Minimal Warped Holography

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work in progress
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Introduction

- A Lorentz invariant QFT couples to a (pseudo)-Riemannian metric $(g_{\mu\nu}, \mathcal{M})$.
- A Galilean invariant QFT couples to a Newton-Cartan geometry $(\tau_\mu, h^{\mu\nu}, v^\mu, A_\mu, \mathcal{M})$. [Son, 2013] [Jensen, 2014]
- In the AdS/CFT context these structures arise as the boundary geometry of some dual space-time.
  - Newton-Cartan arises at the boundary of Lifshitz space-times [Hartong, Kiritsis, Obers, 2014]

The goals

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1. What background geometry does a Warped CFT (WCFT) couple to ?
2. Construct a minimal framework to realize a WCFT holographically.

Warped AdS_3 space-times

AdS_3 has isometry group $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$. We pick a Killing vector $\xi^a \in SL(2,\mathbb{R})_R$. A warped AdS_3 space is obtained by the deformation

$$ds^2_{\text{AdS}_3} \to ds^2_{\text{WAdS}_3} = ds^2_{\text{AdS}_3} + \xi_a \otimes \xi_a$$

- The remaining isometries are $SL(2,\mathbb{R})_L \times U(1)_R$.
- The norm (length) of ξ^a generates a time-like, space-like (stretched or squashed) or null WAdS_3.
- Not a vacuum solution of Einstein gravity but a solution to many theories, e.g. Topologically Massive Gravity.

In the holographic context

- The asymptotic symmetry algebra is Virasoro $\oplus$ Kac-Moody [Compère, Detournay, 2008].

What is a Warped CFT

In 2D, scale invariance implies conformal invariance (for a unitary, Poincaré invariant theory with discrete non-negative scaling spectrum). [Polchinski, 1988]

Consider a 2D QFT invariant under translations and a chiral (left) scaling (assuming locality, unitarity and a discrete non-negative dilatation spectrum)

$$x^- \to \lambda x^- + \epsilon^- , \quad x^+ \to x^+ + \epsilon^+ .$$

- The left moving scaling and translation get enhanced to a full left conformal symmetry.
- The right moving translation becomes either
  - a right conformal symmetry, $\text{Virasoro}_L \oplus \text{Virasoro}_R$, a regular CFT
  - a left current algebra, $\text{Virasoro}_L \oplus \text{Kac-Moody}_L$, a Warped CFT

[Hofman, Strominger, 2011]

- The zero modes generate a global $SL(2,\mathbb{R}) \times U(1)$ symmetry.
  - The zero mode of the $U(1)$ current algebra corresponds to a space-time translation.
  - There is a higher mode that corresponds to a Galilean boost.
- A WCFT is invariant under transformations of the form

$$x^- \to f(x^-)\,, \quad x^+ \to x^+ + g(x^-)$$

for arbitrary functions $f(x^-)$ and $g(x^-)$.

A WCFT can be seen as a sort of geometrization of a $U(1)$ current algebra.

Background geometry I

Consider a 2D QFT with two translations and one Galilean boost

$$[H, P] = 0\,, \quad [H, G] = P\,, \quad [P, G] = 0\,.$$

- Introduce a gauge field τ_μ, e_μ and ω_μ for each symmetry (H, P, G).
  - We think of τ_μ, e_μ as vielbeins and introduce inverses τ^μ, e^μ with

$$\tau_\mu \tau^\mu = 1, \;\; \tau_\mu e^\mu = 0, \;\; e_\mu \tau^\mu = 0, \;\; e_\mu e^\mu = 1.$$

- The transformations of the gauge fields under the Galilean boost are

$$\delta\tau_\mu = 0\,, \quad \delta e_\mu = \lambda\tau_\mu\,, \quad \delta\omega_\mu = \partial_\mu\lambda\,.$$

  - The boost invariant objects are τ_μ, e^μ and $\tau_{[\mu}e_{\nu]}$.
  - The volume form does contain the same information as τ_μ and e^μ.

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- We define a covariant derivative $\mathcal{D}_\mu$ and we impose a vielbein postulate

$$\begin{aligned} 0 &= \mathcal{D}_\mu \tau_\nu = \nabla_\mu \tau_\nu\,, \\ 0 &= \mathcal{D}_\mu e_\nu = \nabla_\mu e_\nu - \omega_\mu \tau_\nu\,, \end{aligned}$$

- $\nabla_\mu e^\nu = 0$ follows, hence ∇_μ is compatible with (τ_μ, e^μ).

Background geometry II

The field strengths are

$$
\begin{aligned}
R_H &= d\tau\,, \\
R_P &= de + \tau \wedge \omega\,, \\
R_G &= d\omega\,.
\end{aligned}
$$

Background geometry II

The field strengths are

$$\begin{aligned} R_H &= d\tau\,, \\ R_P &= de + \tau \wedge \omega\,, \\ R_G &= d\omega\,. \end{aligned}$$

- We keep a non-trivial curvature only in R_P, as a non-trivial torsion.
- The conditions $R_H = 0$ and $R_G = 0$ imply that R_P is fully gauge invariant, $\delta R_P = 0$.
- We solve $R_G = 0$ using a Weitzenböck connection, $\omega_\mu = 0$.
  - Intrinsically no metric formulation.
  - Parallel transport of vectors is path independent.
  - Related to the current algebra of the WCFT.

Proposal

A WCFT couples to a background $(\mathcal{M}, g_{\mu\nu}, e^{\mu})$ in a way that is invariant under reparametrizations, Galilean boosts and chiral Weyl transformations.

- The object $g_{\mu\nu}$ must satisfy $g_{\mu\nu}e^{\nu} = 0$. It is a boost invariant degenerate metric, $g_{\mu\nu} = \tau_{\mu}\tau_{\nu}$ in 2D.
- This geometry can be thought of as a conjugated version of a Newton-Cartan geometry.

Let us consider a small variations of the background, schematically

$$\delta S = \int \det(e_{\mu}^{a}) T^{a}{}_{\mu} \delta e_{a}^{\mu} \,.$$

- Invariance under chiral Weyl rescalings, $\tau_{\mu} \to \Lambda\tau_{\mu}$ $e_{\mu} \to e_{\mu}$, and Galilean boosts imply $T^{\mu}{}_{\nu} = e^{\mu} J_{\nu}$ where $T^{\mu}{}_{\nu} \equiv e_{a}^{\mu} T^{a}{}_{\nu}$.
  - The background couples to the two components of J_{μ}.
  - These are morally the components of the currents that generate the left conformal and left current algebras.

Null Warped AdS_3

The conjugated Newton-Cartan geometry is expected to be the relevant geometry at the boundary of any WAdS space-time.

This is compatible with the boundary conditions on a null $WAdS_3$ space-time. A null $WAdS_3$ space can always be written as

$$ds^2 = \left(-e^+_\mu e^+_\nu - e^+_\mu e^-_\nu - e^+_\nu e^-_\mu\right) dx^\mu dx^\nu + \frac{dr^2}{r^2},$$

with boundary conditions $e^+_\mu = r^{-2} e^+_{(0)\mu} + \ldots, e^-_\mu = e^-_{(0)\mu} + \ldots$.

- Bulk $SO(1,2)$ local Lorentz transformations that leave $e^r_\mu = r^{-1}\delta^r_\mu$ inert act on the boundary as Galilean boosts. [Hartong, Rollier, 2013]
- The chiral scaling follows from the boundary conditions.

Extension of the geometry in the bulk

- We add a space-like direction to form a bulk in a way that is conjugated to Newton-Cartan
  - The additional vielbein “behaves morally” like τ_μ, we form the combinations $\tau_\mu^\pm$.
  - The invariant degenerate metric is extended $g_{\mu\nu} = \tau_\mu^+ \tau_\nu^- + \tau_\nu^+ \tau_\mu^-$ and still satisfies $g_{\mu\nu} e^\nu = 0$.
- The symmetries are extended in a natural way
  - Two Galilean boosts acting on $(\tau_\mu^\pm, e_\mu)$ respectively.
  - One relativistic boost acting on (τ_μ^+, τ_μ^-) leaving e_μ invariant.
- We impose vanishing Galilean curvature and vanishing torsion on $g_{\mu\nu}$
  - Weitzenböck connections are associated to the Galilean boosts.
  - A usual spin connection is associated to the relativistic boost.

- This allows us to solve for the space-time connection

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right) + e^\rho \partial_\mu e_\nu \,,$$

where $g^{\mu\nu} = \tau_+^\mu \tau_-^\nu + \tau_+^\nu \tau_-^\mu$.

- This connection satisfies $\nabla_\mu g_{\nu\rho} = \nabla_\mu e^\nu = 0$.

- The bulk splits into Riemannian/Weitzenböck parts.

- We would like to allow for non-trivial curvature in the Riemannian part ($g_{\mu\nu}$) and non-trivial torsion in the Weitzenböck part (e_μ).

Outlook

- We constructed a new holographic framework to study WCFT's.
- Our framework can be expressed in terms of a Chern-Simons theory.
- For example, if we add a cosmological constant for $g_{\mu\nu}$ and consider vanishing torsion in the Weitzenböck part, the equations of motion become those of an $SL(2,\mathbb{R}) \times U(1)$ Chern-Simons theory.
- In contrast to higher spin Chern-Simons theories we do not have an $sl(2,\mathbb{R}) \oplus sl(2,\mathbb{R})$ subalgebra.

Outlook

- The solutions to this theory will have $SL(2,\mathbb{R}) \times U(1)$ global isometries.
- We expect to recover the charges of the WCFT and to reproduce a Virasoro$\times$Kac-Moody asymptotic symmetry group.
- Can this full holographic framework be embedded in a larger theory such as Topologically Massive Gravity or an other theory of gravity ?