{zz\eta\eta} = -2\,i\,(\bar{z}_a\partial_t z^a\bar{\eta}^b\eta_b - \bar{z}_b\partial_t z^a\bar{\eta}^b\eta_a) + h.c.$$

$$V_{zz\eta\theta} = -2\,i\,\left[|z|^2\eta_a(\partial_s - \tfrac{1}{2})\bar{\theta}^a - \bar{z}_b z^a\eta_a(\partial_s - \tfrac{1}{2})\bar{\theta}^b\right] - h.c.$$

$$V_{x^1 z\eta\eta} = -4\,(\partial_s - \tfrac{1}{2})x^1\epsilon^{abc}\bar{z}_a\eta_b\eta_c - h.c.$$

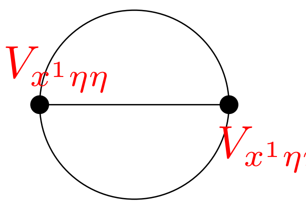
All other terms serve to cancel divergences.

Two loops



$$V_{\varphi x^1 x^1} = -4\,\varphi\,\left[(\partial_s - \tfrac{1}{2})\,x^1\right]^2 \longrightarrow \frac{1}{2} I(1, \tfrac{1}{2}, \tfrac{1}{2})$$

$$-\frac{3}{8} I(\tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}) \longleftarrow V_{x^1 \eta\eta} = -4\, i\, \bar{\eta}^a \eta_a (\partial_s - \tfrac{1}{2}) x^1$$



Always in terms of a particular case of the general class

$$I\left(2\,m^2, m^2, m^2\right) = \frac{K}{8\,\pi^2\,m^2} \qquad\qquad K \equiv \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2}$$

responsible for the appearance of the Catalan constant *K*.

Two-loop result:

$$-\ln Z_2 = \frac{V_2}{T}\left[\frac{1}{2} I\left(1, \tfrac{1}{2}, \tfrac{1}{2}\right) - \frac{3}{8} I\left(\tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}\right)\right] = -\frac{1}{4}\frac{V_2}{T} I\left(1, \tfrac{1}{2}, \tfrac{1}{2}\right) = -\frac{K}{16\,\pi^2}\frac{V_2}{T}$$

Final result: ABJM cusp anomaly

- The two loop **ABJM** cusp anomaly at strong coupling

$$f_{\rm ABJM}(\lambda) = \sqrt{2\lambda} - \frac{5\log 2}{2\pi} - \left(\frac{K}{4\pi^2} + \frac{1}{24}\right)\frac{1}{\sqrt{2\lambda}} + \mathcal{O}\left(\lambda^{-1}\right)$$

Final result: ABJM cusp anomaly

- The two loop **ABJM** cusp anomaly at strong coupling $(\tilde{\lambda} \equiv \lambda - \frac{1}{24})$

$$f_{\rm ABJM}\left(\tilde{\lambda}\right) = \sqrt{2\tilde{\lambda}} - \frac{5\log 2}{2\pi} - \frac{K}{4\pi^2\sqrt{2\tilde{\lambda}}} + \mathcal{O}(\sqrt{\tilde{\lambda}})^{-2}$$

Striking similarity with the **N=4 SYM** result

$$f_{\rm YM}(\lambda_{\rm YM}) = \frac{\sqrt{\lambda_{\rm YM}}}{\pi} - \frac{3\log 2}{\pi} - \frac{K}{\pi\sqrt{\lambda_{\rm YM}}} + \mathcal{O}(\sqrt{\lambda_{\rm YM}})^{-2}$$

Different factors in front of same structures !

Final result: ABJM cusp anomaly

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Different factors in front of same structures **!**

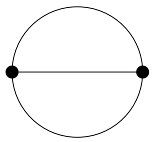
- Starting point (lagrangeans) look rather different, however:

> **Massless** fermions (main difference wrt to AdS_5xS^5 case) behave as effectively decoupled

> Fluctuations in $\mathbf{CP^3}$ behave as effectively decoupled.

> Mechanism of divergence cancellation very similar to AdS_5xS^5 case.

> Relevant cubic vertices are “the same“ in the two cases!



$$\frac{1}{2} I(1, \tfrac{1}{2}, \tfrac{1}{2})$$

Final result

- The two loop **ABJM** cusp anomaly at strong coupling $\quad (\tilde\lambda \equiv \lambda - \frac{1}{24}\,)$

$$f_{\rm ABJM}\left(\tilde\lambda\right) = \sqrt{2\tilde\lambda} - \frac{5\log 2}{2\pi} - \frac{K}{4\pi^2\sqrt{2\tilde\lambda}} + \mathcal{O}(\sqrt{\tilde\lambda})^{-2}$$

Striking similarity with the **N=4 SYM** result

$$f_{\rm YM}(\lambda_{\rm YM}) = \frac{\sqrt{\lambda_{\rm YM}}}{\pi} - \frac{3\log 2}{\pi} - \frac{K}{\pi\sqrt{\lambda_{\rm YM}}} + \mathcal{O}(\sqrt{\lambda_{\rm YM}})^{-2}$$

***Different factors in front of same structures*.**

we get for interpolating function at strong coupling

$$h(\lambda) = \sqrt{\frac{\lambda}{2}} - \frac{\log 2}{2\pi} - \frac{1}{48\sqrt{2\lambda}} + \mathcal{O}(\sqrt{\lambda})^{-2}$$

coincides with strong coupling expansion of [Gromov Sizov 2014] conjecture

$$h(\lambda) = \sqrt{\frac{1}{2}\left(\lambda - \frac{1}{24}\right)} - \frac{\log 2}{2\pi} + \mathcal{O}\left(e^{-2\pi\sqrt{2\lambda}}\right)$$

Concluding remarks & outlook

- ✓ Two-loop calculation of ABJM cusp anomaly at strong coupling.
- ✓ Quantum consistency (UV-finiteness) of this AdS_4xCP^3 action.
- ✓ First non-trivial perturbative check of $h(\lambda)$ at strong coupling.
- ✓ Indirect evidence of quantum integrability of Type IIA string in AdS_4xCP^3

Concluding remarks & outlook

✓ Two-loop calculation of ABJM cusp anomaly at strong coupling.

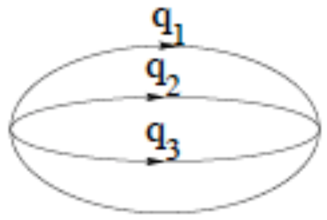
✓ Quantum consistency (UV-finiteness) of this AdS_4xCP^3 action.

✓ First non-trivial perturbative check of $h(\lambda)$ at strong coupling.

✓ Indirect evidence of quantum integrability of Type IIA string in AdS_4xCP^3

★ Three loop calculation: should involve products of $K \ln 2$ and ζ_3
Transcendentality properties studied, but yet unknown integrals.

Interesting for divergence cancellation, quantum integrability,
test of further "mapping" of AdS5xS5 model into AdS4xCP3.



★ Calculate $f(\lambda)$ in backgrounds relevant for the AdS_3/CFT_2 correspondence.

★ Finite coupling "stringy" test of $h(\lambda)$ could be via lattice, à la [McKeown, Roiban, 13]
partition function of the *discretized* AdS light-cone gauge action in the background of the null cusp solution.

EXTRAS

Other string backgrounds: $AdS_3 \times S^3 \times M^4$

- Three light-cone gauge-fixed string theories (Type IIB)

  - $AdS_3 \times S^3 \times T^4$ supported by pure RR flux
  - $AdS_3 \times S^3 \times S^3 \times S^1$ supported by pure RR flux
  - $AdS_3 \times S^3 \times T^4$ supported by a mix RR and NS NS fluxes

  relevant for the AdS_3/CFT_2 correspondence, interesting physics (e.g. BTZ black holes)

  Useful for connecting different working methods (CFT, integrability).

- Super-coset sigma models

$$\frac{PSU(1,1|2) \times PSU(1,1|2)}{SL(2) \times SU(2)} \times U(1)^4 \qquad \frac{D(2,1;\alpha) \times D(2,1;\alpha)}{SL(2) \times SU(2) \times SU(2)} \times U(1)$$

  with Z_4 automorphism -> classical integrability.

[Cagnazzo Zarembo 2011]
[Hoare Tseytlin 2012]

Perturbative structure of worldsheet S-matrix

Expansion of symmetry-determined and phase parts ($\theta^{(0)}$ absorbed in $T^{(0)}$)

$$\hat{S} = \mathbf{1} + i \sum_{n=1} g^{-n} \hat{T}^{(n-1)} \qquad \theta = \sum_{n=1}^{\infty} g^{-n} \theta^{(n-1)}$$

requires one-loop logarithms to contribute only to the diagonal terms

$$S = \mathbf{1} + \frac{i}{g}\hat{T}^{(0)} + \frac{i}{g^2}\left(\hat{T}^{(1)} + \theta^{(1)}\,\mathbf{1}\right) + \frac{1}{g^3}\left(\hat{T}^{(2)} + \frac{i}{2}\theta^{(1)}\hat{T}^{(0)} + \theta^{(2)}\,\mathbf{1}\right)$$

(and two-loop logarithms to be proportional to the tree-level S-matrix
- just the effect of two loop exponentiation - as $\theta^{(2)}$ has no logs)

Goal: compute one loop worldsheet S-matrix
"bootstrapping" it from tree level.

sigma-model

Superstring action

$$S = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \sqrt{-h}\, h^{ab} G_{MN}(X) \partial_a X^M \partial_b X^N + \text{fermions}$$

- Green-Schwarz formulation for fermions $\varrho_a = \partial_a x^\mu E_\mu{}^A \Gamma_A$

  quadratic part $L_F = i(\sqrt{-g} g^{ab} \delta^{IJ} - \epsilon^{ab} s^{IJ}) \bar{\theta}^I \varrho_a D_b \theta^J$

$$D_a \theta^I = \left(\partial_a + \frac{1}{4} \partial_a x^\mu \omega_\mu{}^{AB} \Gamma_{AB} \right) \theta^I + \frac{1}{2} \varrho_a \Gamma_{01234} \epsilon^{IJ} \theta^J$$

- Classical limit: $\lambda \to \infty$ Sigma model coupling constant: $\hat{g} = \frac{2\pi}{\sqrt{\lambda}}$

- Gauge-fixed lagrangean involves rescaling $-\frac{\pi J_+}{\sqrt{\lambda}} < \sigma < \frac{\pi J_+}{\sqrt{\lambda}}$

  Decompactification limit $\frac{J_+}{\sqrt{\lambda}} \to \infty$ and large tension expansion $\hat{g} \to \infty$

Gauge fixing

Use an interpolating lightcone -gauge

[Arutyunov Frolov Plefka Zamaklar 06]

$$X^+ = (1+a)\,t + a\,\varphi \equiv \tau + a\,\sigma$$

AdS5 (t)   S5 (φ)

$a = 1/2$ light-cone gauge

$a = 0$ temporal gauge

Transverse coordinates $z^\mu, \quad \mu = 1 \ldots 4 \qquad y^m, \quad m = 1 \ldots 4$

$$ds^2 = \underbrace{-G_{tt}(z)dt^2 + G_{zz}(z)dz^2}_{AdS_5} + \underbrace{G_{\varphi\varphi}(y)d\varphi^2 + G_{yy}(y)dy^2}_{S^5}$$

$$G_{tt} = \left(\frac{1+\frac{z^2}{4}}{1-\frac{z^2}{4}}\right)^2, \quad G_{zz} = \frac{1}{(1-\frac{z^2}{4})^2}, \quad G_{\varphi\varphi} = \left(\frac{1-\frac{y^2}{4}}{1+\frac{y^2}{4}}\right)^2, \quad G_{yy} = \frac{1}{\left(1+\frac{y^2}{4}\right)^2}.$$

Gauge choice preserves SO(8) at quadratic level, broken by interactions.

Interacting lagrangean

- Bosonic lagrangean to quartic order in the fields $X = (Y, Z)$

$$L = \frac{1}{2}(\partial_{\rm a} X)^2 - \frac{1}{2}X^2 + \frac{1}{4}Z^2(\partial_{\rm a} Z)^2 - \frac{1}{4}Y^2(\partial_{\rm a} Y)^2 + \frac{1}{4}(Y^2 - Z^2)\left(\dot{X}^2 + \acute{X}^2\right)$$
$$- \frac{1-2a}{8}(X^2)^2 + \frac{1-2a}{4}(\partial_{\rm a} X \cdot \partial_{\rm b} X)^2 - \frac{1-2a}{8}\left[(\partial_{\rm a} X)^2\right]^2 \ .$$

Lorentz invariance (quadratic part) broken by interactions.

Massive states with relativistic dispersion relation $\epsilon = \sqrt{1 + p^2}$

$$\epsilon = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}}$$

← loop corrections

Exact dispersion relation known via symmetries

p

(Scattering ws particles, for parametrically large momentum, become solitonic solutions - giant magnons - with $\epsilon \sim \frac{\sqrt{\lambda}}{\pi}\sin\frac{p}{2}$)

- Bosonic part invariant under $SO(4) \times SO(4)$.

Worldsheet fields

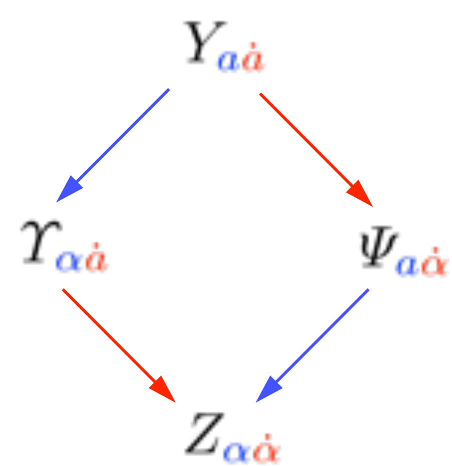
- Worldsheet fields (embedding coordinates in AdS_5xS^5)

$$T,\ \Phi,\ Y^m,\ Z^m,\ \text{fermions}$$

can be represented as bispinors $SO(4) \simeq (SU(2)\times SU(2))/\mathbb{Z}_2$

$$Y_{a\dot{a}} = (\sigma_m)_{a\dot{a}}\, Y^m, \qquad Z_{\alpha\dot{\alpha}} = (\sigma_\mu)_{\alpha\dot{\alpha}}\, Z^\mu \qquad a, \dot{a}, \alpha, \dot{\alpha} = 1, 2$$

- Bosons and fermions form bi-fundamental representation of $\mathrm{PSU}(2|2)_L \times \mathrm{PSU}(2|2)_R$



$\mathrm{PSU}(2,2)_L \times \mathrm{PSU}(2,2)_R$

$\underset{a}{\mathrm{SU}(2)} \times \underset{\dot{a}}{\mathrm{SU}(2)} \times \underset{\alpha}{\mathrm{SU}(2)} \times \underset{\dot{\alpha}}{\mathrm{SU}(2)}$

- Formal definition of a bi-fundamental supermultiplet $\Phi_{A\dot{A}}$, $A = (a|\alpha)$ $\dot{A} = (\dot{a}|\dot{\alpha})$ providing a basis for the definition of the S-matrix.

Worldsheet S-matrix

- Two-particle S-matrix is 256 x 256

$$\mathbb{S}\,|\Phi_{A\dot{A}}(p)\Phi_{B\dot{B}}(p')\rangle = |\Phi_{C\dot{C}}(p)\Phi_{D\dot{D}}(p')\rangle\,\mathbb{S}_{A\dot{A}B\dot{B}}^{C\dot{C}D\dot{D}}(p,p')$$

- Integrability predicts

$$\mathbb{S} = \mathbf{S}\otimes\mathbf{S} \quad , \quad \mathbb{S}_{A\dot{A}B\dot{B}}^{C\dot{C}D\dot{D}}(p,p') = \mathbf{S}_{AB}^{CD}(p,p')\mathbf{S}_{\dot{A}\dot{B}}^{\dot{C}\dot{D}}(p,p')$$

- S-matrices parametrized in terms of the basic SU(2) invariants

$$S_{AB}^{CD} = \begin{cases} A\delta_a^c\delta_b^d + B\delta_a^d\delta_b^c \\ D\delta_\alpha^\gamma\delta_\beta^\delta + E\delta_\alpha^\delta\delta_\beta^\gamma \\ C\epsilon_{ab}\epsilon^{\gamma\delta} \qquad F\epsilon_{\alpha\beta}\epsilon^{cd} \\ G\delta_a^c\delta_\beta^\delta \qquad H\delta_a^d\delta_\beta^\gamma \\ L\delta_\alpha^\gamma\delta_b^d \qquad K\delta_\alpha^\delta\delta_b^c \end{cases}$$

and similar for the dotted one.

Worldsheet S-matrix: explicit perturbative evaluation

- Expansion of worldsheet S-matrix in coupling: defines the T-matrix

$$\mathbb{S} = \mathbb{1} + \frac{1}{\hat{g}}\mathbb{T}^{(0)} + \frac{1}{\hat{g}^2}\mathbb{T}^{(1)} + \ldots = \mathbb{1} + \mathbb{T} \qquad \hat{g} = \frac{\sqrt{\lambda}}{2\pi}$$

- **Tree level result**: first non trivial order in the perturbative expansion
Obtained applying LSZ reduction to quartic vertices of the lagrangean.

[Klose McLoughlin Roiban Zarembo 06]

$$\mathbb{T}^{(0)}_{YY\to YY} = \frac{1}{2}\left[(1-2a)(\varepsilon' p - \varepsilon p') + \frac{(p-p')^2}{\varepsilon' p - \varepsilon p'}\right]\mathbb{1}\otimes\mathbb{1} + \frac{pp'}{\varepsilon' p - \varepsilon p'}(\mathbb{1}\otimes P + P\otimes\mathbb{1})$$

✓ Coincide with the related expansion of the exact spin chain S-matrix.

✓ A test of group factorization

- **One-loop result** via standard Feynman diagrammatics: not existing!
unsuccessful attempts (non-cancellation of UV divergences).

[McLoughlin Roiban 07]

Worldsheet S-matrix at one loop via unitarity cuts: result

$$S_{AB}^{CD}(\mathrm{p}_1,\mathrm{p}_2) = \exp\left(i\varphi_a(\mathrm{p}_1,\mathrm{p}_2)\right)\ \tilde{S}_{AB}^{CD}$$
$$= \exp\left(-\frac{i}{2\hat{g}}(e_2\mathrm{p}_1 - e_1\mathrm{p}_2)(a-\tfrac{1}{2}) + \frac{i}{\hat{g}^2}\tilde{\varphi}(\mathrm{p}_1,\mathrm{p}_2)\right)\ \tilde{S}_{AB}^{CD} + \mathcal{O}\left(\frac{1}{\hat{g}^3}\right)$$

where

Ex. $$A^{(1)} = 1 + \frac{i}{4\,\hat{g}}\frac{(\mathrm{p}_1-\mathrm{p}_2)^2}{\epsilon_2\mathrm{p}_1 - \epsilon_1\mathrm{p}_2} + \frac{1}{4\,\hat{g}^2}\left(\mathrm{p}_1\mathrm{p}_2 - \frac{(\mathrm{p}_1+\mathrm{p}_2)^4}{8(\epsilon_2\mathrm{p}_1-\epsilon_1\mathrm{p}_2)^2}\right)$$

and

$$\tilde{\varphi}(\mathrm{p}_1,\mathrm{p}_2) = \frac{1}{2\pi}\frac{\mathrm{p}_1^2\mathrm{p}_2^2\left((\epsilon_2\mathrm{p}_1-\epsilon_1\mathrm{p}_2) - (\epsilon_1\epsilon_2 - \mathrm{p}_1\mathrm{p}_2)\,\mathrm{arsinh}[\epsilon_2\mathrm{p}_1-\epsilon_1\mathrm{p}_2]\right)}{(\epsilon_2\mathrm{p}_1-\epsilon_1\mathrm{p}_2)^2}$$

- ✓ All logarithmic dependence encoded in the scalar factor (as required from integrability!)
- ✓ All gauge dependence encoded in the scalar factor (as required from physical arguments!)
- ✓ All rational dependence coincides with related expansion of EXACT worldsheet S-matrix