TETRADS IN GEOMETRODYNAMICS.

$$f^{\mu\nu}{}_{;\nu} = 0$$

$$*f^{\mu\nu}{}_{;\nu} = 0$$

$$R_{\mu\nu} = f_{\mu\lambda}\, f_{\nu}{}^{\lambda} + *f_{\mu\lambda}\, *f_{\nu}{}^{\lambda}$$

$f_{\mu\nu} = \left(G^{1/2} / c^{2} \right) F_{\mu\nu}$ geometrized electromagnetic field.

$*f_{\mu\nu} = \frac{1}{2}\, \epsilon_{\mu\nu\sigma\tau}\, f^{\sigma\tau}$ dual tensor of $f_{\mu\nu}$.

WE MAKE THE DEFINITION

$$\xi_{\mu\nu} = \cos\alpha \; f_{\mu\nu} - {}^{*}f_{\mu\nu} \; \sin\alpha$$

THE FIELD HAS UNDERGONE A DUALITY ROTATION BY AN ANGLE $-\alpha$.

$$\xi_{\mu\nu} = e^{-{}^{*}\alpha} \; f_{\mu\nu} \qquad \text{(NOTATION)}$$

ASSUME THAT THE INVARIANTS

$f_{\mu\nu} f^{\mu\nu}$ AND $f_{\mu\nu} \, {}^{*}f^{\mu\nu}$ DO NOT VANISH

- NON NULL FIELD -

CHOOSE THE ANGLE α SO THAT

α : LOCAL SCALAR
THE COMPLEXION

$$\xi_{\mu\nu} \, {}^{*}\xi^{\mu\nu} = 0 \qquad \text{(MISNER AND WHEELER}$$

$$\Rightarrow \quad \tan(2\alpha) = -\left(f_{\mu\nu} {}^{*}f^{\mu\nu}\right) / \left(f_{\sigma\tau} f^{\sigma\tau}\right)$$

STRESS - ENERGY TENSOR

$$T_{\eta\nu} = f_{\eta\lambda} f_{\nu}{}^{\lambda} + {}^{*}f_{\eta\lambda} {}^{*}f_{\nu}{}^{\lambda}$$

$$f_{\eta\nu} = \xi_{\eta\nu} \cos\alpha + {}^{*}\xi_{\eta\nu} \sin\alpha$$ DUALITY ROTATION

$$\Rightarrow T_{\eta\nu} = \xi_{\eta\lambda} \xi_{\nu}{}^{\lambda} + {}^{*}\xi_{\eta\lambda} {}^{*}\xi_{\nu}{}^{\lambda}$$

$\xi_{\eta\nu}$ EXTREMAL FIELD

WE WOULD LIKE TO FIND A TETRAD THAT DIAGONALIZES $T_{\eta\nu}$ COVARIANTLY.

WE CONSIDER THE NON-NULL ELECTROMAGNETIC FIELDS /

$$Q = \xi_{\eta\nu} \xi^{\eta\nu} = -\sqrt{T_{\eta\nu} T^{\eta\nu}} \neq 0 .$$

THEN, WE FIND FOUR VECTORS THAT DIAGONALIZE $T_{\gamma\delta}$ IN GEOMETRODYNAMICS :

$$V^{\alpha}_{(1)} = \xi^{\alpha\lambda}\, \xi_{\rho\lambda}\, X^{\rho}$$

$$V^{\alpha}_{(2)} = \sqrt{\frac{-Q}{2}}\, \xi^{\alpha\lambda}\, X_{\lambda}$$

$$V^{\alpha}_{(3)} = \sqrt{\frac{-Q}{2}}\, {}^{*}\xi^{\alpha\lambda}\, Y_{\lambda}$$

$$V^{\alpha}_{(4)} = {}^{*}\xi^{\alpha\lambda}\, {}^{*}\xi_{\rho\lambda}\, Y^{\rho}$$

WE ARE FREE TO CHOOSE THE TWO VECTOR FIELDS X^{ρ} AND Y^{ρ} AS LONG AS THE FOUR VECTOR FIELDS ARE NOT TRIVIAL

X^{ρ} AND Y^{ρ} : GAUGE VECTORS.

IN A 4-DIM SPACE TIME TWO ANTISYMMETRIC FIELDS SATISFY THE RELATION:

$$A_{\eta\alpha} B^{\nu\alpha} - {}^*B_{\eta\alpha}\, {}^*A^{\nu\alpha} = \frac{1}{2} \delta_\eta^\nu \left(A_{\alpha\beta} B^{\alpha\beta}\right)$$

USING THIS RELATION AND THE EXTREMAL FIELD CONDITION

$$\xi_{\eta\nu}\, {}^*\xi^{\eta\nu} = 0$$

WE FIND:

$$\boxed{\xi_{\alpha\eta}\, {}^*\xi^{\eta\nu} = 0} \quad (1)$$

USING THE SAME RELATION:

$$\boxed{\xi_{\eta\alpha}\, \xi^{\nu\alpha} - {}^*\xi_{\eta\alpha}\, {}^*\xi^{\nu\alpha} = \frac{1}{2} \delta_\eta^\nu\, Q} \quad (2)$$

$$Q = \xi_{\eta\nu}\, \xi^{\eta\nu} \neq 0$$

THEN, USING ① AND ② WE FIND

$$V^{\alpha}_{(1)} \, T_{\alpha}{}^{\beta} = \frac{a}{2} V^{\beta}_{(1)}$$

$$V^{\alpha}_{(2)} \, T_{\alpha}{}^{\beta} = \frac{a}{2} V^{\beta}_{(2)}$$

$$V^{\alpha}_{(3)} \, T_{\alpha}{}^{\beta} = -\frac{a}{2} V^{\beta}_{(3)}$$

$$V^{\alpha}_{(4)} \, T_{\alpha}{}^{\beta} = -\frac{a}{2} V^{\beta}_{(4)}$$

ELECTROMAGNETIC POTENTIALS IN GEOMETRODYNAMICS.

OUR GOAL: SIMPLIFY AS MUCH AS WE CAN THE EXPRESSION OF THE ELECTROMAGNETIC FIELD THROUGH THE USE OF AN ORTHONORMAL TETRAD

→ SO THAT ITS GEOMETRICAL PROPERTIES CAN BE UNDERSTOOD IN AN EASIER WAY.

IN GEOMETRODYNAMICS: $f^{\eta\nu}{}_{;\nu} = 0$

$$*f^{\eta\nu}{}_{;\nu} = 0$$

⇒ EXISTENCE OF TWO POTENTIAL VECTORS:

$$f_{\eta\nu} = A_{\nu;\eta} - A_{\eta;\nu}$$

$$*f_{\eta\nu} = *A_{\nu;\eta} - *A_{\eta;\nu}$$

A_η AND $*A_\eta$ (NOTATION) NOT INDEPENDENT FROM EACH OTHER.

WE CAN MAKE THE CHOICE

$$X^{\rho} = A^{\rho}$$

$$Y^{\rho} = {}^{*}A^{\rho}$$

X^{ρ} AND Y^{ρ} : WE ARE FREE TO CHOOSE THE "GAUGE VECTORS

THEN, THE FOUR VECTORS ARE :

$$V_{(1)}^{\alpha} = \xi^{\alpha\lambda} \, \xi_{\rho\lambda} \, A^{\rho}$$

$$V_{(2)}^{\alpha} = \sqrt{\frac{-Q}{2}} \; \xi^{\alpha\lambda} \, A_{\lambda}$$

$$V_{(3)}^{\alpha} = \sqrt{\frac{-Q}{2}} \; {}^{*}\xi^{\alpha\lambda} \, {}^{*}A_{\lambda}$$

$$V_{(4)}^{\alpha} = {}^{*}\xi^{\alpha\lambda} \, {}^{*}\xi_{\rho\lambda} \, {}^{*}A^{\rho}$$

WE ASSUME FOR SIMPLICITY

$$- V_{(1)}^{\alpha} V_{(1)\alpha} = V_{(2)}^{\alpha} V_{(2)\alpha} > 0 \;\; || \;\; V_{(3)}^{\alpha} V_{(3)\alpha} = V_{(4)}^{\alpha} V_{(4)\alpha} >$$

GAUGE GEOMETRY

ONCE WE MAKE THE CHOICE

$$X^\rho = A^\rho \qquad Y^\rho = {}^*A^\rho$$

WHAT HAPPENS TO THE TETRAD VECTORS WHEN WE MAKE THE TRANSFORMATIONS:

$$A_\alpha \longrightarrow A_\alpha + \Lambda_{,\alpha}$$

$$^*A_\alpha \longrightarrow {}^*A_\alpha + {}^*\Lambda_{,\alpha}$$

NOTATION

$$\Lambda_{,\alpha} = \Lambda_\alpha \qquad {}^*\Lambda_{,\alpha} = {}^*\Lambda_\alpha$$

Λ AND $^*\Lambda$ ARE SCALARS.

SCHOUTEN DEFINED WHAT HE CALLED A TWO-BLADED STRUCTURE IN A SPACETIME.

BLADE ONE : $\left(V_{(1)}^{\alpha},\ V_{(2)}^{\alpha} \right)$

BLADE TWO : $\left(V_{(3)}^{\alpha},\ V_{(4)}^{\alpha} \right)$.

GAUGE TRANSFORMATIONS ON BLADE ONE

$$\begin{cases} \tilde{V}_{(1)}^{\alpha} = V_{(1)}^{\alpha} + \boxed{J^{\alpha\lambda} J_{\rho\lambda} \Lambda^{\rho}} \\ \tilde{V}_{(2)}^{\alpha} = V_{(2)}^{\alpha} + \boxed{\sqrt{\frac{-g}{2}}\, J^{\alpha\lambda} \Lambda_{\lambda}} \end{cases}$$

$$J^{\alpha\lambda} J_{\rho\lambda} \Lambda^{\rho} V_{(3)\alpha} = J^{\alpha\lambda} J_{\rho\lambda} \Lambda^{\rho} V_{(4)\alpha} = 0$$

$$\sqrt{\frac{-g}{2}}\, J^{\alpha\lambda} \Lambda_{\lambda} V_{(3)\alpha} = \sqrt{\frac{-g}{2}}\, J^{\alpha\lambda} \Lambda_{\lambda} V_{(4)\alpha} = 0.$$

WE WRITE THEN,

$$\begin{cases} \tilde{V}^{\alpha}_{(1)} = V^{\alpha}_{(1)} + C\, V^{\alpha}_{(1)} + D\, V^{\alpha}_{(2)} \\ \tilde{V}^{\alpha}_{(2)} = V^{\alpha}_{(2)} + E\, V^{\alpha}_{(1)} + F\, V^{\alpha}_{(2)} . \end{cases}$$

USING RELATIONS ① AND ②.

WE FIND :

$$E = D$$
$$F = C$$

$$\begin{cases} C = \left(-\frac{Q}{2}\right) V_{(1)\sigma} \Lambda^{\sigma} \Big/ \left(V_{(2)\beta} V^{\beta}_{(2)} \right) \\ D = \left(-\frac{Q}{2}\right) V_{(2)\sigma} \Lambda^{\sigma} \Big/ \left(V_{(1)\beta} V^{\beta}_{(1)} \right) . \end{cases}$$

$$\begin{cases} \tilde{V}^{\alpha}_{(1)} \tilde{V}_{(1)\alpha} = \left[(1+C)^2 - D^2 \right] V^{\alpha}_{(1)} V_{(1)\alpha} \\ \tilde{V}^{\alpha}_{(2)} \tilde{V}_{(2)\alpha} = \left[(1+C)^2 - D^2 \right] V^{\alpha}_{(2)} V_{(2)\alpha} \end{cases}$$

$V^{\alpha}_{(1)} V_{(1)\alpha} =$

$= - V^{\alpha}_{(2)} V_{(2}$

CASES: $\left[(1+c)^2 - D^2\right] > 0$

① $1+c > 0$

$$\frac{\tilde{V}_{(1)}^{\alpha}}{\sqrt{-\tilde{V}_{(1)}^{\beta}\tilde{V}_{(1)\beta}}} = \frac{(1+c)}{\sqrt{(1+c)^2 - D^2}}\frac{V_{(1)}^{\alpha}}{\sqrt{-V_{(1)}^{\beta}V_{(1)\beta}}} + \frac{D}{\sqrt{(1+c)^2 - D^2}}\frac{V_{(2)}^{\alpha}}{\sqrt{V_{(2)}^{\beta}V_{(2)\beta}}}$$

$$\frac{\tilde{V}_{(2)}^{\alpha}}{\sqrt{\tilde{V}_{(2)}^{\beta}\tilde{V}_{(2)\beta}}} = \frac{D}{\sqrt{(1+c)^2 - D^2}}\frac{V_{(1)}^{\alpha}}{\sqrt{-V_{(1)}^{\beta}V_{(1)\beta}}} + \frac{(1+c)}{\sqrt{(1+c)^2 - D^2}}\frac{V_{(2)}^{\alpha}}{\sqrt{V_{(2)}^{\beta}V_{(2)\beta}}}$$

AN ELECTROMAGNETIC GAUGE TRANSFORMATION GENERATES A BOOST TRANSFORMATION ON THE NORMALIZED $\left(\frac{V_{(1)}^{\alpha}}{\sqrt{-V_{(1)}^{\beta}V_{(1)\beta}}}, \frac{V_{(2)}^{\alpha}}{\sqrt{V_{(2)}^{\beta}V_{(2)\beta}}}\right)$.

② $1 + c < 0$

$$\frac{\tilde{V}^{\alpha}_{(1)}}{\sqrt{-\tilde{V}^{\beta}_{(1)}\tilde{V}_{(1)\beta}}} = \frac{[-(1+c)]}{\sqrt{(1+c)^2 - D^2}} \frac{(-V^{\alpha}_{(1)})}{\sqrt{-V^{\beta}_{(1)}V_{(1)\beta}}} + \frac{[-D]}{\sqrt{(1+c)^2 - D^2}} \frac{(-V^{\alpha}_{(2)})}{\sqrt{V^{\beta}_{(2)}V_{(2)\beta}}}$$

$$\frac{\tilde{V}^{\alpha}_{(2)}}{\sqrt{\tilde{V}^{\beta}_{(2)}\tilde{V}_{(2)\beta}}} = \frac{[-D]}{\sqrt{(1+c)^2 - D^2}} \frac{(-V^{\alpha}_{(1)})}{\sqrt{-V^{\beta}_{(1)}V_{(1)\beta}}} + \frac{[-(1+c)]}{\sqrt{(1+c)^2 - D^2}} \frac{(-V^{\alpha}_{(2)})}{\sqrt{V^{\beta}_{(2)}V_{(2)\beta}}}$$

AN ELECTROMAGNETIC GAUGE TRANSFORMATION GENERATES THE COMPOSITION OF AN INVERSION AND A BOOST OF THE NORMALIZED

$$\left(\frac{V^{\alpha}_{(1)}}{\sqrt{-V^{\beta}_{(1)}V_{(1)\beta}}} , \frac{V^{\alpha}_{(2)}}{\sqrt{V^{\beta}_{(2)}V_{(2)\beta}}} \right) .$$

$$\left[(1+C)^2 - D^2\right] < 0$$

IMPROPER TRANSFORMATIONS ON BLADE (1).

$$\tilde{V}^{\alpha}_{(1)} \tilde{V}_{(1)\alpha} = \left[-(1+C)^2 + D^2\right] \left(-V^{\alpha}_{(1)} V_{(1)\alpha}\right)$$

$$\left(-\tilde{V}^{\alpha}_{(2)} \tilde{V}_{(2)\alpha}\right) = \left[-(1+C)^2 + D^2\right] \left(V^{\alpha}_{(2)} V_{(2)\alpha}\right).$$

(3) $D > 0$ AND $1+C > 0$ OR $D > 0$ AND $1+C < 0$

$$\frac{\tilde{V}^{\alpha}_{(1)}}{\sqrt{\tilde{V}^{\beta}_{(1)} \tilde{V}_{(1)\beta}}} = \frac{(1+C)}{\sqrt{-(1+C)^2 + D^2}} \frac{V^{\alpha}_{(1)}}{\sqrt{-V^{\beta}_{(1)} V_{(1)\beta}}} + \frac{D}{\sqrt{-(1+C)^2 + D^2}} \frac{V^{\alpha}_{(2)}}{\sqrt{V^{\beta}_{(2)} V_{(2)\beta}}}$$

$$\frac{\tilde{V}^{\alpha}_{(2)}}{\sqrt{-V^{\beta}_{(2)} V_{(2)\beta}}} = \frac{D}{\sqrt{-(1+C)^2 + D^2}} \frac{V^{\alpha}_{(1)}}{\sqrt{-V^{\beta}_{(1)} V_{(1)\beta}}} + \frac{(1+C)}{\sqrt{-(1+C)^2 + D^2}} \frac{V^{\alpha}_{(2)}}{\sqrt{V^{\beta}_{(2)} V_{(2)\beta}}}.$$

BOOST $\begin{pmatrix} \frac{D}{\sqrt{-(1+C)^2 + D^2}} & \frac{(1+C)}{\sqrt{-(1+C)^2 + D^2}} \\ \frac{(1+C)}{\sqrt{-(1+C)^2 + D^2}} & \frac{D}{\sqrt{-(1+C)^2 + D^2}} \end{pmatrix}$ COMPOSED WITH

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ NOT A LORENTZ TRANSF.

(4) $D<0$ AND $1+C<0$ OR $D<0$ AND $1+C>0$

$$\frac{\tilde{V}_{(1)}^{\alpha}}{\sqrt{\tilde{V}_{(1)}^{\beta}\tilde{V}_{(1)\beta}}} = \frac{[-(1+C)]}{\sqrt{-(1+C)^2+D^2}}\frac{(-V_{(1)}^{\alpha})}{\sqrt{-V_{(1)}^{\beta}V_{(1)\beta}}} + \frac{[-D]}{\sqrt{-(1+C)^2+D^2}}\frac{(-V_{(2)}^{\alpha})}{\sqrt{V_{(2)}^{\beta}V_{(2)\beta}}}$$

$$\frac{\tilde{V}_{(2)}^{\alpha}}{\sqrt{-\tilde{V}_{(2)}^{\beta}\tilde{V}_{(2)\beta}}} = \frac{[-D]}{\sqrt{-(1+C)^2+D^2}}\frac{(-V_{(1)}^{\alpha})}{\sqrt{-V_{(1)}^{\beta}V_{(1)\beta}}} + \frac{[-(1+C)]}{\sqrt{-(1+C)^2+D^2}}\frac{(-V_{(2)}^{\alpha})}{\sqrt{V_{(2)}^{\beta}V_{(2)\beta}}}$$

$D = 1+C$ OR $D = -(1+C)$

COULD NOT EXIST.

BOOST $\begin{pmatrix} \frac{[-D]}{\sqrt{-(1+C)^2+D^2}} & \frac{[-(1+C)]}{\sqrt{-(1+C)^2+D^2}} \\ \frac{[-(1+C)]}{\sqrt{-(1+C)^2+D^2}} & \frac{[-D]}{\sqrt{-(1+C)^2+D^2}} \end{pmatrix}$ COMPOSED WITH $-\mathbb{1}_{2\times 2}$ AND COMPOSED WITH $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

NOT A LORENTZ TRANSFORMATION

GAUGE TRANSFORMATIONS ON BLADE TWO

$$\begin{cases} \tilde{V}^{\alpha}_{(3)} = V^{\alpha}_{(3)} + \sqrt{\frac{-\ell}{2}} \; {}^{*}f^{\alpha\lambda} \; {}^{*}\Lambda_{\lambda} \\ \tilde{V}^{\alpha}_{(4)} = V^{\alpha}_{(4)} + {}^{*}f^{\alpha\lambda} \; {}^{*}f_{\rho\lambda} \; {}^{*}\Lambda^{\rho} \end{cases}$$

$$\begin{cases} \sqrt{\frac{-\ell}{2}} \; {}^{*}f^{\alpha\lambda} \; {}^{*}\Lambda_{\lambda} \; V_{(1)\alpha} = \sqrt{\frac{-\ell}{2}} \; {}^{*}f^{\alpha\lambda} \; {}^{*}\Lambda_{\lambda} \; V_{(2)\alpha} = 0 \\ {}^{*}f^{\alpha\lambda} \; {}^{*}f_{\rho\lambda} \; {}^{*}\Lambda^{\rho} \; V_{(1)\alpha} = {}^{*}f^{\alpha\lambda} \; {}^{*}f_{\rho\lambda} \; {}^{*}\Lambda^{\rho} \; V_{(2)\alpha} = 0 \end{cases}$$

$$\begin{cases} \tilde{V}^{\alpha}_{(3)} = V^{\alpha}_{(3)} + K \, V^{\alpha}_{(3)} + L \, V^{\alpha}_{(4)} \\ \tilde{V}^{\alpha}_{(4)} = V^{\alpha}_{(4)} + M \, V^{\alpha}_{(3)} + N \, V^{\alpha}_{(4)} \end{cases}$$

USING RELATIONS (1) AND (2) $\Rightarrow$

$K = N$

$L = -M$

$$M = \left(-\frac{\ell}{2}\right) V_{(3)\sigma} \, {}^{*}\Lambda^{\sigma} / \left(V_{(4)\beta} \, V^{\beta}_{(4)}\right)$$

$$N = \left(-\frac{\ell}{2}\right) V_{(4)\sigma} \, {}^{*}\Lambda^{\sigma} / \left(V_{(3)\beta} \, V^{\beta}_{(3)}\right).$$

$$\begin{cases} \tilde{V}_{(3)}^{\beta} \tilde{V}_{(3)/\beta} = \left[(1+N)^2 + M^2 \right] V_{(3)}^{\beta} V_{(3)/\beta} \\ \tilde{V}_{(4)}^{\beta} \tilde{V}_{(4)/\beta} = \left[(1+N)^2 + M^2 \right] V_{(4)}^{\beta} V_{(4)/\beta} \end{cases}$$

$$\frac{\tilde{V}_{(3)}^{\alpha}}{\sqrt{\tilde{V}_{(3)}^{\beta} \tilde{V}_{(3)/\beta}}} = \frac{(1+N)}{\sqrt{(1+N)^2 + M^2}} \frac{V_{(3)}^{\alpha}}{\sqrt{V_{(3)}^{\beta} V_{(3)/\beta}}} - \frac{M}{\sqrt{(1+N)^2 + M^2}} \frac{V_{(4)}^{\alpha}}{\sqrt{V_{(4)}^{\beta} V_{(4)/\beta}}}$$

$$\frac{\tilde{V}_{(4)}^{\alpha}}{\sqrt{\tilde{V}_{(4)}^{\beta} \tilde{V}_{(4)/\beta}}} = \frac{M}{\sqrt{(1+N)^2 + M^2}} \frac{V_{(3)}^{\alpha}}{\sqrt{V_{(3)}^{\beta} V_{(3)/\beta}}} + \frac{(1+N)}{\sqrt{(1+N)^2 + M^2}} \frac{V_{(4)}^{\alpha}}{\sqrt{V_{(4)}^{\beta} V_{(4)/\beta}}}$$

FOR $(1+N)^2 + M^2 > 0$, THE $*A^{\alpha} \longrightarrow *A^{\alpha} + *\Lambda^{\alpha}$ GAUGE TRANSFORMATION, GENERATES A ROTATION TRANSFORMATION OF THE NORMALIZED VECTORS:

$$\left(\frac{V_{(3)}^{\alpha}}{\sqrt{V_{(3)}^{\beta} V_{(3)/\beta}}} , \frac{V_{(4)}^{\alpha}}{\sqrt{V_{(4)}^{\beta} V_{(4)/\beta}}} \right).$$

GROUP ISOMORPHISM

LB1 (LORENTZ BLADE ONE)

THE GROUP OF SO(1,1) BOOST TETRA[D]
TRANSFORMATIONS ON BLADE ON[E]
PLUS $-\mathbb{1}_{2\times 2}$ PLUS $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ NOT A LORENT[Z] TRANSF.

$\Lambda^{0}_{\ 0} = \Lambda^{1}_{\ 1} = 0 \quad \Lambda^{0}_{\ 1} = \Lambda^{1}_{\ 0} = 1$

LB2 (LORENTZ BLADE TWO)

THE GROUP OF LORENTZ TETRAD
TRANSFORMATIONS (ROTATIONS) SO(2)
ON BLADE TWO.

LB1 ISOMORPHIC TO LB2

ISOMORPHISM ON BLADE ONE

THEOREM: THE MAPPING BETWEEN THE LOCAL GAUGE GROUP AND THE GROUP LB1 IS AN ISOMORPHISM.

ISOMORPHISM ON BLADE TWO

THEOREM: THE MAPPING BETWEEN THE LOCAL GAUGE GROUP AND THE GROUP LB2 IS AN ISOMORPHISM.

ORTHONORMAL TETRAD

$$U^\alpha = \xi^{\alpha\lambda} \xi_{\rho\lambda} A^\rho \Big/ \left(\sqrt{\frac{-Q}{2}} \sqrt{A_\eta \xi^{\eta\sigma} \xi_{\nu\sigma} A^\nu} \right)$$

$$V^\alpha = \xi^{\alpha\lambda} A_\lambda \Big/ \left(\sqrt{A_\eta \xi^{\eta\sigma} \xi_{\nu\sigma} A^\nu} \right)$$

$$Z^\alpha = {}^*\xi^{\alpha\lambda} \, {}^*A_\lambda \Big/ \left(\sqrt{{}^*A_\eta \, {}^*\xi^{\eta\sigma} \, {}^*\xi_{\nu\sigma} \, {}^*A^\nu} \right)$$

$$W^\alpha = {}^*\xi^{\alpha\lambda} \, {}^*\xi_{\rho\lambda} \, {}^*A^\rho \Big/ \left(\sqrt{\frac{-Q}{2}} \sqrt{{}^*A_\eta \, {}^*\xi^{\eta\sigma} \, {}^*\xi_{\nu\sigma} \, {}^*A^\nu} \right)$$

$$-U^\alpha U_\alpha = V^\alpha V_\alpha = Z^\alpha Z_\alpha = W^\alpha W_\alpha = 1.$$

$$g_{\alpha\beta} = -U_\alpha U_\beta + V_\alpha V_\beta + Z_\alpha Z_\beta + W_\alpha W_\beta$$

$$T_{\alpha\beta} = \left(\frac{Q}{2}\right)\left[-U_\alpha U_\beta + V_\alpha V_\beta - Z_\alpha Z_\beta - W_\alpha W_\beta\right]$$

$$f_{\alpha\beta} = -2\sqrt{\frac{-Q}{2}}\cos(\alpha)\; U_{[\alpha} V_{\beta]} +$$

$$+ 2\sqrt{\frac{-Q}{2}}\sin(\alpha)\; Z_{[\alpha} W_{\beta]} \; .$$

NULL TETRAD

$i^2 = -1$.

$$K_\alpha = \frac{1}{\sqrt{2}}\left(U_\alpha + V_\alpha\right) \qquad L_\alpha = \frac{1}{\sqrt{2}}\left(U_\alpha - V_\alpha\right)$$

$$T_\alpha = \frac{1}{\sqrt{2}}\left(Z_\alpha + i W_\alpha\right) \qquad \bar{T}_\alpha = \frac{1}{\sqrt{2}}\left(Z_\alpha - i W_\alpha\right)$$

BIVECTORS

$$U_{\alpha\beta} = \bar{T}_\alpha L_\beta - \bar{T}_\beta L_\alpha$$

$$V_{\alpha\beta} = K_\alpha T_\beta - K_\beta T_\alpha$$

$$W_{\alpha\beta} = T_\alpha \bar{T}_\beta - T_\beta \bar{T}_\alpha - K_\alpha L_\beta + K_\beta L_\alpha$$

SELF-DUAL ELECTROMAGNETIC BIVECTOR

$$\Phi_{\alpha\beta} = f_{\alpha\beta} + i\, {*f_{\alpha\beta}} = \boxed{-\sqrt{\frac{-Q}{2}}\, e^{-i\alpha}}\, W_{\alpha\beta}$$

$$= \phi_1$$

VACUUM EINSTEIN - MAXWELL EQUATIONS

(NEWMAN - PENROSE TETRAD FORMALISM)

$$D\phi_1 = 2\rho\phi_1$$

$$\delta\phi_1 = -2\tau\phi_1$$

$$\bar{\delta}\phi_1 = -2\pi\phi_1$$

$$\Delta\phi_1 = -2\mu\phi_1 .$$

SUMMARY

- NEW ORTHONORMAL TETRAD FOR NON-NULL ELECTROMAGNETIC FIELDS IN CURVED SPACETIMES.

- ISOMORPHISMS BETWEEN THE LOCAL GAUGE GROUP AND LOCAL LB1 AND LB2 GROUPS. (GEOMETRIZATION OF GAUGE THEORIES).

- MAXIMUM SIMPLIFICATION OF RELEVANT TENSORS AND FIELD EQUATIONS.

- NEW TETRADS ENCODE GRAVITATIONAL AND GAUGE INFORMATION.

- EXPLICIT RELATIONSHIP BETWEEN "GAUGE" AND "GRAVITY".

- WE ARE INTRODUCING AND EXPLICIT "LINK" BETWEEN THE "INTERNAL" AND THE "SPACETIME", SO FAR DETACHED FROM EACH OTHER.

- FOR OTHER THEORIES

  OTHER FIELD EQUATIONS

  NON-ABELIAN

→ NEW CHOICES FOR THE "GAUGE VECTORS" ALLOW TO PROVE THEOREMS IN THE NON-ABELIN CASE.

- EXTENSION OR GENERALIZATION TO NON-ABELIAN THEORIES

$SU(2) \times U(1)$

IS IT POSSIBLE TO BUILD THE NEW TETRADS WITH SKELETON - GAUGE VECTOR STRUCTURE IN HIGHER DIMENSIONAL SPACETIMES ?

IS IT POSSIBLE IN HIGHER DIMENSIONAL SPACETIMES TO PROVE NEW RESULTS IN GROUP THEORY FOR THE ASSOCIATED SYMMETRIES ?

NULL FIELD

$$f_{\eta\nu} = K_\eta V_\nu - K_\nu V_\eta$$

$$V_\eta V^\eta = 1 \qquad K_\nu K^\nu = 0$$

$$K_\eta V^\eta = 0$$

$$R_{\eta\nu} = 2 K_\eta K_\nu$$