# The black hole interior in AdS/CFT and the information paradox

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based on work with Suvrat Raju: 1211.6767, 1310.6334, 1310.6335 + in progress
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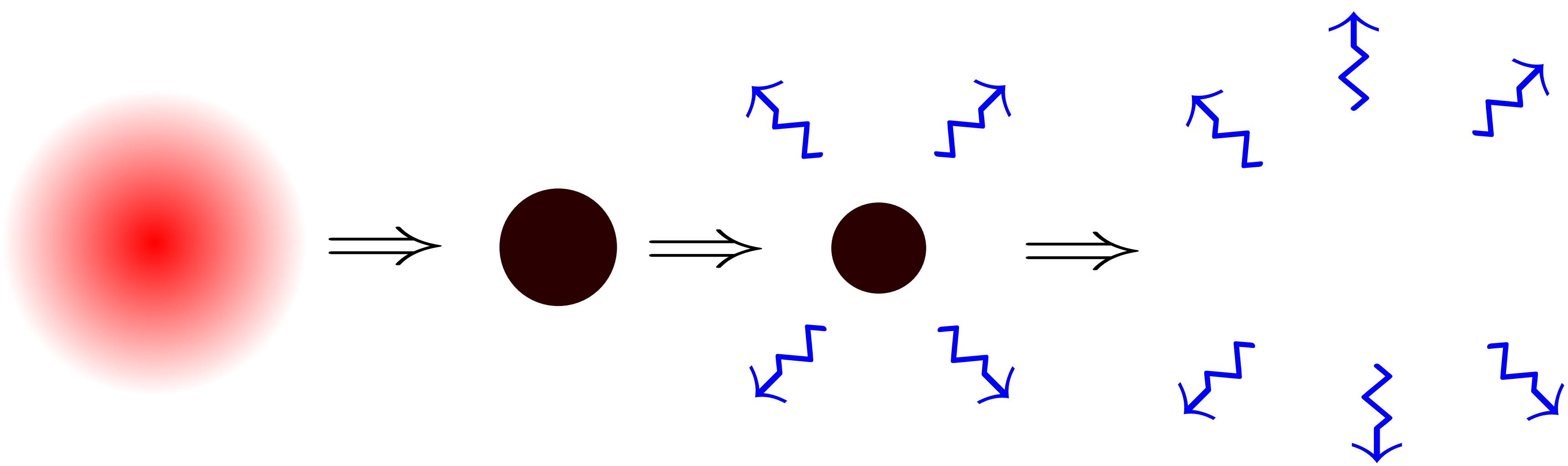
Does CFT describe BH interior?

What happens to the infalling observer?

What can we learn about the information paradox?

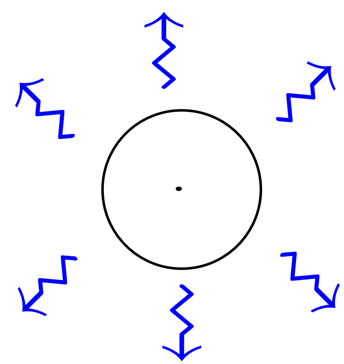


gas cloud
in pure state

Hawking radiation

$$|\Psi_0\rangle \quad \Longrightarrow \quad \cdots \quad \Longrightarrow \quad \rho_{\text{thermal}}$$

**Inconsistent with unitary evolution**



Size of Hilbert space is $e^{S_{BH}}$

Semiclassical-Hawking computation gives "ensemble average" of coarse-grained observables

In this approximation radiation looks thermal

However, in typical pure state, these observables will differ from Hawking's computation by exponentially small deviations

**Exponentially small corrections to Hawking's computation (for simple observables) can restore unitarity**

Pure states vs Ensemble (Lloyd):

define ensemble averages of observable $A$

$$\overline{A} = \mathrm{Tr}(\rho A)$$

and

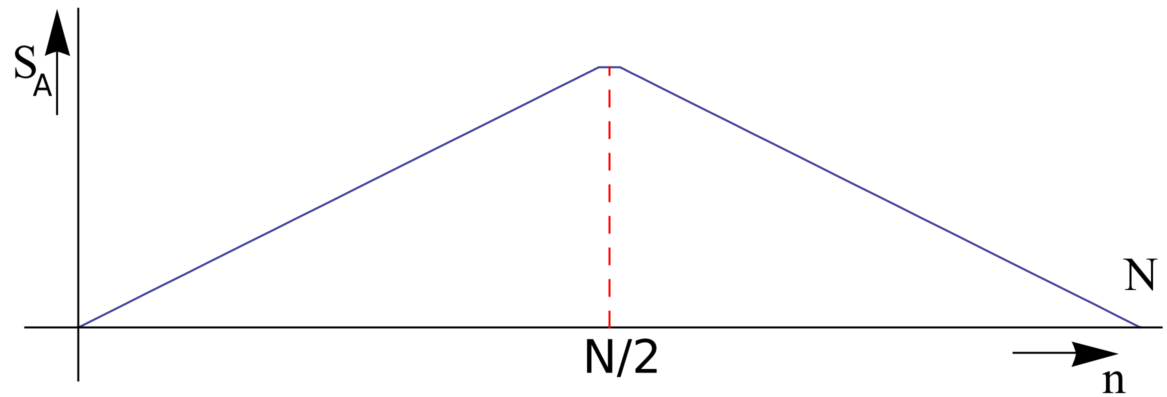
$$\overline{A^2} = \mathrm{Tr}(\rho A^2)$$

then for typical pure microstate $|\Psi\rangle$ we have

$$\textbf{Ensemble Variance}\ \text{of}\ \Big[\langle\Psi|A|\Psi\rangle\Big] = \frac{1}{e^S}\Big[\overline{A^2} - \overline{A}^2\Big]$$

Expectation values of "simple" observables on typical pure states are exponentially close to ensemble averages.

Entropy of subsystem (Page)

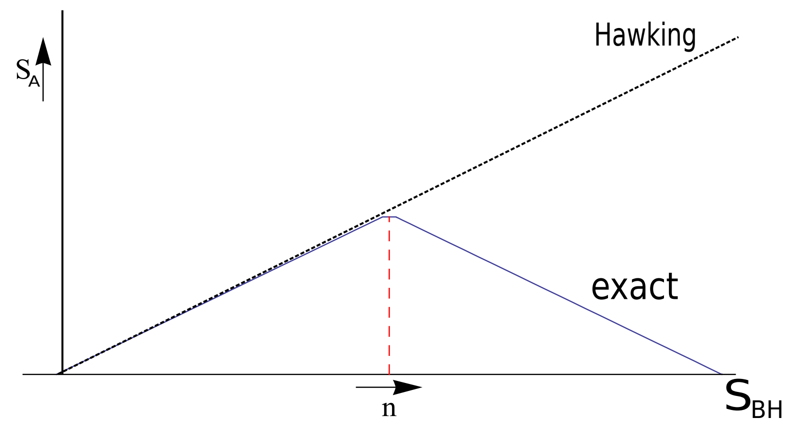



Consider a large system of size $dim(H) = e^N$. The entire system is in a typical pure state $|\Psi\rangle$.

We consider a subsystem $A$ of size $dim(H_A) = e^n$. It will be in mixed state with density matrix

$$\rho_A = \mathrm{Tr}_{A'}(|\Psi\rangle\langle\Psi|)$$

with entanglement entropy $S_A = -\mathrm{Tr}(\rho_A \log \rho_A)$




Define $A = \{$Hawking radiation emitted up to some particular time $\}$.

In general Hawking radiation is entangled with the remaining black hole.

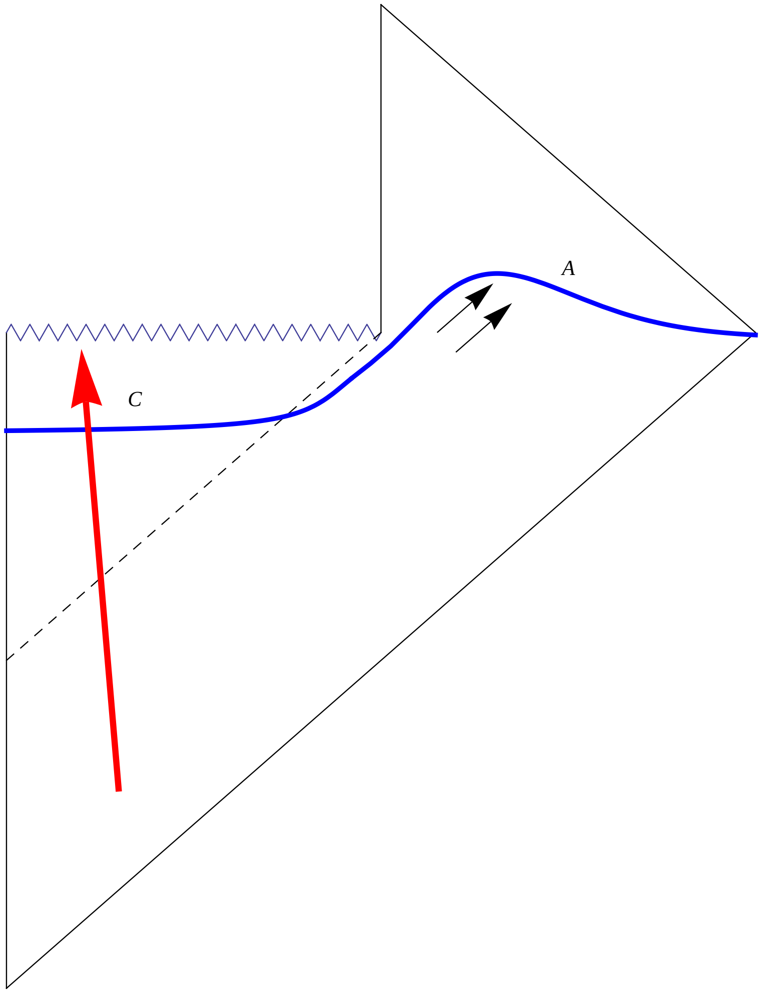
$$S_A = -\mathrm{Tr}(\rho_A \log \rho_A)$$

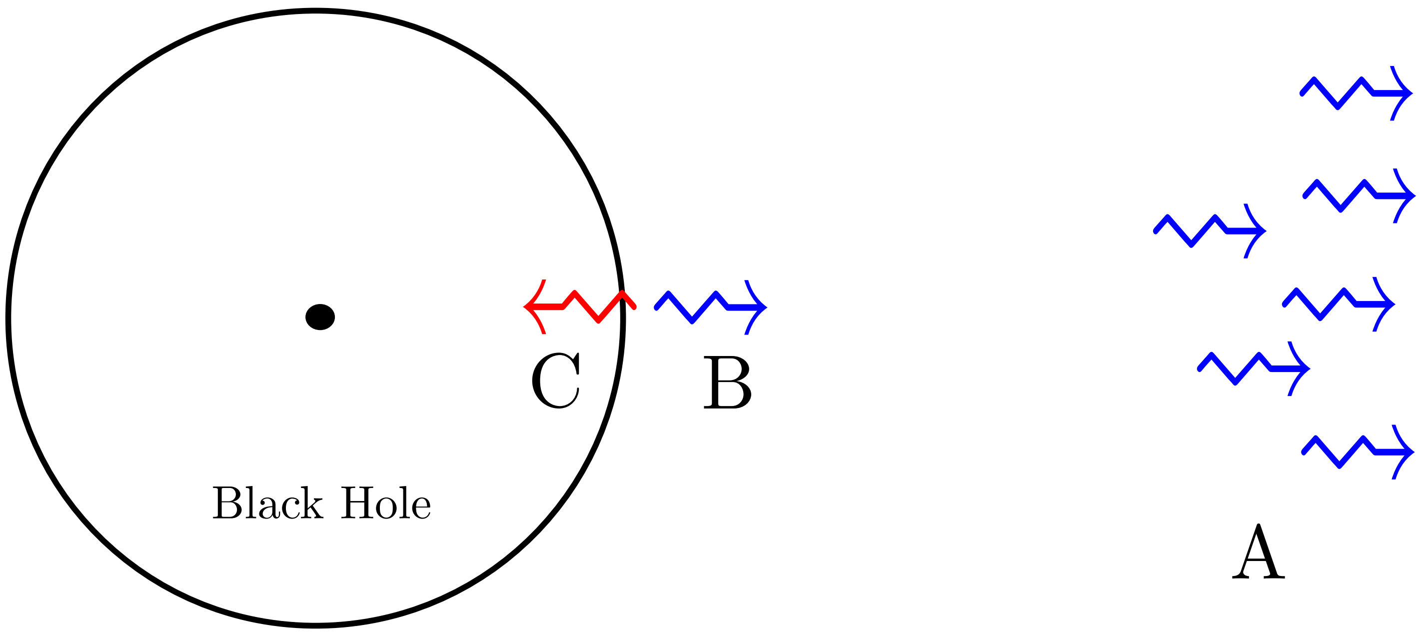
According to Page's general analysis we expect the graph shown above, for $S_A$ a function of $\#$ of emitted particles.

If we only look far from the horizon: there is no sharp information paradox

Exponentially small corrections **to simple observables** can restore unitarity

# Quantum Cloning on “nice slices”






**Strong subadditivity theorem: for 3 independent systems A,B,C we have**

$$S_{AB} + S_{BC} \geq S_A + S_C$$

For the Hawking pair production we have $S_{BC} \approx 0$ and $S_C = \mathcal{O}(1)$ which would imply

$$S_{AB} > S_A$$

Tension between

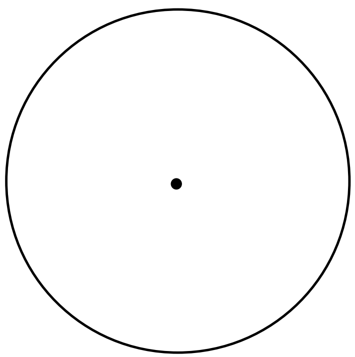
Unitarity

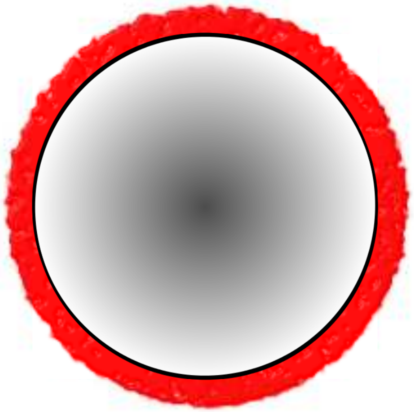
Locality

Smooth horizon

**Can small corrections resolve the paradox?**

Proposals to modify interior of black hole:
-Fuzzball
-Firewall
-....





infalling observer feels deviations from GR/burns-up when crossing the horizon

**(small) Non-locality/Complementarity**

$\Rightarrow$ **can resolve the information paradox**

No need for firewalls, (fuzzballs) or other exotic physics at the horizon....

**BH interior is a scrambled copy of exterior**

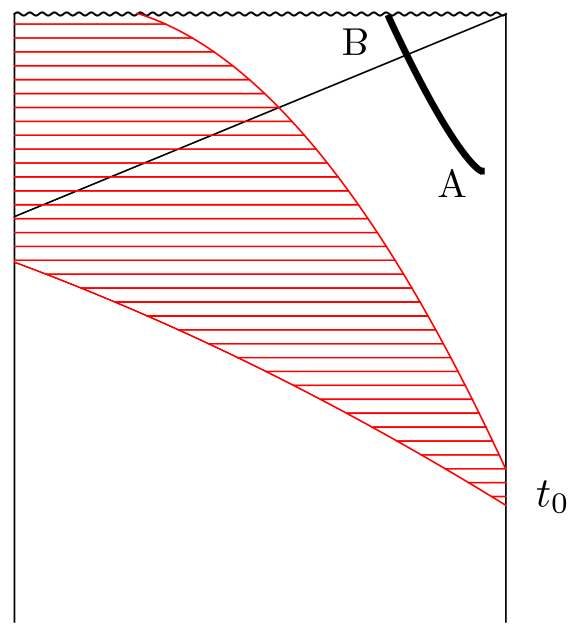
hence

no cloning problem
no subadditivity Mathur/AMPS problem

(since both were based on assumption that Hilbert space factorizes into interior $\times$ exterior)

**Is complementarity consistent with locality in effective field theory?**

YES, we can have BH complementarity with only very small non-locality (not detectable within effective field theory)




Consider the $\mathcal{N} = 4$ SYM on $S^3 \times \mathrm{time}$, at large $N$, large $\lambda$.
and typical pure state $|\Psi\rangle$ with energy of $\mathcal{O}(N^2)$.

What is experience of infalling observer? $\Rightarrow$ Need local bulk observables

Large $N$ factorization allows us to write local* observables in empty AdS as non-local observables in CFT (smeared operators)

$$\phi_{\rm CFT}(t,\vec{x},z) = \int_{\omega>0} d\omega\, d\vec{k}\, \left(\mathcal{O}_{\omega,\vec{k}}\; f_{\omega,\vec{k}}(t,\vec{x},z) + \text{h.c.}\right)$$

where $\phi_{\rm CFT}$ obeys EOMs in AdS, and $[\phi_{\rm CFT}(P_1), \phi_{\rm CFT}(P_2)] = 0$, if points $P_1, P_2$ spacelike with respect to AdS metric

(based on earlier works: Banks, Douglas, Horowitz, Martinec, Bena, Balasubramanian, Giddings, Lawrence, Kraus, Trivedi, Susskind, Freivogel Hamilton, Kabat, Lifschytz, Lowe, Heemskerk, Marolf, Polchinski, Sully...)

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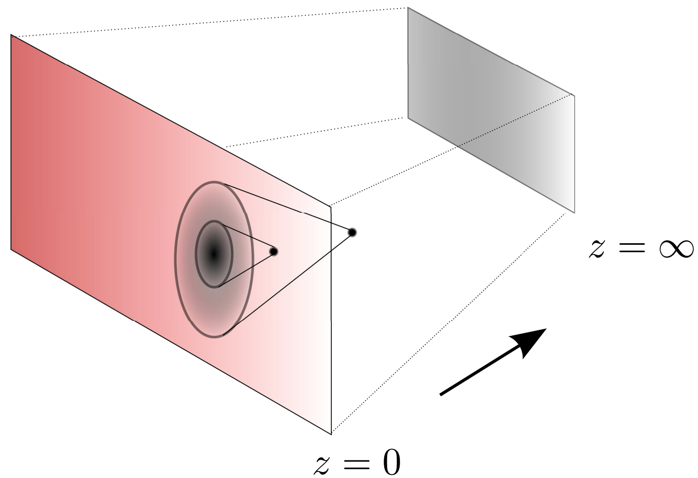
* Locality is approximate:

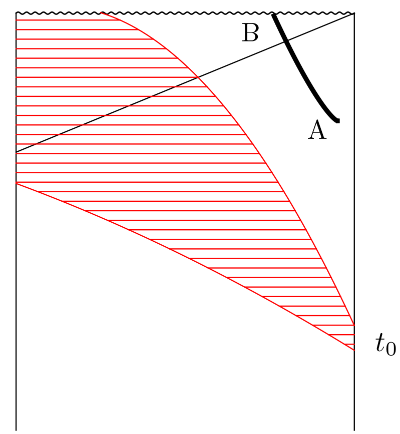
1. (Plausibly) true in $1/N$ perturbation theory
2. Unlikely that $[\phi_{\rm CFT}(P_1), \phi_{\rm CFT}(P_2)] = 0$ to $e^{-N^2}$ accuracy
3. Locality may break down for high-point functions (perhaps no bulk spacetime interpretation)




$$\phi_{\mathrm{CFT}}(t,\vec{x},z)=\int dt' d\vec{x}'\; K(t,\vec{x},z\;;\;t',\vec{x}')\mathcal{O}(t',\vec{x}')$$

where $K$ is some kernel — sometimes called the *smearing function*.

Subtleties: 1/N expansion, gauge invariance....

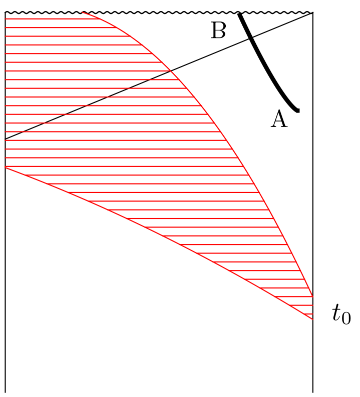



Consider typical QGP pure state $|\Psi\rangle$ (energy $O(N^2)$). Single trace correlators **still factorize** at large $N$

$$\langle\Psi|\mathcal{O}(x_1)...\mathcal{O}(x_n)|\Psi\rangle = \langle\Psi|\mathcal{O}(x_1)\mathcal{O}(x_2)|\Psi\rangle...\langle\Psi|\mathcal{O}(x_{n-1})\mathcal{O}(x_n)|\Psi\rangle + ...$$

The 2-point function in which they factorize is the thermal 2-point function, which is hard to compute, but obeys KMS condition

$$G_\beta(-\omega, k) = e^{-\beta\omega} G_\beta(\omega, k)$$




Local bulk field outside horizon of AdS black hole*

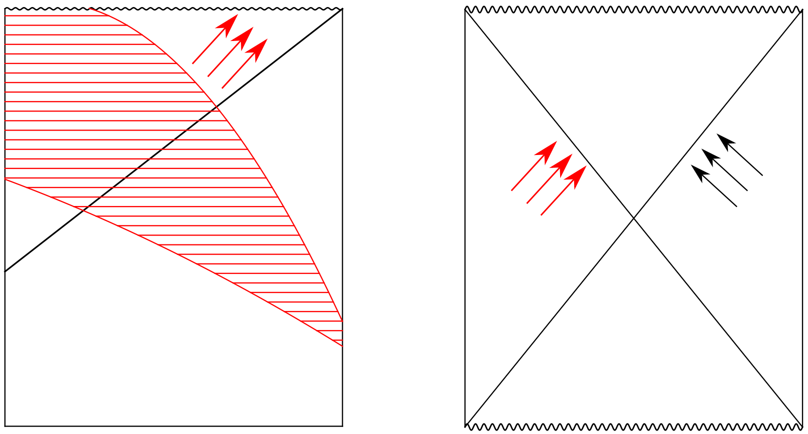
$$\phi_{\mathrm{CFT}}(t,\Omega,z) = \sum_m \int_0^\infty d\omega\, \mathcal{O}_{\omega,m}\, f^{\beta}_{\omega,m}(t,\Omega,z) + \mathrm{h.c.}$$

At large $N$ (and late times) the correlators

$$\langle\Psi|\phi_{\mathrm{CFT}}(t_1,\Omega_1,z_1)...\phi_{\mathrm{CFT}}(t_n,\Omega_n,z_n)|\Psi\rangle$$

reproduce those of semiclassical QFT on the BH background (in AdS-Hartle-Hawking state).

* Subtleties about the convergence of the sum/integral...



### Need new modes

For free infall we expect

$$\phi_{\mathrm{CFT}}(t,\Omega,z) = \sum_m \int_0^\infty d\omega \Big[ \mathcal{O}_{\omega,m}\, e^{-i\omega t} Y_m(\Omega) g^{(1)}_{\omega,m}(z) + \text{h.c.}$$

$$+ \boxed{\widetilde{\mathcal{O}}_{\omega,m}}\, e^{-i\omega t}\, Y_m(\Omega)\, g^{(2)}_{\omega,m}(z) + \text{h.c.} \Big]$$

where the modes $\widetilde{\mathcal{O}}_{\omega,m}$ must satisfy certain conditions

The $\widetilde{\mathcal{O}}_{\omega,m}$'s (*mirror* or *tilde* operators) must obey the following conditions, in order to have smooth interior:

1. For every $\mathcal{O}$ there is a $\widetilde{\mathcal{O}}$
2. The algebra of $\widetilde{\mathcal{O}}$'s is isomorphic to that of the $\mathcal{O}$'s
3. The $\widetilde{\mathcal{O}}$'s commute with the $\mathcal{O}$'s
4. The $\widetilde{\mathcal{O}}$'s are "correctly entangled" with the $\mathcal{O}$'s

Equivalently:

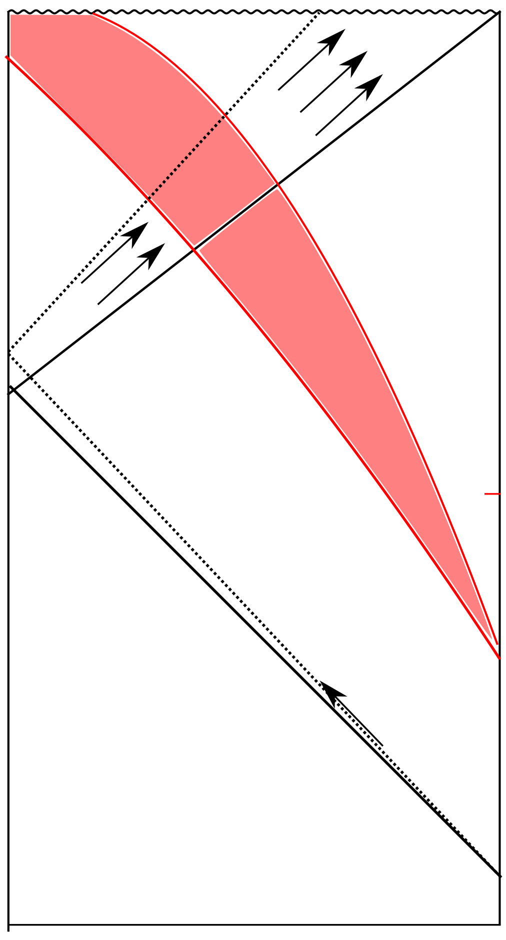
Correlators of all these operators on $|\Psi\rangle$ must reproduce (at large $N$) those of the thermofield-double state

$$|TFD\rangle = \sum_i \frac{e^{-\beta E_i/2}}{\sqrt{Z}} |E_i, \widetilde{E}_i\rangle$$

$$\langle\Psi|\mathcal{O}(t_1)...\widetilde{\mathcal{O}}(t_k)..\mathcal{O}(t_n)|\Psi\rangle \approx \frac{1}{Z}\mathrm{Tr}\big[\mathcal{O}(t_1)...\mathcal{O}(t_n)\mathcal{O}(t_k + i\frac{\beta}{2})...\mathcal{O}(t_m + i\frac{\beta}{2})\big]$$

**Main Question:** Does the CFT contain the operators $\widetilde{\mathcal{O}}$ with the desired properties?

If so, we will declare that the CFT describes the interior of the black hole and that we have free infall through the horizon.



Using bulk EFT evolution to find the $\widetilde{\mathcal{O}}$? $\Rightarrow$ Trans-planckian problem...(?)

Exterior of AdS black hole $\Rightarrow$ Described by “algebra of (products of) single trace operators $\mathcal{O}$”

Why do we get a second **commuting** copy $\widetilde{\mathcal{O}}$?

Exterior of AdS black hole $\Rightarrow$ Described by "algebra of (products of) single trace operators $\mathcal{O}$"

Why do we get a second **commuting** copy $\widetilde{\mathcal{O}}$?

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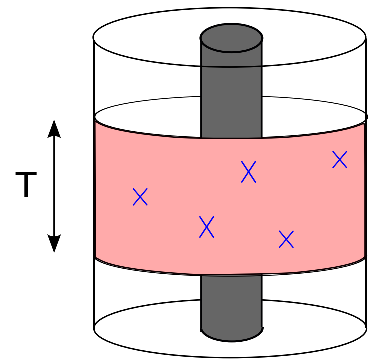
The doubling of the observables is a general phenomenon whenever we have:

A large (chaotic) quantum system in a typical state $|\Psi\rangle$

We are probing it with a small algebra $\mathcal{A}$ of observables

Under these conditions, the small algebra $\mathcal{A}$ is effectively "doubled".

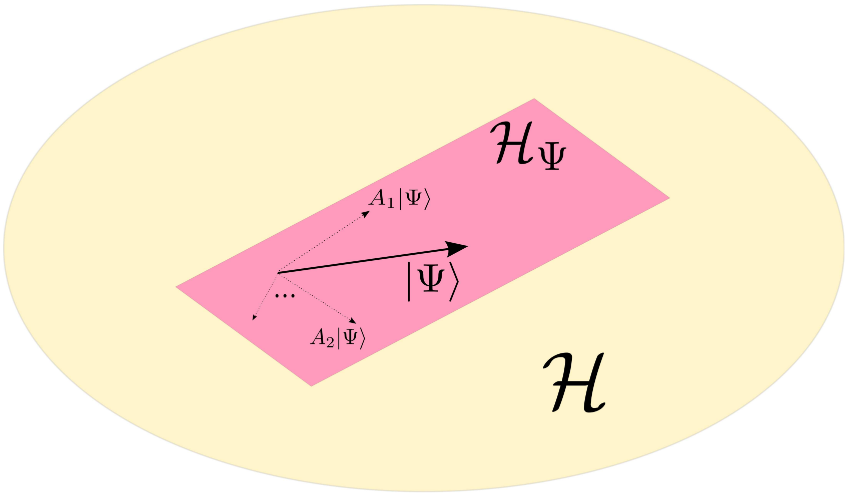



For us, $|\Psi\rangle$= BH microstate (typical QGP state of $E \sim \mathcal{O}(N^2)$)

$\mathcal{A}$= "algebra" of small (i.e. $O(N^0)$) products of single trace operators

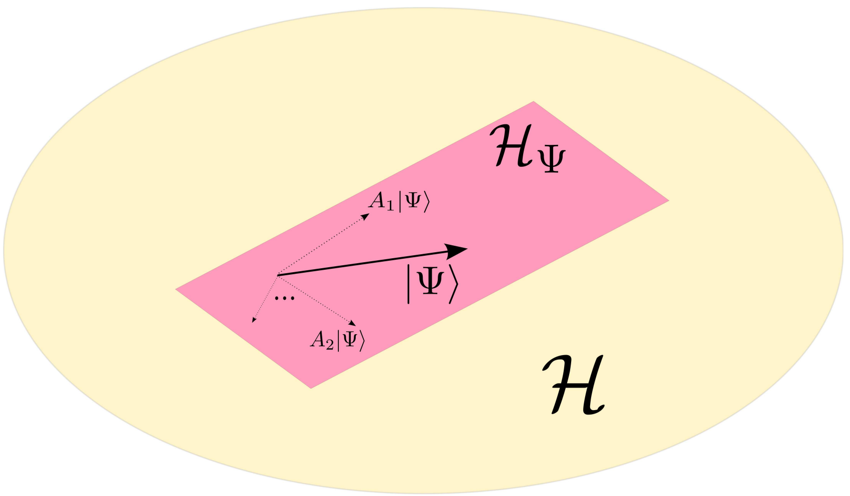
$$\mathcal{A} = \text{span of}\{\mathcal{O}(t_1,\vec{x}_1),\ \mathcal{O}(t_1,\vec{x}_1)\mathcal{O}(t_2,\vec{x}_2), ...\}$$

Here $T$ is a long time scale and also need some UV regularization.




For any given microstate $|\Psi\rangle$ consider the linear subspace $\mathcal{H}_\Psi$ of the full Hilbert space $\mathcal{H}$ of the CFT

$$\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle = \{\text{span of} : \mathcal{O}(t_1, \vec{x}_1)...\mathcal{O}(t_n, \vec{x}_n))|\Psi\rangle\}$$




$\mathcal{H}_\Psi$ depends on $|\Psi\rangle$

$\mathcal{H}_\Psi \Rightarrow$ Contains states of higher and lower energies than $|\Psi\rangle$

Bulk EFT experiments around BH $|\Psi\rangle$ take place within $\mathcal{H}_\Psi$ (bulk observer cannot easily see outside $\mathcal{H}_\Psi$)

**The "doubling" follows from the important property**:

$$\boxed{A|\Psi\rangle \neq 0 \qquad\qquad \text{if} \qquad\qquad A \neq 0, \qquad \forall A \in \mathcal{A}}$$

(we cannot annihilate the QGP microstate by the action of a few single trace operators)

Physical interpretation:

"The state $|\Psi\rangle$ *appears to be entangled when probed by the algebra* $\mathcal{A}$".

Consider the Hilbert space of two spins, and $\mathcal{A} =$ operators acting on the first. If the two spins are in the state

$$|\Psi\rangle = |\uparrow\uparrow\rangle$$

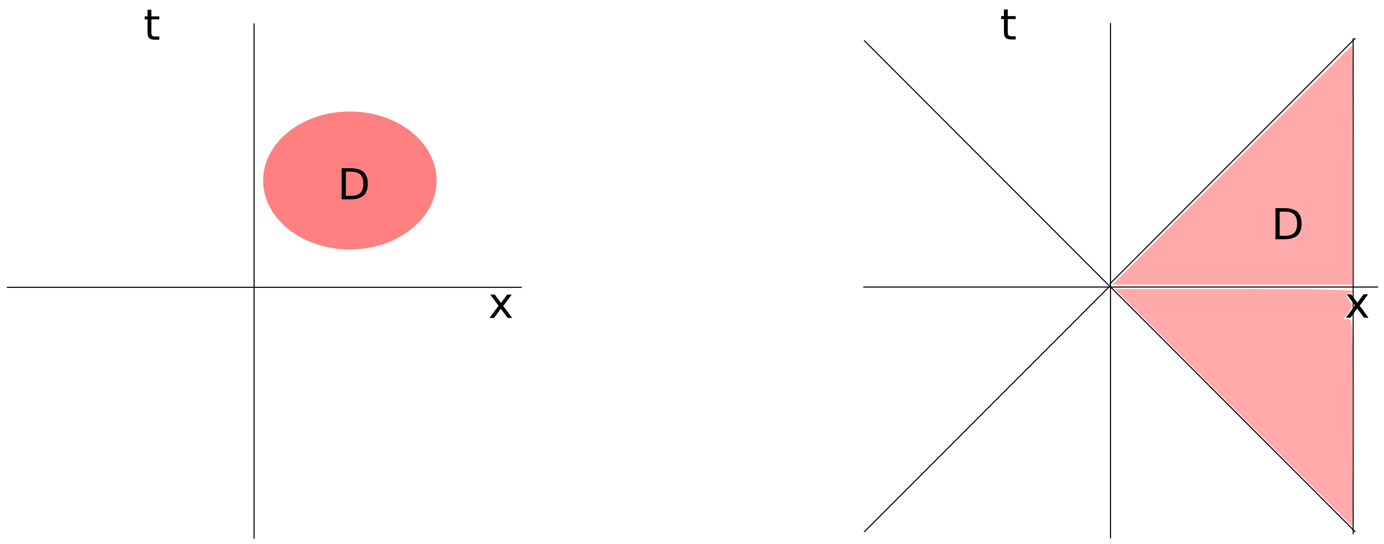
In this case there is no entanglement and indeed the previous condition is violated since

$$s_+^{(1)}|\Psi\rangle = 0 \qquad \text{while} \qquad s_+^{(1)} \neq 0$$

On the other hand consider the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

Now there is entanglement and, relatedly, there is no non-vanishing operator acting on the first spin that annihilates the state.




Reeh-Schlieder theorem: Minkowski vacuum $|0\rangle_M$ cannot be annihilated by acting with local operators in $D$.

$$\Rightarrow$$

In $|0\rangle_M$ local operator algebras are entangled — (though, no proper factorization of Hilbert space due to UV divergences)

Remember the important condition

$$A|\Psi\rangle \neq 0 \qquad \text{for} \qquad A \neq 0 \qquad (1)$$

Suppose that

$$\dim\mathcal{A} = n$$

Then from $(1)$ follows that

$$\dim\mathcal{H}_\Psi = \dim\left(\mathrm{span}\mathcal{A}|\Psi\rangle\right) = n$$

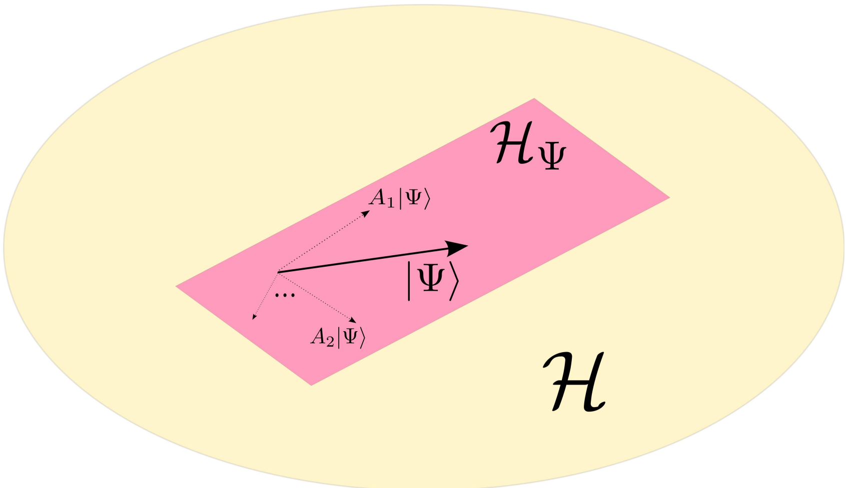
However the algebra $\mathcal{L}(\mathcal{H}_\Psi)$ of *all* operators that can act on $\mathcal{H}_\Psi$ has dimensionality

$$\dim\mathcal{L}(\mathcal{H}_\Psi) = n^2$$

while the original algebra $\mathcal{A}$ had only $\dim\mathcal{A} = n$.
This suggests that

$$\mathcal{L}(\mathcal{H}_\Psi) = A \otimes \widetilde{A}$$

where $\widetilde{A}$ is a "second copy" of $A$. We can choose basis so that $[A, \widetilde{A}] = 0$




$|\Psi\rangle$= BH microstate (QGP microstate)
$\mathcal{A}$ = "algebra" of small products of single trace operators
Black Hole interior operators $\widetilde{\mathcal{O}}$ must commute with $\mathcal{A} \Rightarrow$ They are elements of the "commutant" $\mathcal{A}'$ of the algebra.

**What is $\mathcal{A}'$ for the algebra of single trace operators $\mathcal{A}$ acting on a typical QGP state?**

Consider a von-Neumann algebra $\mathcal{A}$ acting on a Hilbert space $\mathcal{H}$.

Question: what is the commutant $\mathcal{A}'$?

In general, question is difficult. $\mathcal{A}'$ could be trivial. However, if $\exists$ a state $|\Psi\rangle$ in $\mathcal{H}$ for which

i) States $\mathcal{A}|\Psi\rangle$ span $\mathcal{H}$
ii) $A|\Psi\rangle \neq 0$ for all $A \neq 0$

then

**Theorem:** (Tomita-Takesaki) The commutant $\mathcal{A}'$ is isomorphic to $\mathcal{A}$ (**doubling**!). There is a canonical isomorphism $J$ acting on $\mathcal{H}$ such that

$$\widetilde{\mathcal{O}} = J\mathcal{O}J$$

see also early work of A.Connes

On the subspace $\mathcal{H}_\Psi$ we define the *antilinear* map $S$ by

$$\boxed{SA|\Psi\rangle = A^\dagger|\Psi\rangle}$$

This is well defined *because* of the condition $A|\Psi\rangle \neq 0$ for $A \neq 0$.
We manifestly have

$$S|\Psi\rangle = |\Psi\rangle$$

and

$$S^2 = 1$$

For any operator $A \in \mathcal{A}$ acting on $\mathcal{H}_\Psi$ we define a new operator acting on the same space by

$$\hat{A} = SAS$$

The hatted operators commute with those in $\mathcal{A}$:

$$\hat{B}A|\Psi\rangle = SBSA|\Psi\rangle = SBA^\dagger|\Psi\rangle = (BA^\dagger)^\dagger|\Psi\rangle = AB^\dagger|\Psi\rangle$$

and also

$$A\hat{B}|\Psi\rangle = ASBS|\Psi\rangle = AB^\dagger|\Psi\rangle$$

hence

$$[A, \hat{B}]|\Psi\rangle = 0$$

The "hatted" operators $\hat{A} = SAS$ satisfy:

- Their algebra is isomorphic to $\mathcal{A}$
- They commute with $\mathcal{A}$

they are almost the mirror operators, but not quite (the mixed $A$-$\hat{A}$ correlators are not "canonically" normalized)

The mapping $S$ is not an isometry. We define the "magnitude" of the mapping

$$\Delta = S^\dagger S$$

and then we can write

$$J = S\Delta^{-1/2}$$

where $J$ is (anti)-unitary. Then the correct mirror operators are

$$\boxed{\widetilde{O} = JOJ}$$

The operator $\Delta$ is a positive, hermitian operator and can be written as

$$\Delta = e^{-K}$$

where

$$K = \text{“modular Hamiltonian''}$$

For entangled bipartite system $A \times B$ this construction would give $K_A \sim \log(\rho_A)$ i.e. the usual modular Hamiltonian for $A$.

In the large $N$ gauge theory and using the KMS condition for correlators of single-trace operators we find that for equilibrium states

$$K = \beta(H_{CFT} - E_0)$$

To summarize, we have

$$SA|\Psi\rangle = A^\dagger|\Psi\rangle$$

and

$$\Delta = e^{-\beta(H_{CFT} - E_0)}$$

We define the $J$ by

$$J = S\Delta^{-1/2}$$

Finally we define the mirror operators by

$$\boxed{\widetilde{O} = JOJ}$$

Putting everything together we **define** the mirror operators by the following set of linear equations

$$\widetilde{\mathcal{O}}_\omega|\Psi\rangle = e^{-\frac{\beta\omega}{2}}\mathcal{O}_\omega^\dagger|\Psi\rangle$$

and

$$\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle = \mathcal{O}...\mathcal{O}\widetilde{\mathcal{O}}_\omega|\Psi\rangle$$

These conditions are self-consistent because $A|\Psi\rangle \neq 0$, which in turns relies on

1. The algebra $\mathcal{A}$ is not too large
2. The state $|\Psi\rangle$ is complicated (this definition would not work around the ground state of CFT)

These “mirror operators” $\widetilde{O}$ obey the desired conditions mentioned several slides ago, i.e. at large $N$ they lead to

$$\langle\Psi|\mathcal{O}(t_1)...\widetilde{\mathcal{O}}(t_k)..\mathcal{O}(t_n)|\Psi\rangle \approx \frac{1}{Z}\mathrm{Tr}\big[\mathcal{O}(t_1)...\mathcal{O}(t_n)\mathcal{O}(t_k+i\frac{\beta}{2})...\mathcal{O}(t_m+i\frac{\beta}{2})\big]$$

Using the $\mathcal{O}_\omega$'s and $\widetilde{\mathcal{O}}_\omega$'s we can reconstruct the black hole interior by operators of the form

$$\phi_{\rm CFT}(t,\Omega,z) = \sum_m \int_0^\infty d\omega \Big[\mathcal{O}_{\omega,m}\, e^{-i\omega t} Y_m(\Omega) g^{(1)}_{\omega,m}(z) + \text{h.c.}$$

$$+\widetilde{\mathcal{O}}_{\omega,m}\, e^{-i\omega t}\, Y_m(\Omega)\, g^{(2)}_{\omega,m}(z) + \text{h.c.}\Big]$$

Low point functions of these operators reproduce those of effective field theory in the interior of the black hole

$\Rightarrow$

$\exists$ **Smooth interior**

**Nothing dramatic when crossing the horizon**

The operators $\widetilde{\mathcal{O}}$ seem to commute with the $\mathcal{O}$'s

This is only approximate: the commutator $[\mathcal{O}, \widetilde{\mathcal{O}}] = 0$ only inside low-point functions (by construction)

If we consider $N^2$-point functions, then we find that the construction cannot be performed since we will violate

$$A|\Psi\rangle \neq 0, \qquad \text{for} \qquad A \neq 0$$

or equivalently, in spirit, we will find that

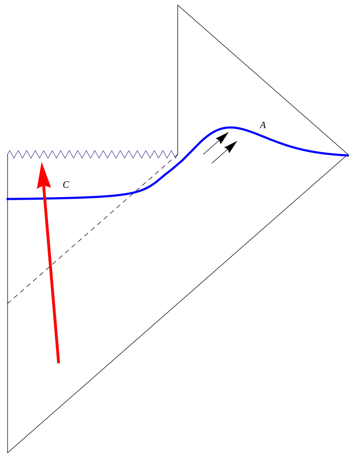
$$[\mathcal{O}, \widetilde{\mathcal{O}}] \neq 0$$

inside complicated correlators.

Relatedly, we can express the $\widetilde{\mathcal{O}}$'s as very complicated combination of $\mathcal{O}$'s.

Spin chain example

Black Hole interior is not independent Hilbert space, but scrambled copy of (part of) the exterior




In our construction:

Exterior of black hole $\Rightarrow$ operators $\phi(x)$
Interior of black hole $\Rightarrow$ operators $\widetilde{\phi}(y)$
In low-point correlators $\phi$, $\widetilde{\phi}$ seem to be independent
If we act with too many (order $S_{BH}$) of $\phi$'s we can "reconstruct" the $\widetilde{\phi}$'s

**Complementarity can be realized consistently with locality in effective field theory** (commutator is small)

Our operators were defined to act on $\mathcal{H}_\Psi$ (they are *sparse* operators).

For given BH microstate and for an EFT observer placed near the BH $|\Psi\rangle$, this part of the Hilbert space is the only relevant (for simple experiments)

For different microstate $|\Psi'\rangle$ the "same physical observables" will be acting on a different part of the Hilbert space $\mathcal{H}_{\Psi'}$ and (a priori) will be different **linear** operators

Is it possible to define the $\widetilde{\mathcal{O}}_\omega$ globally on the Hilbert space?

Why it seems unlikely that $\widetilde{\mathcal{O}}$ can be defined to act on all microstates:

There are certain "counting" arguments against the existence of globally defined $\widetilde{\mathcal{O}}$ operators [Bousso, Almheiri, Marolf, Polchinski, Stanford, Sully]

State-dependence could explain why we automatically get "correct entanglement" for typical states

Even outside the horizon, non-trivial(?) to get rid of state (background) dependence

It may be that in Quantum Gravity all local observables are state-dependent

Hence, in quantum gravity local observables may be state dependent, in general

In any case, no known **sharp** problem with state dependence

Suppose $|\Psi_0\rangle$ is equilibrium state. Consider the state:

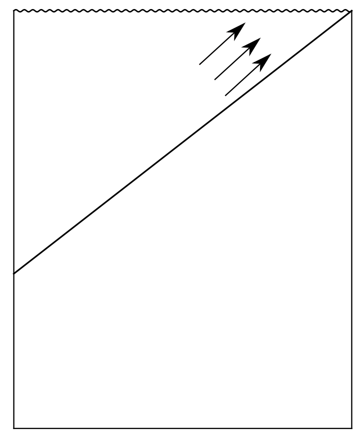
$$|\Psi\rangle = U(\mathcal{O})|\Psi_0\rangle$$

where $U$ is a unitary corresponding to a wavepacket outside the horizon.

It is easy to detect that $|\Psi\rangle$ is non-equilibrium, since correlators of the algebra $\mathcal{A}$ on $|\Psi\rangle$ differ from the thermal ones.

But what about the state

$$|\Psi'\rangle = U(\widetilde{\mathcal{O}})|\Psi_0\rangle$$



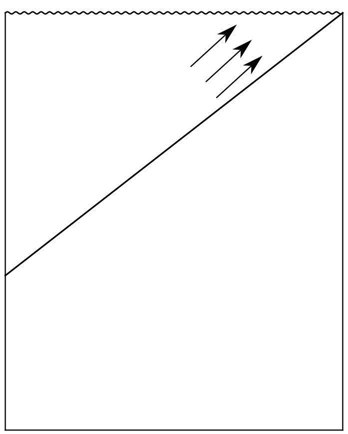
$$\langle\Psi'|\mathcal{O}..\mathcal{O}|\Psi'\rangle = \langle\Psi_0|U^\dagger\mathcal{O}..\mathcal{O}U|\Psi_0\rangle = \langle\Psi_0|U^\dagger U\mathcal{O}..\mathcal{O}|\Psi_0\rangle = \langle\Psi_0|\mathcal{O}..\mathcal{O}|\Psi_0\rangle$$

where we used that $[\mathcal{O},\widetilde{\mathcal{O}}] = 0$ and $U^\dagger U = 1$.

Correlators of $\mathcal{O}$'s on $|\Psi'\rangle$ look like equilibrium correlators, it is very hard to detect the excitation behind the horizon.

Potential ambiguity (Marolf, Wall, Raamsdonk, Maldacena, Harlow..): what happens to the infalling observer in state $|\Psi'\rangle$?

$$|\Psi'\rangle = U(\widetilde{\mathcal{O}})|\Psi_0\rangle$$



If we define the $\widetilde{\mathcal{O}}$ wrt to $|\Psi_0\rangle$, we will predict that infalling observer finds excitation behind the horizon.

If we define the $\widetilde{\mathcal{O}}$ wrt to $|\Psi'\rangle$ we will predict an empty interior.

What is the right answer?

We can use the fact that

$$[H, \widetilde{O}] \neq 0$$

to "detect" the excitation behind the horizon by measuring correlators of the form

$$\langle \Psi' | OO...H^k | \Psi' >$$

This seems to resolve the ambiguity

Consider the state

$$|\Psi'\rangle = U(\widetilde{\mathcal{O}})|\Psi_0\rangle$$

Intuitively, this is a non-equilibrium state — its hard, but possible, to detect the "excitation behind the horizon".

This may look like some kind of peculiar state, whose existence depends on the "state-dependent operator construction".

HOWEVER: remember that

$$\widetilde{\mathcal{O}}_\omega|\Psi_0\rangle \sim e^{-\frac{\beta\omega}{2}}\mathcal{O}_\omega^\dagger|\Psi_0\rangle$$

Hence the state can also be written as

$$|\Psi'\rangle = V(O)|\Psi_0\rangle$$

where $V$ is made out of $O$'s, but not necessarily unitary.

When $|\Psi_0\rangle$ has sharp energy $E_0$ and in the large $N$ limit we can show that

$$|\Psi'\rangle = U(\widetilde{\mathcal{O}})|\Psi_0\rangle = e^{-\beta\frac{(H-E_0)}{2}}U(\mathcal{O})|\Psi_0\rangle$$

Creating a wavepacket behind the horizon (starting from an equilibrium state), can be achieved by considering the state

$$e^{-\beta\frac{(H-E_0)}{2}}U(\mathcal{O})|\Psi_0\rangle$$

Notice that we can create this particle without having to talk about state-dependent operators!
In any stat-mech system (chaotic/ergodic etc.) states of the form

$$e^{-\beta\frac{(H-E_0)}{2}}U(\mathcal{O})|\Psi_0\rangle$$

are subtle non-equilibrium states, which are the analogue of states which contain excitations behind the horizon.

On these states

$$e^{-\beta\frac{(H-E_0)}{2}}U(\mathcal{O})|\Psi_0\rangle$$

correlation functions of usual observables look like they are in equilibrium, but if we include $H$ then we see that there is some non-equilibrium behavior.

Under time evolution the state settles down to equilibrium!

It might be interesting to further study these states from a more general perspective in statistical mechanics

Notice that these are statements about the usual, normal operators (no state dependent tilde operators here)

Observables obeying the Eigenstate Thermalization Hypothesis (ETH)

$$A_{ab} = f(E)\delta_{ab} + R_{ab}$$

where $f(E)$ is a smooth function. $R_{ab} \sim \mathcal{O}(e^{-S/2})$ and have erratic phases. Our modes

$$A_\omega = R_{ab}$$

connect states of energy $E$ to $E + \omega$.

On equilibrium states we have

$$\langle\Psi|A_\omega|\Psi\rangle \approx 0$$

A typical state

$$|\Psi\rangle = \sum_i c_i |E_i\rangle$$

with random phases.

We have

$$\langle\Psi|A_\omega|\Psi\rangle = \sum_{ij} c_i^* c_j R_{ij} \sim \mathcal{O}(e^{-S/2})$$

due to random cancellations of phases.

When we excite the state as

$$|\Psi'\rangle = U(O)|\Psi\rangle$$

we can have

$$\langle\Psi|U(O)^\dagger A_\omega U(O)|\Psi\rangle \neq 0$$

Even if $|\Psi\rangle$ is in the microcanonical, the state $|\Psi'\rangle = U(O)|\Psi\rangle$ has significant spread in energy.

Define projection operators $P_E$ on energies between $E$ and $E + \delta E$. We can decompose our original state as

$$|\Psi'\rangle = \sum_E P_E|\Psi'\rangle = \sum_E |\Psi'_E\rangle$$

then we have

$$\langle\Psi'|A_\omega|\Psi'\rangle = \sum_E \langle\Psi'_{E-\omega}|A_\omega|\Psi'_E\rangle = \sum_E f(E)$$

If $\langle\Psi'|A_\omega|\Psi'\rangle \sim \mathcal{O}(1)$ then we also have $f(E) \sim \mathcal{O}(1)$ (or at least, not exponentially suppressed)

In states of the form

$$|\Psi'\rangle = U(O)|\Psi\rangle$$

we get an $O(1)$ answer for

$$\langle\Psi'|A_\omega|\Psi'\rangle$$

because the phases of different energy bins are correlated — in relation to the matrix element of $A$.

What about states of the form

$$U(\widetilde{O})|\Psi\rangle$$

Since $[A_\omega, \widetilde{O}] = 0$ we also have $[A_\omega, U(\widetilde{O})] = 0$ and

$$\langle\Psi|U(\widetilde{O})^\dagger A_\omega U(\widetilde{O})|\Psi\rangle = \langle\Psi|U(\widetilde{O})^\dagger U(\widetilde{O}) A_\omega|\Psi\rangle = \langle\Psi|A_\omega|\Psi\rangle = 0$$

States of the form

$$U(\widetilde{O})|\Psi\rangle$$

look like equilibrium states for operators in the algebra $\mathcal{A}$, but intuitively we think of them as excited, non-equilibrium states.

Excitation can be detected by correlation functions involving the Hamiltonian but let us also look at the phases. We have

$$U(\widetilde{O})|\Psi\rangle = ... = e^{-\beta(H-E_0)/2}U(O)|\Psi\rangle$$

States created by acting with unitaries of the tildes can be expressed in terms of ordinary unitaries, modulated by a function of the Hamiltonian.

Notice that the definition of these states is "state independent" — and independent of the existence/definition of the tildes.

So the claim is that if $|\Psi\rangle$ is an equilibrium state then on the state

$$|\Psi''\rangle = e^{-\beta(H-E_0)/2}U(O)|\Psi\rangle$$

correlators of $\mathcal{A}$ are the same (to leading order in $1/N$) with those on $|\Psi\rangle$. This follows from the KMS condition of thermal correlators (and large $N$ factorization).

In particular we have

$$\langle\Psi''|A_\omega|\Psi''\rangle \approx 0$$

Remember that in the original state $|\Psi\rangle$ we had

$$\langle\Psi|A_\omega|\Psi\rangle \approx 0$$

due to the erratic phases. The reasons that $\langle\Psi''|A_\omega|\Psi''\rangle \approx 0$ is qualitatively different.

$$\langle\Psi''|A_\omega|\Psi''\rangle = \langle\Psi|U(O)^\dagger\, e^{-\beta(H-E_0)/2}\, A_\omega\, e^{-\beta(H-E_0)/2}\, U(O)|\Psi\rangle$$

Inserting again the projection operators in the coarse energy bins we find

$$\langle\Psi''|A_\omega|\Psi''\rangle = \sum_E e^{-\beta(E-E_0)} f(E)$$

Notice that the $f(E)$ are non-zero! The fact that this correlator is zero is because of cancellations between different energy bins (and the KMS condition). Comparing the states

$$U(O)|\Psi\rangle$$

and

$$e^{\frac{-\beta(H-E_0)}{2}}U(O)|\Psi\rangle$$

we see that they have the same microscopic phases, but in the second state we have some sort of "population inversion".

“Proper” equilibrium states

$$|\Psi\rangle$$

phases between different energy bins (relative to $O$’s) erratic.

States with particles outside the horizon

$$U(O)|\Psi\rangle$$

phases betweeen different energy bins correlated. Under time evolution the phases decohere and the state equilibriates (particles get lost behind the horizon).

States with particles behind the horizon

$$e^{-\beta\frac{H-E_0}{2}}U(O)|\Psi\rangle$$

phases betweeen different energy bins correlated. **cancellations between contributions from different bins**.

We argued that there is a canonical class of "quasi-equilibrium" states, of the form

$$e^{-\beta(H-E_0)/2}U(O)|\Psi\rangle$$

which are parametrized in a similar way as perturbations in region I (i.e. by unitaries $U(O)$) — yet the perturbations are almost undetectable by single trace operators.

This indicates the existence of a seemingly causally disconnected region of spacetime in the bulk.

The tilde operators cause transitions between these states.

But the existence of these states is rather robust (no state-dependence, or need to define the tildes)

1. Big AdS black holes have smooth interior, CFT can describe it
2. An infalling observer does not see any deviations from what is predicted by semiclassical GR
3. By extrapolation, we conjecture the same for flat space black holes
4. Information paradox resolved by exponentially small corrections to EFT
5. Entanglement/cloning related paradoxes resolved by complementarity
6. Progress towards a mathematically precise realization of complementarity
7. Evidence that complementarity and locality in EFT are compatible

Important point to settle: state dependence and observables in Quantum Gravity

**THANK YOU**