Logarithmic Corrections to Extremal Black Hole Entropy

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Many works has been done to understand the microscopic origin of the celebrated formula

$$S_{BH} = \frac{A_H}{4} \tag{1}$$

A_H is the area of horizon.
One of the success of the string theory has been the explanation of the microscopic origin of the entropy in a special class of black holes.

These are BPS black holes which are extremal.

In the extremal limit the temperature is zero.

In the BPS case the mass is determined in terms of the charges carried by the black hole.

One distinguishing feature of an extremal black hole is the appearance of AdS_2 factor in the near horizon geometry.

e.g. RN black hole in 4 dim.

$$ds^2 = -\left(1 - \frac{\rho_+}{\rho}\right)\left(1 - \frac{\rho_-}{\rho}\right) d\tau^2 + \frac{d\rho^2}{\left(1 - \frac{\rho_+}{\rho}\right)\left(1 - \frac{\rho_-}{\rho}\right)} + \rho^2 d\Omega_2^2 \tag{2}$$

In the extremal and near horizon limit, we get

$$ds^2 = \rho_+^2 \left[-\left(r^2 - 1\right) dt^2 + \frac{dr^2}{(r^2 - 1)}\right] + \rho_+^2 d\Omega_2^2 \tag{3}$$

which is $AdS_2 \times S^2$.

This is also a decoupling limit.

Proposal

Let us consider an extremal black hole with near horizon geometry $AdS_2 \times K$.
Now we consider the following Euclidean partition function,

$$\mathcal{Z}^{finite}_{AdS_2\times K}(\vec{q}) = \left\langle \exp\left[-iq_i \oint_{\partial AdS_2} d\theta\, A^{(i)}_\theta\right]\right\rangle^{finite}_{AdS_2\times K} \tag{4}$$

$A^{(i)}$ =gauge fields

- we need to carry out the path integral over all massless fields.
- all field configurations asymptote to near horizon configuration.
- The asymptotic values of the parameters of the metric and the scalars are determined in terms of the charges.

Quantum degeneracy associated to the horizon of extremal black hole with charge $\vec{q}$ is given by,

$$d_{hor}(\vec{q}) = \mathcal{Z}^{finite}_{AdS_2 \times K}(\vec{q}). \tag{5}$$

Thus the quantum corrected entropy is,

$$S(\vec{q}) = \ln d_{hor}(\vec{q}). \tag{6}$$

One can test this proposal by comparing with known examples.

e.g. $\frac{1}{4}$th BPS black hole in $\mathcal{N} = 4$ and $\frac{1}{8}$th BPS black hole in $\mathcal{N} = 8$.

Use this proposal to predict the quantum entropy in unknown cases.

e.g. $\frac{1}{2}$-BPS black hole in $\mathcal{N} = 2$.

These examples provide consistency checks for the proposal.

Saddle points

Equation of motion fixes area in terms of charges of the black hole.

Large charge limit corresponds to semiclassical analysis.

The effective gravitational coupling is small,

$$\frac{1}{g^2} \sim \frac{a^2}{G_N} \tag{7}$$

a is the size of horizon.

In semiclassical limit the path integral receives contributions from all it's saddle point.

Hence we need to know all saddle points of the path integral.

In 4 dim. leading contribution comes from $AdS_2 \times S^2$. It's classical contribution is,

$$d_{hor} \sim Exp\left[\frac{A_H}{4}\right], \qquad A_H \sim a^2. \tag{8}$$

There are other saddle points.

One such class of saddle–points are obtained by taking the $\mathbb{Z}_N$ orbifold of $AdS_2 \times S^2$,

$$L_0 - J_3 : (\theta, \phi) \equiv \left(\theta + \frac{2\pi}{N}, \phi - \frac{2\pi}{N}\right). \tag{9}$$

This has correct boundary condition and it's classical contribution is

$$d_{hor/N} \sim Exp\left[\frac{A_H}{4N}\right]. \tag{10}$$

Logarithmic corrections

We want to go beyond the classical contribution.

$$S = \frac{A_H}{4} + C_1 \log A_H + C_2 + \frac{C_3}{A_H} + ... + A_H^n e^{-A_H} + ... \qquad (11)$$

We compute one loop partition function of the supergravity fields about near horizon background.

We look for logarithmic corrections,

$$\Delta S_{log} \sim c \log A_H . \qquad (12)$$

In the case when charges are uniformly large, these come from two derivative action of massless fields at one loop and indep. of massive fields.

A field with mass m at one loop has

$$\ln \mathcal{Z}_{1-loop}^{\log} \sim \left[\frac{b_0 m^4}{2} - b_2 m^2 + b_4\right] \ln(m^2\epsilon). \tag{13}$$

b_n's are deWitt-Seeley-Gilkey's coefficients and appear in the heat kernel expansion,

$$b_0 \sim \mathcal{O}\left(a^4\right), \quad b_2 \sim \mathcal{O}\left(a^2\right), \quad b_4 \sim \mathcal{O}(1). \tag{14}$$

Thus a particle with $m^2 \sim \frac{1}{a^2}$ will give $c \log a$.

For massless field, there is a natural IR cutoff $\sim \frac{1}{a}$. The massless fields running in the loop generate logarithmic corrections.

Also BH background mixes fluctuations of massless fields and generate potential $\sim \frac{1}{a^2}$. This will generate $\sim \mathcal{O}(1)\ln(a)$ correction.

The one loop partition function can be expressed in terms of heat kernel,

$$\ln \mathcal{Z}_{1-loop} \sim \frac{1}{2}\int_{\epsilon}^{\infty} \frac{dt}{t}\left(K_{int}(t) - n_0\right) + \ln \mathcal{Z}_{zero} \qquad (15)$$

where

$$K_{int}(t) = \sum_{n} d(n) e^{-\frac{t\lambda_n}{a^2}} \qquad (16)$$

On $AdS_2 \times S^2$ background and on it's orbifold, the fluctuations of higher spin bosonic (fermionic) fields can be expressed in terms of scalar (spin half fermionic) fields.
Thus we need to know the degeneracy of scalar and Dirac spinor on $AdS_2 \times S^2$ background and on it's orbifold.

For scalar field, the degeneracy on $AdS_2 \times S^2$ background

$$d^s_{\lambda l} = -\lambda \tanh(\pi\lambda)(2l+1) \tag{17}$$

and the degeneracy on $(AdS_2 \times S^2)/\mathbb{Z}_N$ background

$$d^s_{\lambda l} = -\frac{1}{N}\lambda \tanh(\pi\lambda)(2l+1) + \frac{1}{N}\sum_{s=1}^{N-1} \chi^s_\lambda\left(\frac{\pi s}{N}\right)\chi^s_l\left(\frac{\pi s}{N}\right) \tag{18}$$

The $SL(2;R)$ character is

$$\chi^s_\lambda\left(\frac{\pi s}{N}\right) = \frac{1}{2}\frac{\cosh\left(\pi - \frac{2\pi s}{N}\right)\lambda}{\cosh(\pi\lambda)\sin\left(\frac{\pi s}{N}\right)} \tag{19}$$

$\chi^s_l\left(\frac{\pi s}{N}\right)$ is $SU(2)$ charcater

$$\chi^s_l\left(\frac{\pi s}{N}\right) = \frac{\sin\left(\frac{(2l+1)\pi s}{N}\right)}{\sin\left(\frac{\pi s}{N}\right)} \tag{20}$$

We have similar expression for the degeneracy of Dirac fermion on $AdS_2 \times S^2$ background

$$d^f_{\lambda l} = -\lambda \coth(\pi\lambda)(2l+2) \tag{21}$$

and the degeneracy on $(AdS_2 \times S^2)/\mathbb{Z}_N$ background

$$d^f_{\lambda l} = -\frac{1}{N}\lambda \coth(\pi\lambda)(2l+2) + \frac{4}{N}\sum_{s=1}^{N-1} \chi^f_\lambda\left(\frac{\pi s}{N}\right)\chi^f_{l+\frac{1}{2}}\left(\frac{\pi s}{N}\right) \tag{22}$$

The $SL(2;R)$ character is

$$\chi^f_\lambda\left(\frac{\pi s}{N}\right) = \frac{1}{2}\frac{\sinh\left(\pi - \frac{2\pi s}{N}\right)\lambda}{\sinh(\pi\lambda)\sin\left(\frac{\pi s}{N}\right)} \tag{23}$$

$\chi^f_{l+\frac{1}{2}}\left(\frac{\pi s}{N}\right)$ is $SU(2)$ character.

Zero modes:

Fields	Unquotient	Quotient	β_ϕ
Vector	-1	-1	1
Gravity	-6	-2	2
gravitino	-4	-2	3

Results

We look for the log correction in the one loop partition function about $AdS_2 \times S^2$ background and on it's orbifold,

$$\mathcal{Z}_{1-loop}^{log} \sim c \log A_H \qquad (24)$$

Theory	$AdS_2 \times S^2$	$AdS_2 \times S^2/\mathbb{Z}_N$
$\frac{1}{8}$-BPS in $\mathcal{N}=8$	-4	-4
$\frac{1}{4}$-BPS in $\mathcal{N}=4$	0	0
$\frac{1}{2}$-BPS in $\mathcal{N}=2$	$\left(2-\frac{\chi}{24}\right)$	$\left(2-\frac{N\chi}{24}\right)$

Here

$$\chi = 2(n_V - n_H + 1) \qquad (25)$$

Summary

- We proposed the generalisation of Plancherel measure on the orbifold of $AdS_2 \times S^2$.
- We calculated the logarithmic corrections about the orbifold saddle points and found perfect agreements with the known microscopic results.
- We found non trivial answer for log correction incase of half BPS black hole in $\mathcal{N} = 2$.