Line defects in N=2 QFT

- framed quivers and cluster algebras -

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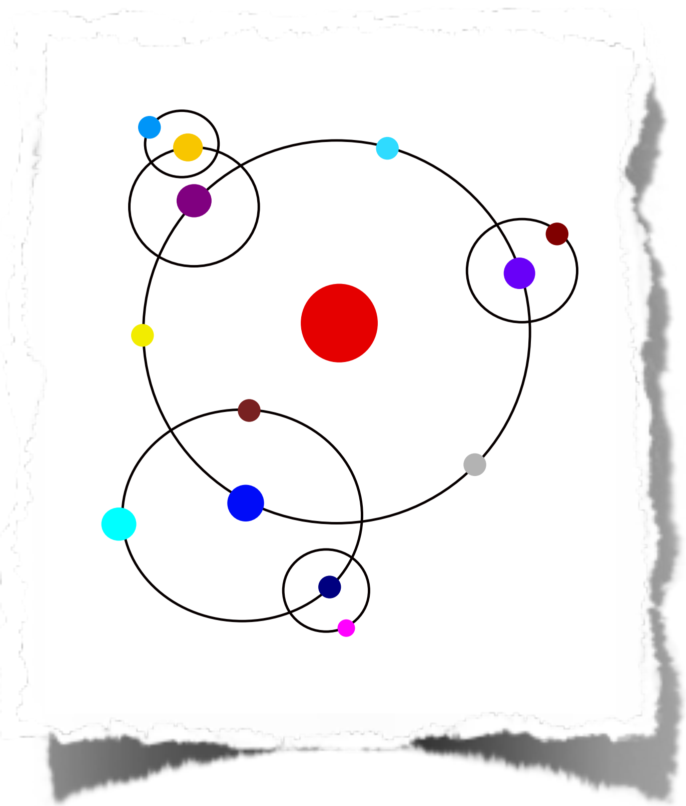
Defects in QFT

- We have learned a lot by studying the BPS sector of susy QFTs
- Susy QFTs have supersymmetric defects: line, surface, domain wall, divisor defects...
- A new BPS sector opens up: BPS states can bound the defect
- Understand the geometrical structures. Today: line defects



Line defects in class S

- Theories of class S[A] arise from the compactification of the 6d N=(0,2) theory on $\mathbb{R}^{1,3} \times \mathcal{C}$
- Compute the BPS spectrum in many cases (aka decomposition of the KS operator)
- Further dim red: $\mathbb{R}^3 \longrightarrow \mathcal{M}(\mathcal{C})$
- Darboux coordinates on $\mathcal{M}(\mathcal{C})$

[Gaiotto, Moore, Neitzke]

Consistency with WCF: hyperkahler metric

$$\mathcal{Y}_\gamma(\zeta) = \mathcal{Y}_\gamma^{\rm sf}(\zeta) \exp\left[-\sum_{\gamma'} \frac{\langle \gamma, \gamma' \rangle \, \Omega(\gamma'; u)}{4\pi i} \int_{\ell_{\gamma'}} \frac{d\zeta'}{\zeta'} \frac{\zeta' + \zeta}{\zeta' - \zeta} \log(1 - \mathcal{Y}_{\gamma'}(\zeta')) \right]$$

Line defects in class S

- Add a line defect: new Hilbert space $\mathcal{H}^{BPS}_{L_\zeta} = \bigoplus_\gamma \mathcal{H}^{BPS}_{L_\zeta,\gamma}$

- Framed BPS states: a new index (PSC)

$\mathfrak{su}(2)_R$

$$\overline{\underline{\Omega}}(u, L, \gamma; q) = \mathrm{Tr}_{\mathcal{H}^{BPS}_{u,L,\gamma}} q^{2J_3} (-q)^{2I_3}$$

Witten index: net number of ground states

$q \longrightarrow -1$

$$\mathrm{Tr}_{\mathcal{H}^{BPS}_{u,L,\gamma}} (-1)^{2J_3}$$

$q \longrightarrow +1$

$$\mathrm{Tr}_{\mathcal{H}^{BPS}_{u,L,\gamma}} (-1)^{2I_3}$$

Dimension of Hilbert space if no exotics

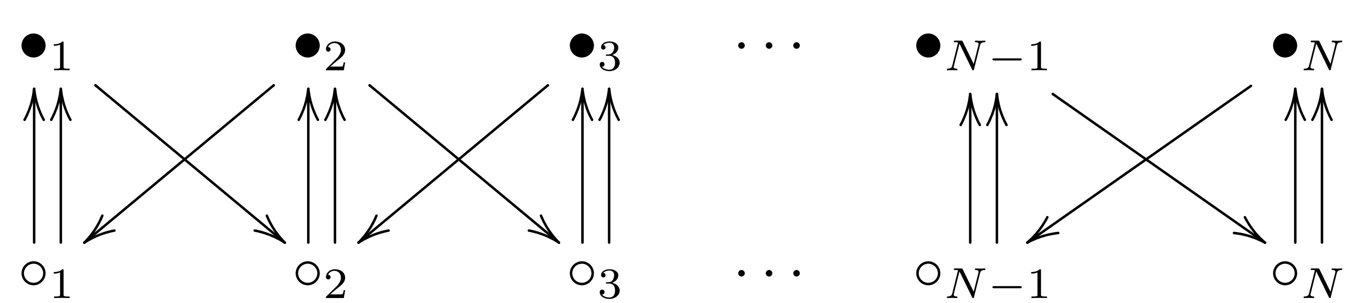
- In 3d the path integral of the QFT with a defect

$$\langle L_\zeta \rangle_{q=+1} = \sum_\gamma \overline{\underline{\Omega}}(L_\zeta, \gamma; q = +1)\ \mathcal{Y}_\gamma$$

[Gaiotto, Moore, Neitzke]

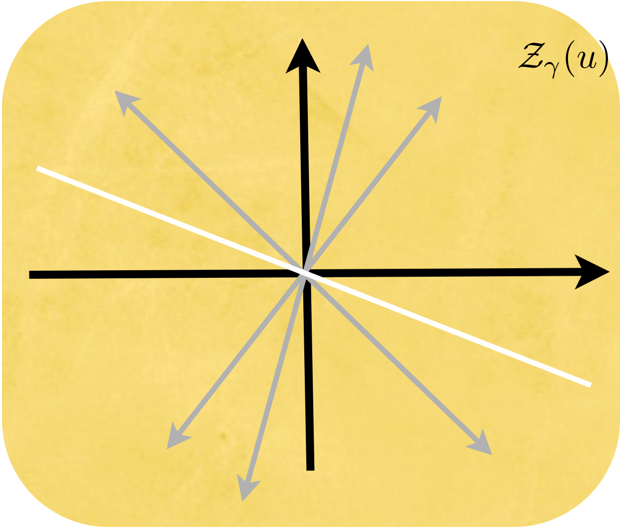
BPS Quivers

- We will consider SU(N+1)
- The ordinary BPS spectrum is captured by a low energy susy quiver quantum mechanics (SQQM)
- lattice of em charges
- The nodes are partonic (a basis of Γ) constituents γ_i which interact via bi-fundamental fields (the arrows, given by $\langle\gamma_i, \gamma_j\rangle$) and a superpotential W
- Ground states of this SQQM correspond to stable BPS particles



Quivers and mutations

- In certain chambers the mutation method gives an algorithm to solve the BPS spectral problem
- The assignment particle vs. antiparticle is arbitrary: white line



- As we rotate it, the "partonic basis" changes by a mutation (Seiberg duality):

$$\mu_{k,+}(\gamma_i) = \begin{cases} -\gamma_k & \text{if } i = k \\ \gamma_i + [\langle \gamma_i, \gamma_k \rangle]_+ \gamma_k & \text{if } i \neq k \end{cases}$$

[Alim, Cecotti, Cordova, Espahbodi, Rastogi, Vafa]

- In finite chambers a full rotation generates the full spectrum (any stable state is fundamental in some duality frame)

Cluster algebras and Q-systems

[Cecotti, Del Zotto]

- Every SU(N) quiver comes with a Q-system, a discrete integrable system (the cluster algebra of the quiver). Assign formal variables y_{γ_i} to each node

- For any mutation we define the "partial evolution"

$$\mathrm{mut}_{k,+}\,\mathsf{y}_{\gamma_i} = \begin{cases} \mathsf{y}_{\gamma_i}^{-1} & i = k \\ \mathsf{y}_{\gamma_i}\,\mathsf{y}_{\gamma_k}^{[\langle\gamma_i,\gamma_k\rangle]_+}\,(1+\mathsf{y}_{\gamma_k})^{-\langle\gamma_i,\gamma_k\rangle} & i \neq k \end{cases}$$

Same transformation as the $\mathcal{Y}_\gamma$

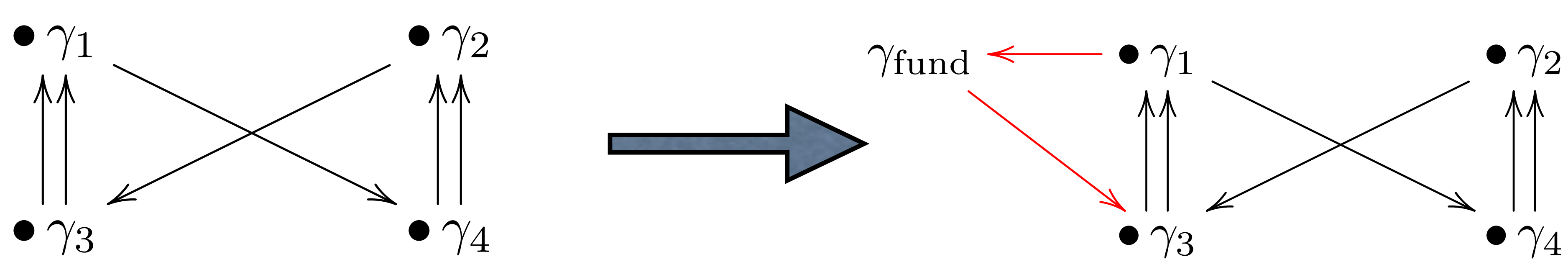
- Massaging this transformations we find the equation of the Q-system: $\mathcal{Q}_{\alpha,t+1}\,\mathcal{Q}_{\alpha,t-1} = \mathcal{Q}^2_{\alpha,t} + \mathcal{Q}_{\alpha+1,t}\,\mathcal{Q}_{\alpha-1,t}$

- The evolution of the Q-system is the sequence of mutation which generates the BPS spectrum

Framed Quivers

- Now incorporate line defects

- We think of a line defect as an infinitely massive particle: modify the quiver adding a framing node



- Construct the formal generating function

$$\mathscr{L} = \sum_{\gamma} \overline{\Omega}(u, L, \gamma; q = +1)\ \mathrm{y}_{\gamma}$$

[MC]
[MC, Del Zotto]

Mutation symmetry of defects

- Consider the sequence of mutations which generates the BPS spectrum

- It generates new defects!

- Line defects come in <u>cluster orbits</u>

$$\begin{tikzcd} \mathrm{Q}^{L_1} \arrow[r, "\mu"] \arrow[d, leftrightsquigarrow] & \mathrm{Q}^{L_2} \arrow[d, leftrightsquigarrow] \\ \mathscr{L}^{(L_1)} \arrow[r, "\mathsf{mut}_\mu^{-1}"] & \mathscr{L}^{(L_2)} \end{tikzcd}$$

- But for a Wilson line the framed quiver is invariant

Wilson lines are the constants of motion of the Q-system – explicit formulas! –

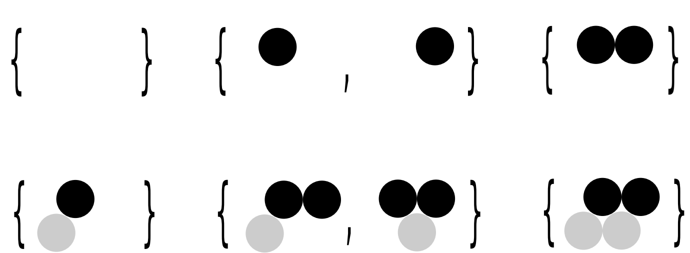
No exotic conjecture automatically true

Equivariant localization

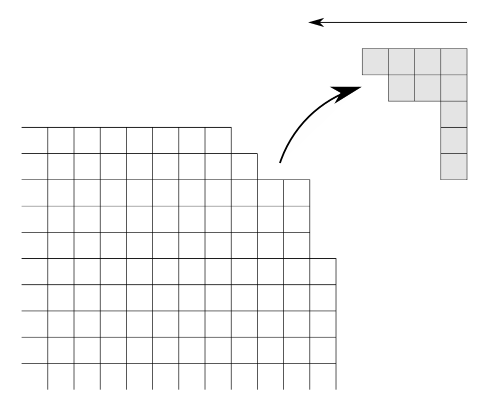
- Now consider the Witten index $\overline{\Omega}(L, \gamma; q = -1)$
- We compute it with localization: fixed points + virtual tangent space (+ stability)

[MC,Sinkovics, Szabo]

- Natural toric action which rescales the fields in the SQQM (the arrows)
- Combinatorial classification of the fixed points: pyramid partitions



is the analog of: in Nekrasov partition function



Equivariant localization

- The weights of the toric action around a fixed point can be read from the deformation complex

$$0 \longrightarrow \mathsf{S}^0_\pi \xrightarrow{\delta_0} \mathsf{S}^1_\pi \xrightarrow{\delta_1} \mathsf{S}^2_\pi \xrightarrow{\delta_2} \mathsf{S}^3_\pi \longrightarrow 0$$

gauge transf

fields

eom

relations between eom

ratio of dets

- We find an explicit formula for the index:

$$\underline{\overline{\Omega}}(L,\gamma;q=-1) = \sum_{\pi\in\mathscr{M}^c_{\mathbf{d}}(\mathsf{Q})^{\mathbb{T}_\mathsf{W}}} (-1)^{\dim\mathsf{S}^0_\pi - \dim\mathsf{S}^1_\pi}$$

Conclusions

- New structures appear with line defects
- Line defects come in (cluster) families
- Claim for $\overline{\underline{\Omega}}(L,\gamma;q=+1)$: Wilson lines are constant of motions of the Q-system
- Claim for $\overline{\underline{\Omega}}(L,\gamma;q=-1)$: compute via localization
- Many open questions: valid for any QFT? role of integrable systems? Hitchin moduli space interpretation?