Tensionless Strings and Carrollian Things

BMS on the Worldsheet

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Outline

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TENSIONLESS STRINGS: WHY BOTHER?

Tensionless strings have been studied since Schild in 1977.

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- Of interest to higher spin holographic dualities. [Klebanov-Polyakov '03, Gaberdiel-Gopakumar '10]
- Our programme: Initial goals.

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  Aim(2): Make connection between tensionless strings and higher spins concrete.

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  Aim(2): Make connection between tensionless strings and higher spins concrete.
- Lacking: A worldsheet organising principle (like 2d CFT for string theory).

SUMMARY OF RESULTS

References:

- A. Bagchi. JHEP 1305 (2013) 141 [arXiv:1303.0291].
- A. Bagchi, S. Chakrabortty, P. Parekh. JHEP 1601 (2016) 158 [arXiv:1507.04361].
- A. Bagchi, S. Chakrabortty, P. Parekh. JHEP 1610 (2016) 113 [arXiv:1606.09628].
- AB, Banerjee, Chakrabortty, Parekh. JHEP 1802 (2018) 065 [arXiv:1710.03482].

Results:

- 3d *Bondi-Metzner-Sachs* algebra arises on the worldsheet as opposed to usual two copies of the Virasoro algebra.
- Classical tensionless strings: properties obtained intrinsically and as limit of relativistic strings and superstrings.
- Quantum tensionless strings: limiting & intrinsic description differ. New physics!
- Interesting connections to strings near the Hagedorn transition.

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CLASSICAL CLOSED STRINGS

Isberg, Lindstrom, Sundborg,Theodoridis 1993

Start with Nambu-Goto action:

$$S = -T \int d^2\xi \sqrt{-\det \gamma_{\alpha\beta}}. \tag{1}$$

To take the tensionless limit, first switch to Hamiltonian framework.

- Generalised momenta: $P_m = T\sqrt{-\gamma}\gamma^{0\alpha}\partial_\alpha X_m$.
- Constraints: $P^2 + T^2\gamma\gamma^{00} = 0, \;\; P_m\partial_\sigma X^m = 0$.
- Hamiltonian: $\mathcal{H}_T = \mathcal{H}_C + \rho^i(\text{constraints})_i = \lambda(P^2 + T^2\gamma\gamma^{00}) + \rho\, P_m\partial_\sigma X^m$.

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Action after integrating out momenta:

$$S = \frac{1}{2}\int d^2\xi\, \frac{1}{2\lambda}\left[\dot{X}^2 - 2\rho\dot{X}^m\partial_\sigma X_m + \rho^2\partial_\sigma X^m\partial_\sigma X_m - 4\lambda^2T^2\gamma\gamma^{00}\right] \quad (2)$$

Identifying

$$g^{\alpha\beta} = \begin{pmatrix} -1 & \rho \\ \rho & -\rho^2 + 4\lambda^2T^2 \end{pmatrix},$$

action takes the familiar Weyl-invariant form

$$S = -\frac{T}{2}\int d^2\xi \sqrt{-g}g^{\alpha\beta}\partial_\alpha X^m\partial_\beta X^n\eta_{mn}. \quad (3)$$

Classical Tensionless Closed Strings

Isberg, Lindstrom, Sundborg,Theodoridis 1993

- Tensionless limit can now be taken systematically.
- $T \to 0 \Rightarrow$
$$g^{\alpha\beta} = \begin{pmatrix} -1 & \rho \\ \rho & -\rho^2 \end{pmatrix}.$$
- Metric is degenerate. $\det g = 0$.
- Replace degenerate metric density $T\sqrt{-g}g^{\alpha\beta}$ by a rank-1 matrix $V^\alpha V^\beta$ where V^α is a vector density
$$V^\alpha \equiv \frac{1}{\sqrt{2}\lambda}(1, \rho) \tag{4}$$

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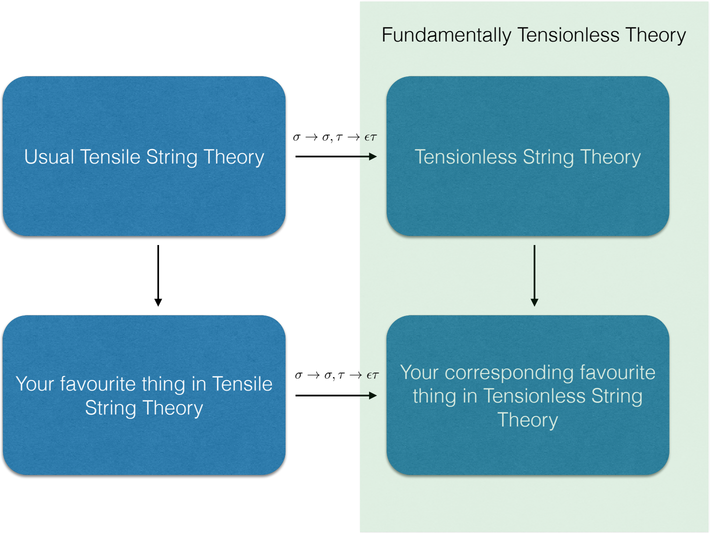
- Action in $T \to 0$ limit

$$\boxed{S = \int d^2\xi \; V^\alpha V^\beta \partial_\alpha X^m \partial_\beta X^n \eta_{mn}.} \tag{5}$$

- Starting point of tensionless strings.
- Need not refer to any parent theory. Treat this as action of fundamental objects.

CLOSURE?


Fundamentally Tensionless Theory
Usual Tensile String Theory
$\sigma \to \sigma, \tau \to \epsilon\tau$
Tensionless String Theory
Your favourite thing in Tensile String Theory
$\sigma \to \sigma, \tau \to \epsilon\tau$
Your corresponding favourite thing in Tensionless String Theory

SYMMETRIES OF TENSIONLESS CLOSED STRINGS

Isberg, Lindstrom, Sundborg,Theodoridis 1993

- Tensionless action is invariant under world-sheet diffeomorphisms.
- Fixing gauge: "Conformal" gauge: $V^\alpha = (v, 0)$ (v: constant).

- Tensile: Residual symmetry after fixing conformal gauge = Vir $\otimes$ Vir. Central to understanding string theory.
- Tensionless: Similar residual symmetry left over after gauge fixing.

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- For world-sheet diffeomorphism: $\xi^\alpha \to \xi^\alpha + \varepsilon^\alpha$, change in vector density:

$$\delta_\varepsilon V^\alpha = -V \cdot \partial \varepsilon^\alpha + \varepsilon \cdot \partial V^\alpha + \frac{1}{2}(\partial \cdot \varepsilon) V^\alpha$$

- Tensionless residual symmetries: for $V^\alpha = (v, 0)$,

$$\varepsilon^\alpha = \{f'(\sigma)\tau + g(\sigma), f(\sigma)\} \tag{6}$$

SYMMETRIES OF TENSIONLESS CLOSED STRINGS

Isberg, Lindstrom, Sundborg,Theodoridis 1993

- Define: $L(f) = f'(\sigma)\tau\partial_\tau + f(\sigma)\partial_\sigma, \quad M(g) = g(\sigma)\partial_\tau.$
- Expand: $f = \sum a_n e^{in\sigma}, \quad g = \sum b_n e^{in\sigma}$
- Therefore we have:

$$L(f) = \sum_n a_n e^{in\sigma}(\partial_\sigma + in\tau\partial_\tau) = \sum_n a_n L_n, \tag{7}$$

$$M(g) = \sum_n b_n e^{in\sigma}\partial_\tau = \sum_n b_n M_n. \tag{8}$$

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- Symmetry algebra in terms of Fourier modes:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c_L}{12}(m^3 - m)\delta_{m+n,0}, \quad [M_m, M_n] = 0.$$

$$[L_m, M_n] = (m-n)M_{m+n} + \frac{c_M}{12}(m^3 - m)\delta_{m+n,0}. \tag{9}$$

  Isberg et al find $c_L = c_M = 0$.
- 3d Bondi-Metzner-Sachs algebra or 2d Galilean Conformal Algebra.
- Various other applications: Holography of 3d flat space, Galilean field theories, non-relativistic limit of AdS/CFT.

Tensionless Strings: Symmetries as a Limit

A Bagchi 2013

- Tensile string: Residual symmetry in conformal gauge $g_{\alpha\beta} = e^{\phi}\eta_{\alpha\beta}$:

$$\begin{aligned} [\mathcal{L}_m, \mathcal{L}_n] &= (m-n)\mathcal{L}_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0} \\ [\mathcal{L}_m, \bar{\mathcal{L}}_n] &= 0, \quad [\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n} + \frac{\bar{c}}{12}m(m^2-1)\delta_{m+n,0} \end{aligned} \tag{10}$$

- World-sheet is a cylinder. Symmetry best expressed as 2d conformal generators on the cylinder.

$$\mathcal{L}_n = ie^{in\omega}\partial_\omega, \quad \bar{\mathcal{L}}_n = ie^{in\bar{\omega}}\partial_{\bar{\omega}} \tag{11}$$

where $\omega, \bar{\omega} = \tau \pm \sigma$. Vector fields generate centre-less Virasoros.

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- Tensionless limit $\Rightarrow$ length of string becomes infinite ($\sigma \to \infty$).
- Ends of closed string identified $\Rightarrow$ limit best viewed as ($\sigma \to \sigma, \tau \to \epsilon\tau, \epsilon \to 0$).

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- Define

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}). \tag{12}$$

- New vector fields (L_n, M_n) well-defined in limit and given by:

$$L_n = ie^{in\sigma}(\partial_\sigma + in\tau\partial_\tau), \quad M_n = ie^{in\sigma}\partial_\tau. \tag{13}$$

- These are *exactly the generators defined previously* . Close to form BMS_3.

TENSIONLESS STRINGS AND CARROLLIAN STRUCTURES

- Tensionless limit on the worldsheet: $\sigma \to \sigma, \tau \to \epsilon\tau, \epsilon \to 0$
- Worldsheet velocities $v = \frac{\sigma}{\tau} \to \infty$. Effectively, $\frac{v}{c} \to \infty$
- Hence worldsheet speed of light $\to 0$. Carrollian limit.

- Degenerate worldsheet metric.
- Riemannian tensile worldsheet $\to$ Carrollian tensionless worldsheet.

- Action for tensionless string $\Rightarrow$ a massless spin-0 particle coupled to a Carrollian background. Compare, e.g. with [Bergshoeff, Gomis, Rollier, Rosseel, Veldhuis 2017]
- BMS symmetries are conformal Carroll symmetries.
- Very clear why the BMS_3 algebra appears here.

- Similar discussions: [Duval, Gibbons, Horvathy 2014]

Tensionless EM-Tensor

A Bagchi 2013

- Spectrum of tensile string theory (in conformal gauge in flat space)
  - Quantise worldsheet theory as a theory free scalar fields.
  - Constraint: vanishing of EOM of metric (which is fixed to be flat).
  - Op form: Physical states vanish under action of modes of E-M tensor.
- EM tensor for 2d CFT on cylinder:

$$T_{cyl} = z^2 T_{plane} - \frac{c}{24} = \sum_n \mathcal{L}_n e^{in\omega} - \frac{c}{24}; \quad \bar{T}_{cyl} = \sum_n \bar{\mathcal{L}}_n e^{in\bar{\omega}} - \frac{\bar{c}}{24} \tag{14}$$

- The Ultra-relativistic EM tensor

$$T_{(1)} = \lim_{\epsilon \to 0} \left(T_{cyl} - \bar{T}_{cyl} \right) = \sum_n (L_n - in\tau M_n) e^{in\sigma} - \frac{c_L}{24} \tag{15}$$

$$T_{(2)} = \lim_{\epsilon \to 0} \epsilon \left(T_{cyl} + \bar{T}_{cyl} \right) = \sum_n M_n e^{in\sigma} - \frac{c_M}{24} \tag{16}$$

- Classical constraint on the tensionless string: $T_{(1)} = 0, \quad T_{(2)} = 0.$
- Quantum version: physical spectrum of tensionless strings restricted by

$$\langle \text{phys}|T_{(1)}|\text{phys}'\rangle = 0, \quad \langle \text{phys}|T_{(2)}|\text{phys}'\rangle = 0. \tag{17}$$

INTRINSIC ANALYSIS: EOM AND SOLUTIONS

AB, Chakrabortty, Parekh 2015

- Equation of motion in $V^a = (v, 0)$ gauge: $\ddot{X}^\mu = 0$.
- Solution:

$$X^\mu(\sigma, \tau) = x^\mu + \sqrt{2c'} A_0^\mu \sigma + \sqrt{2c'} B_0^\mu \tau + i\sqrt{2c'} \sum_{n \neq 0} \frac{1}{n} \left(A_n^\mu - in\tau B_n^\mu\right) e^{in\sigma} \tag{18}$$

- Closed string b.c.: $X^\mu(\sigma, \tau) = X^\mu(\sigma + 2\pi, \tau) \Rightarrow A_0^\mu = 0$.
- Constraints:

$$\dot{X}^2 = 2c' \sum_{m,n} B_{-m} \cdot B_{m+n} \, e^{in\sigma} = 0, \quad \dot{X} \cdot X' = 2c' \sum_{m,n} (A_{-m} - in\tau B_{-m}) \cdot B_{m+n} \, e^{in\sigma} = 0$$

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- Define:

$$L_n = \sum_m A_{-m} \cdot B_{m+n}, \quad M_n = \sum_m B_{-m} \cdot B_{m+n}$$

- Classical constraints in terms of modes:

$$\sum_n (L_n - in\tau M_n) \; e^{in\sigma} = 0 = T_{(1)}, \quad \sum_n M_n \, e^{in\sigma} = 0 = T_{(2)}. \qquad (19)$$

- Familiar form obtained earlier from purely algebraic considerations.

Intrinsic Analysis: EOM and Solutions

AB, Chakrabortty, Parekh 2015

- The algebra of the modes are:

$$\{A_m^\mu, A_n^\nu\} = 0, \quad \{B_m^\mu, B_n^\nu\} = 0, \quad \{A_m^\mu, B_n^\nu\} = -im\delta_{m+n,0}\,\eta^{\mu\nu}. \qquad (20)$$

  Note: this is *not* the algebra of harmonic oscillator modes. (More later.)

- The worldsheet symmetry algebra of tensionless strings, now constructed from the quadratics of the modes:

$$\{L_m, L_n\} = -i(m-n)L_{m+n}, \; \{L_m, M_n\} = -i(m-n)M_{m+n}, \; \{M_m, M_n\} = 0. \quad (21)$$

- Quantization: $\{,\}_{PB} \to -\frac{i}{\hbar}[,]$ leads to the 2d Galilean Conformal Algebra.

Limiting Analysis: Modes

AB, Chakrabortty, Parekh 2015

- Tensile string mode expansion:

$$X^{\mu}(\sigma,\tau) = x^{\mu} + 2\sqrt{2\alpha'}\alpha_0^{\mu}\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}[\tilde{\alpha}_n^{\mu}e^{-in(\tau+\sigma)} + \alpha_n^{\mu}e^{-in(\tau-\sigma)}].$$

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- The limiting procedure: $\tau \to \epsilon\tau,\ \sigma \to \sigma,\ \alpha' = c'/\epsilon$ with $\epsilon \to 0$

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$$= x^\mu + 2\sqrt{2c'}(\sqrt{\epsilon})\alpha_0^\mu\tau + i\sqrt{2c'}\sum_{n\neq 0}\frac{1}{n}\left[\frac{\alpha_n^\mu - \tilde{\alpha}_{-n}^\mu}{\sqrt{\epsilon}} - in\tau\sqrt{\epsilon}(\alpha_n^\mu + \tilde{\alpha}_{-n}^\mu)\right]e^{in\sigma}.$$

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- Thus we get a relation between the tensionless and tensile modes:

$$A_n^\mu = \frac{1}{\sqrt{\epsilon}}(\alpha_n^\mu - \tilde{\alpha}_{-n}^\mu), \quad B_n^\mu = \sqrt{\epsilon}(\alpha_n^\mu + \tilde{\alpha}_{-n}^\mu). \tag{22}$$

- The equivalent of the Virasoro contraints are now related as:

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon\left[\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right] \tag{23}$$

OUTLINE

QUANTUM TENSIONLESS STRINGS FROM THE LIMIT

AB, Chakrabortty, Parekh 2015

- Think of Tensionless Hilbert space as one obtained from the tensile theory. Metric degenerates. So constraints on the system change.
- Impose Tensionless Constraints on the Hilbert space of the Tensile theory.
- Physical Hilbert space of tensile theory: Built on the tensile vacuum.

$$|0\rangle_\alpha : \quad \alpha_n^\mu |0\rangle_\alpha = 0 = \tilde{\alpha}_n^\mu |0\rangle_\alpha \quad \forall n > 0. \tag{24}$$

- The states at a general level: $|\Phi\rangle = \prod_{i,j} \alpha_{-m_i}^{\mu_i} \tilde{\alpha}_{-m_j}^{\mu_j} |0\rangle_\alpha$

Quantum Tensionless Strings from the Limit

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- The states at a general level: $|\Phi\rangle = \prod_{i,j} \alpha^{\mu_i}_{-m_i} \tilde{\alpha}^{\mu_j}_{-m_j} |0\rangle_\alpha$
- The mass^2 operator acting on this state:

$$\begin{aligned} m^2 &= \frac{2}{c'} \left(\sum_{m \neq 0} B_{-m} \cdot B_m \right) \prod_{i,j} \alpha^{\mu_i}_{-m_i} \tilde{\alpha}^{\mu_j}_{-m_j} |0\rangle_\alpha \\ \Rightarrow \quad m^2 &= \frac{2\epsilon}{c'} \left(\sum_{m \neq 0} (\alpha_{-m} + \tilde{\alpha}_m) \cdot (\alpha_m + \tilde{\alpha}_{-m}) \right) \prod_{i,j} \alpha^{\mu_i}_{-m_i} \tilde{\alpha}^{\mu_j}_{-m_j} |0\rangle_\alpha \to 0 \end{aligned}$$

- Massless higher spin states (rather trivially).

Fundamentally Quantum Tensionless Strings

AB, Chakrabortty, Parekh 2015

- Wish to return to harmonic oscillator basis for the tensionless string. Define:

$$C^\mu_n = \frac{1}{2}(A^\mu_n + B^\mu_n), \quad \tilde{C}^\mu_n = \frac{1}{2}(-A^\mu_{-n} + B^\mu_{-n}) \tag{25}$$

- The algebra: $[C^\mu_m, C^\nu_n] = m\delta_{m+n}\eta^{\mu\nu}$, $[\tilde{C}^\mu_m, \tilde{C}^\nu_n] = m\delta_{m+n}\eta^{\mu\nu}$.
- The tensile and tensionless raising and lowering operators are related by

$$C^\mu_n(\epsilon) = \beta_+ \alpha^\mu_n + \beta_- \tilde{\alpha}^\mu_{-n}, \text{ where: } \beta_\pm = \frac{1}{2}\left(\sqrt{\epsilon} \pm \frac{1}{\sqrt{\epsilon}}\right)$$

$$\tilde{C}^\mu_n(\epsilon) = \beta_- \alpha^\mu_{-n} + \beta_+ \tilde{\alpha}^\mu_n. \tag{26}$$

- The fundamental tensionless vacuum $|0\rangle_c$:

$$C^\mu_n|0\rangle_c = 0 = \tilde{C}^\mu_n|0\rangle_c \quad \forall n > 0. \tag{27}$$

- Different from tensile vacuum: mixing of tensile raising & lowering op in $C, \tilde{C}$.
- "Fundamental" states built by acting raising operators on $|0\rangle_c$:

$$|\Psi\rangle = \prod_{i,j} C^{\mu_i}_{-m_i} \tilde{C}^{\mu_j}_{-m_j} |0\rangle_c \tag{28}$$

THE “OTHER” VACUUM

AB, Chakraborty, Parekh 2015

The relation between operators is a Bogoliubov transformation

$$C_n^\mu = e^{iG}\alpha_n e^{-iG} = \cosh\theta\, \alpha_n^\mu - \sinh\theta\, \tilde{\alpha}^\mu_{-n}, \quad G = i\sum_{n=1}^{\infty}\theta\left[\alpha_{-n}.\tilde{\alpha}_{-n} - \alpha_n.\tilde{\alpha}_n\right]$$

$$\tilde{C}_n^\mu = e^{iG}\tilde{\alpha}_n e^{-iG} = -\sinh\theta\, \alpha^\mu_{-n} + \cosh\theta\, \tilde{\alpha}_n^\mu, \quad \tanh\theta = \frac{1-\epsilon}{1+\epsilon} \tag{29}$$

- Relation between the two vacua:

$$|0\rangle_c = \exp[iG]|0\rangle_\alpha = \left(\frac{1}{\cosh\theta}\right)^{1+1+\ldots}\prod_{n=1}^{\infty}\exp[\tanh\theta\alpha_{-n}\tilde{\alpha}_{-n}]|0\rangle_\alpha \tag{30}$$

- Using the regularisation: $1+1+1+\ldots\infty = \zeta(0) = -\frac{1}{2}$

$$\boxed{|0\rangle_c = \sqrt{\cosh\theta}\prod_{n=1}^{\infty}\exp[\tanh\theta\alpha_{-n}\tilde{\alpha}_{-n}]|0\rangle_\alpha} \tag{31}$$

- This new vacuum is a squeezed state w.r.t $|0\rangle_\alpha$.

OUTLINE

TWO DIFFERENT TENSIONLESS SUPERSTRINGS

AB, (Banerjee), Chakrabortty, Parekh 2016-17

- Inspired by the RNS superstring, the tensionless action:

$$S = \int d^2\sigma \left[V^a V^b \partial_a X \cdot \partial_b X + i\bar{\psi} \cdot \rho^a \partial_a \psi \right]. \tag{32}$$

- Fundamental change in the fermionic sector:

$$\{\rho^\alpha, \rho^\beta\} = 2\eta^{\alpha\beta} \to \boxed{\{\rho^\alpha, \rho^\beta\} = 2V^\alpha V^\beta} \tag{33}$$

- In the "conformal" gauge $V^\alpha = (1, 0)$,

$$\{\rho^0, \rho^1\} = \mathbb{O},\ (\rho^0)^2 = \mathbb{I},\ (\rho^1)^2 = \mathbb{O}. \tag{34}$$

- Modified Clifford algebra: invariant under similarity trans $\rho^a \to S^{-1}\rho^a S$.
- Two different classes of solutions, disconnected by similarity transformation.
- Homogeneous and Inhomogeneous tensionless superstrings.

- Different contraction of 2 copies of Super-Virasoros.
- Also called the "democratic" and the "despotic" limits [Lodato, Merbis 2016].
- Algebras appear also as asymptotic symmetries of 3d flat Supergravity.

HOMOGENEOUS TENSIONLESS SUPERSTRINGS

AB, Chakrabortty, Parekh 2016

- Arises from the "trivial" solution:

$$\rho^0 = \mathbb{I}, \quad \rho^1 = \mathbb{O}. \tag{35}$$

- Residual symmetry [Lindstrom et al 1991, Gamboa et al 1989]:

$$\begin{aligned} &[L_n, L_m] = (n-m)L_{n+m}, \quad [L_n, M_m] = (n-m)M_{n+m} \\ &[L_n, Q_r^\alpha] = \left(\frac{n}{2} - r\right)Q_{n+r}^\alpha, \quad \{Q_r^\alpha, Q_s^\beta\} = \delta^{\alpha\beta} M_{r+s} \end{aligned} \tag{36}$$

- This is a "homogeneous" contraction of two copies of Super Virasoro.

$$\begin{aligned} &L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right), \\ &Q_r^+ = \sqrt{\epsilon}\,\mathcal{Q}_r, \quad Q_r^- = \sqrt{\epsilon}\,\bar{\mathcal{Q}}_{-r}. \end{aligned} \tag{37}$$

- Homogeneous as both supercharges are scaled in the same way.
- Also arises as asymptotic symmetries of usual supergravity in 3d flat space. [Barnich et al. 2014; Lodato, Merbis 2016].

INHOMOGENEOUS TENSIONLESS SUPERSTRINGS

AB, Banerjee, Chakrabortty, Parekh 2017

- Arises from the solution:

$$\rho^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \rho^1 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \tag{38}$$

- Residual symmetry:

$$\begin{aligned} &[L_n, L_m] = (n-m)L_{n+m}, \quad [L_n, M_m] = (n-m)M_{n+m}, \\ &\{G_r, G_s\} = 2L_{r+s}, \ \{G_r, H_s\} = 2M_{r+s}, \\ &[L_n, G_r] = \Big(\frac{n}{2} - r\Big)G_{n+r}, \ [L_n, H_r] = \Big(\frac{n}{2} - r\Big)H_{n+r}, \ [M_n, G_r] = \Big(\frac{n}{2} - r\Big)H_{n+r}. \end{aligned} \tag{39}$$

- This is another contraction of the Super Virasoro:

$$\begin{aligned} &L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \ M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}), \\ &G_r = \mathcal{Q}_r - i\bar{\mathcal{Q}}_{-r}, \ H_r = \epsilon(\mathcal{Q}_r + i\bar{\mathcal{Q}}_{-r}). \end{aligned} \tag{40}$$

- Inhomogeneous as supercharges are scaled in the different ways.

- Asymptotic symmetries of "twisted" supergravity in 3d flat space. [Lodato, Merbis 2016].

Outline

HAGEDORN PHYSICS

AB, Chakrabortty, Parekh 2015

- String theory: exponential growth of states. # of states at nth level $\sim \exp(4\pi n^{1/2})$.
- Canonical partition function diverges above Hagedorn temp: $\mathcal{T}_H = \frac{1}{4\pi\sqrt{\alpha'}}$. New phase? New degrees of freedom?
- Effective string tension near Hagedorn temperature [Pisarski, Alverez 1982; Olesen 1985]:

$$T_{eff} = T_0 \sqrt{1 - \frac{\mathcal{T}^2}{\mathcal{T}_H^2}} \Rightarrow T_{eff} \to 0 \text{ as } \mathcal{T} \to \mathcal{T}_H \tag{41}$$

- Our analysis is useful here: we provide a worldsheet description which was previously considered impossible.

- In the approach to $\mathcal{T}_H$, it becomes thermodynamically favourable to form a long string as opposed to heating up a gas of strings.
- Conjecture: Vacuum $|0\rangle_C$ is the worldsheet analogue of this emergent long string.
- New degrees of freedom as the excitations of $|0\rangle_C$.
- Worldsheet description doesn't break down: Carrollian structures appear!

OTHER DIRECTIONS AND APPLICATIONS

- Thermal nature of the tensionless vacuum: result of left-right entanglement of the world-sheet 2d CFT and its limit?
- Bogoliubov transformations: Rindler physics on the worldsheet? Observer in the tensionless vacuum always sees a thermal state.
- Strings near spacetime singularities: another domain for applying tensionless techniques?

OUTLINE

SUMMARY

Classical Tensionless Strings

- Limiting procedure reproduced earlier work by Isberg et al.
- Mode expansions done in fundamental and limit methods matched well.
- Supersymmetric generalisations: two different algebras.

Quantum Tensionless Strings

- Limiting case showed link to massless higher spins.
- Fundamental tensionless strings naturally defined a different vacuum.
- This was a coherent or a squeezed state w.r.t. the tensile vacuum.
- Connections to Hagedorn physics.
- Tensionless vacuum = emergent long string near $\mathcal{T}_H$.

Questions and Future Directions

What happens to spacetime?

QUESTIONS AND FUTURE DIRECTIONS

What happens to spacetime?

- We have taken a weird limit on the worldsheet.
- Doing something drastic to the string would also affect the background where it propagates.
- Should be a deformation of Minkowski spacetime.
- Implement on worldsheet by tensionless analogue of vanishing of beta functions for maintaining conformal invariance.

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Others

- Open strings?
- Exploring Hagedorn physics with Carrollian structures.
- Connections to higher spin holography (do this in AdS!)
- String amplitudes in the tensionless limit.
- Connections to the Ambitwistor string. [Casali-Tourkine 2016-17]
- Many more possible avenues..

Thank you!