The GS string and light-cone gauge in $\mathrm{AdS}_n \times \mathrm{S}^n \times \mathrm{M}^{10-2n}$

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Talk based on arXiv:1407.7883 (with R. Roiban, A. Tseytlin and L. Wulff)

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Motivation and stating the problem

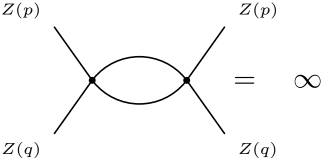
• AdS / CFT relates string theory on $AdS_n \times M^{10-n}$ with boundary CFT

• Surprisingly, many of the dual pairs seem to be *integrable*
Allows for exact algebraic solution for many quantities on both gauge and string theory side. For example, in this talk:

worldsheet S-matrix and magnon dispersion relation fixed by symmetry

$$S^{AB}(p,q), \qquad \epsilon(p) = \sqrt{m^2 + 4h^2(\lambda)\sin^2\frac{p}{2}}$$

• However, for light-cone gauge BMN string @ 1-loop [Roiban, McLoughlin '06]



• Somewhat unexpected since string partition function is finite....

In this talk I will discuss how to properly regularize the worldsheet theory in a way consistent with underlying integrability

• For AdS / CFT, gravitational dual is (sometimes partially) described by a supercoset-model. In decreasing order of symmetry:

$$AdS_5 \times S^5 : \qquad \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}$$

$$AdS_4 \times \mathbb{CP}_3 : \qquad \frac{OSP(4|6)}{SO(1,3) \times SU(3)}$$

$$AdS_3 \times S^3 \times S^3 \times S^1 : \qquad \frac{D(2,1;\alpha)^2}{SL(2) \times SU(2)^2} \times U(1)$$

$$AdS_3 \times S^3 \times T^4 : \qquad \frac{PSU(1,1|2)^2}{SL(2) \times SU(2)} \times U(1)^4$$

$$AdS_2 \times S^2 \times T^6 : \qquad \frac{PSU(1,1|2)}{SO(1,1) \times U(1)} \times U(1)^6$$

General form G/H $\times$ abelian factors. Non-coset directions corresponds to symmetries broken by the background and generally give rise to massless worldsheet excitations [Tseytlin, Metsaev; Babichenko, Stefanski, Zarembo; Sorokin, Tseytlin, Wulff, Zarembo]

Coset models and integrability

• Supergroups decomposes under $\mathbb{Z}_4$ grading [Berkovits, Bershadsky, Hauer, Zukov, Zwiebachl]

For $g \in G$ construct left-invariant (flat) current:

$$J = g^{-1}dg = J^{(0)} + J^{(2)} + J^{(1)} + J^{(3)}, \qquad dJ + J \wedge J = 0$$

which is the basic building block of Green-Schwarz string:

$$S = \sqrt{\lambda} \int d^2\sigma Str\left[\sqrt{-h}h^{ab}J_a^{(2)}J_b^{(2)} + \epsilon^{ab}J_a^{(1)}J_b^{(3)}\right]$$

• Furthermore, equations of motion of current admit a Lax representation

$$L_a = J_a^{(0)} + \frac{\mathbf{x}^2+1}{\mathbf{x}^2-1}J_a^{(2)} - \frac{2\mathbf{x}}{\mathbf{x}^2-1}\frac{1}{\sqrt{-h}}h_{ab}\epsilon^{bc}J_c^{(2)} + \ldots, \qquad \mathbf{x} \neq \pm 1$$

From Lax rep. classical integrability follows via monodromy matrix:

$$M(x) = P\exp\int_C dx^a L_a(x;\mathbf{x}), \qquad \det\left(z - M(x)\right)|_{\mathbf{x}\to\infty} \sim \sum_{i=0}^{\infty} Q_i f_i(\mathbf{x})$$

[See review by Beisert et. al.]

Light-cone gauge and BMN string

For simplicity I restrict to $AdS_5 \times S^5$ now. Discussion similar but a bit more involved for other backgrounds

- GS string highly non-linear. Somewhat linearized in light-cone gauge:

$$x^+ = \tau, \qquad p_- = \text{constant}, \qquad h^{ab} = \eta^{ab} + \mathcal{O}(\frac{1}{\sqrt{\lambda}}), \qquad \Gamma^+\Theta = 0$$

Action is still very complicated some simplifying limit needed.

- Consider fast moving string with large angular momenta J in S^5 such that $\sqrt{\lambda}/J$=constant. Expanding in inverse power of coupling gives [Berenstein, Maldacena, Nastase]

$$\mathcal{L} = \mathcal{L}_2 + \lambda^{-1/2}\mathcal{L}_4 + \lambda^{-1}\mathcal{L}_6 + \ldots$$

subscript denotes number of transverse d.o.f and leading order is a free *relativistic* massive KG + Dirac theory:

$$\mathcal{L}_2 = \frac{1}{2}(\partial X_i)^2 - \frac{1}{2}m_i^2 X_i^2 + \text{fermions}$$

$\Rightarrow$ worldsheet theory interacting 2D QFT.

S-matrix and phase

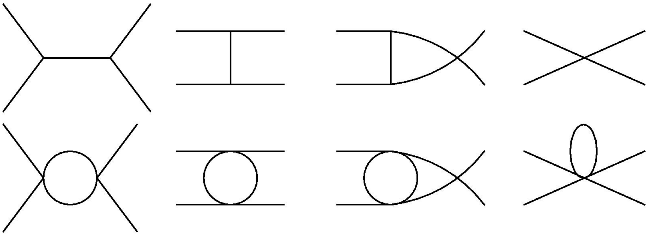
• Integrability implies factorized $2 \to 2$ scattering and two-body S-matrix determined via underlying symmetry:
(modulo a scalar phase factor that can be determined / guessed via crossing equations and unitarity based arguments)

At one loop ($h \sim \sqrt{\lambda}$)

$$\mathbb{S} = e^{i\theta}\hat{\mathbb{S}}, \qquad \theta = \sum_{n=1}^{\infty} h^{-1-n}\theta^{(n)}, \qquad \hat{\mathbb{S}} = 1 + \frac{i}{h}\sum_{n=0}^{\infty}\mathbb{T}^{(n)}$$

$$\mathbb{S} = 1 + \frac{i}{h}\mathbb{T}^{(0)} + \frac{i}{h^2}\left(\mathbb{T}^{(1)} + \theta^{(1)}\right) + \dots$$

• For explicit comparison with worldsheet computations many diagrams contribute:



[Rughoonauth, PS, Wulff; PS, Wulff; Abbott, Murugan, PS, Wulff]

Regularization

Restricting to bosonic in and out-states and writing (z, y) for transverse AdS_5 and S^5 we find (at one-loop) using dimensional regularization:

$$\mathcal{A}^{naive}(zz \to zz) = -\frac{1}{h}\ell_1 + \frac{1}{h^2}\left(\frac{1}{2\pi}\gamma(\epsilon)\ell_1 + \dots\right),$$
$$\mathcal{A}^{naive}(yy \to yy) = \frac{1}{h}\ell_1 + \frac{1}{h^2}\left(\frac{1}{2\pi}\gamma(\epsilon)\ell_1 + \dots\right)$$

while $\mathcal{A}(zy \to zy)$ is UV-finite and yields the expected result.

• Since divergent piece proportional to tree-level elements, wave-function renormalization seem be a good solution. In fact, further evidence from off-shell propagators:

$$\langle zz \rangle = \frac{iZ_z}{p^2 - m^2} + \mathcal{O}(h^{-2}), \quad \langle yy \rangle = \frac{iZ_y}{p^2 - m^2} + \mathcal{O}(h^{-2}),$$
$$Z_z = 1 + \frac{1}{4\pi h}\gamma(\epsilon), \qquad Z_y = 1 - \frac{1}{4\pi h}\gamma(\epsilon)$$

Regularization

- With wave-function renormalization:

$$\mathcal{A}(zz \to zz) = \left(\sqrt{Z_z}\right)^4 \mathcal{A}^{naive}(zz \to zz),$$
$$\mathcal{A}(zy \to zy) = \left(\sqrt{Z_z}\right)^2 \left(\sqrt{Z_y}\right)^2 \mathcal{A}^{naive}(zy \to zy) = \mathcal{A}^{naive}(zy \to zy),$$
$$\mathcal{A}(yy \to yy) = \left(\sqrt{Z_y}\right)^4 \mathcal{A}^{naive}(yy \to yy),$$

all amplitudes are UV-finite. However, finite piece sensitive to the regularization.

For example, textbook QFT dim-reg or Λ-cutoff not consistent with integrability.

- Lets look a bit closer at the integrals that appear:

$$B^{r,s}(P) = \int \frac{d^2k}{(2\pi)^2} \frac{k_+^r k_-^s}{(k^2 - m^2)((k-P)^2 - m^2)}, \quad T^{r,s}(P) = \int \frac{d^2k}{(2\pi)^2} \frac{k_+^r k_-^s}{((k-P)^2 - m^2)}$$

corresponding to bubble and tadpole type typologies

Regularization

- First we isolate all divergencies in terms of tadpole integrals via

$$B^{r,s}(P) = T^{r,s}(P) + m^2 B^{r-1,s-1}(P), \qquad r, s \geq 1$$

which results in tadpoles and bubbles with $B^{r0}(P), B^{0s}(P)$ $(r, s = 0, 1, 2, 3)$

- The remaining bubble integrals are still (potentially) divergent for $r, s \geq 2$. The identity

$$\frac{1}{(k-P)^2 - m^2} = \frac{1}{k^2 - m^2} + \frac{2k \cdot P - P^2}{(k^2 - m^2)((k-P)^2 - m^2)}$$

implies

$$P_- B^{r+1,s}(P) + P_+ B^{r,s+1}(P) = T^{r,s}(P) - T^{r,s}(0) + P^2 B^{r,s}(P)$$

- Since $B^{10}(P)$ and $B^{01}(P)$ are finite we can shift integration variables which gives:

$$B^{10}(P) = \frac{1}{2} P_+ B^{00}(P) \quad \text{and} \quad B^{01}(P) = \frac{1}{2} P_- B^{00}(P)$$

Combining this with the identity above furthermore implies:

$$T^{00}(P) - T^{00}(0) = 0$$

Regularization

• Consistency demands that we should allow shift of tadpole type integrals.
All integrals can be reduced to:

$$B^{00}(P), \qquad T^{00}(0), \qquad T^{11}(0)$$

• Note, this is not the same as evaluating all integrals in dim-reg.
For example, identities on previous slide implies

$$B^{02}(P) = -\frac{P_-}{P_+}\left(m^2 - P^2\right) B^{00}(P)$$

which is *not* true in dim-reg.
(For those in the know: the difference is rational terms which gives a mismatch with the non-transcendental piece of the one-loop phase)

• Comments:
The origin of the discrepancy lies in divergent surface terms which one usually neglect in dim-reg.
Furthermore, the problematic integrals, such as $B^{02}(P)$, only appear in non-relativistic 2D theories

Summary

• With this prescription the one-loop worldsheet S-matrix is in complete agreement with integrability

• While focus here was $AdS_5 \times S^5$, prescription works for less symmetric examples also.
Especially, for $AdS_3 \times S^3 \times T^4$ with mixed NSNS / RR-flux, where no conjecture for the phase exist, our prescription agree with results based on generalized unitarity (see V. Forini talk on Friday)

• For backgrounds with massless worldsheet excitations (which are hard to include in the exact solution), our regularization prescription decouples the massive and massless sectors (at one-loop)

• Not clear how to extend beyond one-loop level. At two loop for example, no standard set of scalar integrals exist. Work in progress.

Vielen Dank!