Torsion, Duality
and the
Wilderness of Orientifolds

MAX-PLANCK-GESELLSCHAFT

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What is duality?

Duality comes in two flavors:

- **Exact duality:** The same quantum theory has various different descriptions, which become tractable in different regions of parameter space:
  - AdS/CFT
  - T-duality and mirror symmetry
  - IIB/$\mathcal{N} = 4$ (Montonen-Olive) S-duality

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We find evidence for the existence of new **Montonen-Olive** dualities in a large class of $\mathcal{N} = 1$ theories.

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In many (but not all) cases we can explicitly identify various dual perturbative descriptions of the same theory. In some cases we can only identify dual descriptions of the IR fixed points.

Montonen-Olive $\mathcal{N} = 4$ duality

Given a 4d $\mathcal{N} = 4$ field theory with gauge group G and gauge coupling $\tau = \theta + i/g^2$, there is a completely equivalent description with gauge group $G^\vee$ and coupling $-1/\tau$ (for $\theta = 0$ this is $g \leftrightarrow 1/g$). Examples:

G	$G^\vee$
$U(1)$	$U(1)$
$U(N)$	$U(N)$
$SU(N)$	$SU(N)/\mathbb{Z}_N$
$SO(2N+1)$	$Sp(2N)$

Very non-perturbative duality, exchanges **gauge bosons** with **monopoles**! (So, the usual field theory tools are not particularly illuminating here.)

Field theories from solitons

If we want to construct four dimensional field theories from string theory, we want solitons with a four dimensional core. These can be constructed in type IIB string theory via $D3$ branes.

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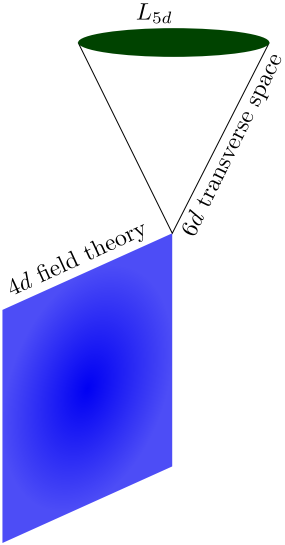


Field theories from solitons

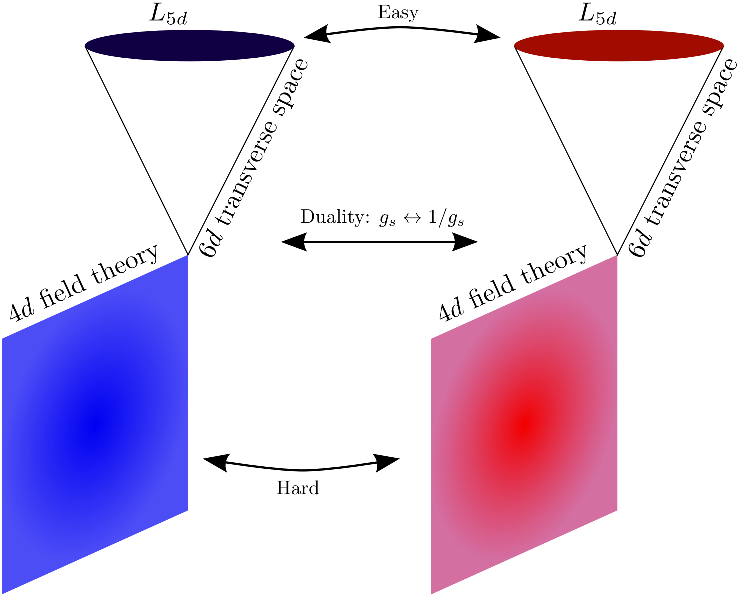
Key idea

Since the resulting theory is determined by the geometry, one can determine robust results without knowing much of the dynamical details of the duality acting on the core of the soliton.

One just needs to know how the duality acts at infinity (which is just the simple action on IIB).

We then reconstruct the dual theory as that living in the soliton with the right (dual) charge as infinity.

The duality as seen from string theory



Montonen-Olive duality from string theory

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Beautiful field theory insights follow trivially from the duality dictionary. For example, the gauge boson $\leftrightsquigarrow$ monopole map follows easily from the $F1 \leftrightsquigarrow D1$ duality dictionary entry.

Montonen-Olive duality from string theory

There are four versions of the $O3$ plane in string theory, distinguished by discrete RR and $NSNS$ 2-form fluxes B_2, C_2 in the transverse space: [Witten:hep-th/9805112]

$$H^3(S^5/\mathbb{Z}_2, \widetilde{\mathbb{Z}}) = \mathbb{Z}_2 \, .$$

- $(0,0) \quad : \quad O3^- + N\, D3s \longrightarrow SO(2N)$
- $(0,1) \quad : \quad \widetilde{O3^-} + N\, D3s \longrightarrow SO(2N+1)$
- $(1,0) \quad : \quad O3^+ + N\, D3s \longrightarrow USp(2N)$
- $(1,1) \quad : \quad \widetilde{O3^+} + N\, D3s \longrightarrow USp(2N)$

IIB $SL(2,\mathbb{Z})$ exchanges the configurations.

Under S-duality

$$\widetilde{O3^-} \longleftrightarrow O3^+ \qquad : \qquad SO(2N+1) \longleftrightarrow USp(2N)$$

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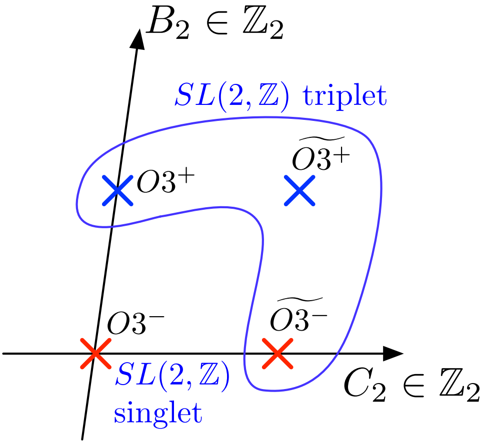

Beyond $\mathcal{N} = 4$

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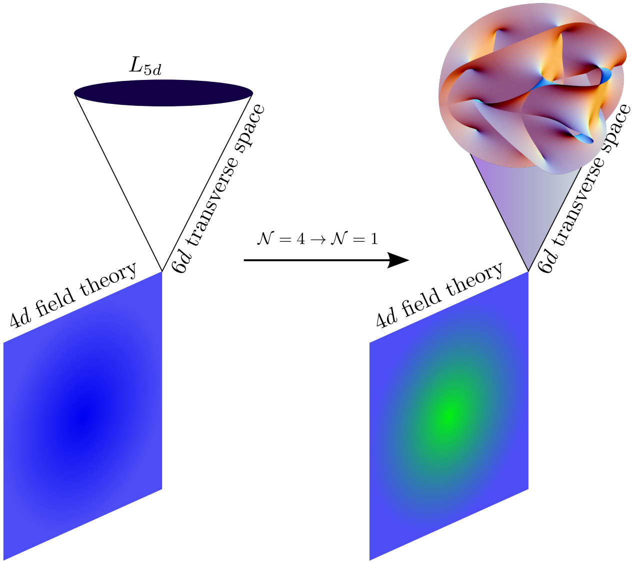
New $\mathcal{N} = 1$ dualities

Engineer certain $\mathcal{N} = 1$ theories in IIB, develop the S-duality dictionary as needed, and read the effect of strong/weak duality on $\mathcal{N} = 1$ theories.

Duality engineering



Outline

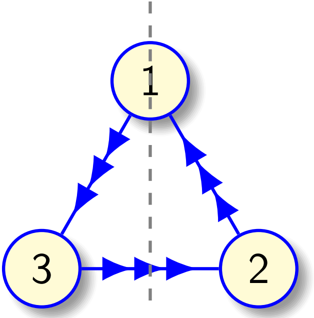
Orientifolding $\mathbb{C}^3/\mathbb{Z}_3$

Orbifolding $\mathcal{N}=4$ duality

Consider the orientifold action with generators $\{\mathcal{R}, \mathcal{I}\,\Omega(-1)^{F_L}\}$:

$$\mathcal{R} : (x, y, z) \longrightarrow (\omega x, \omega y, \omega z)$$
$$\mathcal{I} : (x, y, z) \longrightarrow (-x, -y, -z)$$

with $\omega = \exp(2\pi i/3)$.



A proposed $\mathcal{N}=1$ duality

	$USp(\widetilde{N}+4)$	$SU(\widetilde{N})$	$SU(3)$	$U(1)_R$	$\mathbb{Z}_3$
A^i	$\overline{\square}$	$\square$	$\square$	$\frac{2}{3}-\frac{2}{\widetilde{N}}$	1
B^i	1	$\overline{\square\square}$	$\square$	$\frac{2}{3}+\frac{4}{\widetilde{N}}$	-2

(here $\widetilde{N}\in 2\mathbb{Z}$) is dual to

	$SO(N-4)$	$SU(N)$	$SU(3)$	$U(1)_R$	$\mathbb{Z}_3$
A^i	$\overline{\square}$	$\square$	$\square$	$\frac{2}{3}+\frac{2}{N}$	1
B^i	1	$\overline{\begin{array}{c}\square\\\square\end{array}}$	$\square$	$\frac{2}{3}-\frac{4}{N}$	-2

in both cases with $W=\frac{1}{2}\epsilon_{ijk}\mathrm{Tr}\,A^iA^jB^k$.

Global anomalies, the moduli spaces and the spectrum of operators match if $\boxed{\widetilde{N}=N-3}$. (As far as we have been able to check so far.)

Superconformal index matching

A very powerful and refined indicator of duality comes from putting the theory on $S^3 \times \mathbb{R}$, and computing the index [Di Pietro, Martelli] [Romelsberger:hep-th/0510060,0707.3702], [Kinney, Maldacena, Minwalla, Raju:hep-th/0510251]:

$$\mathcal{I}(t,x,f) = \int dg \, \mathrm{Tr}\, (-1)^F e^{-\beta\mathcal{H}} t^{\mathcal{R}} x^{2\overline{J}_3} f g \,, \tag{1}$$

with $2\mathcal{H} = \{Q, Q^\dagger\}$. Romelsberger gave a procedure for computing the index from weak coupling quantities. Start with the "letter":

$$\begin{aligned} i_{\mathcal{T}}(t,x,g,f) = & \frac{(2t^2 - t(x+x^{-1}))\chi_{\mathsf{Adj}}(g)}{(1-tx)(1-tx^{-1})} \\ & + \frac{\sum_i \left(t^{r_i} \chi_{R_G^i}(g)\, \chi_{R_F^i}(f) - t^{2-r_i} \chi_{\overline{R_G^i}}(g)\, \chi_{\overline{R_F^i}}(f) \right)}{(1-tx)(1-tx^{-1})} \,. \end{aligned}$$

and then take the plethystic exponential:

$$\mathcal{I}_{\mathcal{T}}(t,x,f) = \int dg \, \exp\left[\sum_{k=1}^{\infty} \frac{1}{k} i_{\mathcal{T}}(t^k, x^k, g^k, f^k) \right] .$$

Superconformal index matching

For $SO(3) \times SU(7) \leftrightarrow USp(8) \times SU(4)$ we get:

$$\begin{aligned}\mathcal{I}_{SO/USp}(t,x,f) = 1 &+ t^{\frac{2}{3}}\left[\chi_{0,2}(f) + \chi_{4,0}(f)\right] \\ &+ t^{\frac{4}{3}}\left[2\chi_{0,4}(f) + 2\chi_{2,0}(f) + \chi_{3,1}(f) + 2\chi_{4,2}(f) + \chi_{8,0}(f)\right] \\ &+ t^{\frac{5}{3}}(x + x^{-1})\left[\chi_{0,2}(f) + \chi_{4,0}(f)\right] \\ &+ t^{2}\big[3\chi_{0,6}(f) + \chi_{12,0}(f) + \chi_{1,4}(f) + 5\chi_{2,2}(f) + 3\chi_{3,3}(f) \\ &\qquad + 2\chi_{4,1}(f) + 3\chi_{4,4}(f) + \chi_{5,2}(f) + 4\chi_{6,0}(f) + \chi_{6,3}(f) \\ &\qquad + \chi_{7,1}(f) + 2\chi_{8,2}(f) + 4\big] + \ldots\end{aligned}$$

We have checked up to order $t^{11/3}$ for this value of N, higher orders for other values of N, and to all orders in the large N limit:

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A conjecture about elliptic hypergeometric functions

(See Spiridonov et al.)

$$\mathcal{I}_{USp} = \mathcal{I}_{SO}$$

Stringy interpretation

Forgetting the change in rank, this seems to be essentially a $SO \leftrightarrow USp$ duality, **as in $\mathcal{N} = 4$ under S-duality**.

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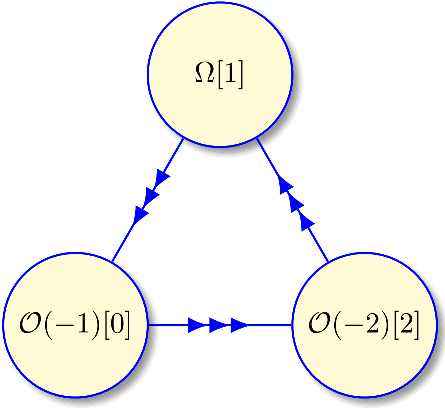
- What is going on microscopically?
- Why the change in rank? $(\widetilde{N} = N - 3)$
- Can we derive the duality from the known properties of IIB under S-duality?

Branes at singularities as large volume objects

We can think of the fractional branes at the singularity as large volume D-branes continued to small volume, receiving strong α' corrections.

These α' corrections affect the conditions for supersymmetry, and the masses of states, but we can still think of the object in large volume terms, and compute the chiral spectrum in that picture. [Douglas:hep-th/0011017]

Compact $\mathbb{C}^3/\mathbb{Z}_3$ orientifold



The three branes become mutually supersymmetric at $B = J = 0$.
[Aspinwall:hep-th/0403166]

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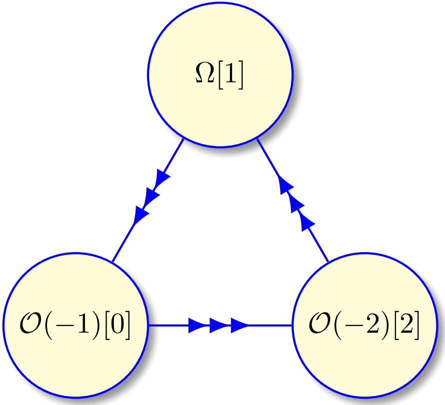


The three branes become mutually supersymmetric at $B = J = 0$. [Aspinwall:hep-th/0403166]

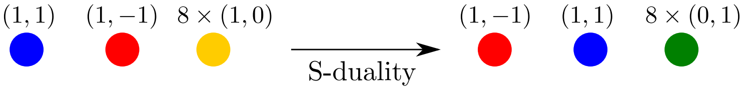
The branes are exchanged exactly as predicted by [Diaconescu,Garcia-Raboso,Karp,Sinha:hep-th/0606180].

$O7^+$ at strong coupling

We need to understand the strongly coupled limit of the $O7^+$ in flat space. In F-theory, the $O7^+$ is given by a (frozen) singularity with D_8 monodromy. [Witten:hep-th/9712028]

Such a monodromy can be achieved by considering a BCA^8 system, where A is a $(1,0)$ 7-brane (a $D7$), B a $(1,1)$ 7-brane, and C a $(1,-1)$ 7-brane.

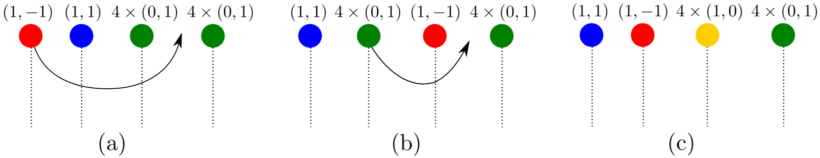
Under S-duality, this configuration becomes $CBX^8_{(0,1)}$. We want to describe this as a $O7^-$ plane plus other 7-branes.



$O7^+$ at strong coupling

S-duality

$$O7^+ \longleftrightarrow O7^- + 4A + 4X_{(0,1)}$$



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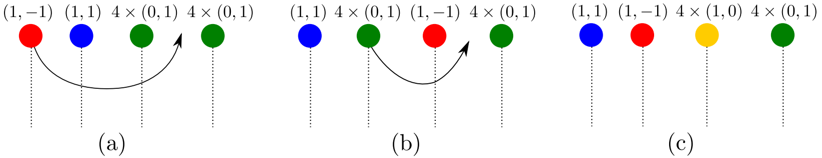


$$USp(\widetilde{N}+4)\times SU(\widetilde{N}) \longleftrightarrow SO(\widetilde{N}-1)\times SU(\widetilde{N}+3)$$

Discrete torsion argument for $\mathcal{N} = 4$

Recall the $\mathcal{N} = 4$ case: $O3^{\pm} + D3\,s$. [Witten:hep-th/9805112] The charge of the system is classified by the cohomology on the $S^5/\mathbb{Z}_2 = \mathbb{RP}^5$ that surrounds the configuration. For fields even under the orientifold action, we have:

$$H^{\bullet}(\mathbb{RP}^5, \mathbb{Z}) = \{\mathbb{Z}, 0, \mathbb{Z}_2, 0, \mathbb{Z}_2, \mathbb{Z}\}\,,$$

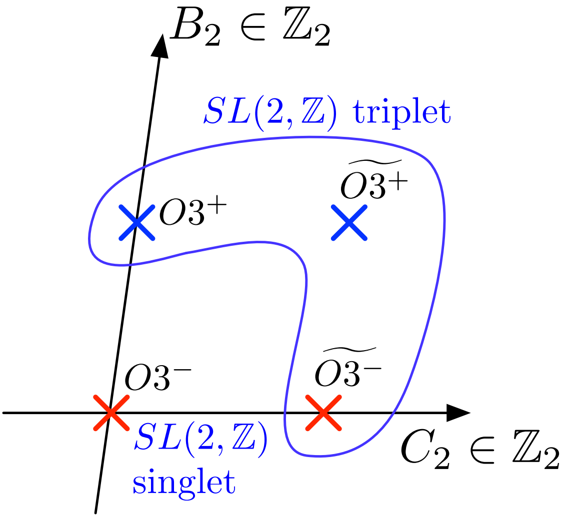
while for fields odd under the orientifold action:

$$H^{\bullet}(\mathbb{RP}^5, \widetilde{\mathbb{Z}}) = \{0, \mathbb{Z}_2, 0, \mathbb{Z}_2, 0, \mathbb{Z}_2\}\,.$$

This is *(co)homology with local coefficients*. Working on the S^5 covering space $k \otimes C \simeq \gamma k \otimes \gamma C$. For coefficients in $\mathbb{Z}$ we have $\gamma k = k$ while for coefficients in $\widetilde{\mathbb{Z}}$ we have $\gamma k = -k$. Ordinary (co)homology theory otherwise: $H^{\bullet} = \ker \partial / \operatorname{im} \partial$.

Discrete torsion argument for $\mathcal{N} = 4$

In particular, $H_3 = dB_{NSNS}$ and $F_3 = dC_2$ belong to $H^3(\mathbb{RP}^5, \widetilde{\mathbb{Z}}) = \mathbb{Z}_2$, classifying the orientifold types.



The $\mathbb{C}^3/\mathbb{Z}_3$ orbifold

We did the computation for the orientifold of $\mathbb{C}^3/\mathbb{Z}_3$, with horizon manifold $X = \mathbb{RP}^5/\mathbb{Z}_3 \sim (S^5/\mathbb{Z}_3)/\widetilde{\mathbb{Z}_2}$.

It is easier to work in homology and use Poincare duality

$$H^i(X, \widetilde{\mathbb{Z}}) = H_{\dim(X)-i}(X, \widetilde{\mathbb{Z}})\,.$$

We are thus looking for elements of $H_2(X, \widetilde{\mathbb{Z}})$. Can be conveniently computed using a long exact sequence: [Hatcher]

$$\begin{tikzcd} \dots \arrow[r] & H_i(X,\widetilde{\mathbb{Z}}) \arrow[r] & H_i(Y,\mathbb{Z}) \arrow[r, "p_*^i"] & H_i(X,\mathbb{Z}) \arrow[dlll, rounded corners] & \\ & H_{i-1}(X,\widetilde{\mathbb{Z}}) \arrow[r] & H_{i-1}(Y,\mathbb{Z}) \arrow[r, "p_*^{i-1}"] & H_{i-1}(X,\mathbb{Z}) \arrow[r] & \dots \end{tikzcd}$$

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$$\begin{array}{l} \ldots \longrightarrow H_i(X,\widetilde{\mathbb{Z}}) \longrightarrow H_i(Y,\mathbb{Z}) \xrightarrow{p_*^i} H_i(X,\mathbb{Z}) \longrightarrow \\ \quad \longrightarrow H_{i-1}(X,\widetilde{\mathbb{Z}}) \longrightarrow H_{i-1}(Y,\mathbb{Z}) \xrightarrow{p_*^{i-1}} H_{i-1}(X,\mathbb{Z}) \longrightarrow \ldots \end{array}$$

$$\boxed{H_\bullet(X, \widetilde{\mathbb{Z}}) = \{\mathbb{Z}_2, 0, \mathbb{Z}_{\mathbf{2}}, 0, \mathbb{Z}_2, 0\}}$$

$2^2 = 4$ choices of torsion $\implies$ $SL(2,\mathbb{Z})$ singlet plus triplet.

Generalization to other orbifolds

The proposal generalizes straightforwardly to $\mathbb{C}^3/\mathbb{Z}_n$ singularities, as long as the singularity is isolated (so $n \in 2\mathbb{Z}+1$). Everything works beautifully in these cases too.
[Bianchi,Inverso,Morales,Pacifi:1307.0466], [I.G.-E., Heidenreich, Wrase:1307.1701]

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What lies beyond susy orbifolds?

General case

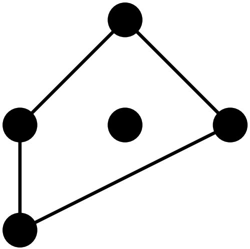
By a computation in algebraic topology one can see that for a toric $O3/O7$ orientifold of a toric CY_3 cone, with

- k sides
- isolated conical singularity of the cone
- fixed points of the orientifold only at the conical singularity

$$H^3(X, \widetilde{\mathbb{Z}}) = \mathbb{Z}_2^{k-2}$$

For example, for $\mathcal{C}_{\mathbb{C}}(dP_1) = \mathcal{C}_{\mathbb{R}}(Y^{2,1})$

$$H^3\left(Y^{2,1}/\mathbb{Z}_2, \widetilde{\mathbb{Z}}\right) = \mathbb{Z}_2 \oplus \mathbb{Z}_2$$

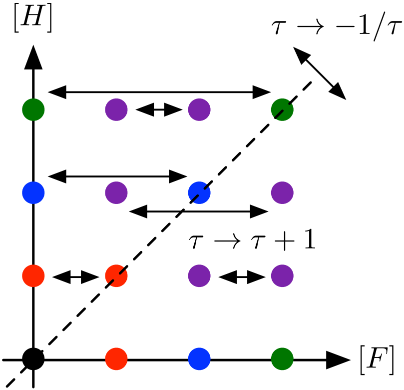


so there are $2^{2\cdot 2} = 16$ orientifold types:

- 1 $SL(2,\mathbb{Z})$ singlet, 3 triplets, 1 sextet.
- $\rightsquigarrow$ 10 different weakly coupled limits.

Orientifold phases for dP_1

More graphically, the duality structure of these theories can be depicted as



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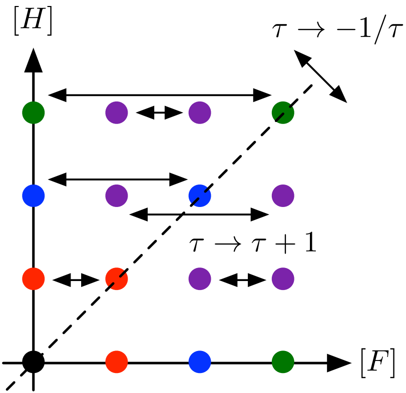
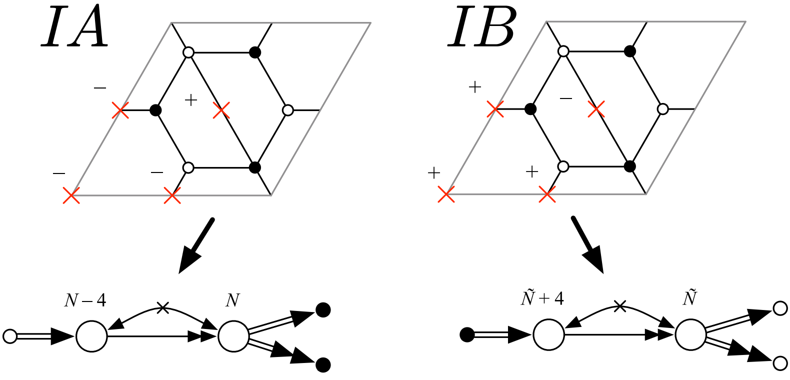


Our task is to map the dots to theories!

Known orientifolds of dP_1

The previously known orientifolds for branes at the dP_1 singularity can be obtained via dimer methods
[Franco,Hanany,Krefl,Park,Uranga,Vegh:0707.0298]



Known orientifolds of dP_1

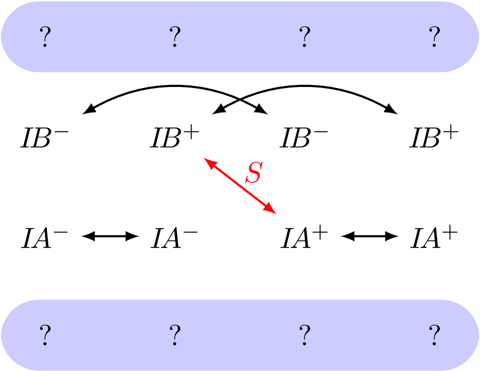
Anomaly and SCI matching tell us that theories IA and IB are dual to each other **iff** N is odd (and $N = \widetilde{N} + 2$). Furthermore, partially resolving $dP_1 \to \mathbb{C}^3/\mathbb{Z}_3 + \mathbb{C}^3$ allows us to read where the type I orientifolds are located in the torsion diagram:

$$
\begin{tikzcd}
? & ? & ? & ? \\
IB^- \arrow[rr, bend left, leftrightarrow] & IB^+ \arrow[rr, bend left, leftrightarrow] \arrow[dr, leftrightarrow, "S", red] & IB^- & IB^+ \\
IA^- \arrow[r, leftrightarrow] & IA^- & IA^+ \arrow[r, leftrightarrow] & IA^+ \\
? & ? & ? & ?
\end{tikzcd}
$$

(Sign $= -(-1)^N$)

Known orientifolds of dP_1

Anomaly and SCI matching tell us that theories IA and IB are dual to each other **iff** N is odd (and $N = \widetilde{N} + 2$). Furthermore, partially resolving $dP_1 \to \mathbb{C}^3/\mathbb{Z}_3 + \mathbb{C}^3$ allows us to read where the type I orientifolds are located in the torsion diagram:



(Sign $= -(-1)^N$)

New orientifolds from "inconsistent" tilings

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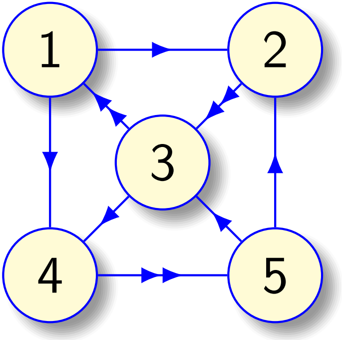
2. Not really known to be inconsistent, maybe just subtle. (Negative a-maximized R-charges.) Orientifolding an inconsistent theory can give a consistent theory. (R charges consistent with unitarity bounds.)
3. Looking to the classification in [0909.2868] we find an "inconsistent" tiling that admits an involution.

New orientifolds from "inconsistent" tilings

The candidate from [Davey,Hanany,Pasukonis:0909.2868]:



This almost works, but we need to add some flavors to make the IR physics match with the predictions of duality. We denote this theory (with flavors) as *Phase II*.

New orientifolds from deconfinement

One can generalize the procedure in [Davey,Hanany,Pasukonis:0909.2868] by reinterpreting it in terms of deconfinement [Berkooz:hep-th/9505067].

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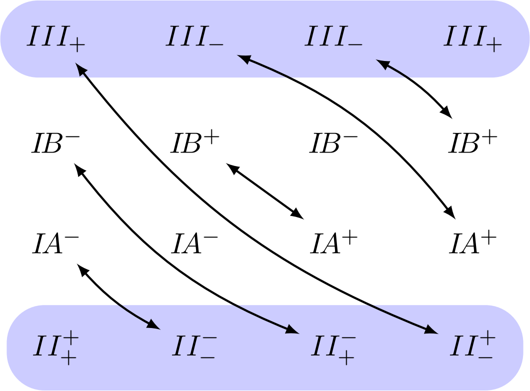
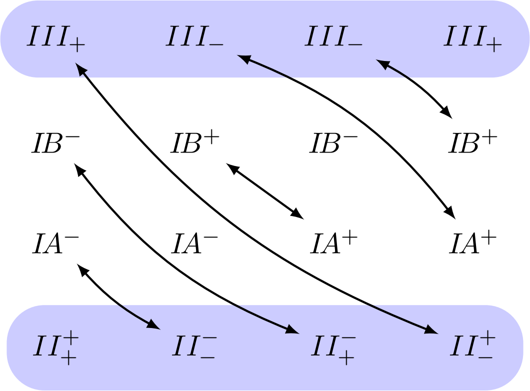
Schematically, if we have a theory with a two index representation, such as $\square\!\square$, $\begin{smallmatrix}\square\\[-2pt]\square\end{smallmatrix}$, or $(\square, \overline{\square})$ we can view this rep as being the meson of a confining theory, and work directly with the confining theory.

Combining this with Seiberg duality, we find a number of new IR equivalent descriptions of the same quiver, which admit orientifold involutions. We have identified, in addition to phase II before, also a *phase III* with the right duality properties.

New orientifolds from deconfinement



New orientifolds from deconfinement



Perfect agreement between SCIs, agrees with partial resolution, etc.

Conclusions

- We find very strong evidence for the existence of an extension of Montonen-Olive duality to interesting $\mathcal{N} = 1$ theories: non-conformal, chiral, . . .
- The best understood cases are isolated orientifolds of orbifolds, and some of the non-orbifold theories.
- We find evidence for the existence of a *large* class of hitherto unknown $\mathcal{N} = 1$ theories for orientifolded singularities.
- For some of these more involved theories we can construct a description of the IR fixed point, but not yet of the UV theory (which must, nevertheless, exist, but it may be intrinsically non-perturbative).

Vielen Dank!