The Exact Renormalization Group: some non-relativistic aspects (maybe)

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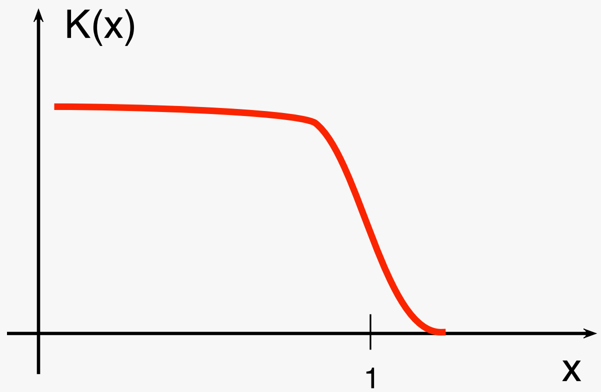
Outline

1. Review of the Exact Renormalization Group (ERG)
2. ERG for *partition function* of free field theories
   - → higher spin gauge theory holography
   - comes about through identification of an enormous non-local symmetry of free field theories
   - holographic fields described by *Cartan connection* (first order formalism)
     - spin 2 part ~ graviton
3. ERG for *wave-functionals* of arbitrary states of free field theories
   - derive explicit flow through space of states
   - ERG is well-designed for *unitary* flow — a *continuous tensor network*
4. Non-relativistic examples

ERG

$$Z = \int [d\phi] e^{-S_o[M,\phi] - S_{int}[\phi]}$$

$$S_o[M,\phi] = \int \phi \; K^{-1}(-\Box/M^2) \; \Box\phi$$



- **path integral is over all modes of field**
  - regulator function gives zero weight to high momentum modes
- **the RG principle is that the choice of K is immaterial**

$$M \frac{d}{dM} Z = 0$$

ERG

- Cutoff independence comes about through the couplings of the theory becoming scale dependent
- Polchinski showed that this gives an *exact* equation

$$M\frac{\partial S_{int}}{\partial M} = \frac{1}{2}\Delta_B(x,y)\int [d\phi]\left[\frac{\delta S_{int}}{\delta\phi(x)}\frac{\delta S_{int}}{\delta\phi(y)} - \frac{\delta^2 S_{int}}{\delta\phi(x)\delta\phi(y)}\right]$$

  - employed a trick: discard a total functional derivative in the path integral
- this single equation can be expanded to extract the scale dependence of each coupling
  - in fact, we will improve on this
    - introduce separate cutoff and renormalization scale
    - complete exact system of equations for sources and correlation functions

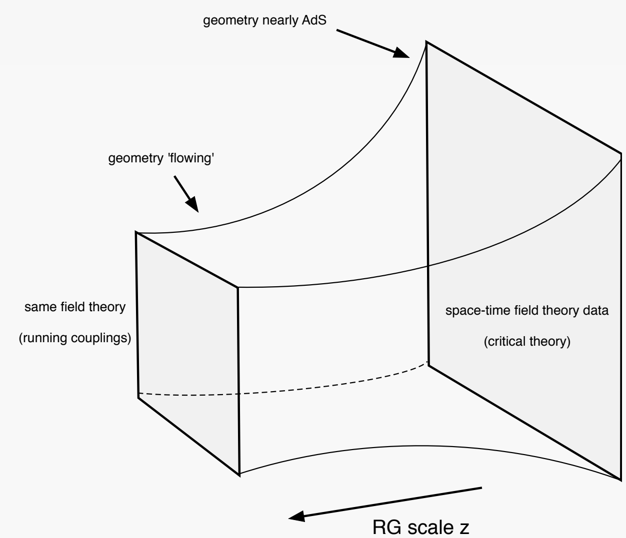
- **will apply this to a special scenario**
    - initially won't turn on explicit interactions, but instead will source 'single trace' operators
    - thus, will first study the RG properties of the generating functional of correlation functions of single trace operators
    - by single trace, we mean local operators of the form

$$\phi_a^* \partial_{\mu_1} \ldots \partial_{\mu_s} \phi^a$$

    - that is, organize elementary fields into an N-vector, and consider only U(N) (or O(N)) singlets
        - these are bilinears, and so path integral for generating functional is Gaussian
    - the ERG equations are a system of first order equations for the scale dependence of the sources and the corresponding expectation values
        - this is the same data tracked by a holographic system

Holography in First-Order

- given a *conserved* current $\hat{j}^{\mu_1 \cdots \mu_s}$ there is a corresponding *massless* gauge field $A_{\mu_1 \dots \mu_s}$ in the bulk (obtained by gauge-fixing bulk tensor)
  - at linearized level, satisfies a second order PDE
  - packages together info about CFT: source and vev of $\hat{j}^{\mu_1 \cdots \mu_s}$
- we will be led to package this info together in a sort of Hamiltonian formalism, in which RG scale plays the role of time, and the bulk gauge field appears as a canonical pair, satisfying a pair of first order PDEs — the ERG equations



The Exactness of ERG

- **usually, we think of RG flows as irreversible, associated with coarse graining**
  - this is a practicality, rather than a necessity
  - it comes about because we throw away information
- **if we could track everything, in principle we could have unitarity**
  - typical this is impractical
  - we must track an infinite number of operators, not just the relevant subset
- **there is one case where we *must* do so**
  - in free field theories, such truncations correspond to explicit breaking of gauge symmetries (from a holographic point of view)

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Holography and Higher Spin Theories

- **free field theories possess an infinite number of conserved currents**
  - holographically dual to gauge fields of various spins $\phi_a^* \partial_{\mu_1} \ldots \partial_{\mu_s} \phi^a$
    - the usual gauge transformations act diagonally on these currents, but more generally, higher spin symmetries act off-diagonally
      - they transform between different currents
  - so if we truncate to a few operators, we are explicitly breaking the symmetry of the free fixed point
    - these symmetries would also be broken by the introduction of any interactions in the field theory
- **so we will keep them all, and determine a lossless RG**
  - the general principle is that RG must act on a **complete closed set** of 'observables' (in free theories, this set is {"single trace ops"})
- **RG can be presented as an exact Ward identity**

Symmetries of Free Fixed Points

- this symmetry acts linearly, but non-locally $|\phi^a\rangle \mapsto \mathcal{L}|\phi^a\rangle$
- or, in the space-time basis

$$\phi^a(x) \mapsto \int d^d y \ \mathcal{L}(x,y)\phi^a(y) \qquad (*)$$

- this encodes diffeomorphisms as well as higher-spin analogues

$$\mathcal{L}(x,y) = \delta^{(d)}(x-y) + \zeta^\mu(x)\partial_\mu^{(x)}\delta^{(d)}(x-y) + \zeta^{\mu\nu}(x)\partial_\mu^{(x)}\partial_\nu^{(x)}\delta^{(d)}(x-y) + \ldots$$

- we implement (*) as a change of variables in the path integral
  - this generates an exact Ward identity — in the *background* sense
  - will be an important ingredient in RG, and is the *origin of higher spin symmetry* in the holographic bulk
  - geometry of the bulk is associated with symmetry of the fixed point

Generating Functionals and Ward Identities

- a standard tool is a generating functional

$$Z[A_\mu(x)] = \langle e^{i\int d^dx A_\mu(x)\hat{j}^\mu(x)}\rangle$$

- if we can compute it, it encodes all of the correlation functions of the operator $\hat{j}^\mu(x)$ that is sourced

- if the quantum theory is such that the current is conserved, we have an exact Ward identity

$$Z[A_\mu^g] = Z[A_\mu]$$

- given a path integral rep'n of Z, derive by the Fujikawa method
  - make a change of integration variables $\phi \mapsto \phi^g$
    - measure invariant (or anomalous), action transforms

$$S[\phi] + \int A_\mu j^\mu \mapsto S[\phi^g] + \int A_\mu^g j^\mu \qquad \text{(Noether)}$$

Generating Functionals for Free QFTs

- we have local operators $\{1, \phi^2(x), j^\mu(x), T^{\mu\nu}(x), \ldots\}$
- would introduce sources ('couplings') $\{U, b(x), a_\mu(x), h_{\mu\nu}(x), \ldots\}$
- in the case of free field theory, all of these operators are bilinear in the elementary fields, and they can be collected together into a bi-local expression

$$\int d^dx \int d^dy \; \langle\phi_a|x\rangle\langle x|B|y\rangle\langle y|\phi^a\rangle = \int d^dx \int d^dy \; \phi_a^*(x)\underline{B(x,y)}\phi^a(y)$$

- we can think of expanding the bi-local source quasi-locally

$$B(x,y) = b_0(x)\delta^{(d)}(x,y) + b_1^\mu(x)\partial_\mu\delta^{(d)}(x,y) + \ldots$$

- this then gives local sources for the infinite collection of spin-s currents that are conserved at the free fixed point

Free Majoranas

- fixed point action

$$S_0 = \int_{x,y} \psi^m(x)\gamma^\mu P_{F;\mu}(x,y)\psi^m(y) \qquad m = 1, 2, \ldots, N$$

$$P_{F;\mu}(x,y) = \partial_\mu^{(x)}\delta^{(d)}(x,y)$$

- introduce sources for *all* single-trace operators

$$S_{int} = U + \frac{1}{2}\int_{x,y} \psi^m(x)\Big(A(x,y) + \gamma^\mu W_\mu(x,y) + \gamma^{\mu\nu} A_{\mu\nu}(x,y) + \ldots\Big)\psi^m(y)$$

- the list of sources terminates, depending on space-time dimension
  - e.g., d=3: *just* $A(x,y)$ *and* $W_\mu(x,y)$
- now perform the non-local change of variables $\psi^a(x) \mapsto \int d^d y\ \mathcal{L}(x,y)\psi^a(y)$

$$S \to \psi^m \cdot \mathcal{L}^T \cdot [\gamma^\mu(P_{F;\mu} + W_\mu) + A] \cdot \mathcal{L} \cdot \psi^m$$

$$= \psi^m \cdot \gamma^\mu \mathcal{L}^T \cdot \mathcal{L} \cdot P_{F;\mu} \cdot \psi^m + \widetilde{\psi}^m \cdot \left[\gamma^\mu(\mathcal{L}^T \cdot [P_{F;\mu}, \mathcal{L}] + \mathcal{L}^T \cdot W_\mu \cdot \mathcal{L}) + \mathcal{L}^T \cdot A \cdot \mathcal{L}\right] \cdot \psi^m$$

- if $\mathcal{L}^T \cdot \mathcal{L} = 1$ then the fixed point action remains unchanged, while the sources transform. That is

$$Z\Big[U, A, W_\mu\Big] = Z\Big[U, \underbrace{\mathcal{L}^{-1} \cdot A \cdot \mathcal{L}}_{\text{tensor}}, \underbrace{\mathcal{L}^{-1} \cdot W_\mu \cdot \mathcal{L} + \mathcal{L}^{-1} \cdot [P_{F,\mu}, \mathcal{L}]}_{\text{connection}}.\Big]$$

- we call this group $O(L^2(\mathbb{R}^{1,d-1}))$
  - $D_\mu = P_{F,\mu} + W_\mu$ plays the role of covariant derivative
  - the fixed point theory corresponds to

$$(A, W_\mu) = (0, W_\mu^{(0)})$$

  - that is, because W_μ is a connection, the QFT is unsourced whenever A is zero and W_μ is a flat connection

Dilatations and ERG

- we extend this to RG by asking how the theory responds to a (homogeneous) dilatation $x^\mu \mapsto \lambda x^\mu$
- one can combine $O(L^2)$ with the dilatation in a simple way, by simply allowing $\mathcal{L}^T \cdot \mathcal{L} = \lambda^{2\Delta_\psi} 1$ (we refer to this as $CO(L^2)$)
- this has the effect

$$Z[M, g; U, A, W_\mu] = Z[\lambda^{-1} M, \lambda^2 g, U^{\mathcal{L}}, A^{\mathcal{L}}, W_\mu^{\mathcal{L}}]$$

metric seen by field theory

$$K = K[-z^2 D^{(0)2}/M^2]$$

- if we parameterize $g_{\mu\nu} = z^{-2}\eta_{\mu\nu}$, we can write this equivalently as

$$Z[M, z; U, A, W_\mu] = Z[\lambda^{-1} M, \lambda^{-1} z, U^{\mathcal{L}}, A^{\mathcal{L}}, W_\mu^{\mathcal{L}}]$$

- we regard $z \in [\epsilon, \infty)$ as the renormalization scale

The Exact RG

- **we perform the ERG in two steps**
  - 1. lower the cutoff $M \mapsto \lambda M$

$$Z[M, z; U, A, W_\mu] = Z[\lambda M, z, \tilde{U}, \tilde{A}, \tilde{W}_\mu]$$

(à la Polchinski)

  - 2. bring M back to its original value via a $CO(L^2)$ transformation

$$Z[\lambda M, z; \tilde{U}, \tilde{A}, \tilde{W}_\mu] = Z[M, \lambda^{-1} z, \tilde{U}^{\mathcal{L}}, \tilde{A}^{\mathcal{L}}, \tilde{W}_\mu^{\mathcal{L}}]$$

- **there is a freedom in the choice of $\mathcal{L}$**
- **comparing the two, we arrive at a relation between the generating functionals at the same scale M, but different renormalization scale z**

$$\boxed{Z[M, z; U, A, W_\mu] = Z[M, \lambda^{-1} z, \tilde{U}^{\mathcal{L}}, \tilde{A}^{\mathcal{L}}, \tilde{W}_\mu^{\mathcal{L}}]}$$

$$Z[M, z; U, A, W_\mu] = Z[M, \lambda^{-1} z, \tilde{U}^{\mathcal{L}}, \tilde{A}^{\mathcal{L}}, \tilde{W}_\mu^{\mathcal{L}}]$$

a well chosen name

- or by taking $\varepsilon \to 0 \qquad (\lambda \simeq 1 - \varepsilon, \qquad \mathcal{L} \simeq 1 + \varepsilon z W_z)$

$$\partial_z \mathcal{W}_\mu^{(0)} - [P_{F;\mu}, \mathcal{W}_z^{(0)}] + [\mathcal{W}_z^{(0)}, \mathcal{W}_\mu^{(0)}] = 0$$

$$\partial_z \mathcal{A} + [\mathcal{W}_z, \mathcal{A}] = \beta^{(\mathcal{A})}$$

$$\partial_z \mathcal{W}_\mu - [P_{F;\mu}, \mathcal{W}_z] + [\mathcal{W}_z, \mathcal{W}_\mu] = \beta_\mu^{(\mathcal{W})}$$

recall:
everything bi-local in x,y

output of ERG

- this is what is obtained from dilatations; we suppose that more generally, these are components of covariant equations

$$d\mathcal{W}^{(0)} + \mathcal{W}^{(0)} \wedge \mathcal{W}^{(0)} = 0$$

$$d\mathcal{A} + [\mathcal{W}, \mathcal{A}] = \beta^{(\mathcal{A})}$$

$$d\mathcal{W} + \mathcal{W} \wedge \mathcal{W} = \beta^{(\mathcal{W})}$$

other components
not determined by RG

but determined by consistency
'Bianchi identities'

(Classical) Bulk Action

- **this is only half of the bulk system**
  - repeat analysis for the vevs (Callan-Symanzik equations)
  - give rise to bulk 'momenta'
- **the resulting system of equations is a Hamiltonian system with respect to z**
  - the ERG analysis determines the Hamiltonian
  - correspondingly, there is an action

$$Z = e^{-S_{HJ}[z;B]} \to e^{-I[\mathcal{P},\mathcal{B}]} = e^{-\int dz \;\; Tr(\mathcal{P}\cdot\partial_z\mathcal{B} - H(\mathcal{P},\mathcal{B}))}$$

$$I = \int dz \;\; Tr\left\{\mathcal{P}^I \cdot \left(\mathcal{D}_I^{(0)}\mathcal{B} - \beta_I^{(\mathcal{B})}\right) + N\Delta_B \cdot \mathcal{B}\right\}$$

$$\beta^{(\mathcal{B})} = \mathcal{B}\cdot\Delta_B\cdot\mathcal{B}$$

(here I've switched to O(N) scalar theory)

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$$I = \int dz\ Tr\left\{\mathcal{P}^I \cdot \left(\underline{\mathcal{D}_I\mathcal{B} - \beta_I^{(\mathcal{B})}}\right) + N\ \underline{\Delta_B} \cdot \mathcal{B}\right\}$$

$$\beta^{(\mathcal{B})} = \mathcal{B} \cdot \Delta_B \cdot \mathcal{B}$$

zero on-shell

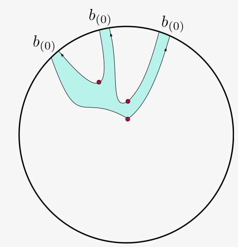
determined by cutoff function

Holographic Higher Spins

- all of the usual holographic machinery can be employed here
- a classical solution corresponds to an RG flow
- the trivial solution corresponds to the free fixed point
  - i.e., turn off all sources — $\mathcal{W}_A$ is flat
  - if we choose "spin-2 gauge", this connection encodes the geometry of
  - AdS_{d+1} , $\mathcal{W}^{(0)} \to \{e^a, \omega^a{}_b\}$

$$\mathcal{W}^{(0)}(x,y) = -\frac{dz}{z} D(x,y) + \frac{dx^\mu}{z} P_\mu(x,y)$$

    - at least when the free fixed point has relativistic symmetry
- all correlation functions can be systematically computed



  - look like "bi-local Witten diagrams"
  - these resum to the determinant — proof that no information has been lost

Conformal details

- the usual conformal group $SO(2,d) \subset CO(L^2)$
- each local operator transforms in a short conformal module $U(\Delta, s)$
- the corresponding sources transform in the dual module $U(d-\Delta, s)$
- the bulk degrees of freedom transform in

$$\oplus_s \Big(U(d-\Delta, s) \oplus U(\Delta, s) \Big)$$

- linearizing around AdS_{d+1}, one can write the equations of motion as decoupled second order PDEs
  - these are nothing but $Casimir = s(s+d-2)$
  - "Fronsdal equations" of higher spin theory

The Role of Time

- The partition function is a functional integral over the modes of fields in *all of space-time* (Euclidean, really)
- We assumed the fixed point had relativistic symmetry (z=1)
- So there are two directions that one can go
  - Consider other structures, such as wave-functionals or density matrices, that fix, say, space-like hypersurfaces (and thus break time-translational invariance)
  - Consider other fixed points with non-relativistic symmetries
- Each requires answering two questions:
  - How do we regulate the functional integral in the ERG fashion?
  - What operators do we source?
    - Must be a closed set under action of RG if exactness is to be maintained

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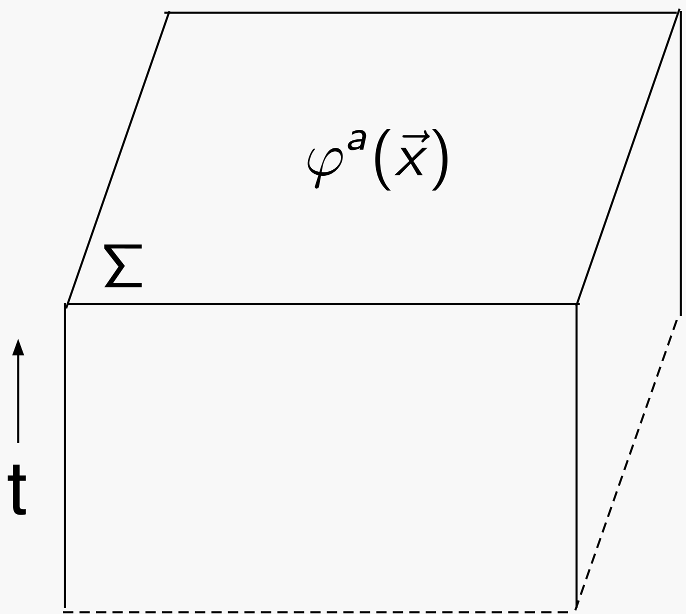
ERG and Wave-functionals

- **in continuum QFT, we do not typically consider explicit wave functionals for states**
- **however, the ground state wave-functional of a free field theory is well-known, being Gaussian**
    - much less understood is how the renormalization group acts on the ground state, as well as all other states
- **we can in fact use ERG methods to study this**
    - builds on similar concepts to those introduced previously
    - the ERG construction naturally retains 'ancillary' degrees of freedom
    - generates a flow in the space of states
        - **equivalent to a <u>continuous tensor network</u> whose properties we can study**

ERG for Wave-functionals

- to employ these exact methods for wave-functionals, we need to carefully construct familiar concepts
- wave-functional in 'position basis' $\langle \varphi^a(\vec{x}) | \Psi \rangle$ obtained by path integral over half space-time in Euclidean time, with specified boundary condition on a space-like hypersurface



$$\langle \varphi^a(\vec{x}) | \Psi \rangle \equiv Z[M_-; \varphi^a]$$

- extract ground state by usual limiting procedure

- generate large class of states by operator insertions in Euclidean time

ERG for Wave-functionals

- various technical points to manage
  - convergence
  - normalizability
  - well-defined canonical structure (boundary conditions)
- here we are interested in states corresponding to insertions of O(N)-invariant operators

$$|\psi\rangle = e^{-\delta\hat{H}}\hat{\mathcal{O}}_1(0,\vec{x}_1)\hat{\mathcal{O}}_2(0,\vec{x}_2)\cdots\hat{\mathcal{O}}_n(0,\vec{x}_n)|\Omega\rangle$$

- it is useful to introduce the *generating functional of states*

$$|\psi[b]\rangle = \mathcal{T}_E e^{-\frac{1}{2}\int_{M_-} d^dx \int_{M_-} d^dy\, \hat{\phi}^a(x) b(x,y) \hat{\phi}^a(y)}|\Omega\rangle$$

- (can generalize to contour ordering in complex time)
- *b(x,y)* plays a similar role to the sources considered earlier

ERG for Wave-functionals

- **again, there is a large non-local symmetry present**
  - restrict $\mathcal{L}(x, y)$ to preserve Σ
  - need to regulate appropriately

$$S_\phi = \frac{1}{2}\int_M \phi(x) \circ K^{-1}\left(-\vec{D}^2/M^2\right) \circ D^2 \circ \phi(x) + \frac{1}{2z^{d-2}}\int_\Sigma \varphi(\vec{x}) \cdot K^{-1}\left(-\vec{D}^2/M^2\right) \cdot D_t \cdot \phi|_\Sigma(\vec{x})$$

  - cannot introduce arbitrary number of time derivatives
    - sufficient to introduce 'spatial' regulator $K\left(-\vec{D}^2/M^2\right)$

- **recall for partition function, we implemented ERG as a 2-step process**
  - lower cutoff $M \mapsto \lambda M$
  - use CO(L2) to take $\lambda M \mapsto M, \quad z \mapsto \lambda^{-1} z$

ERG for Wave-functionals

- **for the generator of states, we employ a similar process**
  - the novelty is that we have to take care with boundary terms
  - there is dependence on a bulk kernel Δ_B but now also a boundary kernel Δ_Σ , and a CO(L[2]) transformation given by W_z and w_z
    - **these know about the details of the regulator**
- **for the partition function, we required M-independence as the basic RG requirement**
  - for wave-functionals, this would be too strong
  - instead we just eliminate M-derivatives from the ERG equations

$$z\frac{\partial}{\partial z}\Psi = \left(zTr_{\Sigma\times C}\left(([W_z, b]_\circ + b\circ\Delta_B\circ b)\circ\frac{\delta}{\delta b}\right) + z\frac{N}{2}Tr_\Sigma g + z\int_\Sigma \varphi\cdot g^T\cdot\frac{\delta}{\delta\varphi}\right)\Psi$$

$$g(z;\vec{x},\vec{y}) := \left(\tfrac{1}{2}\Delta_\Sigma + w_z\right)(\vec{x},\vec{y})$$

ERG for Ground State

- **the ground state is obtained by setting b(x,y)=0**
  - then, the $\delta/\delta b$ terms disappear
    - the ground state does not mix with other states, and satisfies

$$z\frac{d}{dz}\Big|\Omega(z)\Big\rangle = i\Big(\mathbf{K}(z)+\mathbf{L}(z)\Big)\Big|\Omega(z)\Big\rangle$$

$$\mathbf{K}(z) = \frac{z}{2}\left(\hat{\pi}\cdot\Delta_\Sigma(z)\cdot\hat{\phi}+\hat{\phi}\cdot\Delta_\Sigma^T(z)\cdot\hat{\pi}\right)$$ “disentangler”

$$\mathbf{L}(z) = \frac{z}{2}\left(\hat{\pi}\cdot w_z(z)\cdot\hat{\phi}+\hat{\phi}\cdot w_z^T(z)\cdot\hat{\pi}\right)$$ scale transformation

- **K** **and** **L** **are both Hermitian**

- **can be solved in terms of path-ordered exponential**

$$\Psi_\Omega[z_*,\varphi] = \Big\langle\varphi\Big|\Omega(z_*)\Big\rangle = \Big\langle\varphi\Big|\mathcal{P}e^{\frac{i}{2}\int_\epsilon^{z_*}dz\int_\Sigma\left(\hat{\pi}\cdot g(z)\cdot\hat{\phi}+\hat{\phi}\cdot g^t(z)\cdot\hat{\pi}\right)}\Big|\Omega(\epsilon)\Big\rangle.$$

ERG for States

- for any other state, we have

$$z\partial_z |\Psi_C[b]\rangle = \left(-Tr\, \boldsymbol{\beta} \circ \frac{\delta}{\delta b} + i\mathbf{K} + i\mathbf{L} \right) |\Psi_C[b]\rangle$$

- **K** and **L** are state independent, β causes mixing of states along the flow

- if we think of $|\Psi_C[b]\rangle$ as a family of states in the space of *b*, we can regard this equation as a flow along the integral curves of β
  - that is, we introduce a "running" source $\mathcal{B}(z; x, y)$ satisfying

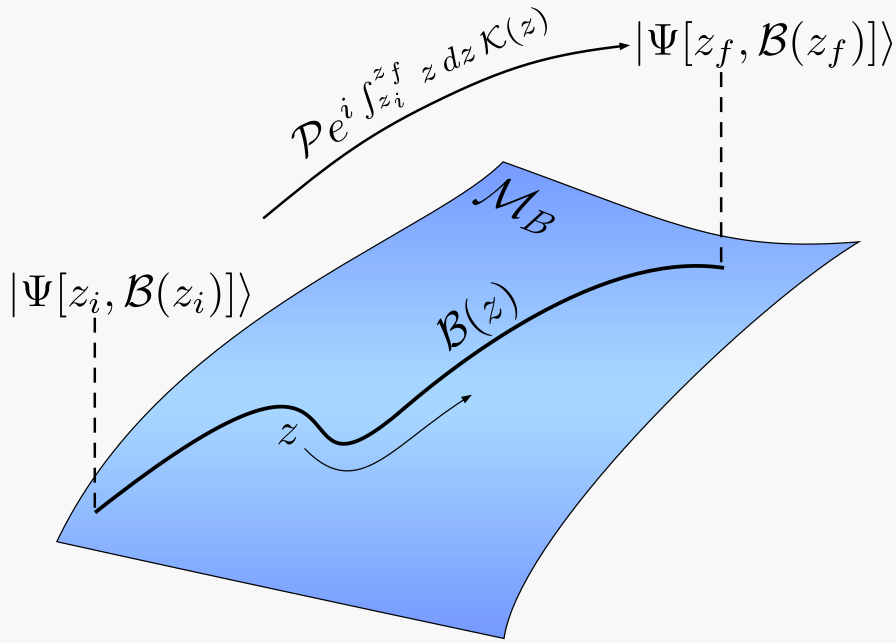
$$z\partial_z \mathcal{B} = \beta[\mathcal{B}]$$

$$z\frac{d}{dz} |\Psi_C[z, \mathcal{B}(z)]\rangle = i\,(\mathbf{K} + \mathbf{L})\, |\Psi_C[z, \mathcal{B}(z)]\rangle$$

  - (same equation as ground state)

ERG for States

- **claim: RG principle for states should be that the flow is along integral curves of β**
  - then, state changes by a unitary operator
  - consistent with RG-invariance of norm (~ partition function)



MERA?

- All of the concepts that I've presented are in fact present in the tensor network story, no more, no less
- Thus we have *derived* a tensor network directly in the continuum
- however, the disentangling is happening in *momentum space*
  - ground state is a product state in momentum space
    - excited states are typically not
  - by looking at non-trivial states, can show that **K** disentangles states above and below RG scale
  - MERA, by hand, implements disentangling *in coordinate space*
- **K is given by the choice of regulator**
  - optimization, as in MERA, then explores different choices of regulator

The Big Questions

- **This optimization is the new ingredient**
  - Usually in QFT, we choose a regulator and believe any choice is equivalent. That is true for the full path integral, as Polchinski emphasized.
  - What is optimization then? Should we minimize some notion of ***complexity*** along RG flows?
- **So we've seen that ERG is about entanglement in momentum space — we can also study *real space entanglement* using ERG**
  - the flow of reduced density matrices, entanglement entropy, etc., apparently requires even more sophisticated regulators

4

ERG for Schrodinger

- One can repeat the analysis for *any* free fixed point, including those with non-relativistic symmetries
  - the z=2 Schrodinger fixed point is particularly simple
- One can extract the non-relativistic ERG from the relativistic version by employing DLCQ
  - Introduce coordinates $(\xi, t, \vec{x})$ with $\Box = \partial_\xi \partial_t + \vec{\nabla}^2$ and assign scaling

$$(\xi, t, \vec{x}) \mapsto (\xi, \lambda^2 t, \lambda \vec{x})$$

- Then the generator $\underline{N} = \underline{\partial_\xi}$ is central, the partition function decomposes into superselection sectors of definite eigenvalue *n*, and within such a sector,

$$\Box \to \mathcal{S}_n = in\partial_t + \vec{\nabla}^2$$

- Thus we can use the same regulator as before, but within a superselection sector, it becomes effectively a function of $\mathcal{S}_n$

ERG for Schrodinger

- A central question is how the DLCQ descends to the bulk theory
- The trick is that one must source operators appropriately

$$\varphi(\xi, t, \vec{x}) = e^{in\xi}\varphi(t, \vec{x})$$

$$B(\xi, t, \vec{x}; \xi', t', \vec{x}') = e^{in(\xi-\xi')}B_n(t, \vec{x}; t', \vec{x}')$$

- This corresponds to a usual result in non-relativistic holography
  - Local bulk fields dual to local conserved currents have n=0.
- here, we define bulk DLCQ to mean that the bulk bi-local fields have this specific ξ-dependence
  - And then one can extract all correlation functions of (higher spin) currents in the free non-relativistic theory

Remarks / Questions

- **free field theories can be interpreted directly as holographic higher spin theories**
    - bulk geometry is encoded in the nature of the fixed point theory
    - the bulk theory gives everything we can expect of a holographic theory
        - does Vasiliev=Leigh?
- **unitary networks emerge from ERG applied to states**
- **natural to ask what happens when QFT interactions are turned on**
    - for 'multi-trace' interactions, can show that a *precise quantum bulk higher-spin theory* emerges
    - higher spin symmetry is largely Higgsed (currents have anomalous dimensions)
    - hope: systematically understand effect on tensor networks