Rigour & Rigidity:

Systematics of Intersecting D6-brane Model Building on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6$ with Discrete Torsion

Wieland Staessens

based on 1409.1236 [hep-th] with J. Ecker & G. Honecker

PRiSMA

PRISMA Cluster of Excellence
Mainz Institute for Theoretical Physics
Theoretical High Energy Physics,
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THEP

mitp Mainz Institute for Theoretical Physics

JG|U JOHANNES GUTENBERG UNIVERSITÄT MAINZ

25 September 2014, COST Workshop Mainz

Introduction

Challenges in String Model Building

- Microscopic realisation of MSSM, GUTS,...
  - ☞ Appearance of $SU(N)$, $SO(N)$, E_n gauge groups
  - ☞ 3 Chiral Generations of left/right-handed quarks/leptons
- Microscopic explanation of parameters (gauge couplings, Yukawa couplings, etc)
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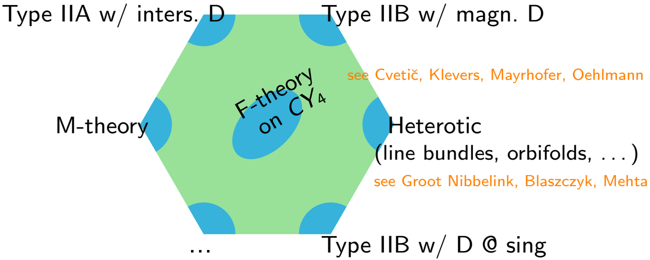

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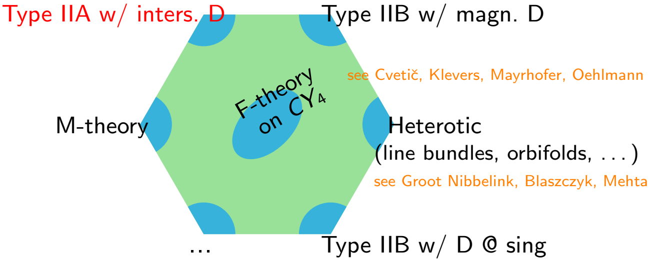

Intersecting D6-branes

Blumenhagen-Cvetič-Langacker-Shiu ('05); Blumenhagen-Körs-Lüst-Stieberger ('06); Ibañez-Uranga ('12); + other reviews

- $U(N)$ gauge groups: $N \uparrow\uparrow$ coincident D6-branes on $\mathcal{M}_{1,3} \times \Pi_a^{\text{3-cycle}}$
- @ $\Pi_a \cap \Pi_b \neq \emptyset$: $\mathcal{N} = 1$ chiral multiplet in $(N_a, \overline{N}_b)$ under $U(N_a) \times U(N_b)$
- On compact space $(T^6,\ T^6/\mathbb{Z}_N,\ T^6/\mathbb{Z}_N \times \mathbb{Z}_M)$:
  If $\Pi_a \cap \Pi_b > 1 \rightsquigarrow$ multiple generations (purely topological)
- Problem: On T^6 & $T^6/\mathbb{Z}_N$ D-branes can be displaced continuously

$\rightsquigarrow \langle$**Adjoint**$\rangle \neq 0$ induces spontaneous breaking of $U(N)$ ↯ $SU(3)_{QCD} \times SU(2)_L$

- Solution: $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2N}$ with discrete torsion contain additional $\mathbb{Z}_2$ twisted sectors
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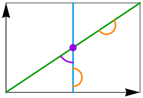

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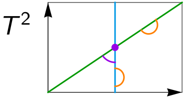

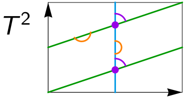

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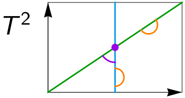

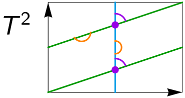

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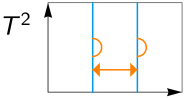

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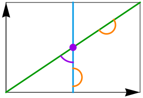

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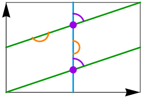

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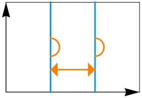
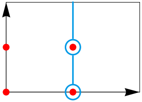


$\Longrightarrow T^2/\mathbb{Z}_2$



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$T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2N}$ with discrete torsion & D6-branes

- $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ with discrete torsion $\rightarrow$ Pati-Salam 4 gen.
  Blumenhagen-Cvetič-Marchesano-Shiu [hep-th/0502095]
- Machinery for other $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2N}$ with discrete torsion
  Förste-Honecker [1010.6070], Honecker [1109.3192]
- $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$ with discrete torsion $\rightarrow$ classes of Pati-Salam 3 gen.
  Honecker-(Ripka)-Staessens [(1209.3010), 1303.6845]
- $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6$ with discrete torsion $\rightarrow$ In this talk
  Ecker-Honecker-Staessens [1409.1236]

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_6 \times \Omega\mathcal{R}$: the geometry

$\mathbb{Z}_2 \times \mathbb{Z}_6$ action on $T^2_{(1)} \times T^2_{(2)} \times T^2_{(3)}$: $\quad z^k \stackrel{\theta^m \omega^n}{\longrightarrow} e^{2\pi i (m\, v_k + n\, w_k)} z^k$

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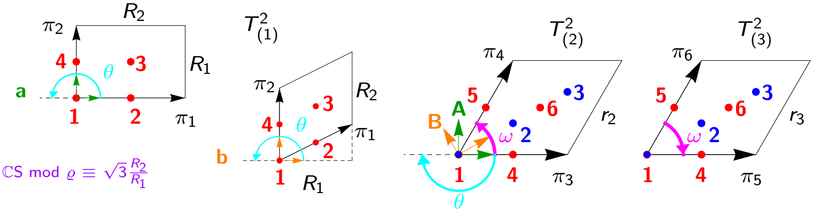

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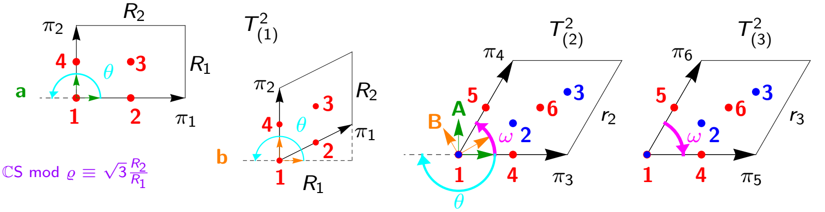

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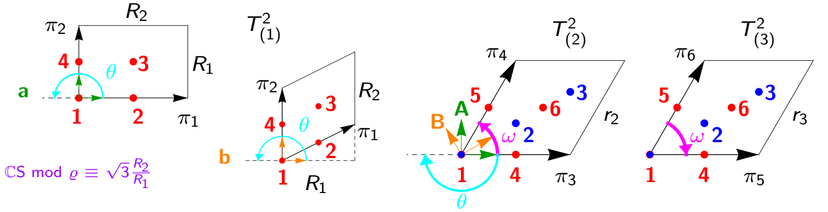

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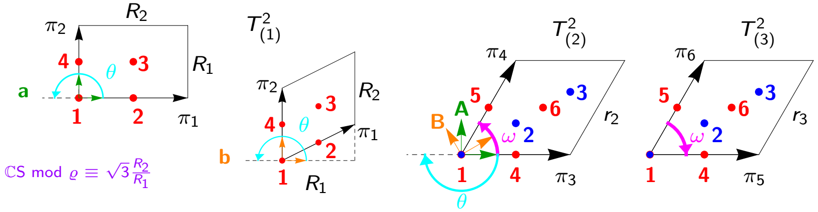

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$\rightsquigarrow$ construction of SUSY fractional 3-cycles stuck at $\mathbb{Z}_2^{(i)}$ fixed points:

$$\Pi_a^{\text{frac}} = \frac{1}{4}\left(\Pi_a^{\text{bulk}} + \sum_{i=1}^{3} \Pi_a^{\mathbb{Z}_2^{(i)}} \right)$$

Geometric representation (pictorially)

Parameters (schematically) on ambient T^2

- ⋆ wrapping numbers (n^i, m^i) with $n^i, m^i \in \mathbb{Z}$
- ⋆ displacement $\sigma \in \{0, 1\}$
- ⋆ $\tau^{\mathbb{Z}_2}$ eigenvalue $\in \{0, 1\}$
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Note: also $\mathbb{Z}_6$-orbifold images $\in \Pi_a^{\text{frac}}$

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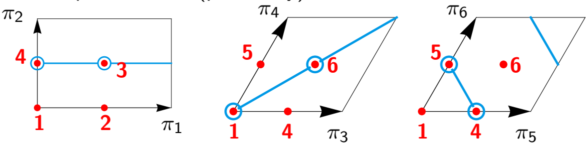

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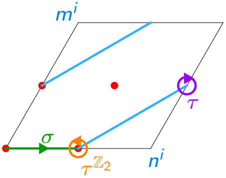

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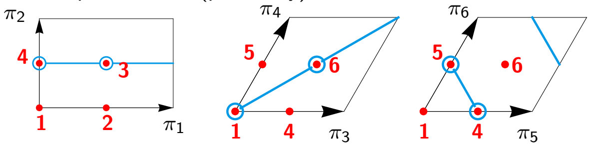

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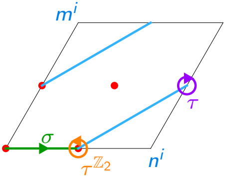
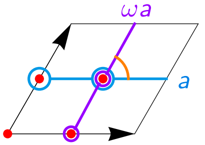


Note: also $\mathbb{Z}_6$-orbifold images $\in \Pi_a^{\text{frac}}$

Rigid 3-cycles

- fractional 3-cycles stuck @ $\mathbb{Z}_2$ fixed points $\Rightarrow$ ~~Position moduli~~
- If $a \cap (\omega^k a)_{k=1,2} \neq \emptyset \rightsquigarrow$ deformation moduli (**Adjoint**)



- Requiring $a \cap (\omega^k a)_{k=1,2} = \emptyset \quad \Rightarrow \quad$ geometric conditions on Π^{frac}
- Results: Completely rigid 3-cycles
  - ⋆ bulk orbit: $\uparrow\uparrow \Omega\mathcal{R}$ or $(n_a^1, m_a^1; 1, 0; 1, -1)$
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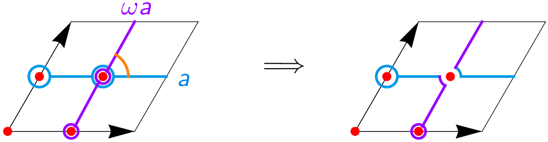

- If $\langle$**Adjoint**$\rangle \neq 0 \longrightarrow$ D-brane recombination ↯ $SU(3)_{QCD}$

  Blumenhagen-Görlich-Ott [hep-th/0211059], Honecker [hep-th/0303015], Cremades-Ibáñez-Marchesano [hep-th/0203160]

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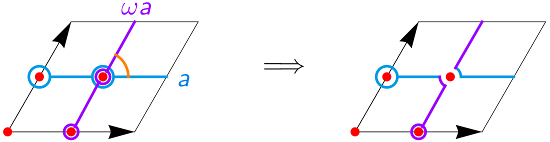

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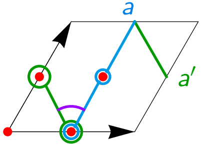
Symmetric and Antisymmetric Matter

- $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6$ compact. $\rightsquigarrow$ $\Omega\mathcal{R}$ projection preserves $\mathcal{N}=1$ SUSY
  $\Rightarrow$ Every Π_a^{frac} comes with $\Omega\mathcal{R}$-image $\Pi_{a'}^{\text{frac}}$



@ $(\omega^k a) \cap (\omega^k a)'_{k=0,1,2}$: $\exists$ chiral multipl.
in **Sym** and/or **Anti** repr.

- Chiral Matter in **Sym** ↯ $SU(3)_{QCD} \times SU(2)_L$
  Requiring absence of **Sym** $\Longrightarrow$ extra geometric conditions on Π^{frac}

- Results: Completely rigid 3-cycles free of chiral **Sym**
  - ⋆ bulk orbit: $\uparrow\uparrow \Omega\mathcal{R}$ or $(n_a^1, m_a^1; 1, 0; 1, -1)$ with constraints on (n_a^1, m_a^1)
    aAA: 31 combi. (n_a^1, m_a^1), **bAA**: 15 combi. (n_a^1, m_a^1)
  - ⋆ Choice of exotic O6-plane & discrete parameters

$$
\begin{array}{lll}
(1) & \eta_{\Omega\mathcal{R}} = -1 : & \sigma_a^2\tau_a^2 = \sigma_a^3\tau_a^3 = 0 \\
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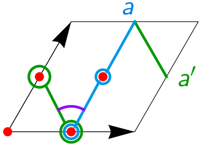

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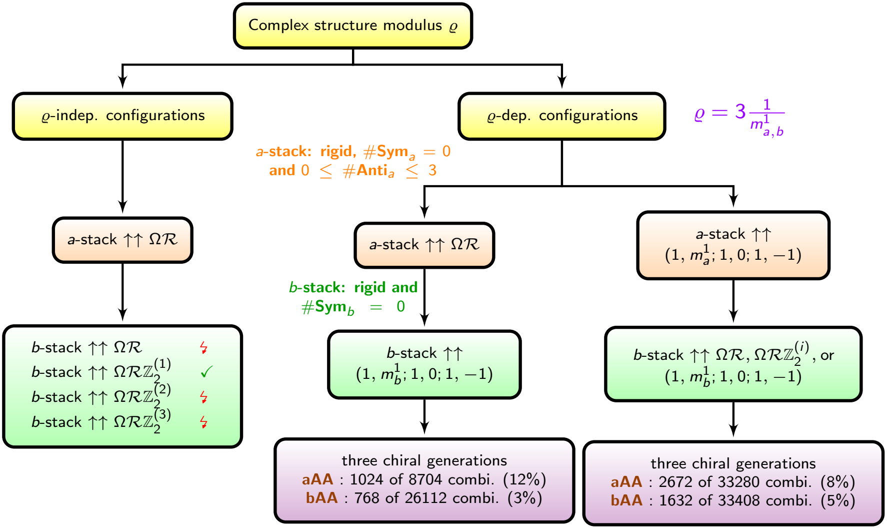
$$

Rigorous Roadmap for 3 gen. under $SU(3)_a \times SU(2)_b$



Explicit Model Building: Pati-Salam models

(SUSY) Pati-Salam Model

- $U(1)_Y \longrightarrow$ left-right symmetric w/ $SU(2)_R \times U(1)_{B-L}$ (+ 3 RH ν's)
- lepton $\sim$ 4th colour $\Rightarrow SU(3)_{QCD} \times U(1)_{B-L} \longrightarrow SU(4)_{\text{strong}}$

- **bAA** lattice: 2 classes of *local* 3-stack D6-brane models for $\varrho = 2$
  - ⋆ gauge group: $U(4)_a \times U(2)_b \times U(2)_c$ with 3 gen. (Q_L, L) & (Q_R, R)
  - ⋆ model I: Higgsless model II: 4 Higgses from non-chiral bc sector
  - ⋆ $SU(4)_a \times SU(2)_c \to SU(3)_{QCD} \times U(1)_Y$ with $(\mathbf{4}, 1, \mathbf{2}) + (\overline{\mathbf{4}}, 1, \mathbf{2})$

    Antoniadis-Leontaris (1989)
  - ⋆ RR tadpole conditions violated + extra matter in non-chiral sectors
- **aAA** lattice: 2 classes of *global* 4-stack D6-brane models, ϱ-indep
  - ⋆ gauge group: $U(4)_a \times USp(2)_b \times USp(2)_c \times U(6)_d || U(2)_d$ with 3 gen. (Q_L, L) & (Q_R, R)
  - ⋆ 10 Higgses (H_u, H_d)
  - ⋆ limited amount exotic matter in chiral and non-chiral sectors
  - ⋆ No GUT Higgses $(\mathbf{4}, 1, \mathbf{2}) + h.c. \rightsquigarrow$ different mechanism for GUT-breaking

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Conclusions & Outlook

Conclusions

- Intersecting D6-brane model building on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6 \times \Omega\mathcal{R})$ with discrete torsion using rigid 3-cycles
- Full classification of rigid 3-cycles (~~**Adj**~~) without chiral **Sym**
- Systematic method to find models w./ 3 chiral quark generations
- Only *global* 4-stack Pati-Salam models on **aAA** lattice (ϱ-indep)
- Note covered: reduction from 6 to 2 lattices (**aAA** & **bAA**) by virtue of symmetries + full classification of SUSY bulk orbits
- Not covered: exclusion of (global) $SU(5)$ GUT

Prospects

- Continue ϱ-dep search for *global* Pati-Salam models on **aAA** lattice
- Continue search for *global* left-right symm. models + MSSM
- In-depth analysis of the EFT (Yukawa + cubic couplings, gauge unification, etc)

Herzlichen Dank

Local Pati-Salam model type I

on **bAA** lattice, $\varrho = 2$, exotic $\Omega\mathcal{R}$-plane w/ $\eta_{\Omega\mathcal{R}} = -1$

Spectrum of the prototype I local Pati-Salam model on bAA		
State	Sector	$(SU(4)_a \times SU(2)_b \times SU(2)_c)_{U(1)_a \times U(1)_b \times U(1)_c}$
(Q_L, L)	ab	$2 \times (\mathbf{4}, \mathbf{2}, \mathbf{1})_{(1,-1,0)}$
(Q_L, L)	ab'	$(\mathbf{4}, \mathbf{2}, \mathbf{1})_{(1,1,0)}$
(Q_R, R)	ac'	$3 \times (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})_{(-1,0,-1)}$
$A, \tilde{A}$	aa'	$2 \times [(\mathbf{6}_{\mathbf{Anti}}, \mathbf{1}, \mathbf{1})_{(2,0,0)} + h.c.]$
$B, \tilde{B}$	bb'	$(\mathbf{1}, \mathbf{1}_{\mathbf{Anti}}, \mathbf{1})_{(0,2,0)} + h.c.$
$C, \tilde{C}$	cc'	$2 \times [(\mathbf{1}, \mathbf{1}, \mathbf{1}_{\mathbf{Anti}})_{(0,0,2)} + h.c.]$
$G_H, \tilde{G}_H$	ac	$2 \times \left[(\mathbf{4}, \mathbf{1}, \mathbf{2})_{(1,0,-1)} + h.c.\right]$

Local Pati-Salam model type II

on **bAA** lattice, $\varrho = 2$, exotic $\Omega\mathcal{R}$-plane w/ $\eta_{\Omega\mathcal{R}} = -1$

Spectrum of the prototype II local Pati-Salam model on bAA		
State	Sector	$(SU(4)_a \times SU(2)_b \times SU(2)_c)_{U(1)_a \times U(1)_b \times U(1)_c}$
(Q_L, L)	ab'	$3 \times (\mathbf{4}, \mathbf{2}, \mathbf{1})_{(1,1,0)}$
(Q_R, R)	ac'	$3 \times (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})_{(-1,0,-1)}$
(H_u, H_d)	bc'	$3 \times (\mathbf{1}, \mathbf{2}, \mathbf{2})_{(0,-1,-1)}$
$A, \tilde{A}$	aa'	$2 \times [(\mathbf{6}_{\mathbf{Anti}}, \mathbf{1}, \mathbf{1})_{(2,0,0)} + h.c.]$
	bb'	$2 \times [(\mathbf{1}, \mathbf{1}_{\mathbf{Anti}}, \mathbf{1})_{(0,2,0)} + h.c.]$
	cc'	$2 \times [(\mathbf{1}, \mathbf{1}, \mathbf{1}_{\mathbf{Anti}})_{(0,0,2)} + h.c.]$
	ab	$(\mathbf{4}, \mathbf{2}, \mathbf{1})_{(1,-1,0)} + h.c.$
	ab'	$(\mathbf{4}, \mathbf{2}, \mathbf{1})_{(1,1,0)} + h.c.$
$G_H, \tilde{G}_H$	ac	$2 \times [(\mathbf{4}, \mathbf{1}, \mathbf{2})_{(1,0,-1)} + h.c.]$
(H_u, H_d)	bc	$2 \times [(\mathbf{1}, \mathbf{2}, \mathbf{2})_{(0,1,-1)} + h.c.]$

Global Pati-Salam model type I

on **aAA** lattice, ϱ-independent, exotic $\Omega\mathcal{R}\mathbb{Z}_2^{(2)}$-plane w/ $\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(2)}} = -1$

Spectrum of the prototype I global Pati-Salam model on aAA		
State	Sector	$(SU(4)_a \times USp(2)_b \times USp(2)_c \times SU(6)_d)_{U(1)_a \times U(1)_d}$
(Q_L, L)	$ab = ab'$	$3 \times (\mathbf{4}, \mathbf{2}, \mathbf{1}, \mathbf{1})_{(1,0)}$
(Q_R, R)	$ac = ac'$	$3 \times (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}, \mathbf{1})_{(-1,0)}$
(H_u, H_d)	$bc = bc'$	$10 \times (\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1})_{(0,0)}$
	$bd = bd'$	$3 \times (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{6})_{(0,1)}$
	$cd = cd'$	$3 \times (\mathbf{1}, \mathbf{1}, \mathbf{2}, \overline{\mathbf{6}})_{(0,-1)}$
$A, \tilde{A}$	aa'	$2 \times \left[(\mathbf{6}_{\mathbf{Anti}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{(2,0)} + h.c.\right]$
	bb'	$5 \times (\mathbf{1}, \mathbf{1}_{\mathbf{Anti}}, \mathbf{1}, \mathbf{1})_{(0,0)}$
	cc'	$5 \times (\mathbf{1}, \mathbf{1}, \mathbf{1}_{\mathbf{Anti}}, \mathbf{1})_{(0,0)}$
	dd'	$2 \times [(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{15}_{\mathbf{Anti}}))_{(0,2} + h.c.]$
	ad	$2 \times [(\mathbf{4}, \mathbf{1}, \mathbf{1}, \overline{\mathbf{6}})_{(1,-1)} + h.c.]$
	ad'	$(\mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{6})_{(1,1)} + h.c.$

Global Pati-Salam model type II

on **aAA** lattice, ϱ-independent, exotic $\Omega\mathcal{R}\mathbb{Z}_2^{(2)}$-plane w/ $\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(2)}} = -1$

Spectrum of the prototype II global Pati-Salam model on aAA		
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$A, \tilde{A}$	aa'	$2 \times \left[(\mathbf{6}_{\mathbf{Anti}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{(2,0)} + h.c.\right]$
	bb'	$5 \times (\mathbf{1}, \mathbf{1}_{\mathbf{Anti}}, \mathbf{1}, \mathbf{1})_{(0,0)}$
	cc'	$5 \times (\mathbf{1}, \mathbf{1}, \mathbf{1}_{\mathbf{Anti}}, \mathbf{1})_{(0,0)}$
	dd'	$6 \times [(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}_{\mathbf{Anti}})_{(0,2)} + h.c.]$
	ad	$2 \times [(\mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{(1,-1)} + h.c.]$
	ad'	$(\mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{(1,1)} + h.c.$
	$bd = bd'$	$3 \times [(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2})_{(0,-1)} + h.c.]$
	$cd = cd'$	$3 \times [(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2})_{(0,-1)} + h.c.]$