The String Theory Universe, Mainz
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# Global F-theory models with U(1)'s and discrete gauge symmetries

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arXiv:1408.4808: D.K., D. Mayorga Peña, P. Oehlmann, H. Piragua, J. Reuter

⟶ see Paul Oehlmann's poster

# Motivation

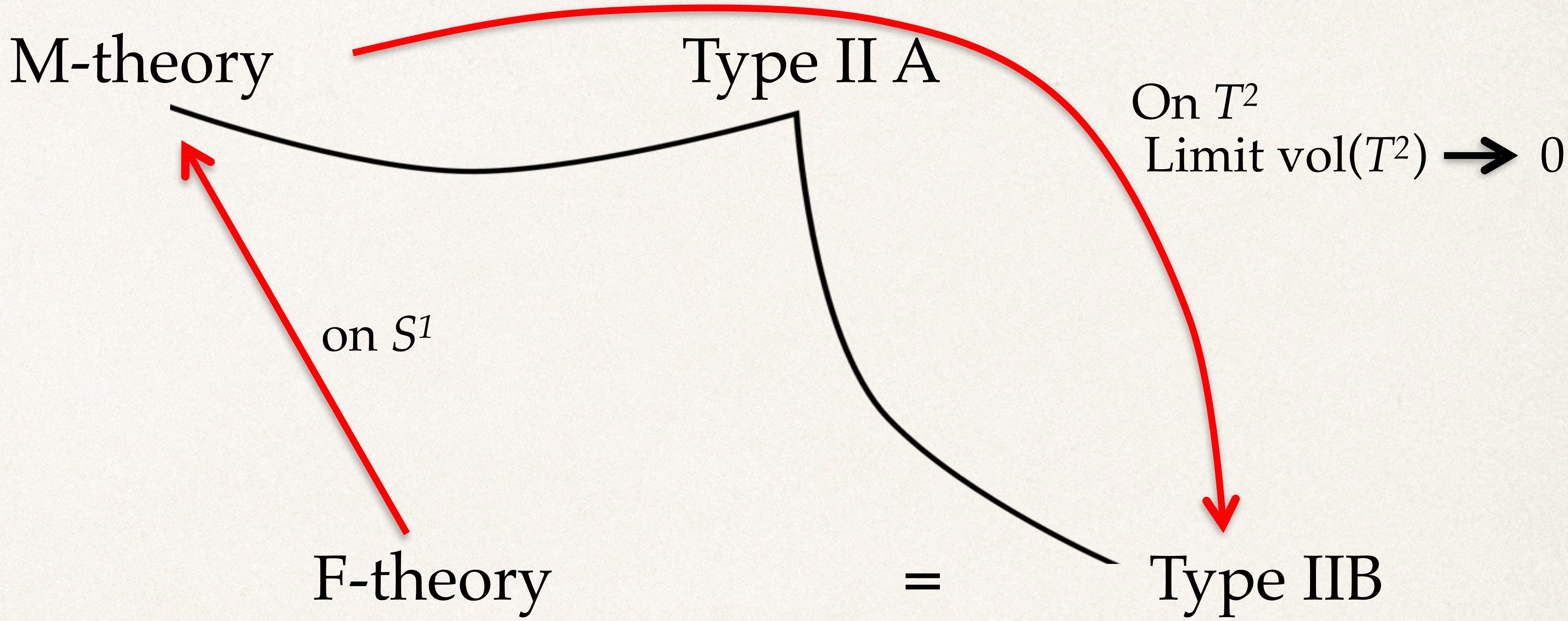
# Why F-theory?

F-theory = Type IIB

F-theory
- elliptically fibered Calabi-Yau manifold

Type IIB
- non-perturbative: regions of large $g_s$
- consistent:
  - back-reaction ✔
  - tadpoles ✔

# Why F-theory?

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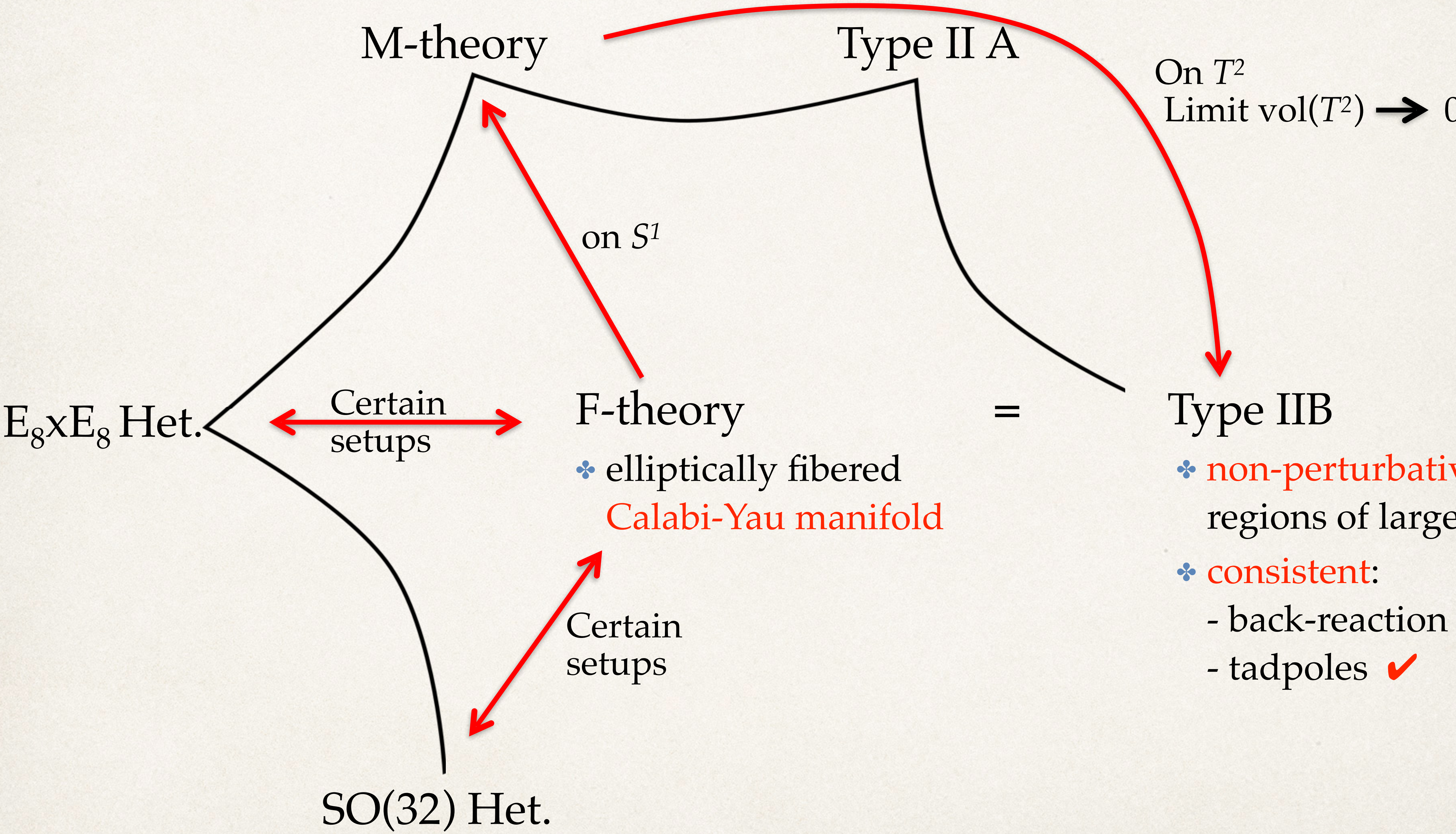


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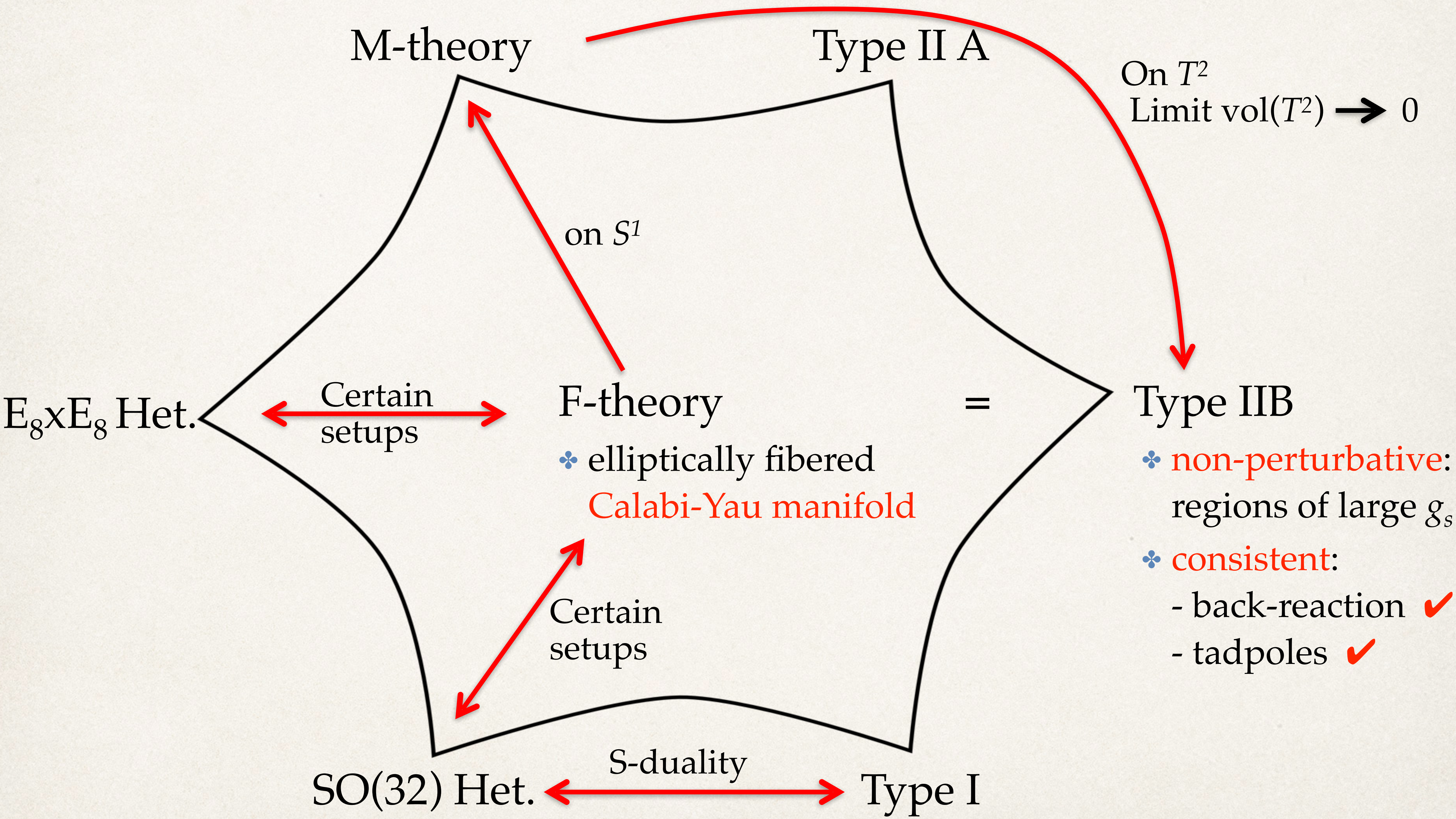

M-theory
Type II A
On $T^2$
Limit vol($T^2$) → 0
on $S^1$
$E_8$x$E_8$ Het.
Certain setups
F-theory
=
Type IIB
elliptically fibered Calabi-Yau manifold
non-perturbative: regions of large $g_s$
consistent:
- back-reaction ✔
- tadpoles ✔
Certain setups
SO(32) Het.
S-duality
Type I

# Effective theories from F-theory

<u>F-theory engineers effective theories of quantum gravity:</u>


➡ Use F-theory for classification of gauge theory sectors in *N=1* SUGRA theories.

- Need to develop and extend geometry / physics dictionary of F-theory

  ➡ F-theory realization of many consistent SUGRA theories still unknown:
  More than four U(1)'s, discrete gauge groups, singlets with U(1)-charges $q>2$…

- Have to understand the constraints imposed by quantum gravity

  ➡ Theories, that are consistent according to QFT, may violate currently unknown quantum gravity consistency constraints: are automatically obeyed in F-theory.

# Goals of this talk

1. Enlarge the space of known F-theory vacua
   - construct models with $\mathbb{Z}_n$ discrete gauge groups.
   - provide models with higher U(1)-charges of singlets.
   - highlight compliance with known quantum gravity constraints.

→ Vacua found by compactifying F-theory on all Calabi-Yau manifolds constructed as fibrations of the 16 toric hypersurface.

2. Investigate moduli space of these F-theory compactifications
   - all these Calabi-Yau manifolds connected by network of extremal transitions/Higgs effects in effective theories.

# Outline

1) A brief review of F-theory

2) Construction of toric hypersurface fibration for F-theory

3) The effective theories of F-theory on toric hypersurface fibrations

   - Determine full 6D (+ non-chiral 4D) effective theory

4) Global F-theory models with discrete gauge groups

5) The Higgs network

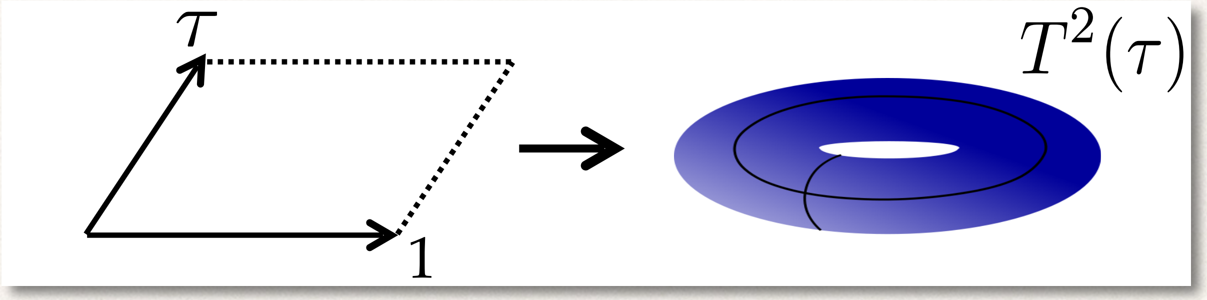
6) Conclusions & Outlook

# 1) A brief review of F-theory

# F-theory vacua: the basic idea

F-theory = geometric, $SL(2,\mathbb{Z})$-invariant formulation of Type IIB. [Vafa]

- View Type IIB axio-dilaton $\tau \equiv C_0 + ig_s^{-1}$ as modular parameter of $T^2$




- Two-torus $T^2(\tau)$ is invariant under modular transformations $SL(2,\mathbb{Z})$.

➡ S-duality invariance achieved by $\tau \longrightarrow T^2(\tau)$

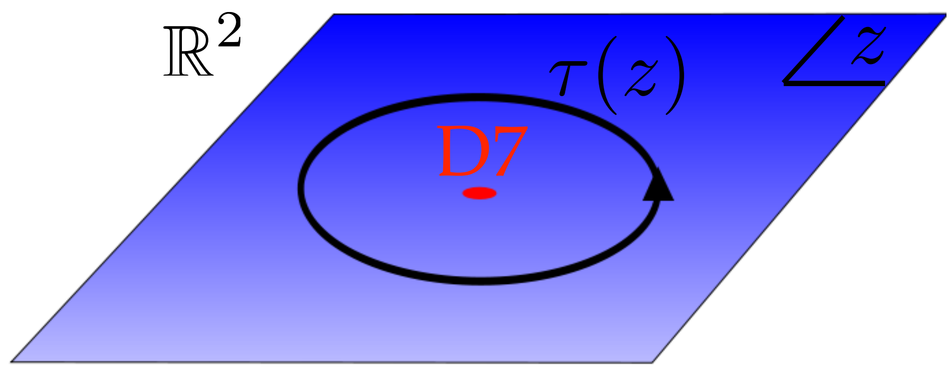
- "Size" of $T^2$ unphysical in Type IIB: formally set $\mathrm{vol}(T^2) \longrightarrow 0$.

- Non-trivial backgrounds of $\tau$

➡ in general singular $T^2$-fibrations over space-time $B$.

# F-theory vacua: the basic idea

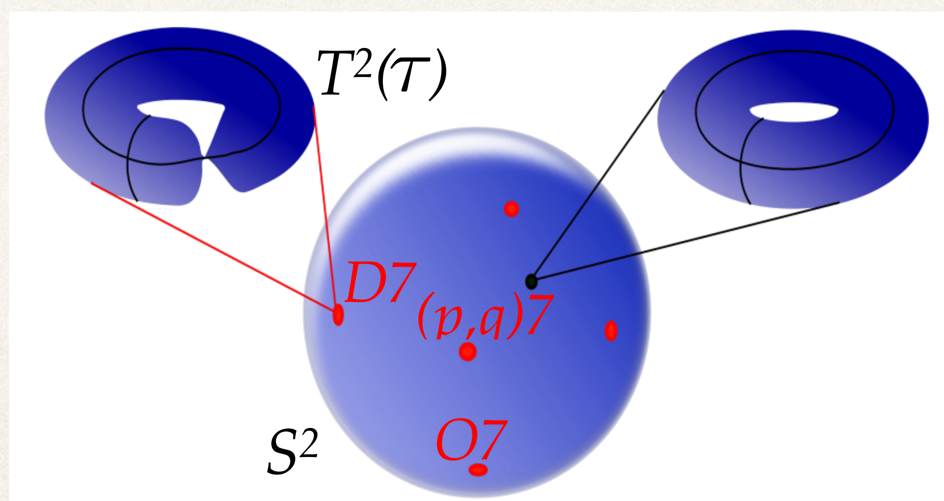
Non-trivial profile of $\tau$ in the presence of SUSY 7-branes.




$$\tau \mapsto \tau + 1$$
$$\Rightarrow \tau(z) = \frac{1}{2\pi i}\ln(z),$$

Singularity at $z$=0.

- 7-branes are global defects of space-time: deficit angle $\pi/6$.
- 24 7-branes deficit angle $4\pi$: $\mathbb{R}^2$ compactified to $S^2$.




[Greene,Shapere,Vafa,Yau;Vafa]

➡ $T^2(\tau)$-fibration over $S^2$ is torus-fibered Calabi-Yau twofold K3.

# F-theory vacua in 6D, 4D

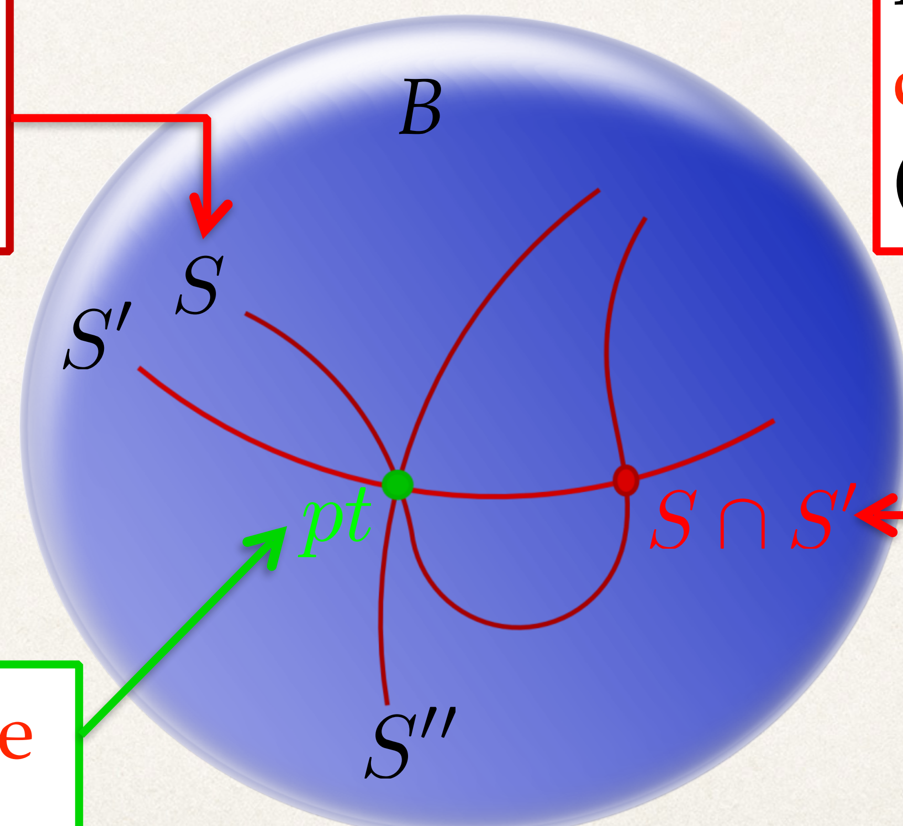
$S^2 \longrightarrow B$: $T^2(\tau)$-fibration is singular torus-fibered Calabi-Yau $X$ over $B$

Singularities of Calabi-Yau $X$ ⟺ setup of intersecting 7-branes

**Gauge theory** in **8D:**
co-dim. one sing. over $S$
(**7-branes**)

**Matter** in **6D :**
co-dim. two sing.
(**intersec. 7-branes**)

[Katz, Vafa]




**4D Yukawa:** co-dim three
$pt = S \cap S' \cap S''$

4D **chiral** matter:
**$G_4$-flux**

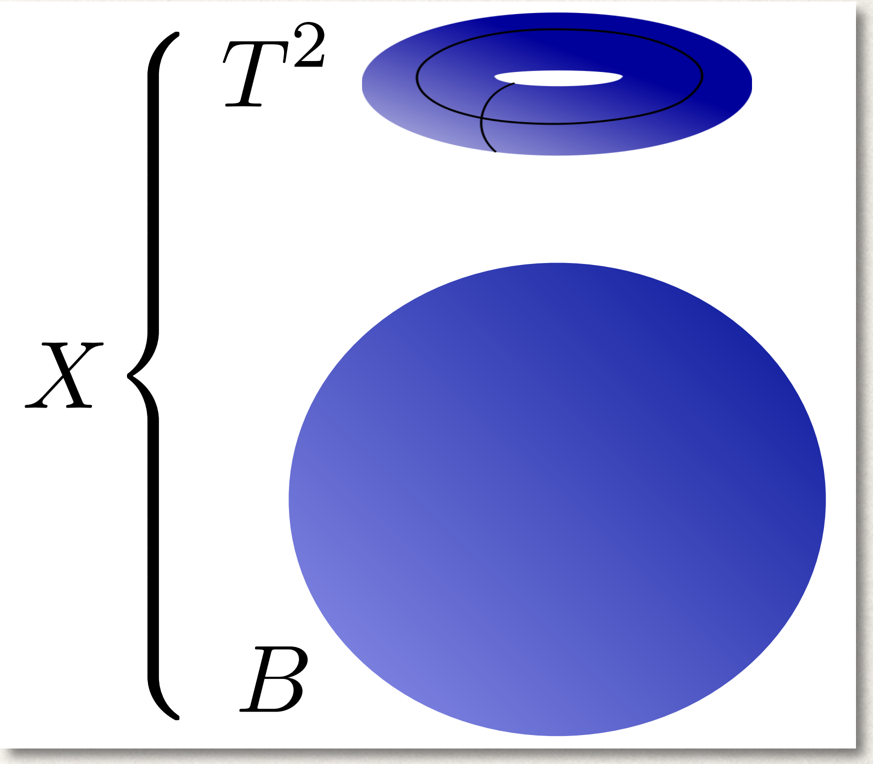
# 2) Construction of toric hypersurface fibrations for F-theory

# Building blocks of torus-fibered Calabi-Yau $X$

1. Base $B$ of $X$
   - here: do not choose specific $B$
   - ➡ analysis base-independent
2. Torus fiber $T^2$ of $X$
   - for applications: $T^2$ = algebraic curve $\mathcal{C}$ of genus one
   - can be brought into *Weierstrass normal form*

   ➡ $$y^2 = x^3 + fxz^4 + gz^6$$

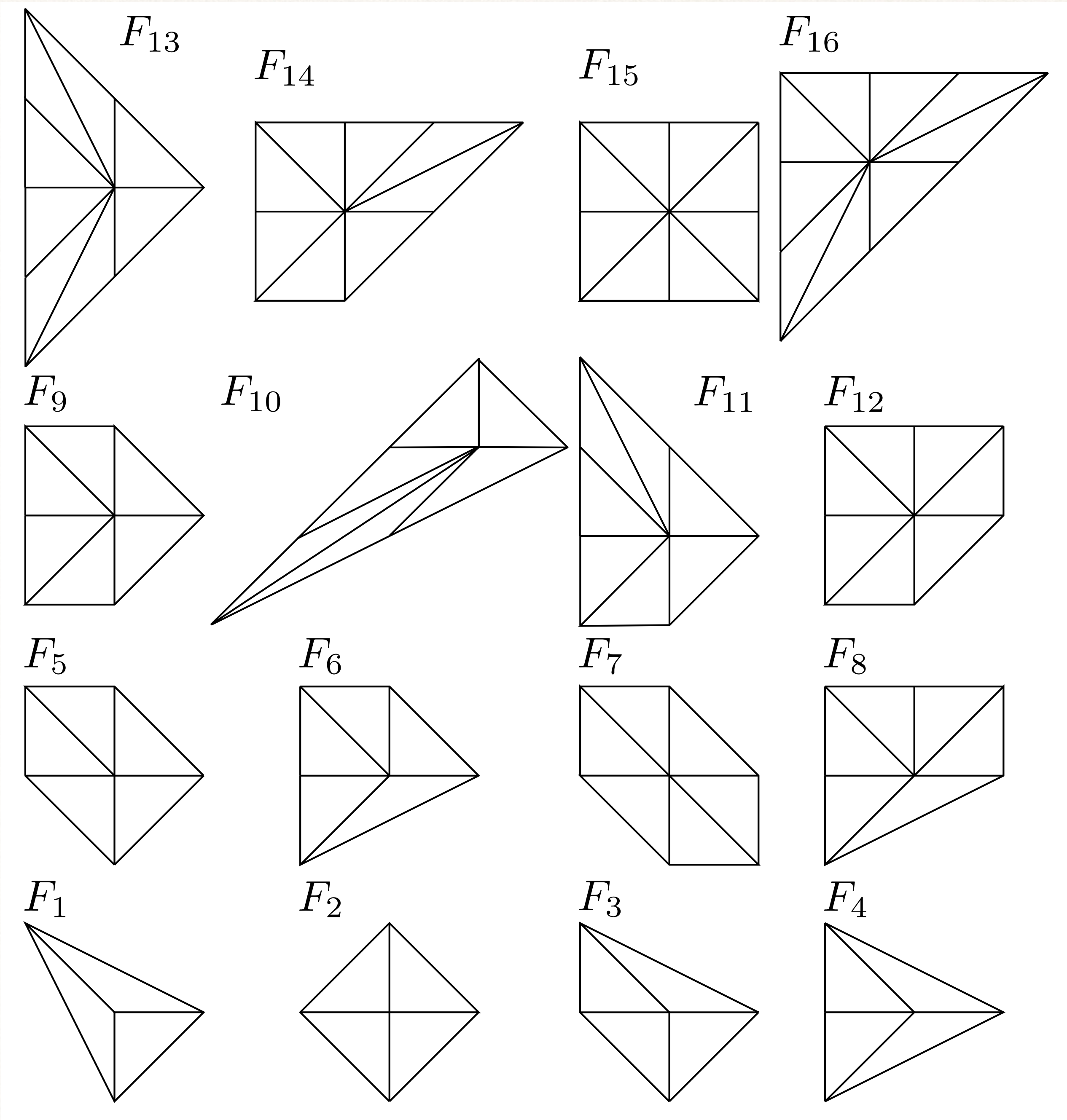
   - here: $\mathcal{C}$ has natural presentation as Calabi-Yau hypersurface in of the 2D toric varieties associated to reflexive polyhedron.




Related works: [Braun,Grimm,Keitel] $Bl_1P(112)$&$dP_2$: [Aldazabal,Font,Ibanez,Uranga; Klemm,Mayr,Vafa]

# Toric varieties from reflexive polytopes

Toric variety $\mathbb{P}_{F_i}$ associated to 16 reflexive polytopes $F_i$ in 2D:



# Toric varieties from reflexive polytopes

- Combinatorics of $F_i$ encodes geometry of toric variety $\mathbb{P}_{F_i}$.

➡ Representation as generalized projective space

$$\mathbb{P}_{F_i} = \frac{\mathbb{C}^{m+2} \backslash \mathsf{SR}}{(\mathbb{C}^*)^m}$$

- Three different types of toric varieties $\mathbb{P}_{F_i}$

  1) blow-ups of $\mathbb{P}^2$ up to $dP_6$ (13 cases),

  2) $\mathbb{P}^2(1,1,2)$ & its blow-up (2 cases),

  3) $\mathbb{P}^1 \times \mathbb{P}^1$ (1 case).

- Each $\mathbb{P}_{F_i}$ has corresponding genus-one curve $\mathcal{C}_{F_i}$.

# Genus-one curve as toric hypersurfaces

Calabi-Yau hypersurfaces $\mathcal{C}_{F_i} = \{p_{F_i} = 0\}$ in $\mathbb{P}_{F_i}$

➡ three different types of genus-one curves $\mathcal{C}_{F_i}$.

1) cubic in blow-ups of $\mathbb{P}^2$ (13 cases)

- most general cubic for $\mathbb{P}^2$, remove one term from $p_{F_i} = 0$ for each blow-up,

2) quartic in $\mathbb{P}^2(1,1,2)$ and its blow-up (2 cases),

3) biquadric in $\mathbb{P}^1 \times \mathbb{P}^1$ (1 case).

# Construction of toric hypersurface fibration $X_{F_i}$

1. Ambient space:

$$\begin{array}{ccc} \mathbb{P}_{F_i} & \longrightarrow & \mathbb{P}^B_{F_i}(\mathcal{S}_7, \mathcal{S}_9) \\ & & \downarrow \\ & & B \end{array}$$

- Fibration completely determined by two divisors $\mathcal{S}_7$ and $\mathcal{S}_9$ on $B$
  - parametrize divisor classes of the two local coordinates on the fiber.

2. Calabi-Yau hypersurface eq. of $X_{F_i}$

$$\begin{array}{ccc} \mathcal{C}_{F_i} \subset \mathbb{P}_{F_i} & \longrightarrow & X_{F_i} \\ & & \downarrow \\ & & B \end{array}$$

- impose CY-eq. $p_{F_i} = 0$: cut out $\mathcal{C}_{F_i} \subset \mathbb{P}_{F_i}$
- impose CY condition on total space $X_{F_i}$

➡ get families of Calabi-Yau manifolds $X_{F_i}(\mathcal{S}_7, \mathcal{S}_9)$

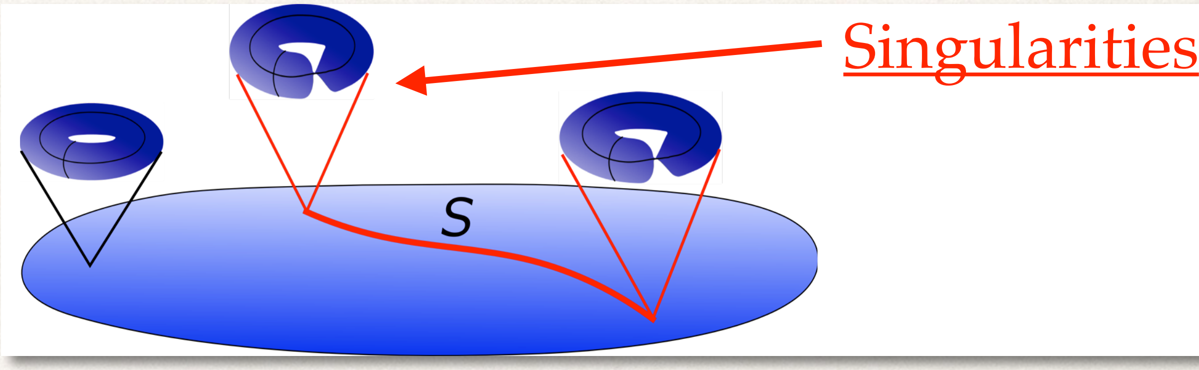
3. Derive the effective theory of F-theory for all these $X_{F_i}$.

# 3) The effective theories of F-theory on toric hypersurface fibrations

# Non-Abelian Gauge Group

- Gauge theory located at zeros of discriminant $\Delta = 4f^3 + 27g^2$:

$\Delta \sim w^n \Delta', \; n \geq 2$
➡ gauge symm. at $S = \{w = 0\}$




[Vafa;Morrison,Vafa;Bershadsky,Intriligator,Kachru,Morrison,Sadov,Vafa]

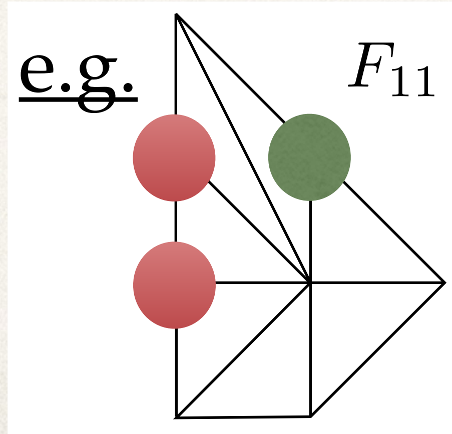
- Type of gauge group $G$ determined using Kodaira classification

[Kodaira;Tate]

➡ Cartan matrix of $G$ realized by intersections in resolution.

- $X_{F_i}$ has intrinsic gauge group $G_{F_i}$: read off from toric diagram

➡ Points inside edges = nodes in Dynkin diagram

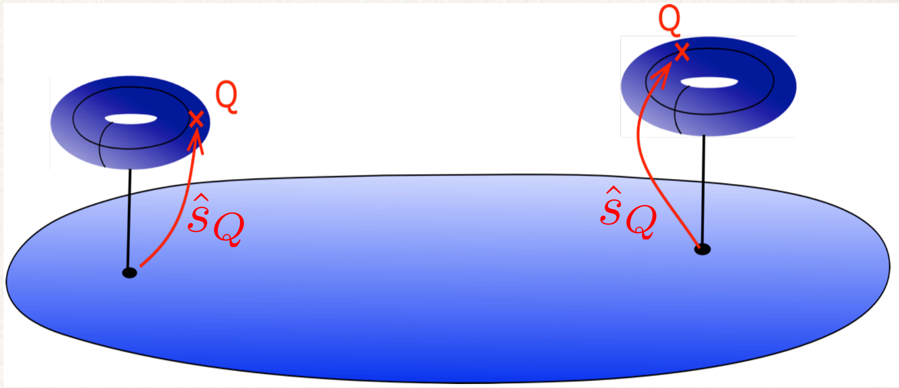



SU(3)xSU(2) $\subset G_{F_{11}}$

# Abelian Gauge Group

U(1)-symmetries ⟷ Mordell-Weil group of rational sections of elliptic fibrations $X_{F_i}$: ↗ see Cvetič's talk

[Morrison,Vafa]

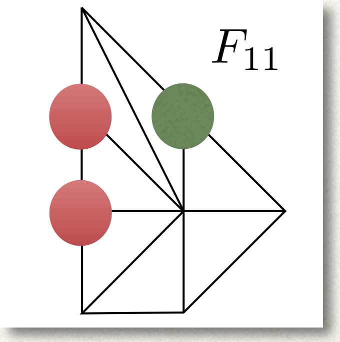



- rational section is map $\hat{s}_Q : B \to X_{F_i}$ induce by rational point Q on $\mathcal{C}_{F_i}$.
- number of U(1)'s / rational sections from toric diagram:

➡ number of U(1)'s = #(vertices of $F_i$)−3

Toric MW-group: [Braun,Grimm,Keitel]

Example:




➡ $4 - 3 = 1$ U(1):
$G_{F_{11}} =$ SU(3)xSU(2)xU(1)

- Some cases are more involved: for $X_{F_3}$ section is non-toric; for $X_{F_2}$ section exist only in its Jacobian.

# Effective theories of the 16 toric hypersurface fibrations

Gauge group $G_{F_i}$ of all 16 toric hypersurface fibrations $X_{F_i}$

| | | | | | |
|---|---|---|---|---|---|
| $G_{F_1}$ | $\mathbb{Z}_3$ | $G_{F_7}$ | $\mathrm{U}(1)^3$ | | |
| $G_{F_2}$ | $\mathrm{U}(1)\times\mathbb{Z}_2$ | $G_{F_8}$ | $\mathrm{SU}(2)^2\times\mathrm{U}(1)$ | $G_{F_{13}}$ | $(\mathrm{SU}(4)\times\mathrm{SU}(2)^2)/\mathbb{Z}_2$ |
| $G_{F_3}$ | $\mathrm{U}(1)$ | $G_{F_9}$ | $\mathrm{SU}(2)\times\mathrm{U}(1)^2$ | $G_{F_{14}}$ | $\mathrm{SU}(3)\times\mathrm{SU}(2)^2\times\mathrm{U}(1)$ |
| $G_{F_4}$ | $(\mathrm{SU}(2)\times\mathbb{Z}_4)/\mathbb{Z}_2$ | $G_{F_{10}}$ | $\mathrm{SU}(3)\times\mathrm{SU}(2)$ | $G_{F_{15}}$ | $\mathrm{SU}(2)^4/\mathbb{Z}_2\times\mathrm{U}(1)$ |
| $G_{F_5}$ | $\mathrm{U}(1)^2$ | $G_{F_{11}}$ | $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ | $G_{F_{16}}$ | $\mathrm{SU}(3)^3/\mathbb{Z}_3$ |
| $G_{F_6}$ | $\mathrm{SU}(2)\times\mathrm{U}(1)$ | $G_{F_{12}}$ | $\mathrm{SU}(2)^2\times\mathrm{U}(1)^2$ | | |

- up to three U(1)'s, non-simply connected & discrete gauge groups.

  **Non-simply connected groups: [Aspinwall,Morrison;Mayrhofer,Morrison,Till,Weigand]**

- Key observations:

$$\mathrm{rk}(G_{F-i}) = \#(\text{points} \in F_i) - 4$$

$$\mathrm{rk}(G_{F_i}) + \mathrm{rk}(G^*_{F_i}) = 6 \quad \text{with } F_i^* \text{ dual to } F_i.$$

- 6D matter (= 4D non-chiral) spectrum & 4D Yukawas derived
  - ➡ ideal techniques: primary decomposition, Gröbener basis, etc.
- all theories anomaly-free & obey quantum gravity constraints.

# Interesting examples

1. Standard-Model-like theory: $X_{F_{11}}$

| Representation | $(\mathbf{3},\mathbf{2})_{1/6}$ | $(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$ | $(\bar{\mathbf{3}},\mathbf{1})_{1/3}$ | $(\mathbf{1},\mathbf{2})_{-1/2}$ | $(\mathbf{1},\mathbf{1})_{-1}$ |
|---|---|---|---|---|---|
| **Multiplicity** | $\mathcal{S}_9([K_B^{-1}]+\mathcal{S}_7-\mathcal{S}_9)$ | $\mathcal{S}_9(2[K_B^{-1}]-\mathcal{S}_7)$ | $\mathcal{S}_9(5[K_B^{-1}]-\mathcal{S}_7-\mathcal{S}_9)$ | $([K_B^{-1}]+\mathcal{S}_7-\mathcal{S}_9)$ $\times(6[K_B^{-1}]-2\mathcal{S}_7-\mathcal{S}_9)$ | $(2[K_B^{-1}]-\mathcal{S}_7)$ $\times(3[K_B^{-1}]-\mathcal{S}_7-\mathcal{S}_9)$ |

- $U(1)_Y$ from rank one MW-group of $X_{F_{11}}$.
- All gauge invariant 4D Yukawas realized.

2. Pati-Salam-like theory: $X_{F_{13}} \longrightarrow$ correct $G_{F_{14}}$, reps & Yukawas.

3. Trinification-like theory: $X_{F_{16}} \longrightarrow$ correct $G_{F_{16}}$, reps & Yukawas.

- Singlet of charge $q=3$: $X_{F_3}$ with non-toric MW-group

➡ Quantum gravity constraint: charge lattice fully populated ✔

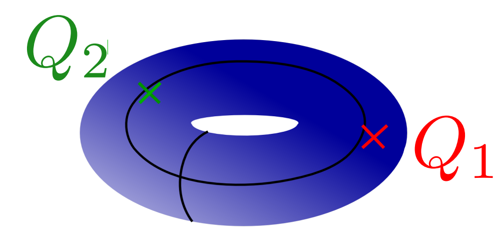
# 4) Global F-theory models with discrete gauge groups

# Global Models with discrete gauge groups

If genus one curve $\mathcal{C}$ has no rational points, only point of degree $n$

➡ $X_{F_i}$ genus-one fibrations without section, only multi-section.

- locally (over $\mathbb{C}$): $n$ distinct points $Q_1, \ldots, Q_n$ on $\mathcal{C}$.




- globally: points are interchanged
  ➡ only sum well-defined globally
  $$Q^{(n)} = Q_1 + \ldots + Q_n$$

Obstruction to gluing points together globally: Tate-Shafarevich group
➡ subset of discrete gauge group of F-theory.

[DeBoer,Dijkgraaf,Hori,Keurentjes,Morgan,Morrison,Sethi]

Three toric hypersurface fibrations have discrete groups $\mathbb{Z}_n$ with matter carrying only discrete charges.

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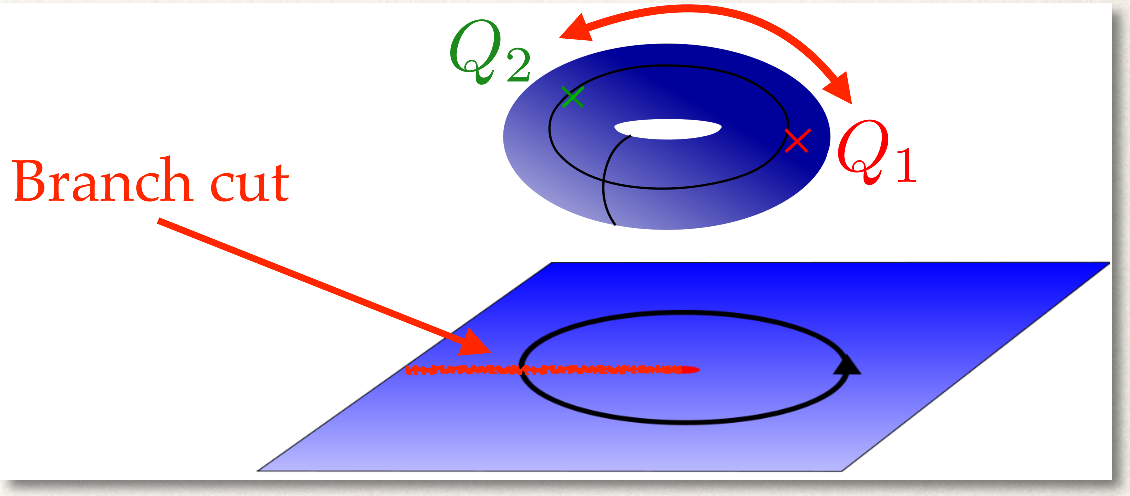


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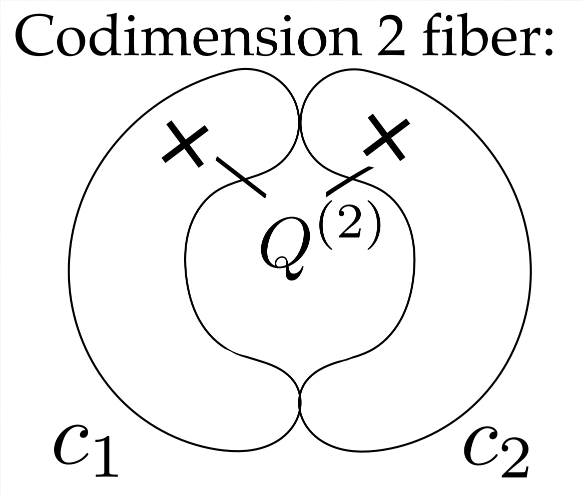
# Global Models with discrete gauge groups

Example: $X_{F_2}$ has $G_{F_2} = \mathrm{U}(1) \times \mathbb{Z}_2$ .

- Fiber $\mathcal{C}_{F_2}$ is general biquadric in $\mathbb{P}^1 \times \mathbb{P}^1$: only degree two pt. $Q^{(2)}$
  - ➡ construct Jacobian fibration: continuous gauge symmetry is U(1).
- Find codimension two singularities (matter): Massless M2-branes there do not carry U(1)-charge. Carry any quantum numbers?

Try to assign quantum number $q$ to M2-branes on curves $c_i$




- charge conjugation: $q(c_1) \overset{!}{=} -q(c_2)$
- monodromy: $c_1 \leftrightarrow c_2 \Rightarrow q(c_1) = q(c_2) \equiv q$

➡ $q + q = 0$ , i.e. $q \in \mathbb{Z}_2$ .

- M2-branes carries $\mathbb{Z}_2$ quantum number & $q$ should be non-trivial:
  - ➡ $\mathbb{Z}_2$ -gauge symmetry associated to pt. $Q^{(2)}$.

# Global Models with discrete gauge groups

Full spectrum $G_{F_2} = \mathrm{U}(1) \times \mathbb{Z}_2$ of $X_{F_2}$ worked out:

| Representation | $\mathbf{1}_{(0,-)}$ | $\mathbf{1}_{(1,+)}$ | $\mathbf{1}_{(1,-)}$ |
|---|---|---|---|
| Multiplicity | $6[K_B^{-1}]^2 + 4[K_B^{-1}](\mathcal{S}_7 - \mathcal{S}_9) -2\mathcal{S}_7^2 - 2\mathcal{S}_7^2$ | $6[K_B^{-1}]^2 + 4[K_B^{-1}](\mathcal{S}_9 - \mathcal{S}_7) +2\mathcal{S}_7^2 - 2\mathcal{S}_9^2$ | $6[K_B^{-1}]^2 + 4[K_B^{-1}](\mathcal{S}_7 - \mathcal{S}_9) -2\mathcal{S}_7^2 + 2\mathcal{S}_9^2$ |

- $\mathbb{Z}_2$-charge is denoted by $\pm$.
- all gauge invariant Yukawas exist, including $\mathbb{Z}_2$ selection rules.

Similar explicit results (spectra, $\mathbb{Z}_n$-selection rules) for

- $X_{F_1}$: $G_{F_1} = \mathbb{Z}_3$,
- $X_{F_4}: G_{F_4} = (\mathrm{SU}(2) \times \mathbb{Z}_4)/\mathbb{Z}_2$.

**For $\mathbb{Z}_2$, related works: [Braun,Morrison; Morrison,Taylor Anderson,García-Etxebarria,Grimm,Keitel; García-Etxebarria,Grimm,Keitel;Mayrhofer,Palti,Till,Weigand]**

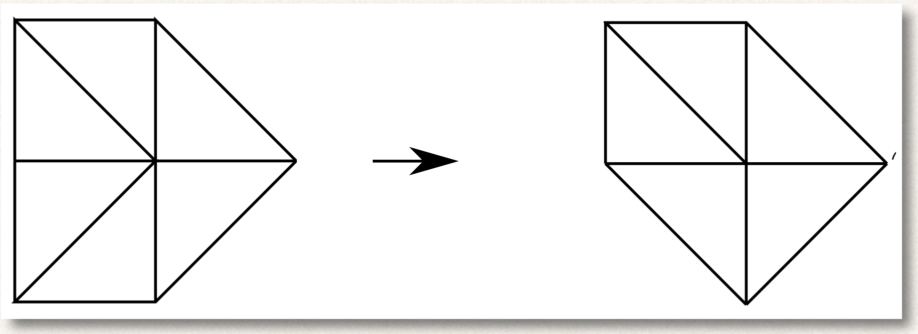
# 5) The Higgs network

# Higgs transitions between toric hypersurface fibrations

All toric hypersurface fibrations $X_{F_i}$ are connected by extremal transitions in fiber $\mathcal{C}_{F_i}$

➡ induced by blow-down in toric ambient space $\mathbb{P}_{F_i}$ of fiber $\mathcal{C}_{F_i}$ & subsequent complex structure deformation.

- Toric diagram: Cutting corners



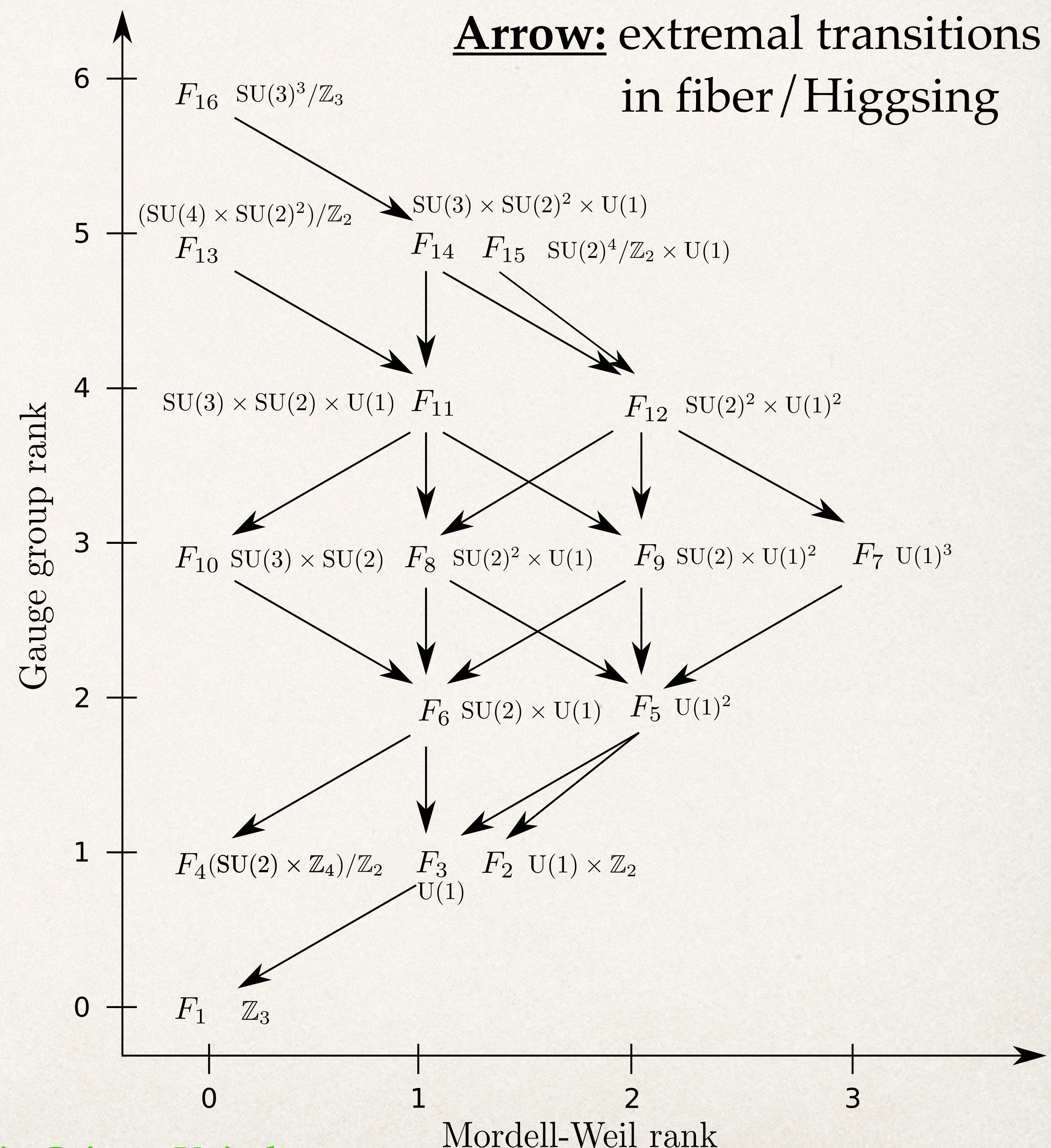
Corresponds to Higgsing in effective field theory

➡ worked out full network of all such Higgsings,

➡ generates only subbranch of moduli space of field theory: "toric Higgs branch".

# Toric Higgs branch

- matched full 6D spectrum (charged & uncharged).
- all theories obtained from maximal ones from $F_{13}$, $F_{15}$, $F_{16}$.
- all models with discrete gauge groups arise from Higgsing gauged U(1)'s:
  - ➡ e.g. $\mathbb{Z}_3$ in $X_{F_1}$ from $X_{F_3}$ with Higgs of charge $q$=3.
  - ➡ quantum gravity constraint: any global symmetry has to be gauged ✔




**For Z$_2$-case:** [Morrison,TaylorAnderson,García-Etxebarria,Grimm,Keitel; García-Etxebarria,Grimm,Keitel;Mayrhofer,Palti,Till,Weigand]

# 6) Conclusions & Outlook

1. Summary

- Constructed & analyzed all genus-one fibrations with fiber $\mathcal{C}_{F_i}$ in toric varieties associated to 16 2D reflexive polytopes $F_i$.
  - ➡ Full effective theory in 6D (= non-chiral 4D) determined
  - ➡ non-trivial gauge groups & matter content: discrete gauge groups, singlets with charge $q$=3, SM, Pati-Salam, Trinification
- Network of Higgsings relating all effective theories studied

2. Outlook

- Construction of 4D chiral models
  - ➡ $G_4$-flux constructions following e.g. [Cvetič,Grassi,DK,Piragua]
- Explore phenomenology of toric hypersurface fibrations.