A quantum mechanical model for Holography

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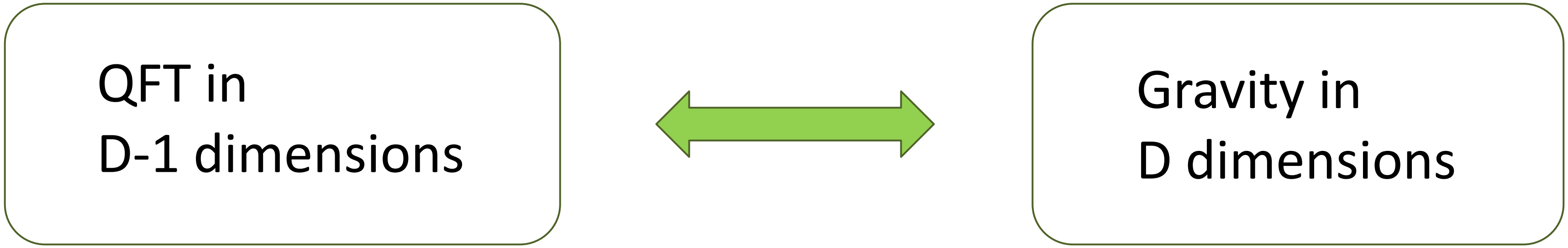
Talk mainly based on:

- **TH, Hartong and Obers,** Phys. Rev. D96 no. 8, 086019 (2017) (ArXiv:**1705:03535** [hep-th])
- **TH**, Phys. Rev. D94 (2016) no. 6, 066001 (ArXiv:**1606.06296** [hep-th])
- **TH and Orselli**, JHEP 1411:134 (ArXiv:**1409.4417** [hep-th])
- **TH, Orselli and Kristjansson**, JHEP 0902:027 (ArXiv:**0806.3370** [hep-th])

Introduction

Question: How do space, time and gravity emerge from quantum theory?

Answer: Holographic duality



QFT in
D-1 dimensions

Gravity in
D dimensions

Space, time and gravity in D dimensional should emerge from a D-1 dimensional QFT

Can one give a quantitative description of how this works?

The AdS/CFT correspondence:

$\mathcal{N}$=4 SYM theory
with gauge group SU(N)

't Hooft coupling:

$$\lambda = g_{\rm YM}^2 N$$

⟷

$$g_s = \frac{\lambda}{N}$$

$$R^4 = \lambda l_s^4$$

Type IIB string theory
on AdS_5 x S^5

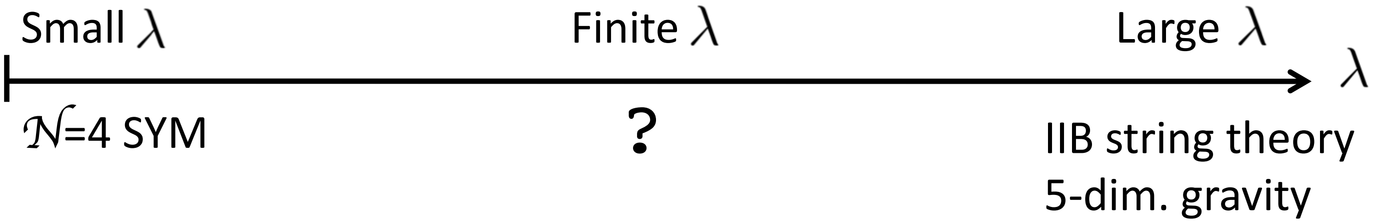
R: Radius of AdS_5 and S^5
g_s: string coupling
l_s: string length

Strict planar limit: N = ∞ ⟷ Tree-level string theory: $g_s = 0$

1/N corrections ⟷ Perturbative expansion in g_s

Finite-N effects ⟷ Non-perturbative string theory: D-branes, Black holes

When are the two dual sides a good description?



Small λ | Finite λ | Large λ | λ

$\mathcal{N}$=4 SYM

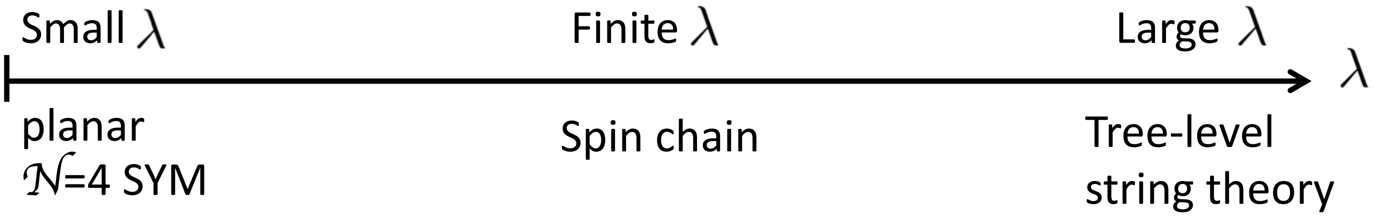
?

IIB string theory
5-dim. gravity

How can we make a quantitative connection between the two sides ?

We need a unifying framework to interpolate between weak and strong coupling

In strict planar limit (N = ∞ and g_s = 0) we have a unifying framework:



Small λ | Finite λ | Large λ | λ

planar
$\mathcal{N}$=4 SYM

Spin chain

Tree-level
string theory

Can we find a unifying framework that generalizes the spin chain beyond this?

Can we find a unifying framework of AdS/CFT for finite, large N?

A finite N generalization of the spin chain?

$\mathcal{N}$=4 SYM simplifies near unitarity bounds / zero-temperature critical points:

Effective description by **Spin Matrix Theory**

TH & Orselli 2014

What is Spin Matrix theory? A well-defined quantum mechanical theory

Hilbert space built from harmonic oscillators:

$$(a_s^\dagger)^i{}_j$$

s: Index for representation of (super) Lie group (the “spin” group)
i,j: Matrix indices for adjoint representation of U(N)

Extra demand:
Only singlets of U(N)

$$\mathrm{Tr}(a^\dagger_{s_1} a^\dagger_{s_2} \cdots a^\dagger_{s_k})\,\mathrm{Tr}(a^\dagger_{s_{k+1}} \cdots) \cdots \mathrm{Tr}(\cdots a^\dagger_{s_L})|0\rangle$$

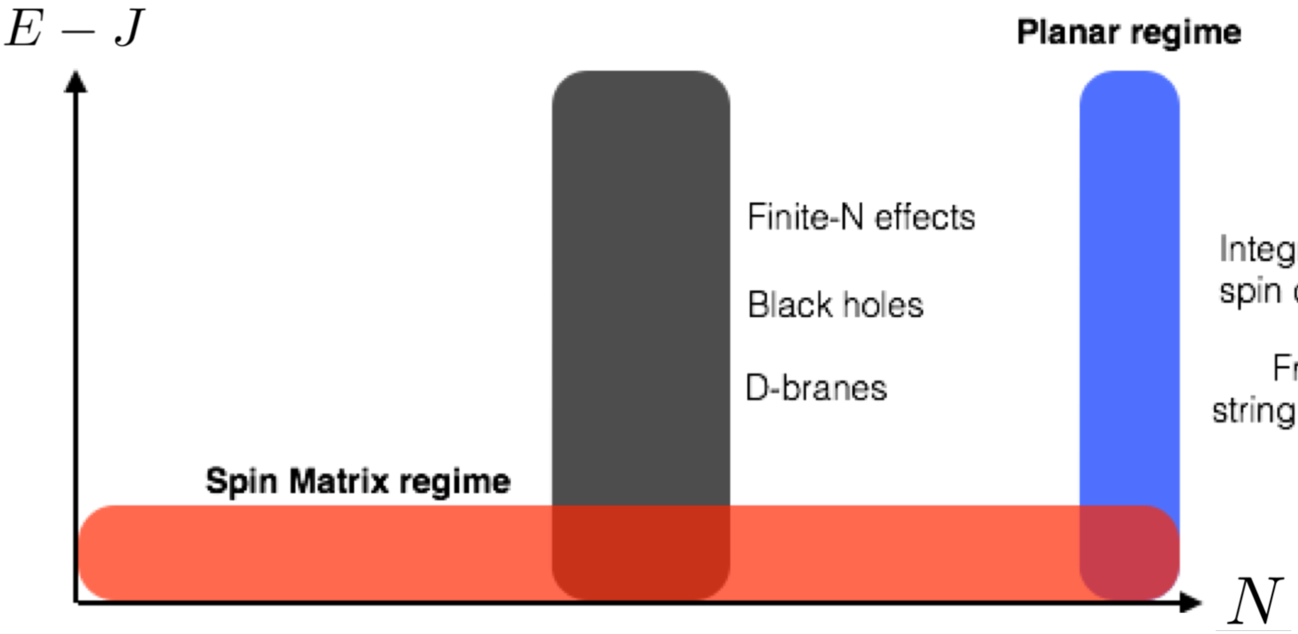
Interaction Hamiltonian: 1) Annihilates 2 excitations, creates 2 new ones.
2) Commutes with “spin” generators. 3) ”spin” and “matrix” parts factorize.

For $N \to \infty$: Spin Matrix Theory reduces to a nearest neighbor spin chain

For a given unitarity bound: $E \geq J$ (linear combo of charges)

The planar regime: $N \to \infty$ with $E - J$ fixed

The Spin Matrix regime: $E - J \to 0$ with N fixed



Spin Matrix regime includes SUSY states with E = J and finite N

Spin Matrix Theory from $\mathcal{N}$=4 SYM near unitarity bound:

For a given unitarity bound: $E \geq J$

SMT limit:

$$H = J + \lim_{\lambda \to 0} \frac{g}{\lambda}(E - J)$$

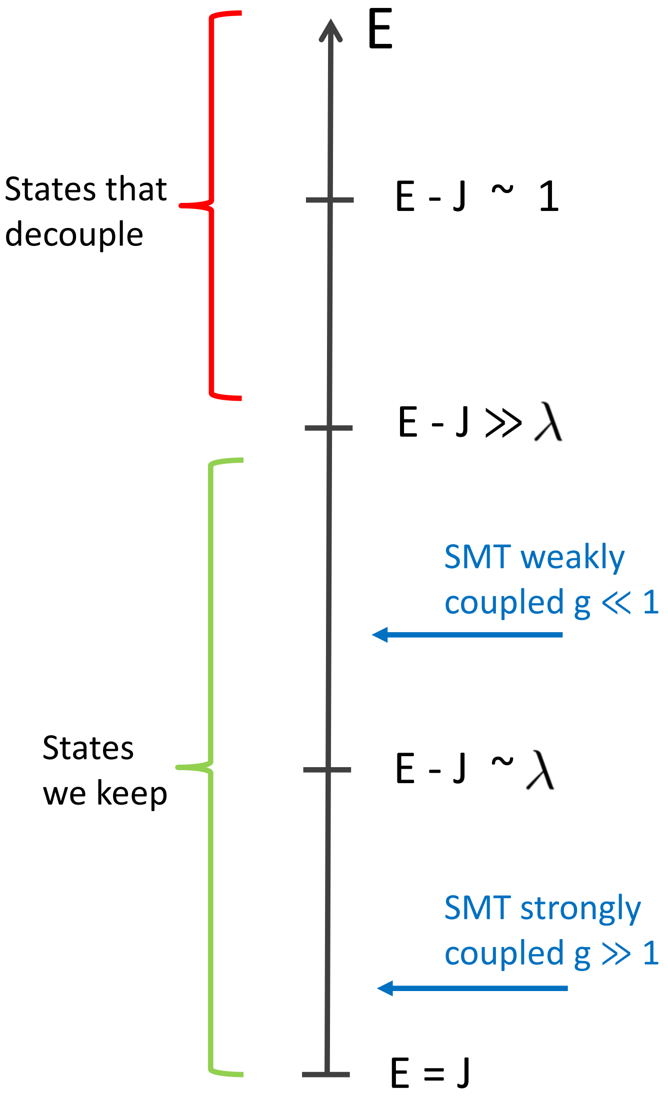
g: Coupling constant of Spin Matrix theory

E: Energy of states in $\mathcal{N}$=4 SYM on R x S^3

(in units of inverse radius of S^3)

= Scaling dim. of operator of $\mathcal{N}$=4 SYM on R^4

N is fixed in limit



E

States that decouple

E - J ~ 1

E - J ≫ λ

SMT weakly coupled g ≪ 1

States we keep

E - J ~ λ

SMT strongly coupled g ≫ 1

E = J

Spin Matrix Theory limits:

Angular momenta on S^3 : S_1 , S_2

R-charges: J_1 , J_2 , J_3

Several unitary bounds in $\mathcal{N}$=4 SYM → Several different Spin Matrix theories

Unitarity bound	Spin group G_s	Cartan diagram for algebra	Representation R_s
$E \geq J_1$	$U(1)$		
$E \geq J_1 + J_2$	$SU(2)$	○	$[1]$
$E \geq J_1 + J_2 + J_3$	$SU(2\|3)$	○—⊗—○—○	$[0, 0, 0, 1]$
$E \geq S_1 + J_1 + J_2$	$SU(1, 1\|2)$	⊗—○—⊗	$[0, 1, 0]$
$E \geq S_1 + S_2 + J_1$	$SU(1, 2\|2)$	○—⊗—○—⊗	$[0, 0, 0, 1]$
$E \geq S_1 + S_2 + J_1 + J_2 + J_3$	$SU(1, 2\|3)$	○—⊗—○—○—⊗	$[0, 0, 0, 1, 0]$

U(1) case: Berenstein's toy model for AdS/CFT $H = \mathrm{Tr}(a^\dagger a)$

SU(2) case: Simplest SMT limit with interactions

This talk:

SU(2) Spin Matrix theory

SMT and non-relativistic geometry

Conclusions and outlook

SU(2) Spin Matrix theory

SU(2) case: $E \geq J_1 + J_2$

Angular momenta on S^3: S_1, S_2

R-charges: J_1, J_2, J_3

SU(2) Spin Matrix Theory

$$H = \mathrm{Tr}(a_1^\dagger a_1 + a_2^\dagger a_2) - \frac{g}{8\pi^2 N}\mathrm{Tr}([a_1^\dagger, a_2^\dagger][a_1, a_2])$$

Singlet condition:

$$\Phi^i{}_j|\phi\rangle = 0 \quad \text{with} \quad \Phi^i{}_j = \sum_{s=1}^{2}\sum_{k=1}^{N}\left[(a_s^\dagger)^i{}_k(a^s)^k{}_j - (a_s^\dagger)^k{}_j(a^s)^i{}_k\right]$$

Two tractable regimes (also at large coupling g):

The "planar regime": N large and $H \ll N$

Described by the spin ½ ferromagnetic Heisenberg spin chain

The "matrix regime": $H \gg N^2$

Described by classical matrix model

SU(2) Spin Matrix theory

and the emergence of non-relativistic strings

Planar regime:

Single-traces → Spin chains

Minahan & Zarembo 2002

SU(2) SMT → Spin ½ ferromagnetic Heisenberg spin chain

Strong coupling limit g ≫ 1:

Low energy spectrum of spin chain for J ≫ 1

Lowest excitations = magnons

In classical limit (many magnons): Described by Landau-Lifshitz sigma-model

$$\boxed{I = \frac{J}{4\pi}\int dt \int_0^{2\pi} d\sigma \left[\sin\theta\dot{\phi} - \frac{\theta'^2 + \sin^2\phi'^2}{4}\right]}$$

Amazingly, one gets the same action from the string theory side, but seemingly in a different regime:

Kruczenski 2003
TH, Orselli & Kristjansson 2008

Gauge theory/SMT side: $g_s N \ll 1$ and $J \gg 1$

String theory side: $g_s N \gg 1$ and $J^2 \gg g_s N$

$\lambda = 4\pi g_s N$

→ The famous “one-loop match” in early post-BMN days

A coincidence? No, it is not! TH, Orselli & Kristjansson 2008

We can take the SMT limit also on the string theory side

We can take the SMT limit also on the string theory side TH, Orselli & Kristjansson 2008

$$H = J + \lim_{g_s \to 0} \frac{g}{4\pi g_s N}(E - J)$$

Consider the planar regime: We should take limit of the string sigma-model on AdS_5 x S^5 background

- Naively: We enter the quantum string regime is string tension goes like $\sqrt{g_s N}$

  However, in the actual limit, the sigma-model action remains large for large J and one gets a different effective string tension

- What about corrections to sigma-model? It is protected by 32 SUSY
- What about other modes? → They become infinitely heavy and decouple
- Zero-mode fluctuation contribution? → Absent due to SUSY of unitarity bound

A match between strongly coupled SU(2) SMT and string theory

SU(2) Spin Matrix theory

and the emergence of D-branes

Matrix regime:

High energy limit: Classical matrix model (from coherent states)

Hamiltonian:

TH & Orselli 2014

$$H = \frac{1}{2}\sum_{s=1}^{2}\mathrm{Tr}(P_s^2 + X_s^2) - \frac{g}{32\pi^2 N}\mathrm{Tr}\Big([X_1,X_2]^2 + [P_1,P_2]^2 + [X_1,P_1]^2$$

$$+[X_2,P_2]^2 + [X_1,P_2]^2 + [X_2,P_1]^2\Big)$$

Gauss constraint: $\sum_{s=1}^{2}[X_s,P_s] = 0$

Matches SMT limit of D-branes on string theory side

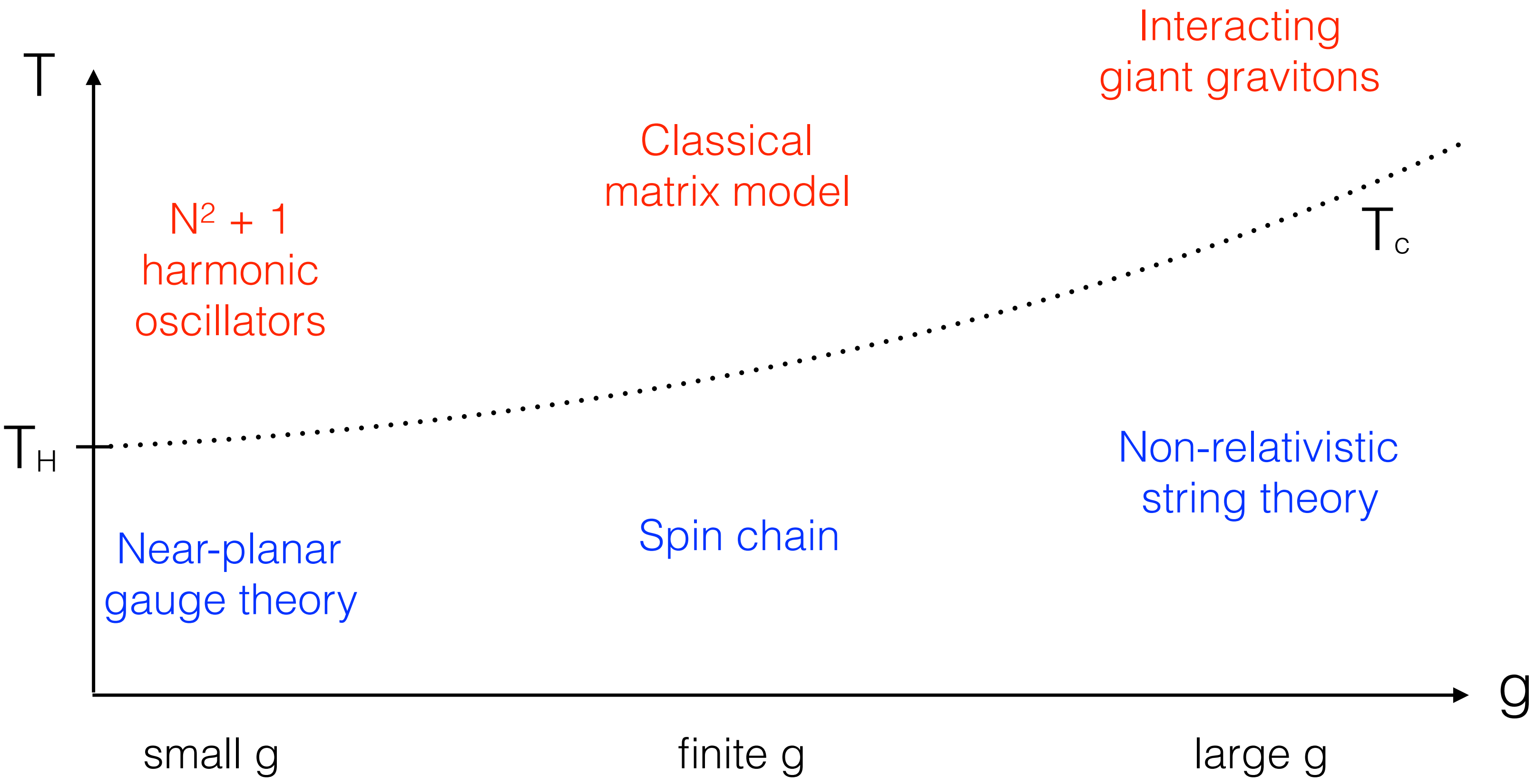
TH 2016

D-branes with E – J small → Giant Gravitons (D3-branes)

For high energy limit: AdS Giant Gravitons: D3-branes on 3-spheres in AdS_5

One gets same matrix model using non-abelian DBI action on D3-branes

We have matched SU(2) SMT for g ≫ 1 both in the planar regime and in the matrix regime



Interacting giant gravitons

Classical matrix model

$N^2 + 1$ harmonic oscillators

Near-planar gauge theory

Spin chain

Non-relativistic string theory

SMT and non-relativistic geometry

SMT limit of string theory is non-relativistic

Consider the general dispersion relation for a magnon in AdS/CFT

TH, Orselli and Kristjansson 2008

$$\begin{tikzcd}
\boxed{E - J = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}} - 1} \arrow[r, Rightarrow, "\text{SMT}", "\lambda \to 0"'] \arrow[d, Rightarrow, "\text{BMN}"', "p \to 0"] & \boxed{\frac{E - J}{\lambda} = \frac{1}{2\pi^2}\sin^2\frac{p}{2}} \arrow[d, Rightarrow, "p \to 0"] \\
\boxed{E - J = \sqrt{1 + \frac{\lambda'}{4\pi^2}p'^2} - 1} \arrow[r, Rightarrow, "\text{SMT}", "\lambda \to 0"'] & \boxed{\frac{E - J}{\lambda'} = \frac{1}{8\pi^2}p'^2}
\end{tikzcd}$$

Can one see the SMT limit as a non-relativistic limit of the geometry and/or the world-sheet string theory?

Geometric understanding of SMT limits?

SU(2) SMT limit of Polyakov action gives:

Kruczenski 2003
TH, Orselli & Kristjansson 2008

$$I = \frac{J}{4\pi}\int dt \int_0^{2\pi} d\sigma \left[\sin\theta\dot{\phi} - \frac{\theta'^2 + \sin^2\phi'^2}{4} \right]$$

(Dot: Time derivative, Prime: Space derivative)

Action non-lorentzian on the world-sheet

But what is the target space?
This should reveal the emerging geometry from SU(2) SMT
But we don’t know (yet) how to read it off

To understand this, we have revisited the limit of the Polyakov action

TH, Hartong and Obers 2017

Starting point: String on AdS$_5$ x S^5:

AdS$_5$ x S^5 geometry is 10D Lorentzian:
One can get it by gauging the Poincare group

$$[p_\mu, p_\nu] = 0 \ , \quad [p_\rho, J_{\mu\nu}] = -i(\eta_{\rho\mu} p_\nu - \eta_{\rho\nu} p_\mu)$$

$$[J_{\mu\nu}, J_{\rho\sigma}] = -i(\eta_{\nu\rho} J_{\mu\sigma} + \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\nu\sigma} J_{\mu\rho})$$

Polyakov action for string:

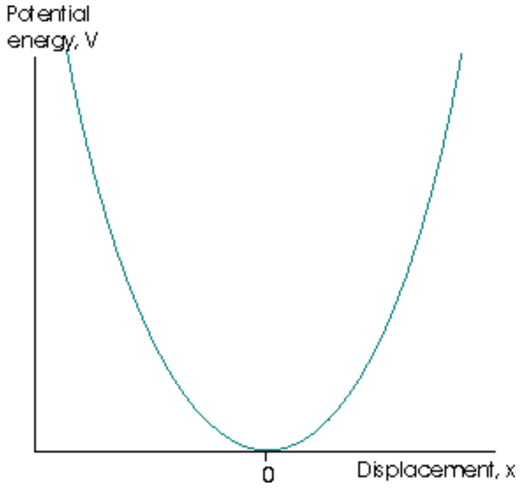
$$S = -\frac{T}{2} \int d^2\xi \sqrt{-g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}$$

Worldsheet theory: Relativistic two-dimensional CFT

$G_{\mu\nu}$ the 10D target space metric

$g_{\alpha\beta}$ is 2D worldsheet metric

SMT limit:



Unitarity bound → BPS bound $E \geq J$

Part of 10D AdS$_5$ x S^5 decouples as 2n directions get an infinitely steep potential → Reduction to 10 – 2n dimensional Lorentzian space-time

Write metric for d+2 = 10-2n dim. space-time as

a = 0,1,2,…,d

d=8-2n

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = 2\tau(du - m) + h_{ab}dx^a dx^b$$

$$\tau = \tau_a dx^a \ , \quad m = m_a dx^a$$

for a null isometry u (choice follows from bound), with x^0 chosen such that

$$E - J = i\partial_{x^0}$$

What is the interpretation of τ, m and h$_{ab}$?
→Torsional Newton-Cartan (TNC) geometry

What is torsional Newton-Cartan (TNC) geometry?

→ A particular type of non-Lorentzian geometry

Local Poincare-invariance → Lorentzian geometry

Bargmann group: Galilean boost + central element + space and time translations + spatial rotations

a,b = 1,2,…,d

$$[H, G_a] = P_a\,, \qquad [P_a, G_b] = \delta_{ab} N\,,$$
$$[J_{ab}, P_c] = \delta_{ac} P_b - \delta_{bc} P_a\,, \qquad [J_{ab}, G_c] = \delta_{ac} G_b - \delta_{bc} G_a\,,$$
$$[J_{ab}, J_{cd}] = \delta_{ac} J_{bd} - \delta_{ad} J_{bc} - \delta_{bc} J_{ad} + \delta_{bd} J_{ac}\,.$$

Local Bargmann invariance → Torsional Newton-Cartan geometry

τ: Clock one-form (Gauge-field for time translations)
m: Gauge-field for central charge
h_{ab}: Spatial metric

Local boosts changes both h and m:

$$\delta h_{ab} = \tau_a \lambda_{\bar{c}} e_b^{\bar{c}} + \tau_b \lambda_{\bar{c}} e_a^{\bar{c}}\,, \quad \delta m_a = \lambda_{\bar{b}} e_a^{\bar{b}}\,, \quad \delta \tau_a = 0 \qquad \text{where} \quad h_{ab} = \delta_{\bar{c}\bar{d}} e_a^{\bar{c}} e_b^{\bar{d}}$$

Part 1: Null-reduction of Polyakov action

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = 2\tau(du - m) + h_{ab}dx^a dx^b$$

Conserved momentum current along u: $P_u^\alpha = \frac{\partial L_{\text{pol}}}{\partial(\partial_\alpha u)} = -T\sqrt{-\gamma}\gamma^{\alpha\beta}\tau_\beta$

Remove u from description by putting P_u on-shell

Need to Legendre transform: $L_{\text{TNC}} = L_{\text{pol}} - P_u^\alpha \partial_\alpha u$

Find $\sqrt{-\gamma}\gamma^{\alpha\beta}$ in terms of P_u and τ

TH, Hartong and Obers 2017

$$\boxed{L_{\text{TNC}} = -P_u^\alpha m_\alpha - \frac{1}{2}\left[-\frac{P_u^\alpha P_u^\beta}{\tau_\gamma P_u^\gamma} + T^2\frac{\epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\tau_\gamma\tau_\delta}{\tau_\epsilon P_u^\epsilon}\right]h_{\alpha\beta}}$$

We have found a new sigma-model for a string moving on a TNC geometry!

Different from earlier ”stringy” NC proposal by Andringa, Bergshoeff, Gomis, de Roo (2012)

Part 2: SMT limit

Spin Matrix theory (SMT) limit for BPS bound $E \geq J$ is:

$$T = \frac{\tilde{T}}{c} \ , \quad x^0 = c^2 \tilde{t}$$

with c → ∞. Choose

$$c^2 = \frac{P^2}{\lambda} = \frac{P^2}{4\pi g_s N}$$

P = total momentum along u

SMT limit in string theory: g_s → 0 with N and $(E-J)/g_s$ kept fixed

Resulting target space geometry:

Rescaled clock one-form: $\tilde{\tau} = d\tilde{t}$

Spatial metric: $h_{ab} dx^a dx^b$

Gauge-field one-form: $m = m_a dx^a$

What kind of geometry is this?

A d+1 dimensional Galilean geometry specified by:

Rescaled clock one-form: $\tilde{\tau} = d\tilde{t}$

Spatial metric: $h_{ab}dx^a dx^b$

A gauge-field one-form living on this geometry: $m = m_a dx^a$

We call this d+1 dimensional U(1)-Galilean geometry

New type of non-Lorentzian geometry, in addition to TNC geometry

Local invariance is Galilei symmetry + U(1) symmetry:

$$\left[\tilde{H}, \tilde{G}_a\right] = P_a\,, \qquad \left[P_a, \tilde{G}_b\right] = 0\,,$$
$$[J_{ab}, P_c] = \delta_{ac}P_b - \delta_{bc}P_a\,, \qquad \left[J_{ab}, \tilde{G}_c\right] = \delta_{ac}\tilde{G}_b - \delta_{bc}\tilde{G}_a\,,$$
$$[J_{ab}, J_{cd}] = \delta_{ac}J_{bd} - \delta_{ad}J_{bc} - \delta_{bc}J_{ad} + \delta_{bd}J_{ac}\,,$$

Local boosts changes only h_{ab}:

$$\delta h_{ab} = \tau_a \lambda_{\bar{c}} e^{\bar{c}}_b + \tau_b \lambda_{\bar{c}} e^{\bar{c}}_a\,, \quad \delta m_a = \delta\tau_a = 0 \quad \text{where} \quad h_{ab} = \delta_{\bar{c}\bar{d}} e^{\bar{c}}_a e^{\bar{d}}_b$$

This is the kind of geometry that emerges from SMT!

What happens in the sigma-model?

Goes to zero

Goes like T^2c^2

τ goes like c^2

$$L_{\mathrm{TNC}} = -P_u^\alpha m_\alpha + \frac{1}{2}\frac{P_u^\alpha P_u^\beta}{\tau_\gamma P_u^\gamma} h_{\alpha\beta} - \frac{T^2}{2}\frac{\epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\tau_\gamma\tau_\delta}{\tau_\epsilon P_u^\epsilon} h_{\alpha\beta}$$

To have a non-trivial sigma-model we need $\tilde{T} = Tc$ fixed for $c \to \infty$ so T → 0

One gets:

$$\boxed{L = -P_u m_a \dot{X}^a - \frac{\tilde{T}^2}{2P_u} h_{ab}(X^a)'(X^b)'}$$

Note: Here we choose a "lightcone" gauge: $P_u = P_u^0 = \mathrm{const.}\ ,\quad P_u^1 = 0\ ,\quad t = \xi^0$

String sigma-model on the "SMT geometry"

Fits with Landau-Lifshitz model, as well as other sigma-models related to nearest neighbor spin chains → We can assign a target space geometry to the sigma-model

SMT limit of string theory on AdS$_5$ x S^5
→ Corresponds to taking both a **non-relativistic limit of the geometry** and a **non-relativistic limit of the string world-sheet theory**

Non-relativistic geometry:

$$\tilde{\tau} = dt \ , \quad m = m_a dx^a \ , \quad h_{ab} dx^a dx^b$$

Non-relativistic string world-sheet theory on this geometry:

$$L = -P_u m_a \dot{X}^a - \frac{\tilde{T}^2}{2P_u} h_{ab} (X^a)' (X^b)'$$

Can we derive a non-relativistic theory of gravity to go with the non-relativistic geometry?
Can we get it from consistency of the string world-sheet theory?

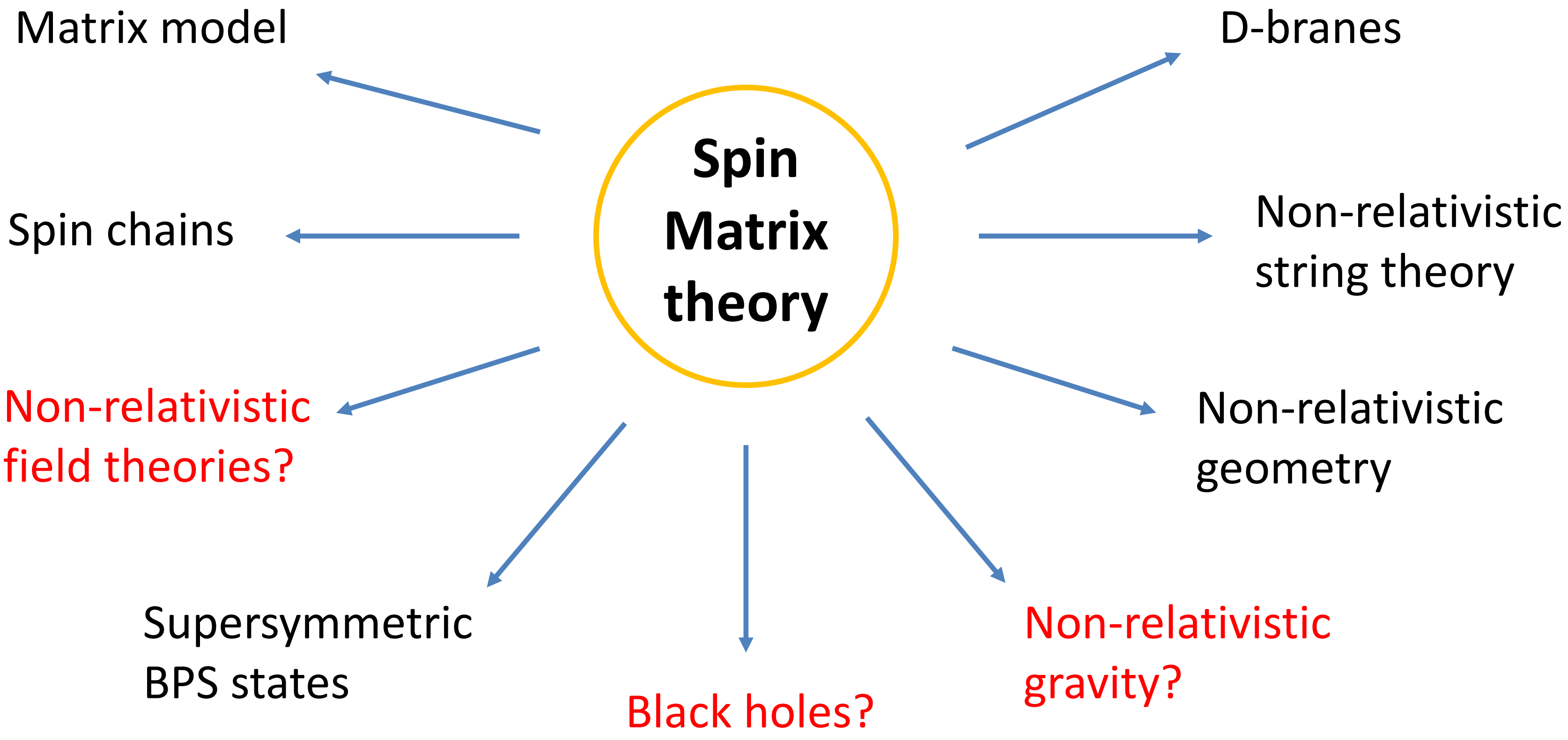
→ Would mean we have found the geometry and gravity that emerges from SMT

Conclusions and outlook

Other Spin Matrix theories from $\mathcal{N}$=4 SYM?

Unitarity bound	Spin group G_s	Cartan diagram for algebra	Representation R_s
$E \geq J_1$	$U(1)$		
$E \geq J_1 + J_2$	$SU(2)$	○	$[1]$
$E \geq J_1 + J_2 + J_3$	$SU(2\|3)$	○—⊗—○—○	$[0,0,0,1]$
$E \geq S_1 + J_1 + J_2$	$SU(1,1\|2)$	⊗—○—⊗	$[0,1,0]$
$E \geq S_1 + S_2 + J_1$	$SU(1,2\|2)$	○—⊗—○—⊗	$[0,0,0,1]$
$E \geq S_1 + S_2 + J_1 + J_2 + J_3$	$SU(1,2\|3)$	○—⊗—○—○—⊗	$[0,0,0,1,0]$

What are the analogues of the classical matrix model for the SU(1,1|2) SMT and SU(1,2|3) SMT?
The free spectra suggest 2D and 3D field theories
New (supersymmetric) non-lorentzian field theories?



Matrix model

D-branes

Spin chains

Spin Matrix theory

Non-relativistic string theory

Non-relativistic field theories?

Non-relativistic geometry

Supersymmetric BPS states

Black holes?

Non-relativistic gravity?

Could SMT be a unified framework for new and more easily solvable holographic dualities?

Thank you!