Non-Relativistic Scaling in Semimetals

Applied Newton-Cartan Geometry,
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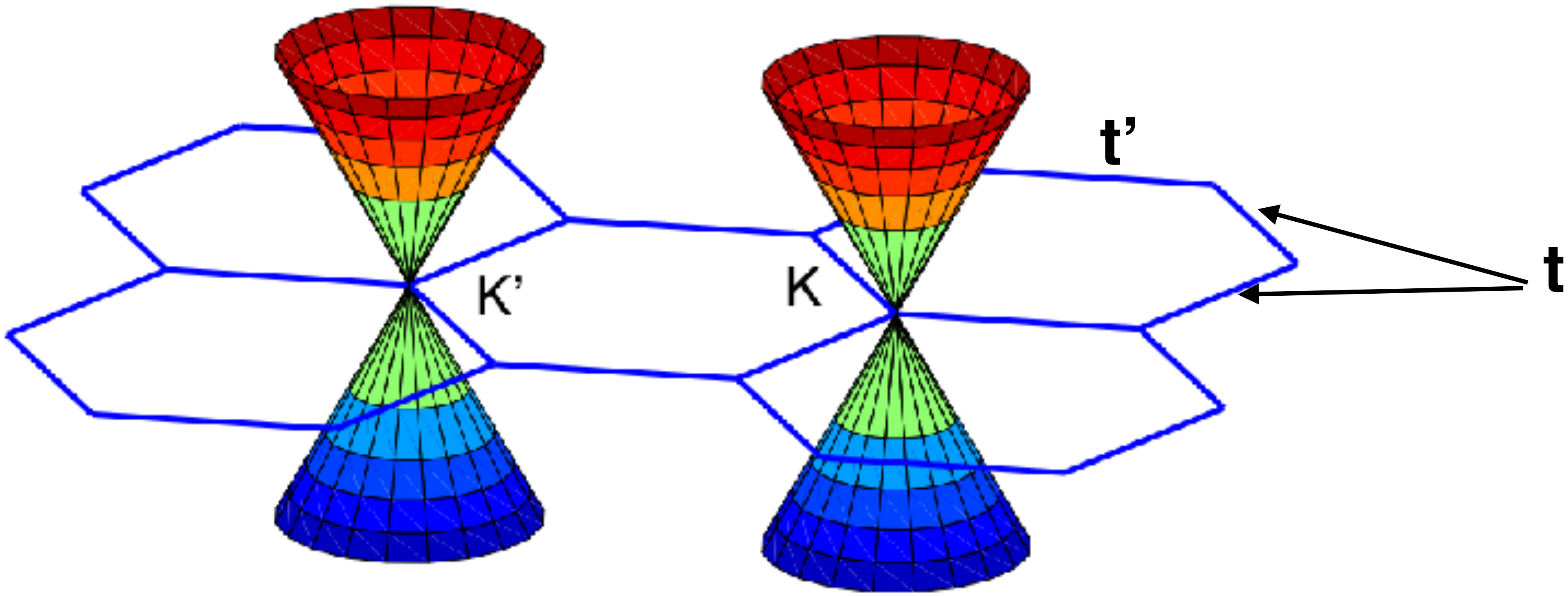
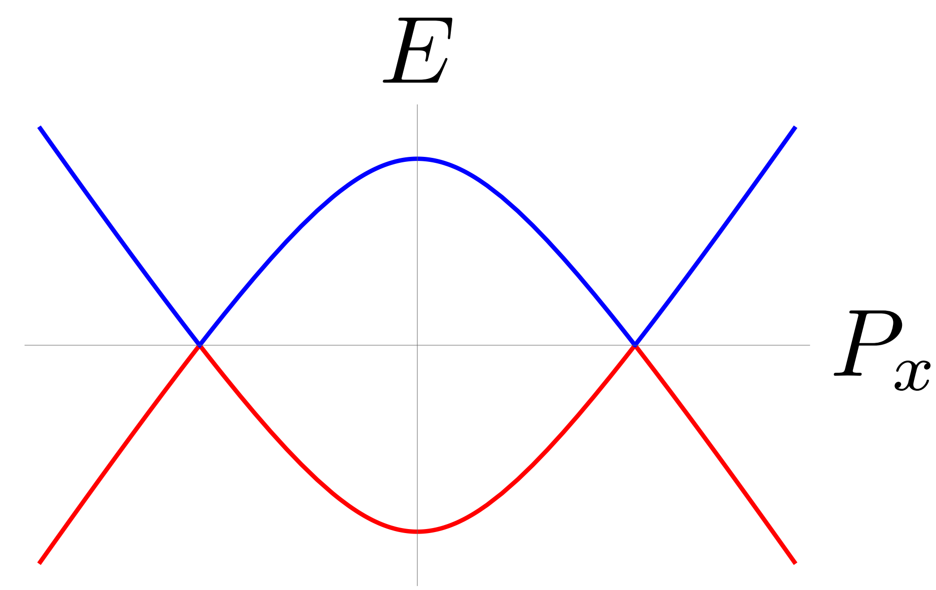
- Transport coefficients can be split in term of dissipative and non-dissipative ones. To compute them it is necessary to couple the system to external background fields.
- For the non-dissipative case symmetries are enough to get universal results related to transport coefficients.
  - In relativistic systems, see for example Son, Surowka 0906.5044, N. Banerjee, et al 1203.3544.
  - For Schrödinger systems, see Son 1306.0638 also see Moroz' talk.
  - …
- To systematically construct effective field theories, is crucial to know the proper symmetries of the system and how to couple it with external fields.

outline

- Band touching semimetals
  - Example in 2+1 dimensions
  - Examples in 3+1 dimensions
- Holographic semimetals
- Summary

2+1 dimensions

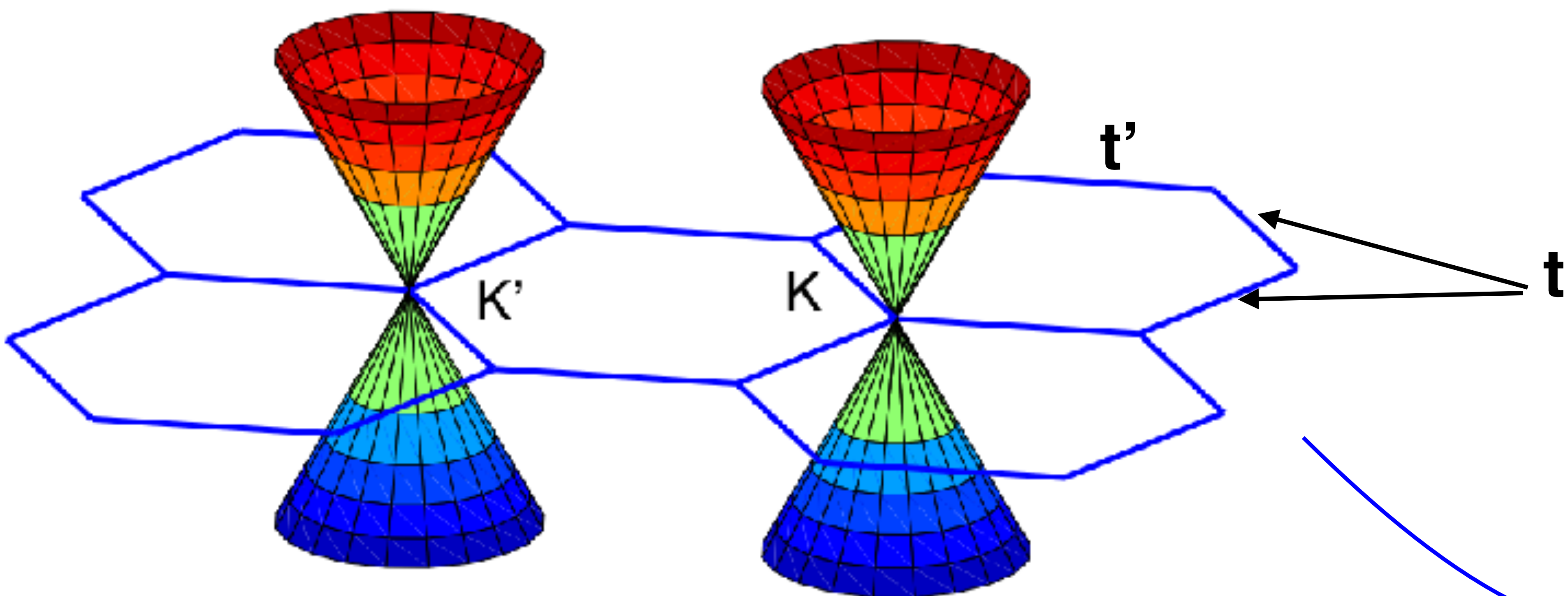


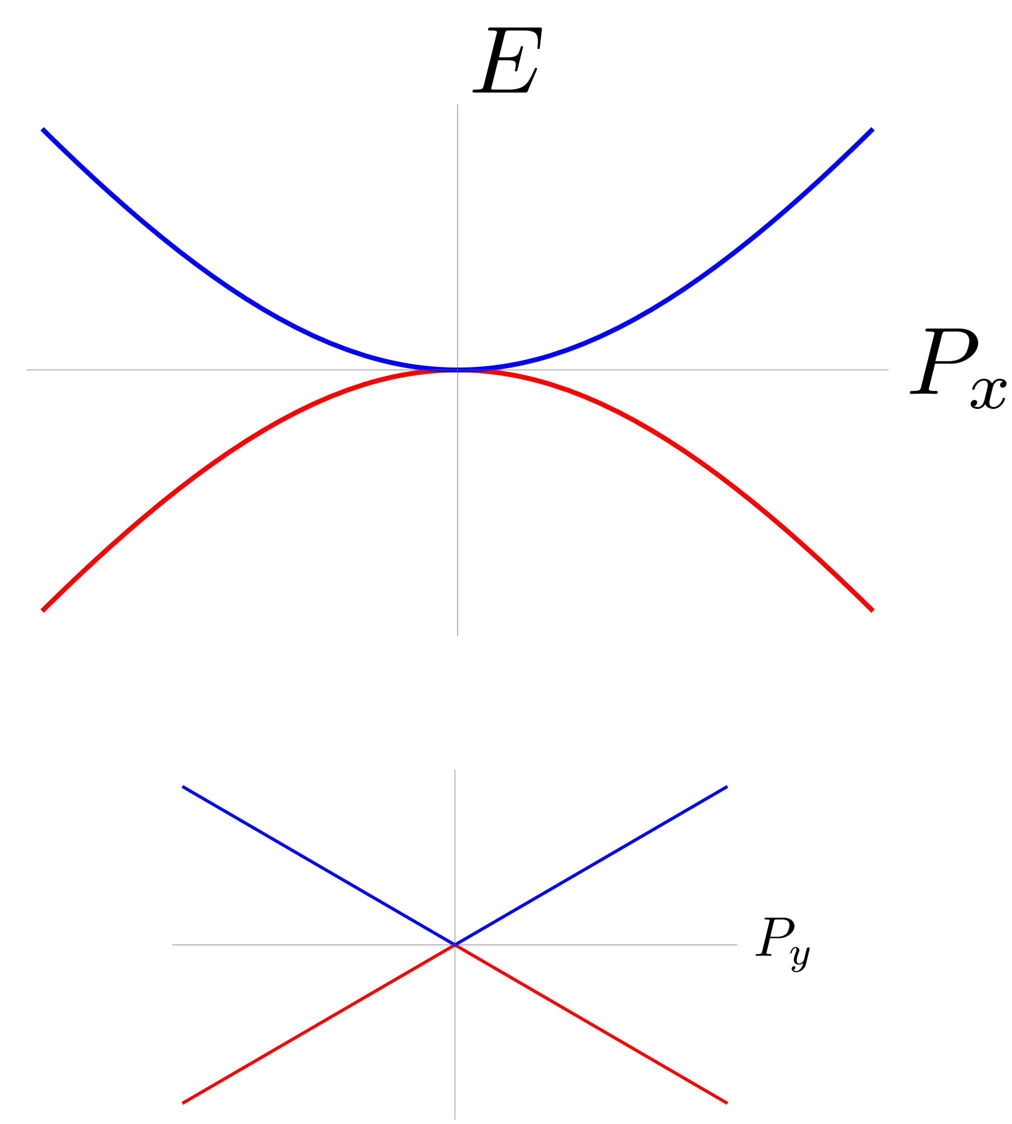


$$H = \frac{1}{2M}\left(P_x^2 - \delta^2\right)\sigma_x + P_y\sigma_y$$

[G. Montambaux, F. Piechon, J.-N. Fuchs, and M.O. Goerbig]

2+1 dimensions



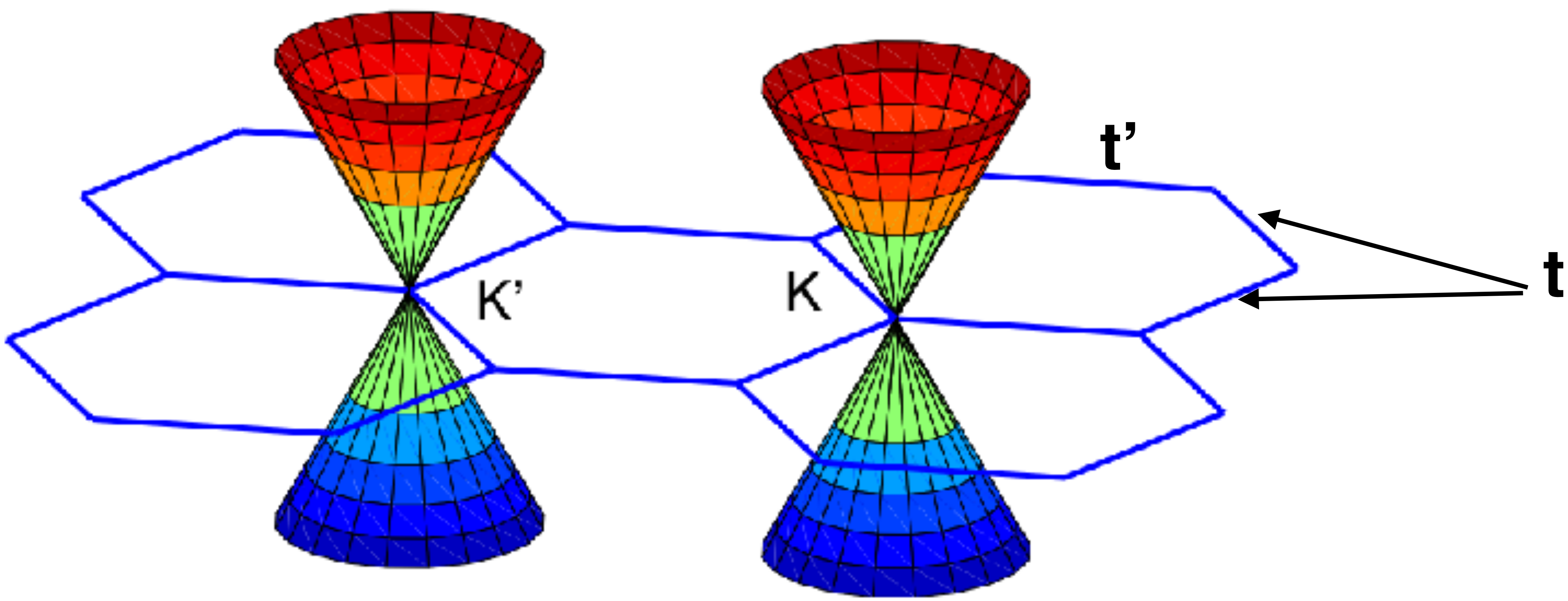


$$H = \frac{1}{2M}\left(P_x^2 - \delta^2\right)\sigma_x + P_y\sigma_y$$

$$t' = 2t \implies \delta = 0$$

[G. Montambaux, F. Piechon, J.-N. Fuchs, and M.O. Goerbig]

2+1 dimensions



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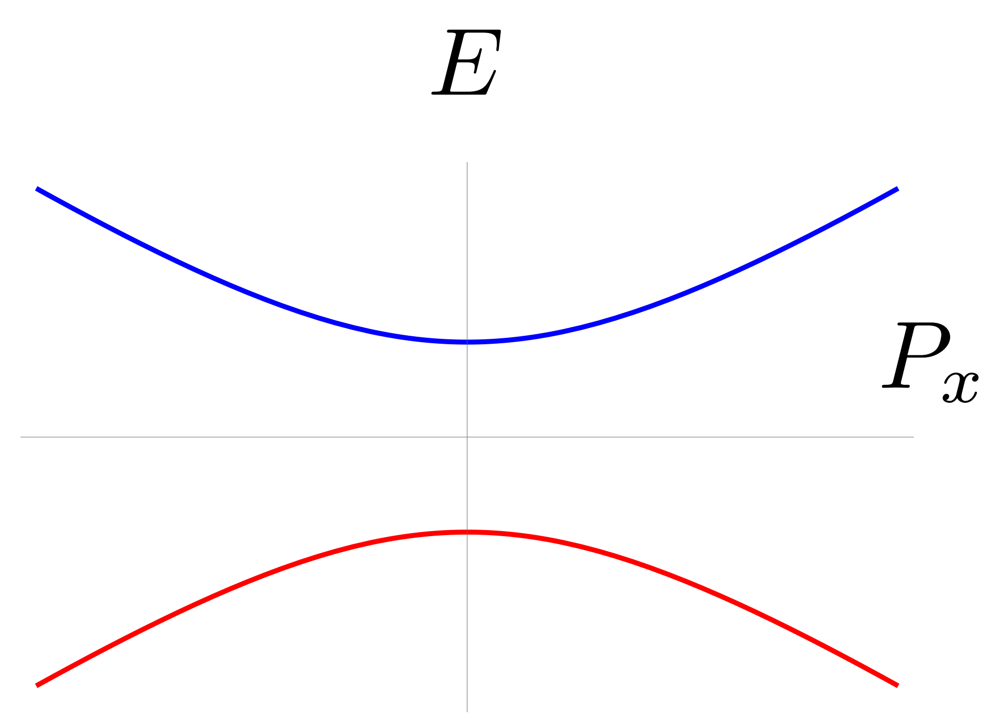


[G. Montambaux, F. Piechon, J.-N. Fuchs, and M.O. Goerbig]

3+1 dimensions

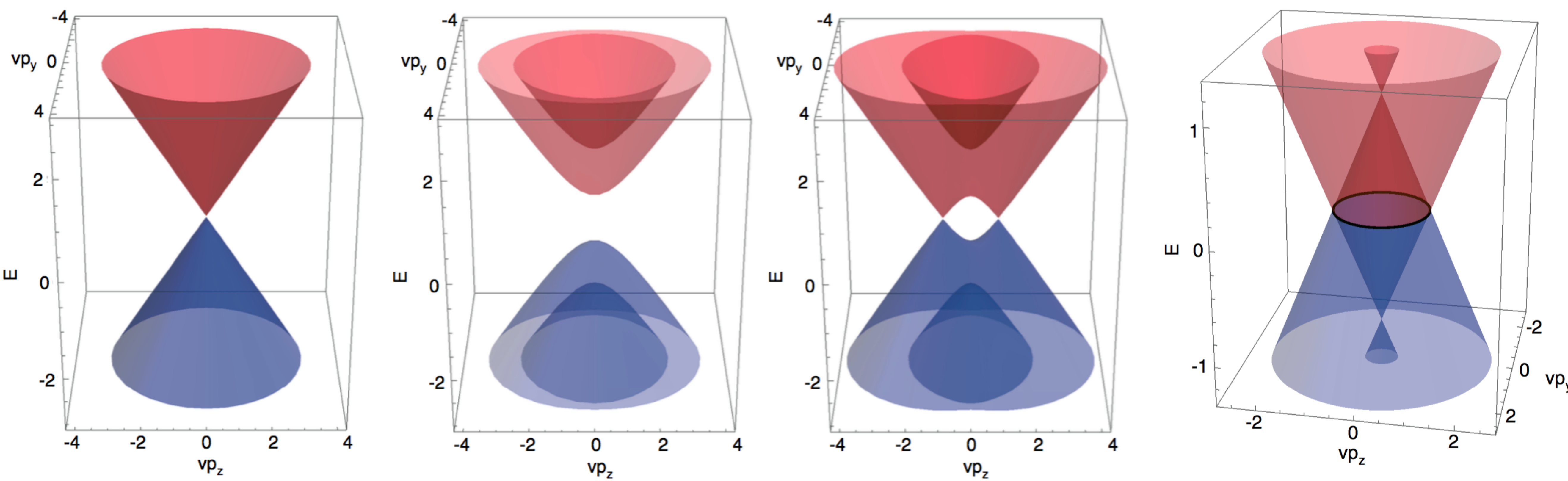
$$L = \bar{\psi}\left(\gamma^\mu \partial_\mu - m - b\gamma^z\gamma_5 - b'\gamma^{xy}\right)\psi$$

$b' = b = 0$
$m = 0$

$b^2 > M^2$

$b^2 < M^2$

$b' \neq 0$
$b = m = 0$



$m_{eff} = \sqrt{m^2 - b^2}$

$b' = 0$

$b_{eff} = \sqrt{b^2 - m^2}$

$b' = 0$

[Burkov, Hook, Balents]
[Armitage, Mele, Wishwanath]

3+1 dimensions

$$L = \bar{\psi}\left(\gamma^\mu \partial_\mu - m - b\gamma^z\gamma_5 - b'\gamma^{xy}\right)\psi$$

- the topological nature of the nodes imply interesting observable effects
  - Chiral magnetic effet

$$\mathbf{j} = -\frac{e^2}{4\pi^2}\partial_t\theta\,\mathbf{B}.$$

  - Fermi arcs (surface states)
  - anomalous Hall conductivity

$$\theta(\mathbf{r}, t) = 2\mathbf{b}\cdot\mathbf{r} - 2b_0 t,$$

$$\mathbf{j} = \frac{e^2}{4\pi^2}\boldsymbol{\nabla}\theta \times \mathbf{E}.$$

Although the chiral magnetic effect always vanishes at equilibrium in Weyl semimetals!

There is a long literature on anomaly induced transport in Weyl semimetals
For a review see: [Landsteiner, 1610.04413]

multi-Weyl semimetals

Weyl semimetals are characterized by having a monopole Berry curvature in momentum space.

The best known examples correspond to monopole charge $n=1$.

$n=1$ Weyl semimetals have linear dispersion relations and suffer of chiral anomalies.

Chiral anomalies imply new non-dissipative transport coefficients and negative magnetoresistance.

Are there Weyl semimetals with higher monopole charge?

The answer is: **YES!**

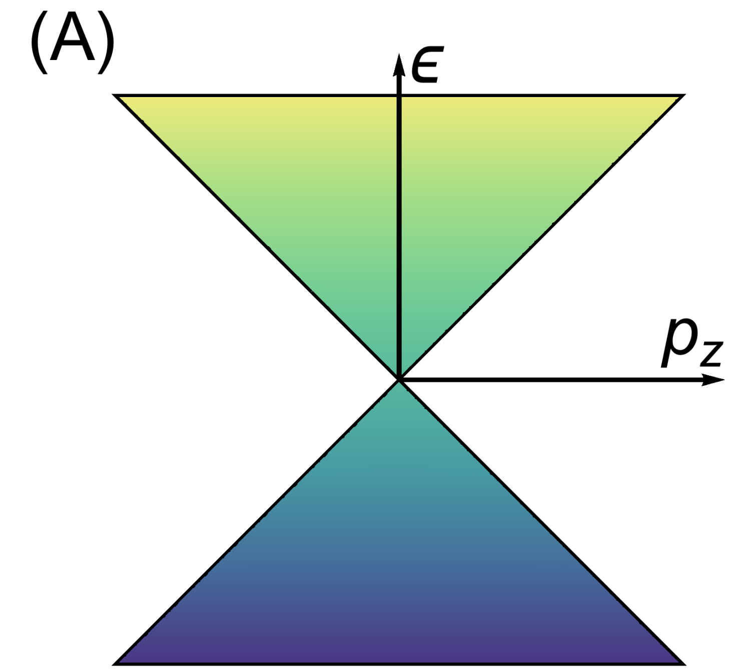
multi-Weyl semimetals

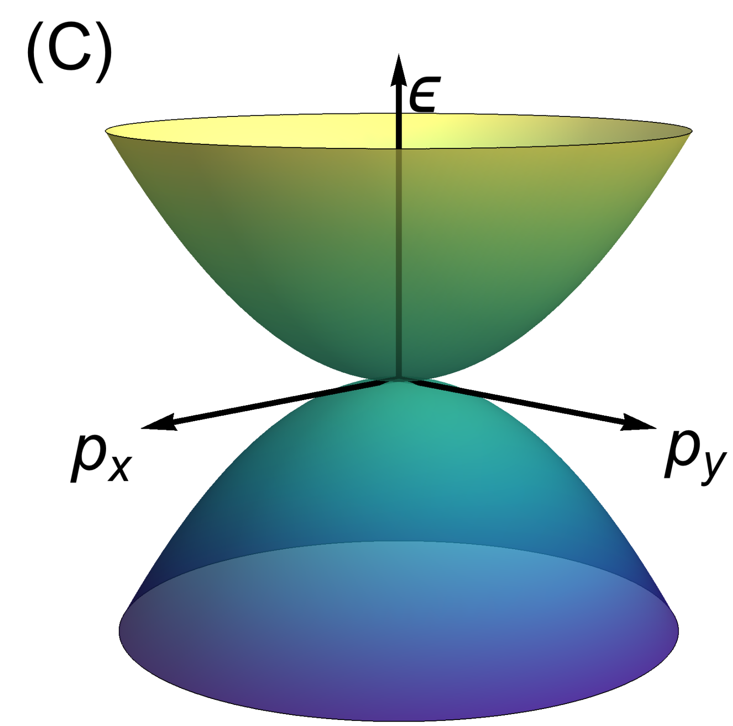
$$H_n(\mathbf{p}) = \alpha_n p_\perp^n \left[\cos(n\phi_p)\,\sigma_x + \sin(n\phi_p)\,\sigma_y\right] + v p_z \sigma_z$$

The Berry curvature around a Weyl point is:

$$\mathbf{\Omega_p} = \frac{1}{2}\frac{n v \alpha_n^2 (p_x^2+p_y^2)^{n-1}}{\left[\alpha_n^2(p_x^2+p_y^2)^n + v^2 p_z^2\right]^{3/2}}\ (p_x, p_y, n p_z)$$

$$n = \frac{1}{2\pi}\oint_\Sigma \Omega_\mathbf{p}\cdot d\mathbf{S}.$$





Kinetic theory predicts

$$\partial\rho + \nabla\cdot\vec{J} = \frac{en}{4\pi^2}\vec{E}\cdot\vec{B} - \frac{\delta\rho}{\tau}$$

And a *n* dependent anisotropic negative magnetoresistance

[Dantas, Roy, P-B, Surowka]

For a QFT analysis of the pure gauge anomaly see also 1705.04576, 1803.01684

So far the pictured I have presented is just for free fermions or at least weakly interacting

Actually at strong coupling (Holography) these systems can be realized and some predictions can be extracted

strong coupling

Ingredients:

- Gauge field dual to the vector current
- Gauge field dual to the axial current
- Axial anomaly (Chern-Simons terms)
- background axial field
- mass deformation (Scalar field)

$$\mathcal{L} = \bar{\Psi}\left(i\partial\!\!\!/ - eA\!\!\!/ - \gamma_5\gamma_z b + M\right)\Psi\,.$$

$$\partial_\mu J_5^\mu = \frac{1}{16\pi^2}\epsilon^{\mu\nu\rho\lambda}F_{\mu\nu}F_{\rho\lambda} + 2M\bar{\Psi}\gamma_5\Psi$$

the model

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{12}{L^2}\right) - \frac{1}{4}F^2 - \frac{1}{4}F_5^2 + \frac{\alpha}{3}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu\left(F^5_{\nu\rho}F^5_{\sigma\tau} + 3F_{\nu\rho}F_{\sigma\tau}\right) + \right.$$
$$\left. + (D_\mu\Phi)^*(D^\mu\Phi) - V(\Phi)\right., \qquad (2.1)$$

$$V = -\tfrac{12}{L^2} + m^2\phi^2 + \tfrac{\lambda}{2}\phi^4$$

IR metric

$$ds^2 = B_0 r^2(-dt^2 + dr^2 + dx^2 + dy^2) + C_0 r^{2\beta} dz^2$$

$$A_z = r^c \qquad , \qquad \phi = \phi_{IR}$$

Anomalous Hall conductivity

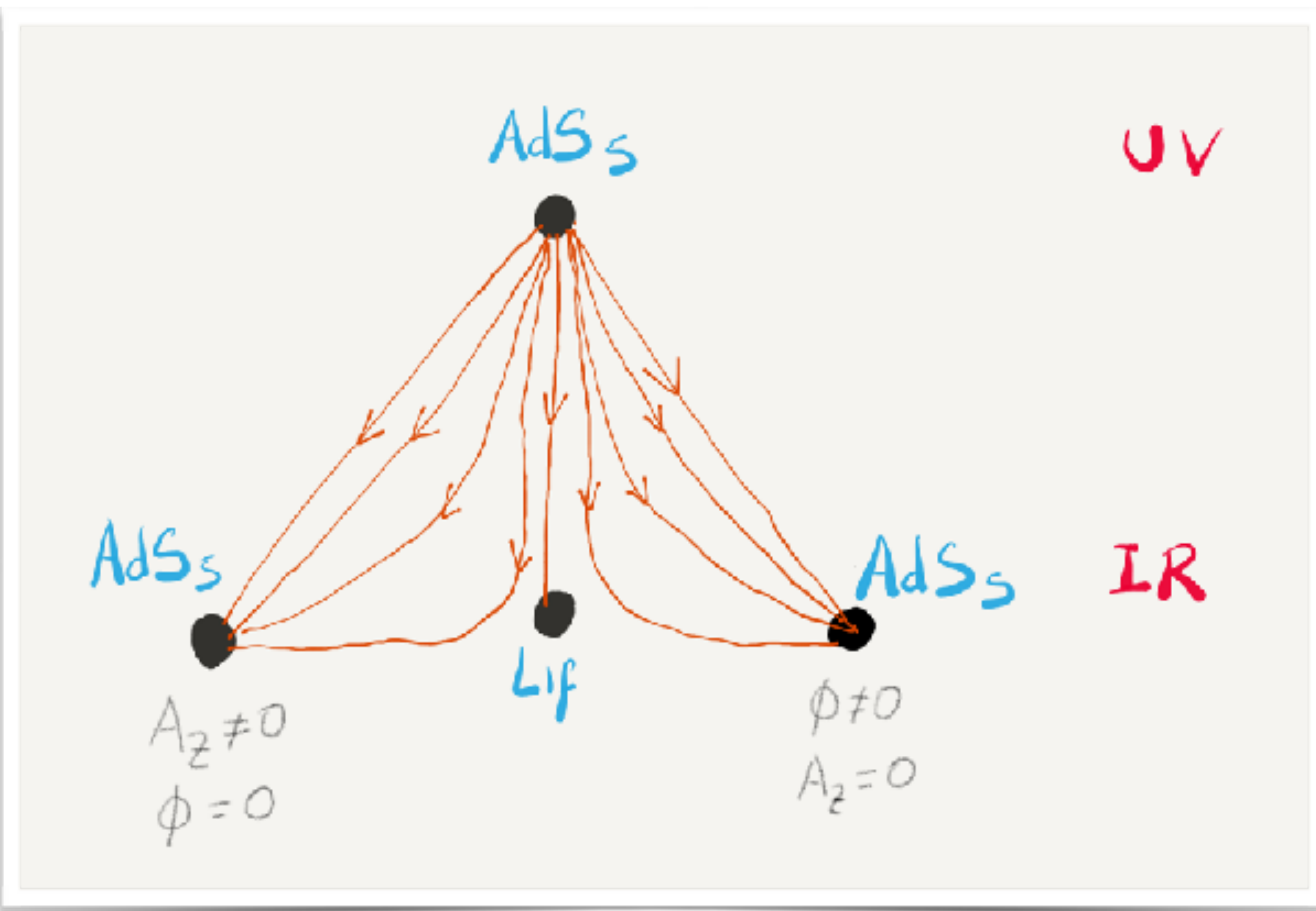
$$\sigma_{xy} \sim \alpha A_z(r_0)$$

[Landsteiner, Liu, Sun]

zero temperature geometries

$$ds^2 = B_0 r^2(-dt^2 + dr^2 + dx^2 + dy^2) + C_0 r^{2\beta} dz^2$$

$$A_z = r^c \qquad , \qquad \phi = \phi_{IR}$$



consistency requires

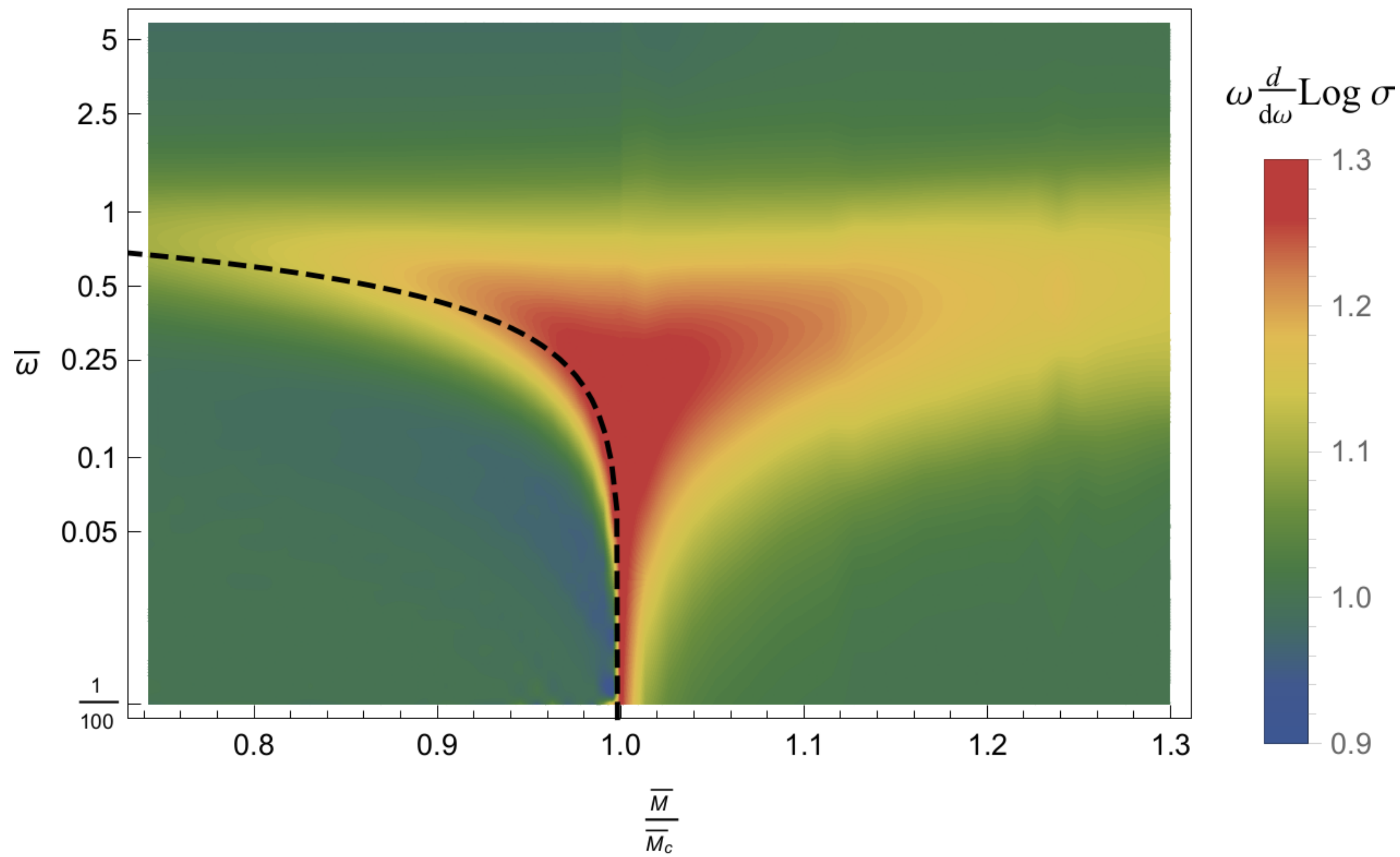
$$0 < \beta \leq 1$$

[Landsteiner, Liu, Sun]
[Grignani, Marini, Speziali, P-B]

optical conductivity

longitudinal conductivity $\Re(\sigma) \propto \omega^{2-\beta}$

transvers conductivity $\Re(\sigma_{\pm}) \propto \omega^{\beta}$

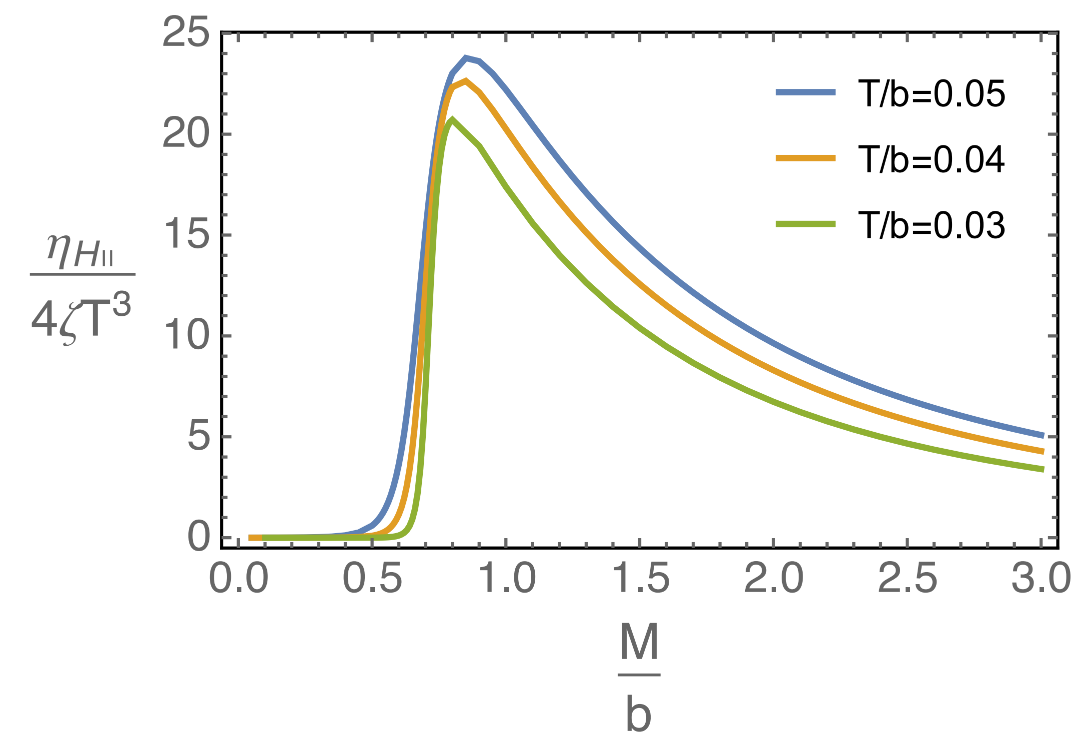


[Grignani, Marini, Speziali, P-B]

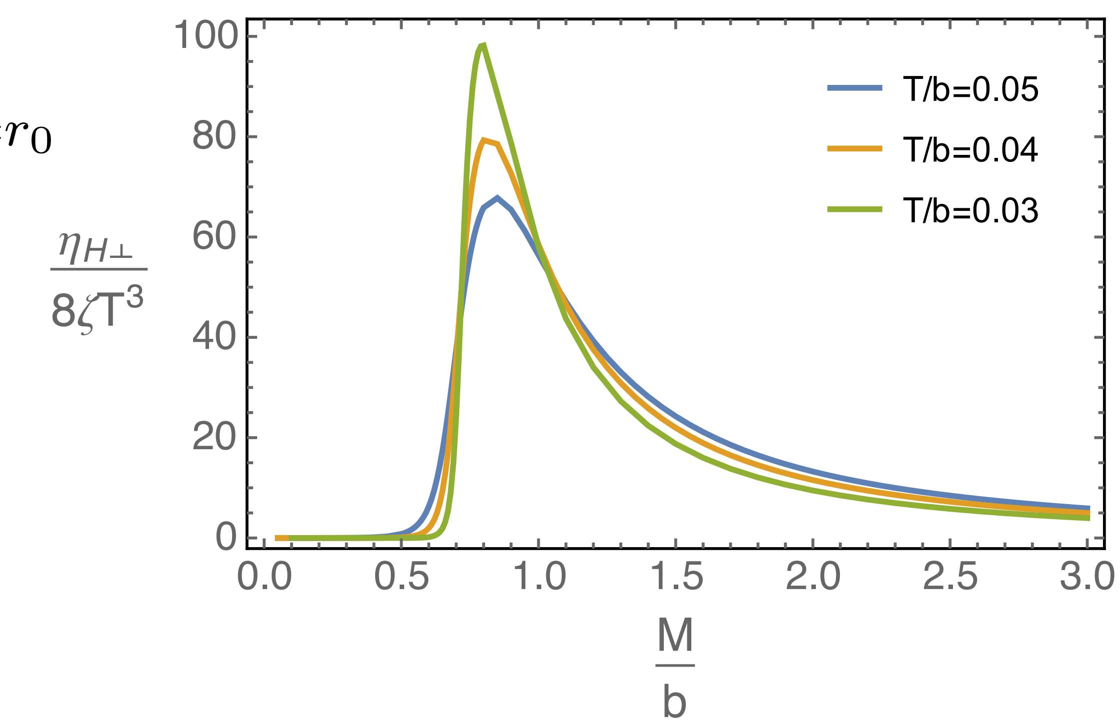
(dissipationless) Odd Viscosities

If rotational invariance is broken odd viscosities are allowed in 3+1 dimensions

$$\delta S \sim \zeta \int A \wedge R \wedge R$$



$$\eta_{H_\parallel} = \eta_{yz,xz} = -\eta_{xz,yz} = 4\zeta \frac{q^2 A_z \phi^2 f^2}{h}\bigg|_{r=r_0}$$

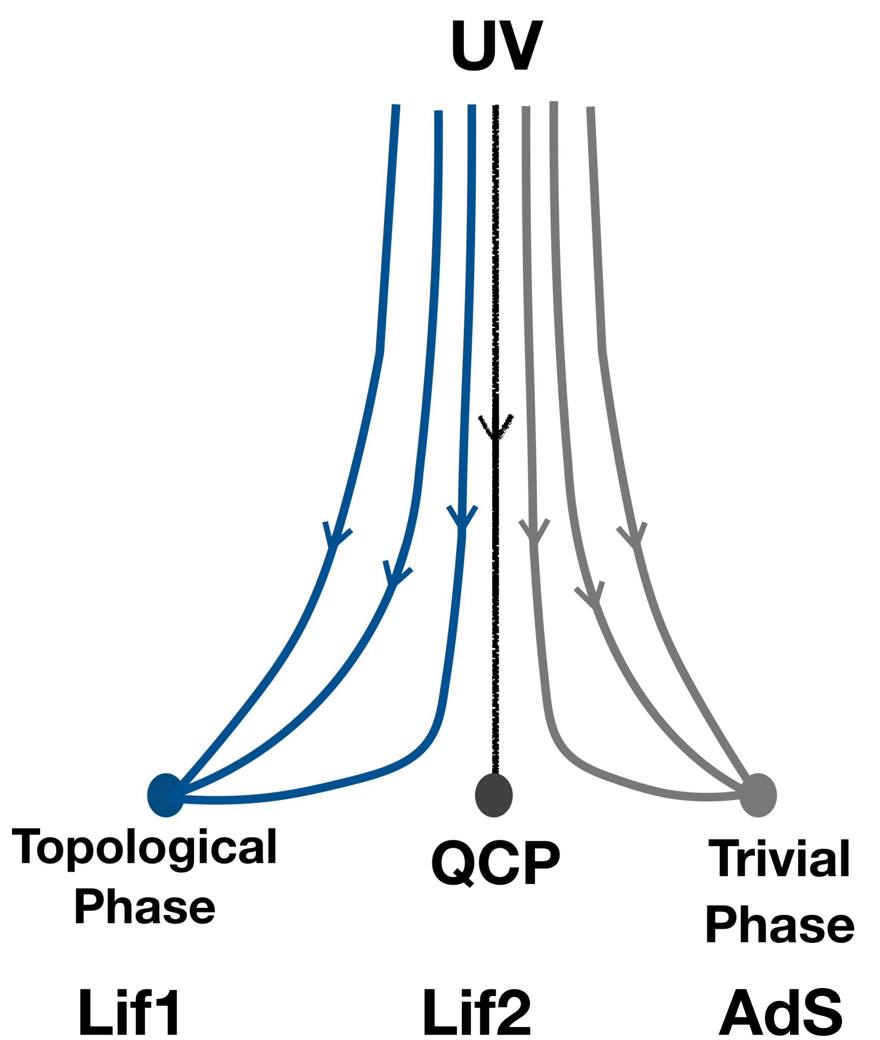


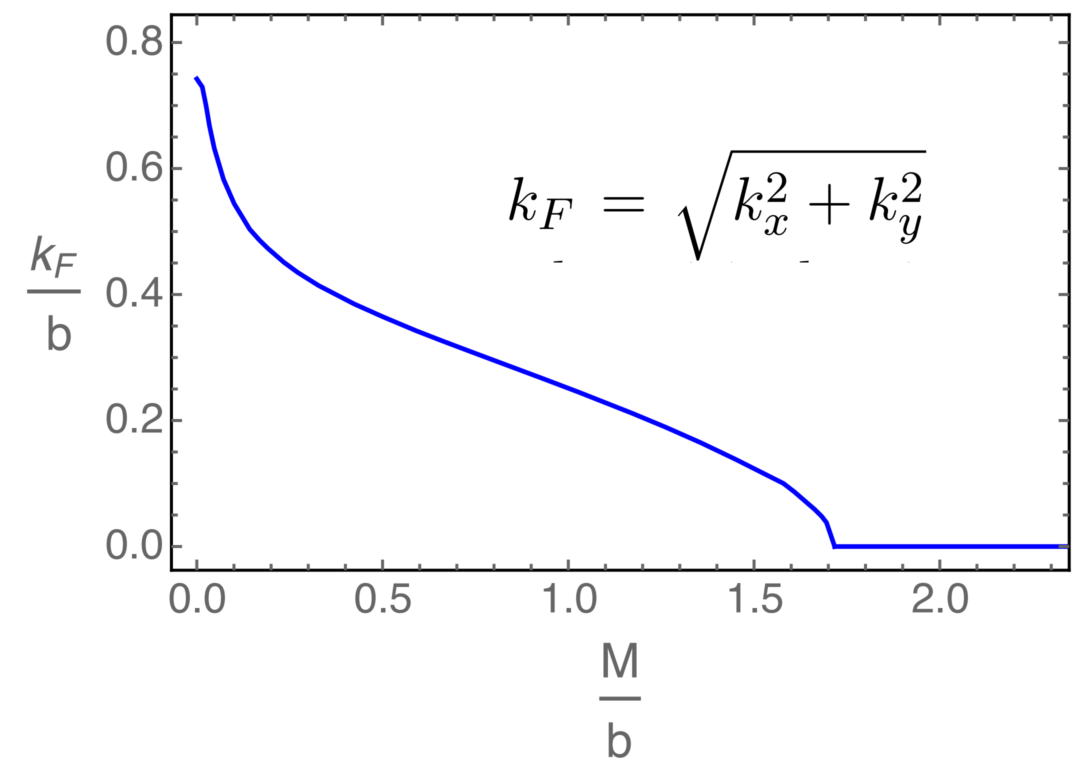
$$\eta_{H_\perp} = 8\zeta q^2 \phi^2 f A_z\bigg|_{r=r_0}$$

[Landsteiner, Liu, Sun]

Holographic nodal semimetal

$$S = \int d^5x\sqrt{-g}\bigg[\frac{1}{2\kappa^2}\Big(R+\frac{12}{L^2}\Big)-\frac{1}{4}\mathcal{F}^2-\frac{1}{4}F^2+\frac{\alpha}{3}\epsilon^{abcde}A_a\Big(3\mathcal{F}_{bc}\mathcal{F}_{de}+F_{bc}F_{de}\Big)$$
$$-(D_a\Phi)^*(D^a\Phi)-V_1(\Phi)-\frac{1}{3\eta}\big(\mathcal{D}_{[a}B_{bc]}\big)^*\big(\mathcal{D}^{[a}B^{bc]}\big)-V_2(B_{ab})-\lambda|\Phi|^2B^*_{ab}B^{ab}\bigg]$$





In this context
Lifshitz means: $(t,z,r^{-1})\to c(t,z,r^{-1})\,,\quad (x,y)\to c^{\alpha/2}(x,y)$

[Liu, Sun]

Summary

- Condensed matter systems are a playground for studying QFT in curved backgrounds. Maybe the only place to get answers related to fundamental physics at present time.
- Experimentalist can test the predictions!
- Big variety of anisotropic systems demanding to be coupled to curved backgrounds.
- Quantum anomalies are relevant not only in relativistic systems.
- Holography is also an useful tool to describe topological states of matter.