EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

Mainz Institute for Theoretical Physics

BITS Pilani
Hyderabad Campus

“The String Theory Universe”

JGU Mainz, 22-26 September 2014

innovate achieve lead

BITS Pilani
Hyderabad Campus

Higher dimensional Bianchi type-III universe with strange quark matter attached to string cloud in general relativity

Dr. P K Sahoo
Department of Mathematics
sahoomaku@rediffmail.com,
pksahoo@hyderabad.bits-pilani.ac.in

Abstract

In this paper, our intention is to construct 5-dimensional Bianchi type-III cosmological models for quark matter attached to a string cloud in general relativity. Different cases for the metric potentials are considered and studied. The physical and kinematical behaviors of all the models are discussed. It is observed that most of the models admit initial singularity.

Introduction

Strange Quark Matter

Typically, strange quark matter is modeled with an equation of state based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In this model, quarks are thought of as degenerate Fermi gases, which exist only in a region of space endowed with vacuum energy density B_C (called the bag model). Additionally, in the framework of this model, the quark matter is composed of massless *u*, *d* quarks,-; massive *s* quarks; and electrons. In the simplified version of this model, on which our study is based, quarks are massless and non-interacting.

Strange Quark Matter…

We then have quark pressure $p_q = \frac{\rho_q}{3}$ (ρ_q is the quark energy density); the total energy density is

$$\rho = \rho_q + B_C \qquad (1.1)$$

while total pressure is

$$p = p_q - B_C \qquad (1.2)$$

Metric and energy momentum tensor

We consider the 5-dimensional Bianchi type-III metric in the form of

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2ax}dy^2 - C^2dz^2 - D^2du^2 \qquad (2.1)$$

where A, B, C, and D are the functions of time t only.

The energy momentum tensor for string cloud is given by

$$T_{ij} = \rho u_i u_j - \rho_s x_i x_j \qquad (2.2)$$

Here, ρ is the rest energy density for the cloud of strings with particles attached to them and ρ_s is the string tension density. They are related by

Metric and energy momentum tensor…

$$\rho = \rho_p + \rho_s \qquad (2.3)$$

where ρ_p is the particle energy density.

We know that string is free to vibrate. The vibration models of the string represent different types of particles because these models are seen as different masses or spins. Therefore, here we consider quarks instead of particles in the string cloud. Moreover, we consider here quark matter energy density instead of particle energy density in the string cloud.

Metric and energy momentum tensor…

In this case, from Eq. (2.3), we get

$$\rho = \rho_q + \rho_s + B_C \qquad (2.4)$$

From Eqs. (2.3) and (2.4), we have the energy momentum tensor for strange quark matter attached to the string as follows:

$$T_{ij} = \left(\rho_q + \rho_s + B_C\right) u_i u_j - \rho_s x_i x_j \qquad (2.5)$$

where u_i is the 5 velocity of the particles and x_i is the unit space-like vector representing the direction of string.

They are related by

$$u^i u_i = -x^i x_i = -1 \text{ and } u^i x_i = 0$$

Metric and energy momentum tensor…

We have taken the direction of the string along the *z*-axis. The components of the energy momentum tensor are then

$$T_1^1 = T_2^2 = T_4^4 = 0,\ T_3^3 = \rho_s\,,\, T_5^5 = \rho \qquad (2.6)$$

where ρ and ρ_s are functions of t only.

Field equations and their solutions

Einstein's field equations read as

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T_{ij} \qquad (3.1)$$

The field equations of Eq. (3.1) for the metric of Eq. (2.1) can be written as follows:

$$\frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{D_{55}}{D} + \frac{B_5 C_5}{BC} + \frac{B_5 D_5}{BD} + \frac{C_5 D_5}{CD} = 0 \qquad (3.2)$$

$$\frac{A_{55}}{A} + \frac{C_{55}}{C} + \frac{D_{55}}{D} + \frac{A_5 C_5}{AC} + \frac{A_5 D_5}{AD} + \frac{C_5 D_5}{CD} = 0 \qquad (3.3)$$

Field equations and their solutions….

$$\frac{A_{55}}{A}+\frac{B_{55}}{B}+\frac{D_{55}}{D}+\frac{A_5B_5}{AB}+\frac{A_5D_5}{AD}+\frac{B_5D_5}{BD}-\frac{a^2}{A^2}=8\pi\rho_s \qquad (3.4)$$

$$\frac{A_{55}}{A}+\frac{B_{55}}{B}+\frac{C_{55}}{C}+\frac{A_5B_5}{AB}+\frac{A_5C_5}{AC}+\frac{B_5C_5}{BC}-\frac{a^2}{A^2}=0 \qquad (3.5)$$

$$\frac{A_5B_5}{AB}+\frac{A_5C_5}{AC}+\frac{B_5C_5}{BC}+\frac{A_5D_5}{AD}+\frac{B_5D_5}{BD}+\frac{C_5D_5}{CD}-\frac{a^2}{A^2}=8\pi\rho \qquad (3.6)$$

$$\frac{A_5}{A}-\frac{B_5}{B}=0 \qquad (3.7)$$

Field equations and their solutions….

where the subscript 5 after A, B, C, and D denotes ordinary differentiation with respect to t.

From Eq. (3.7), we get

$$A = \alpha B$$

Without loss of generality we take the arbitrary constant $\alpha = 1$ such that we have

$$A = B \tag{3.8}$$

Using Eq. (3.8), the field equations of Eqs. (3.2) through (3.6) reduce to the following:

Field equations and their solutions….

$$\frac{B_{55}}{B}+\frac{C_{55}}{C}+\frac{D_{55}}{D}+\frac{B_5C_5}{BC}+\frac{B_5D_5}{BD}+\frac{C_5D_5}{CD}=0 \qquad (3.9)$$

$$2\left(\frac{B_{55}}{B}\right)+\frac{D_{55}}{D}+\left(\frac{B_5}{B}\right)^2+2\left(\frac{B_5D_5}{BD}\right)-\frac{a^2}{B^2}=8\pi\rho_s \qquad (3.10)$$

$$2\left(\frac{B_{55}}{B}\right)+\frac{C_{55}}{C}+\left(\frac{B_5}{B}\right)^2+2\left(\frac{B_5C_5}{BC}\right)-\frac{a^2}{A^2}=0 \qquad (3.11)$$

$$\left(\frac{B_5}{B}\right)^2+2\left(\frac{B_5C_5}{BC}\right)+2\left(\frac{B_5D_5}{BD}\right)+\frac{C_5D_5}{CD}-\frac{a^2}{B^2}=8\pi\rho \qquad (3.12)$$

Field equations and their solutions….

Thus, we have 4 equations with 5 unknowns, B, C, D, ρ and ρ_s

Since these equations are highly nonlinear in nature, in order to get a deterministic solution we need one assumption. We shall explore physically meaningful solutions of the field equations of Eqs. (3.9) through (3.12) by considering a simplifying assumption of the field variables *B*, *C*, and *D*.

In order to obtain a simple but physically realistic solution, let us choose a simple power-law form of the scale factor

$$B = t^n \tag{3.13}$$

where *n* is an arbitrary constant.

Field equations and their solutions….

Using Eq. (3.13) in Eq. (3.11), we get

$$\frac{C_{55}}{C}+2\frac{2nC_5}{tC}+\frac{2n(n-1)+n^2}{t^2}-\frac{a^2}{t^{2n}}=0 \qquad (3.14)$$

Eq. (3.14) is solvable for *n*=1. Hence, Es. (3.13) and (3.14) reduce to

$$B=t \qquad (3.15)$$

and

$$t^2C_{55}+2tC_5+\left(1-a^2\right)C=0 \qquad (3.16)$$

Integrating Eq. (3.16) yields

$$C=t^{\frac{-1+\sqrt{4a^2-3}}{2}} \qquad (3.17)$$

or

$$C=t^{\frac{-1-\sqrt{4a^2-3}}{2}} \qquad (3.18)$$

Field equations and their solutions….

Using Eq. (3.17) in Eq. (3.9), we get

$$t^2 D_{55} + t\left(1+\sqrt{4a^2-3}\right)D_5 + \frac{\left(-1+\sqrt{4a^2-3}\right)^2}{4}D = 0 \qquad (3.19)$$

which on integration yields

$$D = t^{m_1} \quad (3.20) \qquad \text{or} \qquad D = t^{m_2} \quad (3.21)$$

where

$$m_1 = \frac{-\sqrt{4a^2-3}+\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2}}{2}$$

and

$$m_2 = \frac{-\sqrt{4a^2-3}-\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2}}{2}$$

Field equations and their solutions….

Again using Eq. (3.18) in Eq. (3.9), we get

$$t^2 D_{55} + t\left(1-\sqrt{4a^2-3}\right)D_5 + \frac{\left(-1-\sqrt{4a^2-3}\right)^2}{4}D = 0 \qquad (3.22)$$

which on integration yields

$$D = t^{k_1} \qquad (3.23) \qquad \text{or} \qquad D = t^{k_2} \qquad (3.24)$$

where

$$k_1 = \frac{\sqrt{4a^2-3}+\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2}}{2}$$

and

$$k_2 = \frac{\sqrt{4a^2-3}-\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2}}{2}$$

Field equations and their solutions.....

Now the above solutions give 4 different models.

Case I: When $C = t^{\frac{-1+\sqrt{4a^2-3}}{2}}$ **and** $D = t^{m_1}$

The 5-dimensional string cosmological model corresponding to Eqs. (3.15), (3.17), and (3.20) is written as

$$ds^2 = dt^2 - t^2dx^2 - t^2e^{-2ax}dy^2 - t^{-1+\sqrt{4a^2-3}}dz^2 - t^{2m_1}du^2 \quad (3.25)$$

From Eq.(3.12), we get the following rest energy density.

$$\rho = \frac{1}{32\pi t^2}\left[\begin{array}{l} 3\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2} + \sqrt{4a^2-3} \\ +\sqrt{4a^2-3}\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2} - 8a^2 + 3 \end{array}\right] \quad (3.26)$$

Field equations and their solutions….

From Eq. (3.10), we get the following:

string tension density $= \rho_s = \dfrac{m_1^2 + m_1 + 1 - a^2}{8\pi t^2}$ (3.27)

string particle density $= \rho_p = \rho - \rho_s = \dfrac{2n_1 + m_1 + n_1 m_1 - m_1^2}{8\pi t^2}$ (3.28)

where $n_1 = \dfrac{-1 + \sqrt{4a^2 - 3}}{2}$

quark energy density $= \rho_q = \rho - B_C$

$$= \frac{1}{32\pi t^2}\left[\begin{array}{l} 3\sqrt{4a^2 - 3 - \left(-1 + \sqrt{4a^2 - 3}\right)^2} \\ + \sqrt{4a^2 - 3} + \sqrt{4a^2 - 3}\sqrt{4a^2 - 3 - \left(-1 + \sqrt{4a^2 - 3}\right)^2} - 8a^2 + 3 \end{array}\right] - B_C \qquad (3.29)$$

Field equations and their solutions….

quark pressure $= p_q = \dfrac{\rho_q}{3}$

$$= \frac{1}{96\pi t^2}\left[3\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2} + \sqrt{4a^2-3} + \sqrt{4a^2-3}\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2} - 8a^2+3 \right] - \frac{B_C}{3} \qquad (3.30)$$

The volume element of the model in Eq. (3.25) is given by

$$V = \sqrt{(-g)} = t^{n_1+m_1+2} e^{-ax} \qquad (3.31)$$

The expression for the scalar expansion θ is given by

$$\theta = u^i_{;i} = \frac{n_1+m_1+2}{t} \qquad (3.32)$$

Field equations and their solutions….

and the shear σ is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{2}\left[2\left(\frac{1}{3}-\frac{1}{t}\right)^2 + \left(\frac{1}{3}-\frac{n_1}{t}\right)^2 + \left(\frac{1}{3}-\frac{m_1}{t}\right)^2\right] \qquad (3.33)$$

The deceleration parameter q is given by

$$q = \frac{-\left(1+\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2}\right)}{3+\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2}} \qquad (3.34)$$

Field equations and their solutions….

The rest energy density, string tension density, string particle density, quark energy density, quark pressure, expansion scalar, and shear become infinite for t=0, which indicates that the universe starts at t=0.

Hence, the model of Eq. (3.25) admits initial singularity.

The scalar expansion $\theta \to 0$ as $t \to \infty$.

Since $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$, the model does not approach isotropy for large values of t.

The spatial volume V is zero when t=0 and it becomes infinite when $t \to \infty$.

The deceleration parameter is negative. Hence, the model of Eq. (3.25) is inflationary.

Case II: When $C = t^{\frac{-1+\sqrt{4a^2-3}}{2}}$ and $D = t^{m_2}$

The 5-dimensional string cosmological model corresponding to Eqs. (3.15), (3.17), and (3.21) is written as

$$ds^2 = dt^2 - t^2 dx^2 - t^2 e^{-2ax} dy^2 - t^{-1+\sqrt{4a^2-3}} dz^2 - t^{2m_2} du^2 \qquad (3.35)$$

From Eq. (3.12), we get the following rest energy density.

$$\rho = \frac{1}{32\pi t^2}\left[\begin{array}{l} -3\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2} \\ +\sqrt{4a^2-3}-\sqrt{4a^2-3}\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2}-8a^2-3 \end{array}\right] \qquad (3.36)$$

From the above equation, it is seen that the rest energy density does not satisfy the reality conditions, i.e. $(\rho > 0)$.

Hence. the model of Eq. (3.35) is physically unrealistic and not physically acceptable.

Case III: When $C = t^{\frac{-1-\sqrt{4a^2-3}}{2}}$ and $D = t^{k_1}$

The metric of Eq. (2.1) corresponding to Eqs. (3.15), (3.18), and (3.23) is written as

$$ds^2 = dt^2 - t^2 dx^2 - t^2 e^{-2ax} dy^2 - t^{-1-\sqrt{4a^2-3}} dz^2 - t^{2k_1} du^2 \qquad (3.37)$$

The physical and kinematical quantities for the model of Eq. (3.37) have the following expressions:

rest energy density =

$$\rho = \frac{1}{32\pi t^2}\left[3\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2} - \sqrt{4a^2-3} - \sqrt{4a^2-3}\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2} - 8a^2+3\right] \qquad (3.38)$$

string tension density = $\rho_s = \dfrac{k_1^2 + k_1 + 1 - a^2}{8\pi t^2}$ (3.39)

string particle density = $\rho_p = \dfrac{2n_2 + k_1 + n_2 k_1 - k_1^2}{8\pi t^2}$ (3.40)

where $n_2 = \dfrac{1+\sqrt{4a^2-3}}{2}$

quark energy density =

$$\rho_q = \frac{1}{32\pi t^2}\left[\begin{array}{l} 3\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2} - \sqrt{4a^2-3} \\ -\sqrt{4a^2-3}\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2} - 8a^2 + 3 \end{array}\right] - B_C \quad (3.41)$$

quark pressure =

$$p_q = \frac{1}{96\pi t^2}\left[\begin{array}{l} 3\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}-\sqrt{4a^2-3} \\ -\sqrt{4a^2-3}\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}-8a^2+3 \end{array}\right]-\frac{B_C}{3} \quad (3.42)$$

volume = $V = t^{n_2+k_1+2}e^{-ax}$ (3.43)

scalar expansion $= \theta = \dfrac{n_2+k_1+2}{t}$ (3.44)

shear = $\sigma^2 = \frac{1}{2}\left[2\left(\frac{1}{3}-\frac{1}{t}\right)^2+\left(\frac{1}{3}-\frac{n_2}{t}\right)^2+\left(\frac{1}{3}-\frac{k_1}{t}\right)^2\right]$ (3.45) deceleration parameter =

$$q=\frac{-\left(1+\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}\right)}{3+\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}} \qquad (3.46)$$

The rest energy density, string tension density, string particle density, quark energy density, quark pressure, expansion scalar, and shear become infinite for t=0, which indicates that the universe starts at t=0. Hence, the model of Eq. (3.37) admits initial singularity. The scalar expansion $\theta \to 0$ as $t \to \infty$.

Since $\lim_{t\to\infty}\frac{\sigma^2}{\theta^2}\neq 0$, the model does not approach isotropy for large values of t.

The spatial volume V is zero when $t=0$
and it becomes infinite when $t \to \infty$.

The deceleration parameter is negative. Hence, the model of Eq. (3.37) is inflationary.

Case IV: When $C = t^{\frac{-1-\sqrt{4a^2-3}}{2}}$ and $D = t^{k_2}$

The metric of Eq. (2.1) corresponding to Eqs. (3.15), (3.18), and (3.24) is written as

$$ds^2 = dt^2 - t^2 dx^2 - t^2 e^{-2ax} dy^2 - t^{-1-\sqrt{4a^2-3}} dz^2 - t^{2k_2} du^2 \qquad (3.47)$$

The physical and kinematical quantities for the model of Eq. (3.47) are obtained as follows:

rest energy density =

$$\rho = \frac{1}{32\pi t^2}\left[\begin{array}{l} -3\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2} - \sqrt{4a^2-3} \\ +\sqrt{4a^2-3}\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2} - 8a^2+3 \end{array}\right] \qquad (3.48)$$

string tension density = $\rho_s = \dfrac{k_2^2 + k_2 + 1 - a^2}{8\pi t^2}$ (3.49)

string particle density = $\rho_p = \dfrac{2n_2 + k_2 + n_2 k_2 - k_2^2}{8\pi t^2}$ (3.40)

quark energy density =

$$\rho_q = \frac{1}{32\pi t^2}\left[\begin{array}{l} -3\sqrt{4a^2 - 3 - \left(1+\sqrt{4a^2-3}\right)^2} - \sqrt{4a^2-3} \\ +\sqrt{4a^2-3}\sqrt{4a^2 - 3 - \left(1+\sqrt{4a^2-3}\right)^2} - 8a^2 + 3 \end{array}\right] - B_C \qquad (3.51)$$

quark pressure =

$$p_q = \frac{1}{96\pi t^2}\left[\begin{array}{l} -3\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}-\sqrt{4a^2-3} \\ +\sqrt{4a^2-3}\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}-8a^2+3 \end{array}\right]-\frac{B_C}{3} \quad (3.52)$$

volume = $V = t^{n_2+k_2+2}e^{-ax}$ (3.53)

scalar expansion = $\theta = \frac{n_2+k_2+2}{t}$ (3.54)

shear = $\sigma^2 = \frac{1}{2}\left[2\left(\frac{1}{3}-\frac{1}{t}\right)^2+\left(\frac{1}{3}-\frac{n_2}{t}\right)^2+\left(\frac{1}{3}-\frac{k_2}{t}\right)^2\right]$ (3.55) innovate achieve lead

deceleration parameter =

$$q=\frac{-\left(1+\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}\right)}{3+\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}} \qquad (3.56)$$

The reality condition $(\rho>0)$ is satisfied for

$$3\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}+\sqrt{4a^2-3}+8a^2<\sqrt{4a^2-3}\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}+3$$

The rest energy density, string tension density, string particle density, quark energy density, quark pressure, expansion scalar, and shear become infinite for t=0, which indicates that the universe starts at t=0. It is observed that the model of Eq. (3.47) admits initial singularity.

Conclusion

We have constructed 5-dimensional Bianchi type-III cosmological models with strange quark matter attached to the string cloud in general relativity. The following results were obtained for different cases:

(i) When $C = t^{\frac{-1+\sqrt{4a^2-3}}{2}}$ **and** $D = t^{m_1}$

the model of Eq. (3.25) admits initial singularity and does not approach isotropy for large values of *t*. The deceleration parameter is negative and hence the model is inflationary.

(ii) When $C = t^{\frac{-1+\sqrt{4a^2-3}}{2}}$ **and** $D = t^{m_2}$

the rest energy density does not satisfy the reality conditions and hence the model of Eq. (3.35) leads to unphysical situations.

(iii) When $C = t^{\frac{-1-\sqrt{4a^2-3}}{2}}$ **and** $D = t^{k_1}$

the universe starts at t=0 and the model does not approach isotropy for large values of t.

(iv) When $C = t^{\frac{-1-\sqrt{4a^2-3}}{2}}$ **and** $D = t^{k_2}$

all the physical parameters become infinite for t=0. Hence, it is concluded that the model of Eq. (3.47) admits initial singularity.

In summary, it is observed that of the 4 models discussed here, 3 are inflationary and possess initial singularity while 1 is physically unrealistic.

For detail, please visit the following link

http://journals.tubitak.gov.tr/havuz/fiz-1403-5.pdf

doi:10.3906/fiz-1403-5

THANK YOU