Generalized Entropy and higher derivative Gravity

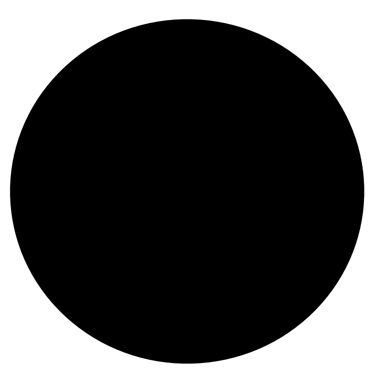
Joan Camps
DAMTP, Cambridge

based on 1310.6659, see also 1310.5713 by Xi Dong

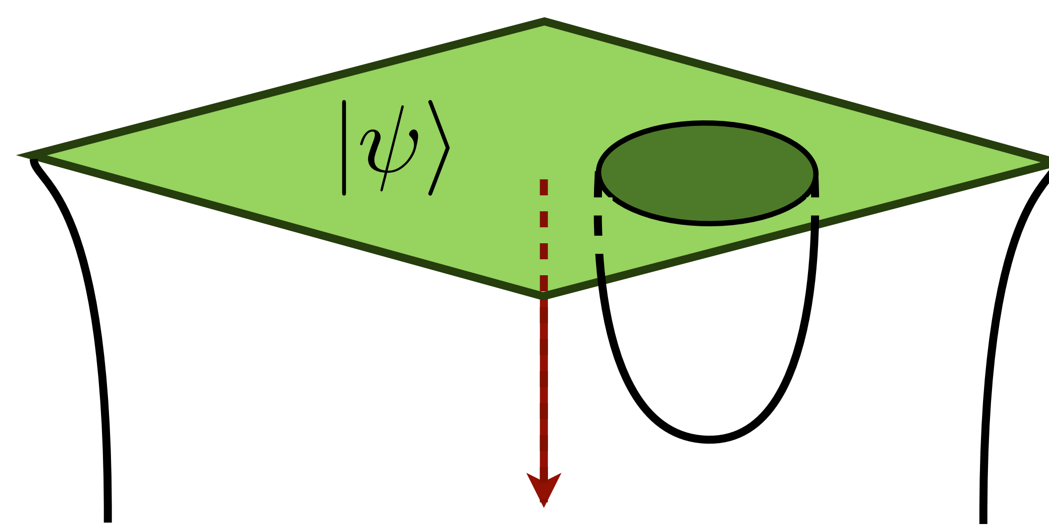
The String Theory Universe, Mainz, 23 September 2014

Entropy in General Relativity

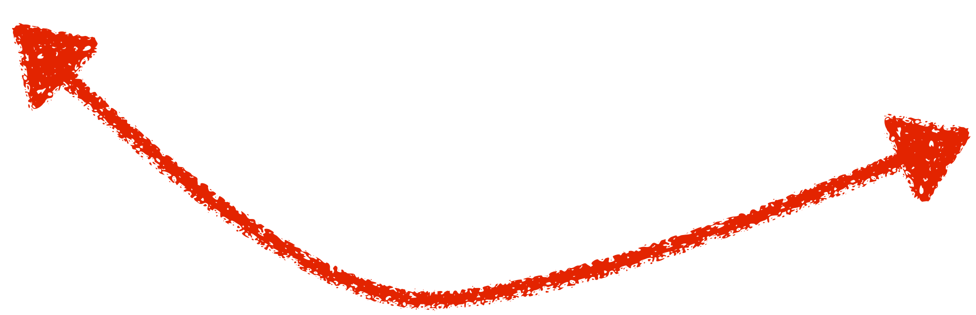
$$I = \frac{1}{16\pi G}\int \sqrt{-g}d^D x\, R$$



$$S = \frac{(\text{Area})_{D-2}}{4G}$$



Bekenstein,
Hawking



Ryu, Takayanagi

Maldacena, Lewkowycz

Why higher derivatives

- Such corrections are predicted in String Theory (essential to the great success in microscopic accounts of entropy)

$$I = \frac{1}{16\pi G}\int \sqrt{-g}d^D x\ \left(R + \alpha'\mathrm{Riem}^2 + \dots\right)$$

- Deformations of AdS/CFT: playground, robustness, universality $\quad \eta/s \qquad c \neq a$
- It is an interesting problem *per se*

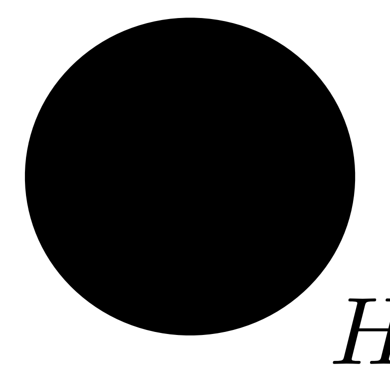
BH Entropy for higher derivative theories

Wald

$$I = \int \sqrt{-g} d^D x \, \mathcal{L}(R_{\mu\nu\rho\sigma}, \nabla_\mu, g_{\mu\nu})$$

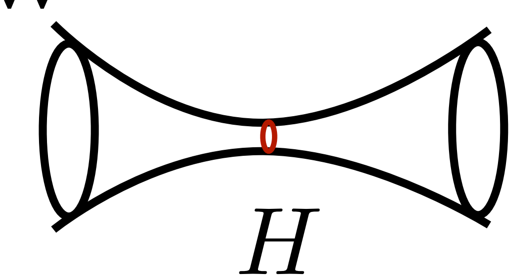
- Black holes enjoy a first law, with entropy

$$S \sim \int_H \sqrt{\gamma} d^{D-2}\sigma \, \frac{\partial \mathcal{L}}{\partial \mathrm{Riem}}$$



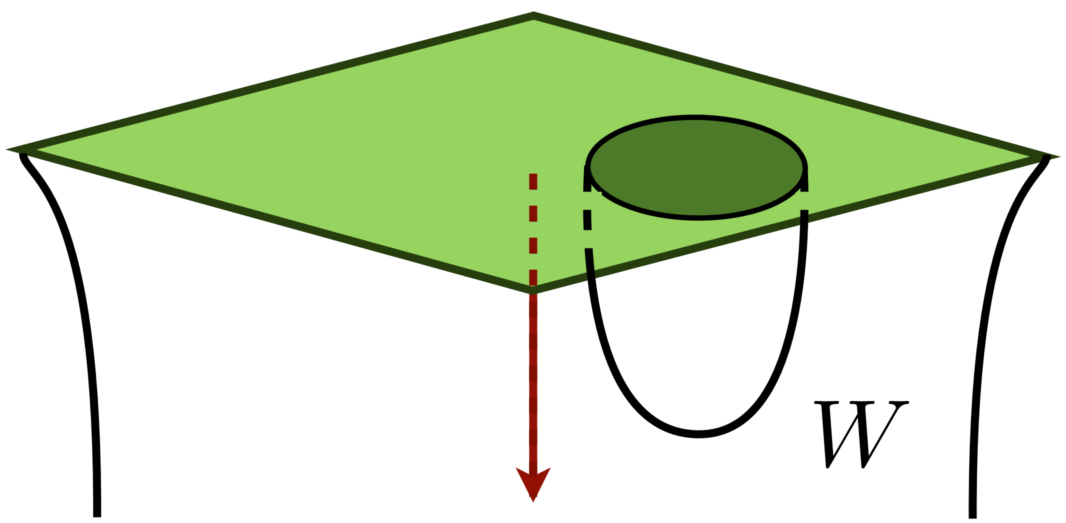
- Not known if they satisfy a second law
- Some proposals for non-stationary

Wald-Iyer



HEE & H Derivatives

- Wald entropy is not a good Holographic entanglement entropy (wrong divergences)



$$S \neq \int_W \sqrt{\gamma} d^{D-2}\sigma \, \frac{\partial \mathcal{L}}{\partial \text{Riem}}$$

- For Lovelock theories, there is a proposal with the correct structure

de Boer et al
Myers et al
Fursaev et al

- That proposal coincides with Wald-Iyer's

Result

$$S \sim \int_W \sqrt{\gamma} d^{D-2}\sigma \left(\frac{\partial \mathcal{L}}{\partial \mathrm{Riem}} + \frac{\partial^2 \mathcal{L}}{\partial \mathrm{Riem}^2} K^2 \right)$$

- It reduces to Wald's for stationary cases (trivially)
- For Lovelock Gravity, it gives the Jacobson-Myers entropy functional

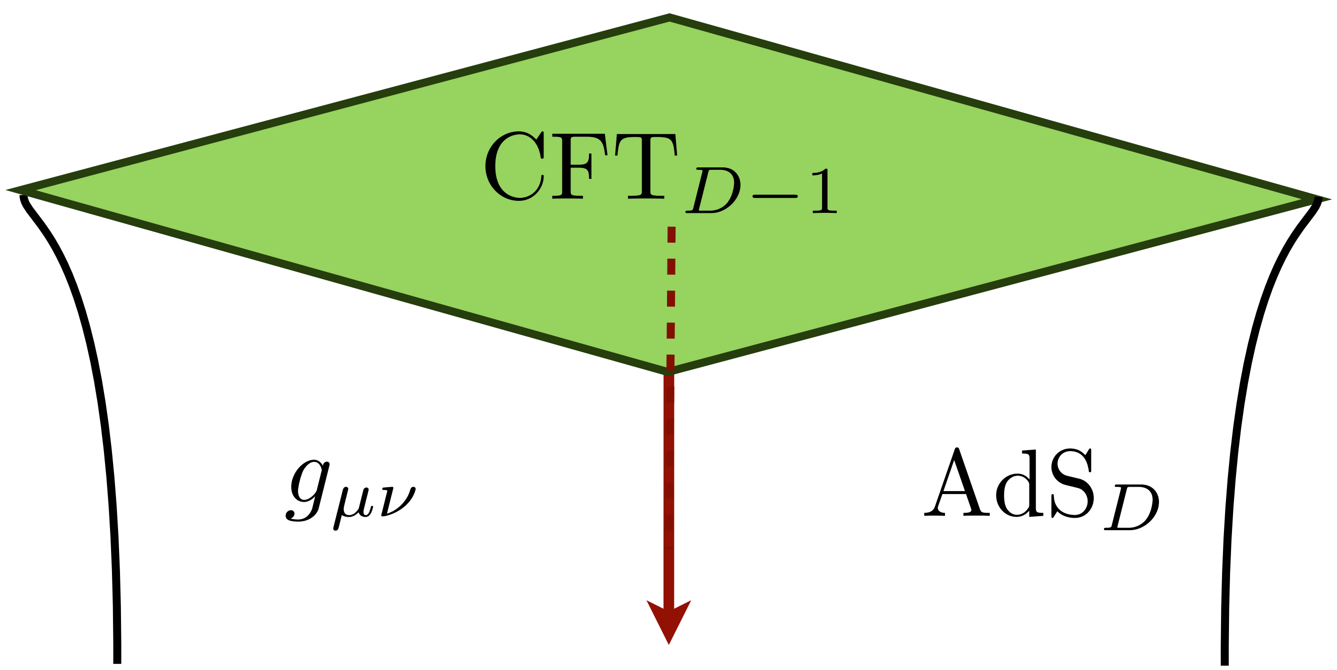
de Boer et al
Myers et al
Fursaev et al

- More generally, disagrees with Wald-Iyer's

Ingredients

Maldacena, Lewkowycz

- Saddle point approximation to holography



$$Z_{\mathrm{CFT}_{D-1}} \approx e^{-I_E[g_{\mu\nu}]}$$

- Replica trick

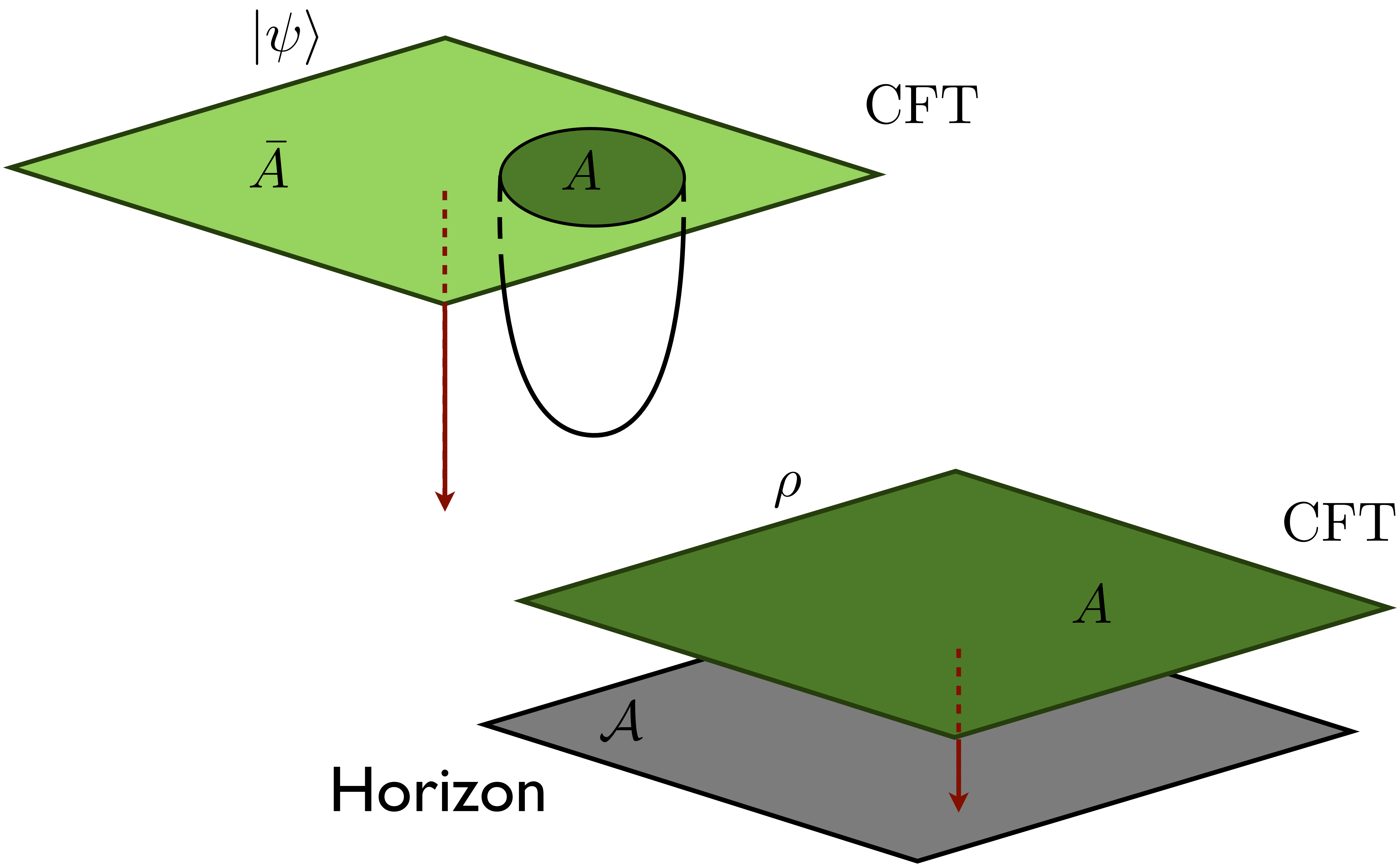
$$\mathrm{tr}(\rho \log \rho) = \lim_{n \to 1} \frac{\log(\mathrm{tr}\rho^n)}{n-1}$$

How to get there

$$S \sim \int_W \sqrt{\gamma} d^{D-2}\sigma \left(\frac{\partial \mathcal{L}}{\partial \mathrm{Riem}} + \frac{\partial^2 \mathcal{L}}{\partial \mathrm{Riem}^2} K^2 \right)$$

Setup

Casini, Huerta, Myers

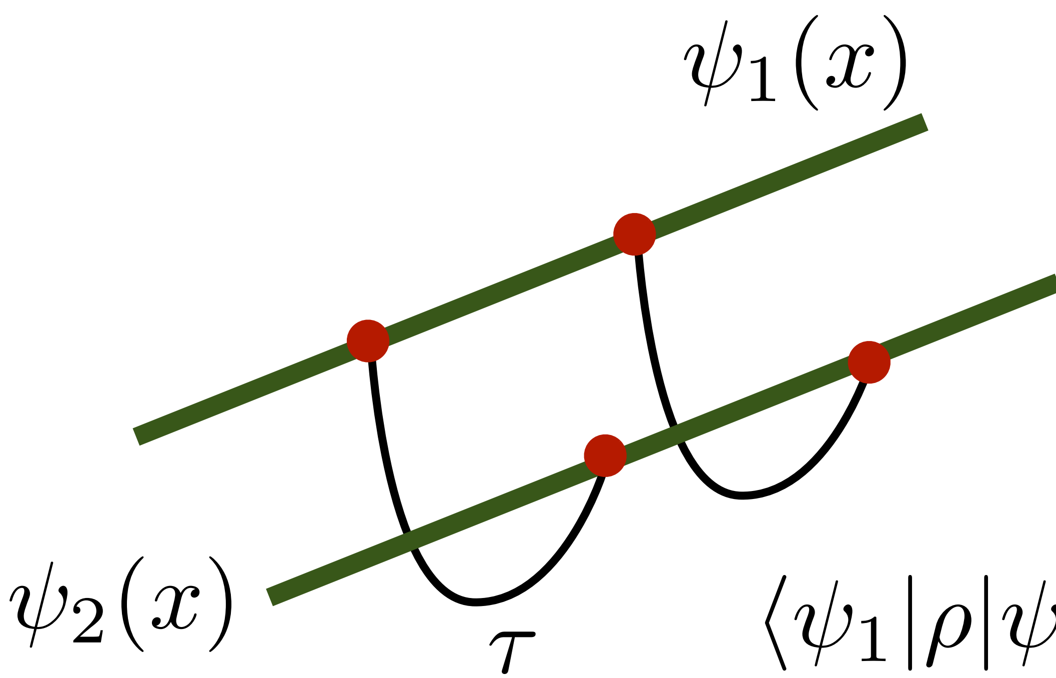


States considered

- Generated by Euclidean paths integrals

$$\rho = P e^{-\int H(\tau) d\tau}$$

$$\rho_T = \sum_k |k\rangle e^{-E_k \beta} \langle k|$$



Field theory directions

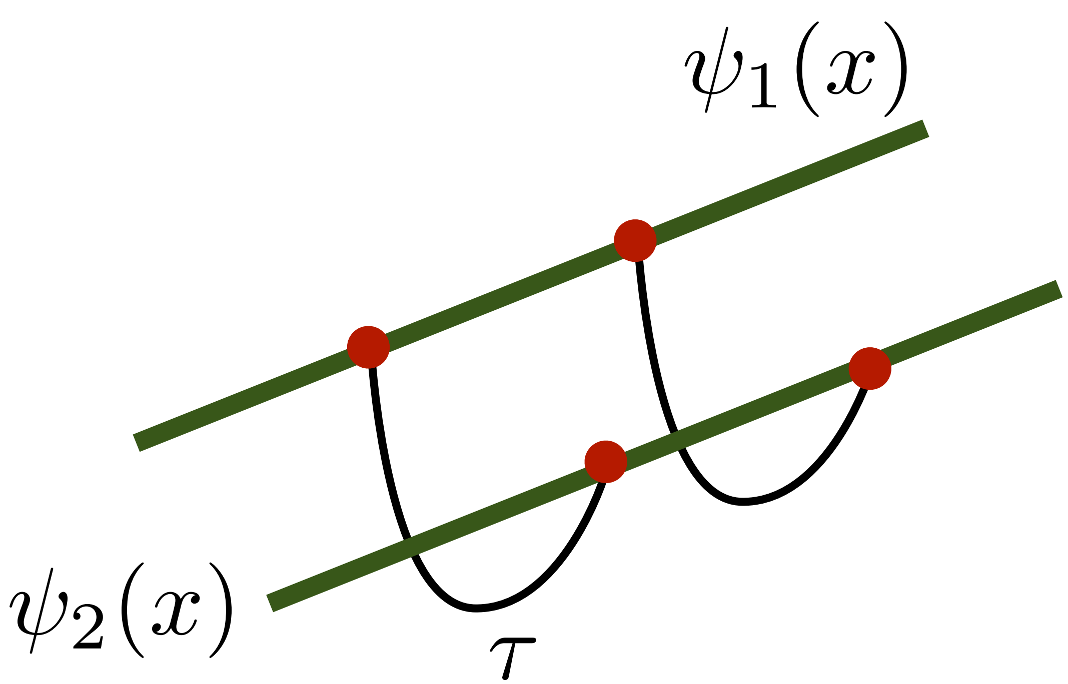
$\langle \psi_1 | \rho | \psi_2 \rangle$ Quantum amplitude

Euclidean time

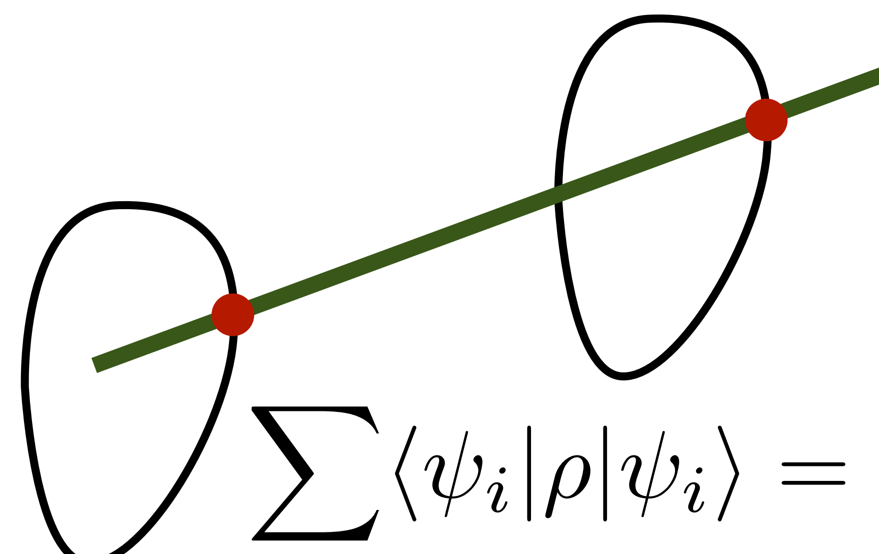
States considered

- Generated by Euclidean paths integrals

$$\rho = Pe^{-\int H(\tau)d\tau}$$

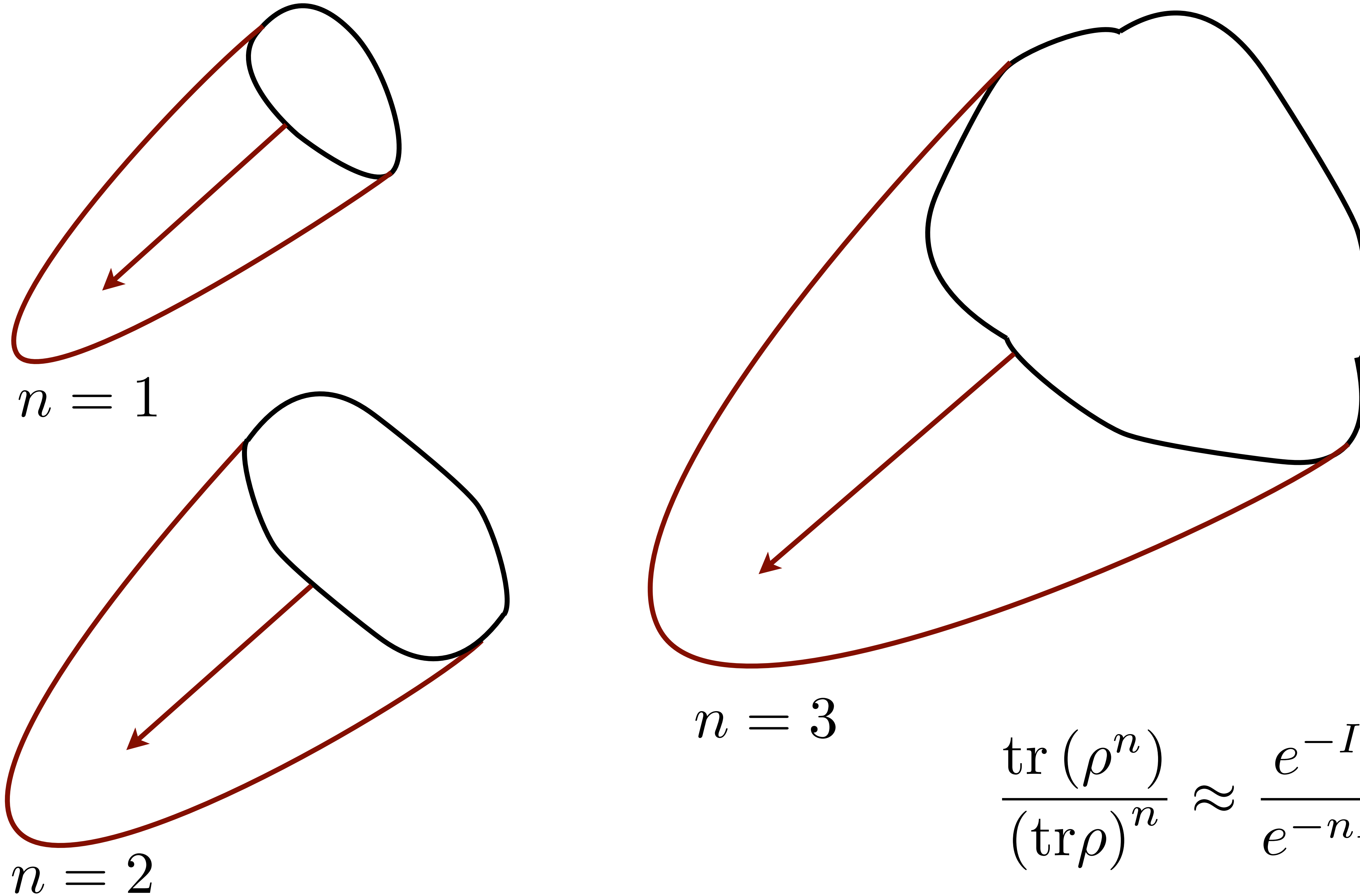


$$\rho_T = \sum_k |k\rangle e^{-E_k\beta}\langle k|$$



$$\sum_i \langle\psi_i|\rho|\psi_i\rangle = \mathrm{tr}\rho$$

Gravity dual



$$\frac{\mathrm{tr}\left(\rho^n\right)}{\left(\mathrm{tr}\rho\right)^n} \approx \frac{e^{-I_n}}{e^{-nI_1}}$$

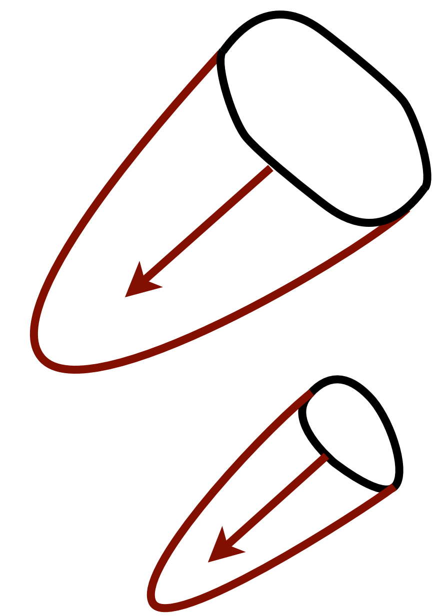
Generalized entropy

Entanglement entropy:
Replica trick

$$S = \lim_{n \to 1} S_n = - \, \partial_n \log \frac{\mathrm{tr}\,(\rho^n)}{(\mathrm{tr}\rho)^n}\bigg|_{n=1}$$

Gravity saddlepoint

$$\frac{\mathrm{tr}\,(\rho^n)}{(\mathrm{tr}\rho)^n} \approx \frac{e^{-I_n}}{e^{-nI_1}}$$



I_n

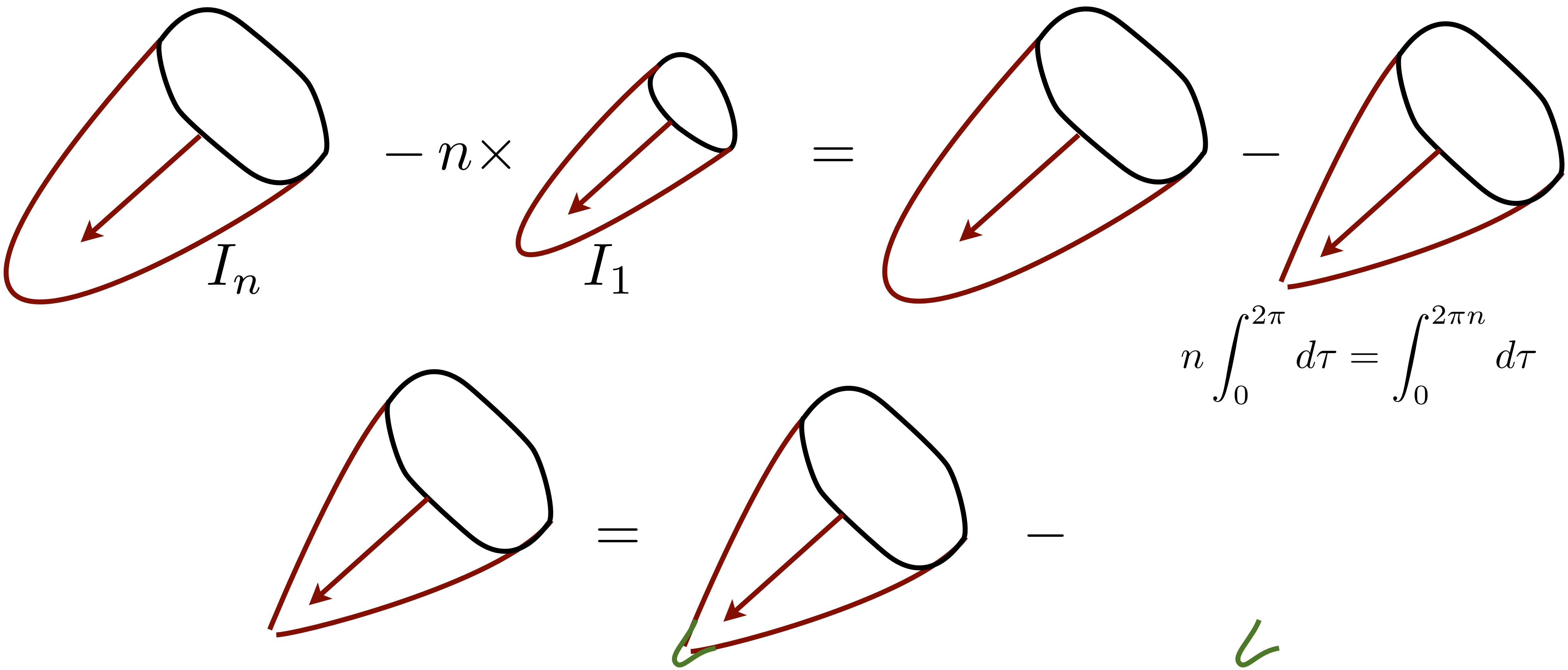
I_1

Gravitational entropy

$$\boxed{S = \partial_n \left(I_n - n I_1 \right)\big|_{n=1}}$$

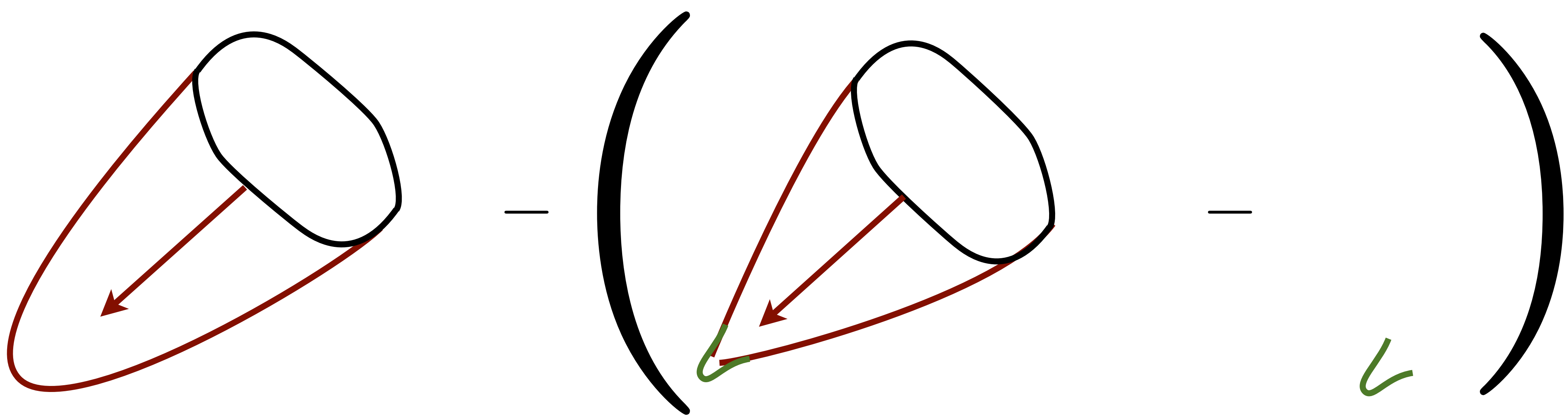
Replica manipulations

$$S = \partial_n \left(I_n - n I_1 \right)\Big|_{n=1}$$



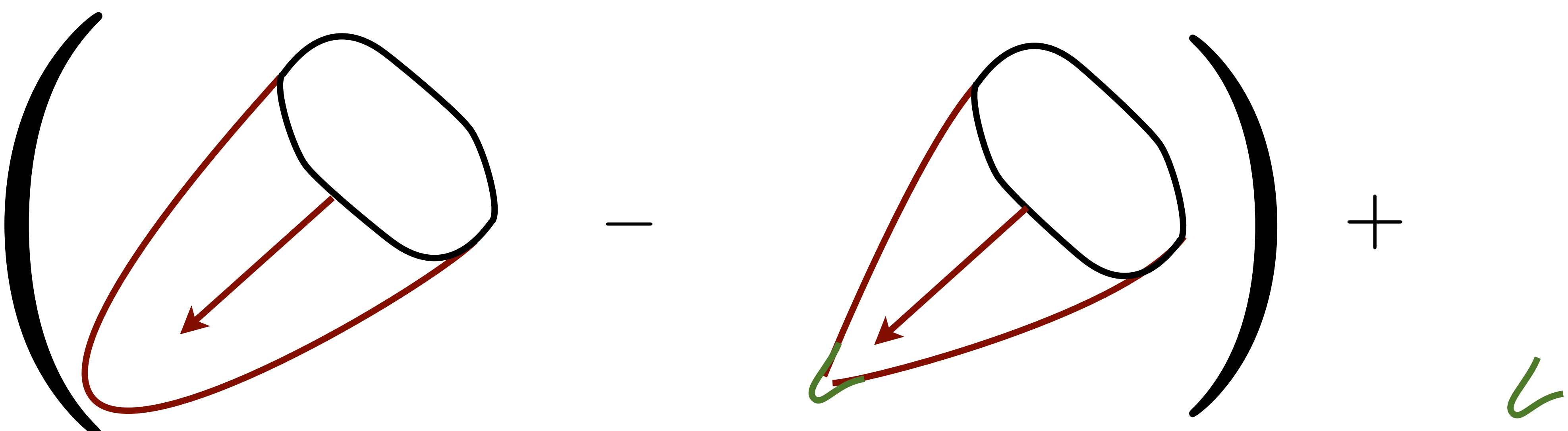
Replica manipulations

$$S = \partial_n \left(I_n - n I_1 \right)\big|_{n=1}$$



Replica manipulations

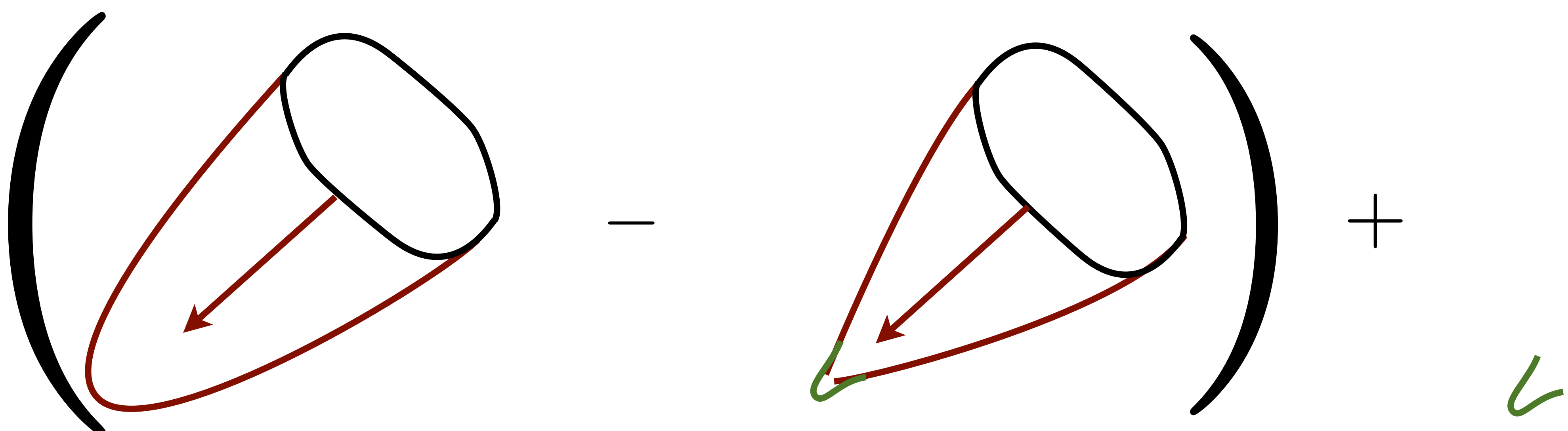
$$S = \partial_n \left(I_n - n I_1 \right) \Big|_{n=1}$$



$O\left((n-1)^2\right)$

Replica manipulations

$$S = \partial_n \left(I_n - n I_1 \right)\big|_{n=1}$$



$$O\left((n-1)^2\right)$$

$$S = \partial_n \left(I[\ ] \right)\big|_{n=1}$$

Recap

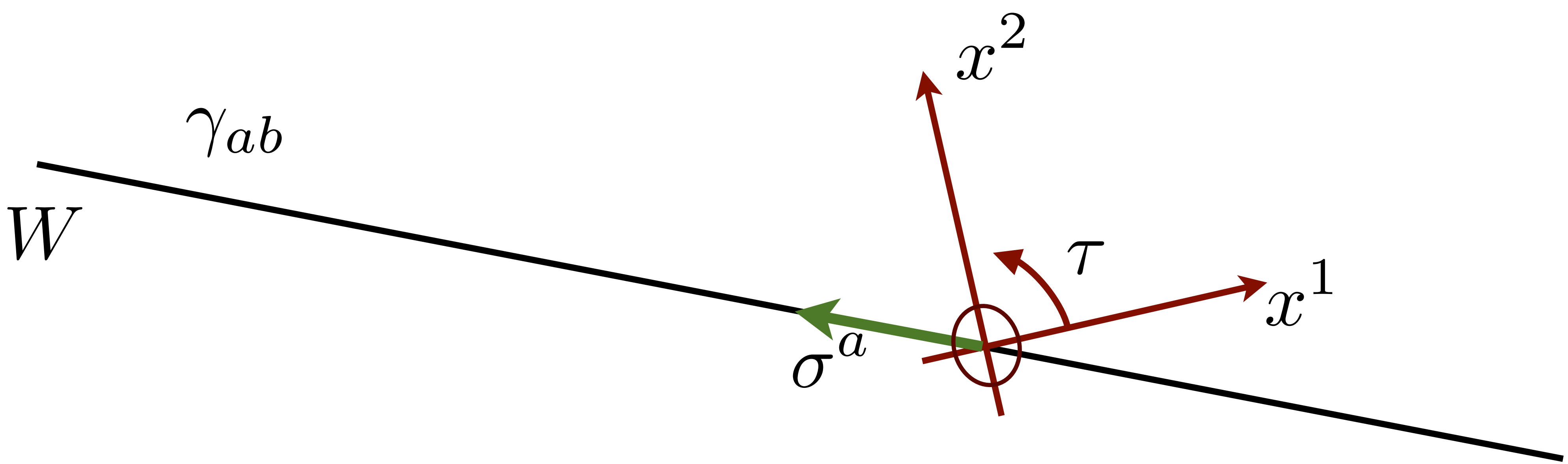
- Replica trick: EE and loops in eucl. time
- Assume 'holography' $Z_{\text{'FT'}} \approx e^{-I_n}$
- Entanglement entropy becomes:

$S = \partial_n \left(I[$  $]) \right|_{n=1}$

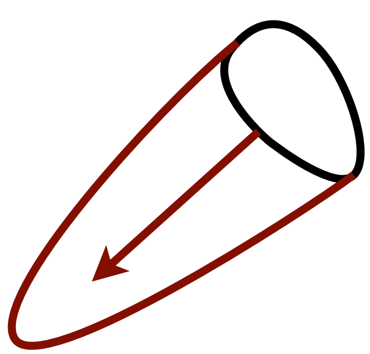
The calculation

$$S = \partial_n \left(I[\quad] \right)\Big|_{n=1}$$

Adapted coordinates

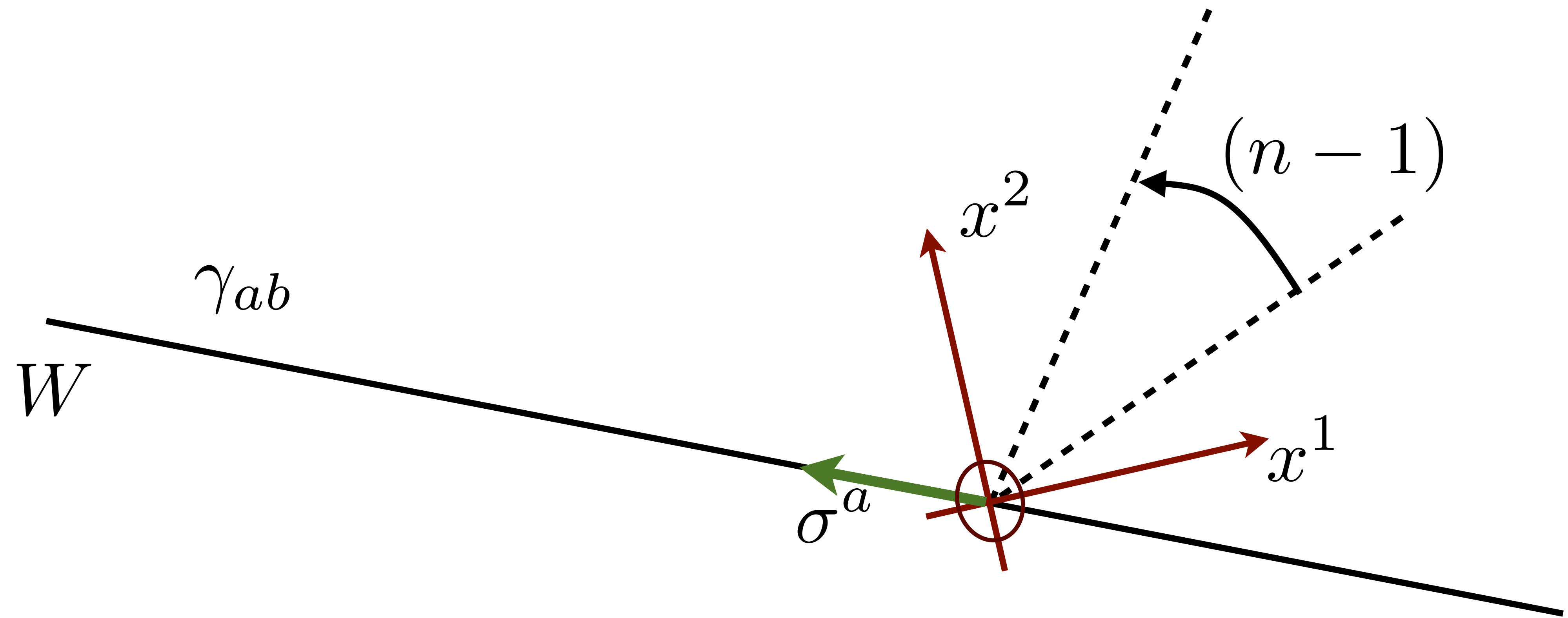


$$ds^2 = dr^2 + r^2 d\tau^2 + \gamma_{ab} d\sigma^a d\sigma^b + \ldots$$

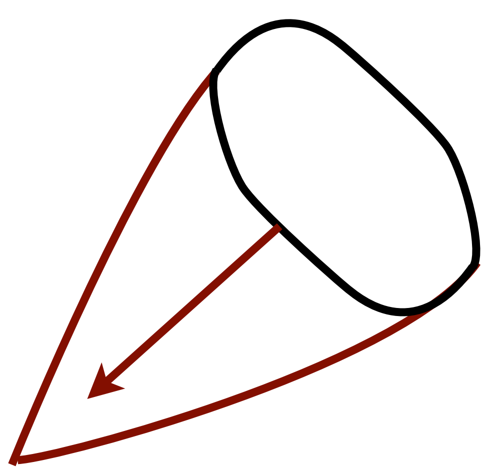


$$\tau \sim \tau + 2\pi$$

Adapted coordinates

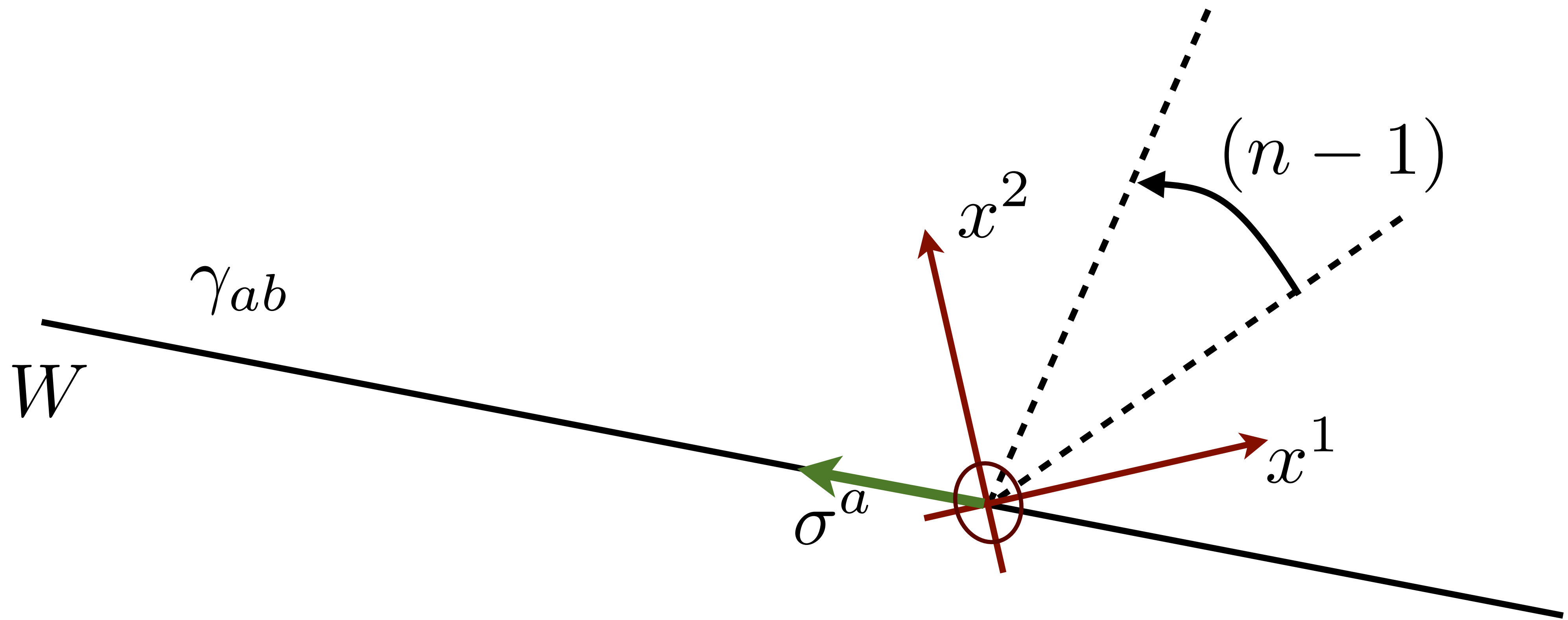


$$ds^2 = dr^2 + r^2 d\tau^2 + \gamma_{ab} d\sigma^a d\sigma^b + \ldots$$

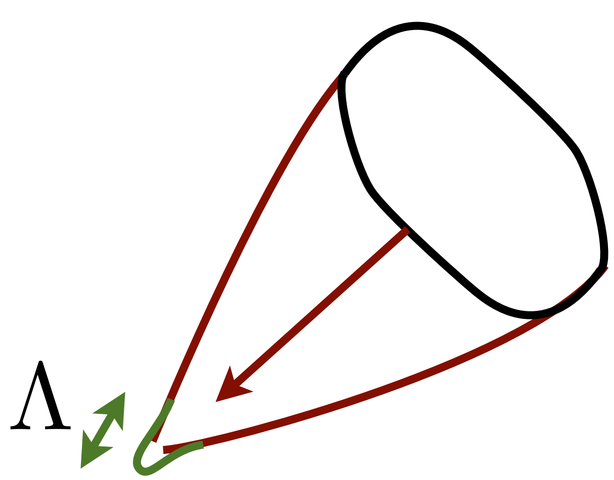


$$\tau \sim \tau + 2\pi n$$

Adapted coordinates



$$ds^2 = dr^2 + r^2 d\tau^2 + \gamma_{ab} d\sigma^a d\sigma^b + \ldots$$

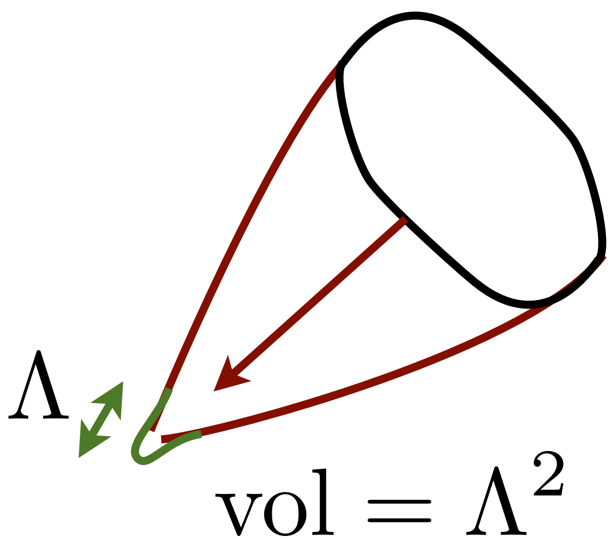


$$\tau \sim \tau + 2\pi n$$

Entropy

$$I = \int d^D x \mathcal{L}[\text{Riem}]$$

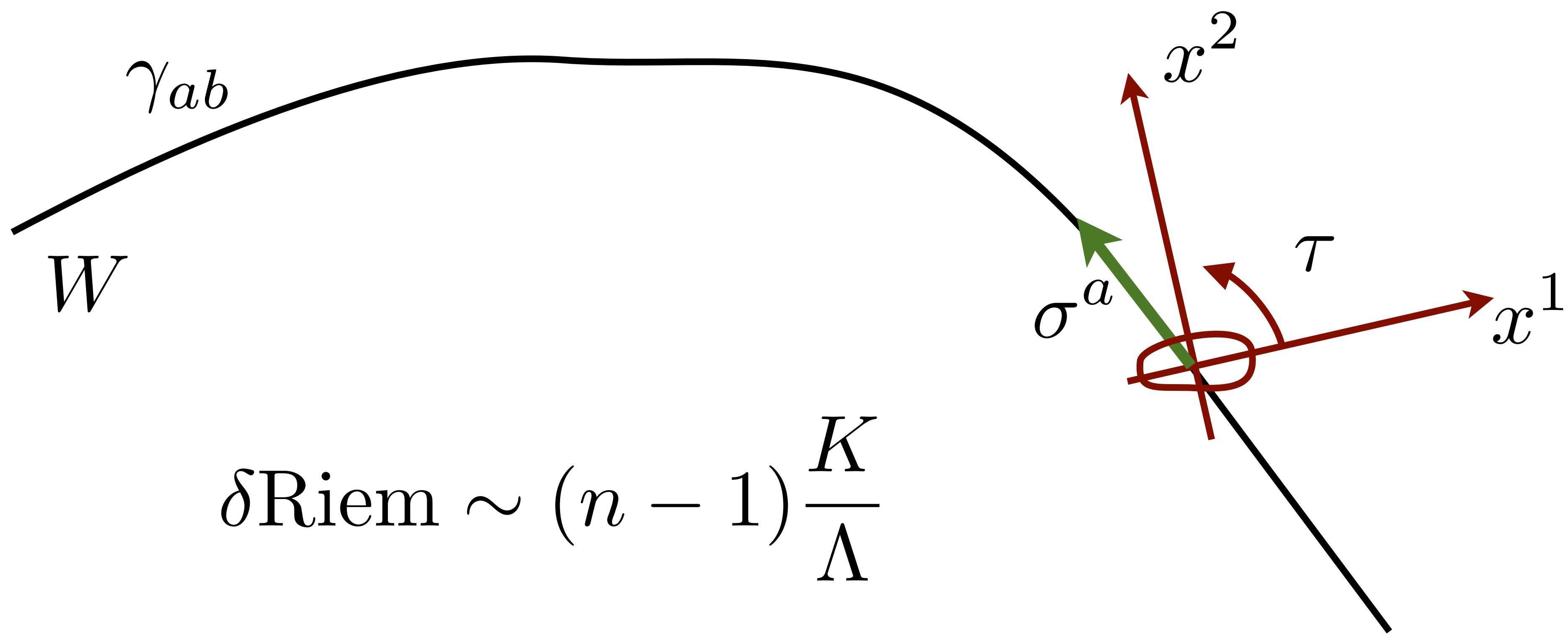
$$\Lambda^2 \int \sqrt{\gamma} d^{D-2}\sigma$$



$$\delta\text{Riem} \sim \frac{n-1}{\Lambda^2}$$

$$S = \partial_n \left(I[\;]\right)\big|_{n=1} = \int_W \sqrt{\gamma} d^{D-2}\sigma \frac{\partial \mathcal{L}}{\partial \text{Riem}}$$

Extrinsic curvature



$$\delta\mathrm{Riem} \sim (n-1)\frac{K}{\Lambda}$$

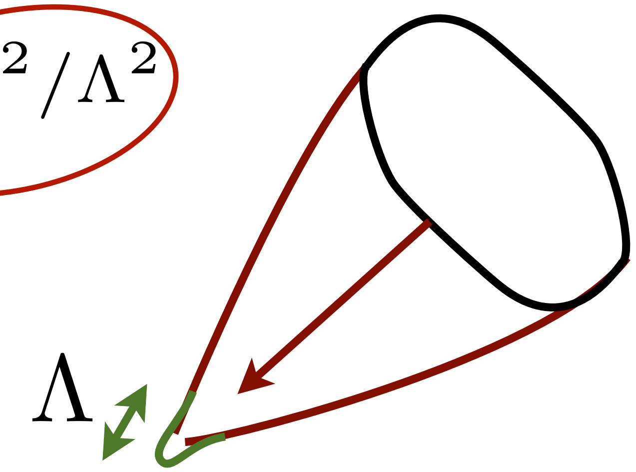
$\Lambda \to 0$

$$\delta S = \partial_n(\delta I[\;])|_{n=1} \sim \int_W \sqrt{\gamma}\, d^{D-2}\sigma\, \frac{\partial^2 \mathcal{L}}{\partial \mathrm{Riem}^2} K^2$$

The subtlety

$S = \partial_n \left(I[$  $]) \right|_{n=1}$

$$\lim_{n\to 1} \partial_n \int_0^\infty dr\, (n-1)^2 r^{2n-3} e^{-r^2/\Lambda^2}$$



$$\lim_{n\to 1} \partial_n \left((n-1)^2 \frac{\Gamma(n-1)}{2} \right) = \frac{1}{2}$$

$$\delta S \sim \int_W \sqrt{\gamma} d^{D-2}\sigma \, \frac{\partial^2 \mathcal{L}}{\partial \mathrm{Riem}^2} K^2$$

Comments on the new formula

$$S \sim \int_W \sqrt{\gamma} d^{D-2}\sigma \left(\frac{\partial \mathcal{L}}{\partial \text{Riem}} + \frac{\partial^2 \mathcal{L}}{\partial \text{Riem}^2} K^2 \right)$$

- It reduces to Wald's for stationary cases (trivially)
- For Lovelock Gravity, it gives the Jacobson-Myers entropy functional

de Boer et al
Myers et al
Fursaev et al

- More generally, disagrees with Wald-Iyer's

Final remarks & Open questions

- First principles derivation of Ryu-Takayanagi
- Euclidean space essential: Lorentzian?
- Non-stationary entropy? Second law?
- Nothing we did relies on replica symmetry; Where should we evaluate the entropy?

Cheng et al; Bhattacharyya et al; Dong; Erdmenger et al

Extra slides

On the location of W

- For Einstein Gravity, the LM construction locates W in GR by demanding regularity of a replica-symmetric ansatz, to $O(n-1)$
- A similar argument is not fully successful for more general theories
Cheng et al; Bhattacharyya et al; Dong; Erdmenger et al
- That may hint to Replica Symmetry Breaking
JC & William Kelly. In progress