New Carroll gravity boundary conditions

Higher spin Carroll gravity

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~~Applied~~ Newton–Cartan geometry
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Summary and acknowledgments

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Carroll algebra ($i, j = 1, 2$):

$$[J,\, P^i] = \epsilon^i{}_j\, P^j$$
$$[J,\, G^i] = \epsilon^i{}_j\, G^j$$
$$[P^i,\, G^j] = -\epsilon^{ij}\, H$$

H: time-translations, P^i: spatial translations
J: rotations, G^i: Carrollian boosts

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- Carroll gravity solutions can have entropy for loose set of bc's
- Focus on 2+1 bulk dimensions: Chern–Simons with Carroll gauge group

Based on collaborations with

- Eric Bergshoeff
- Wout Merbis
- Stefan Prohazka
- Max Riegler
- Jan Rosseel

Outline

Introduction

Most general AdS_3 boundary conditions

Carroll gravity as double contraction from AdS_3

Carroll gravity entropy

Outlook

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  Example:

$$\Phi(x \to \infty) = 0$$

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Example: Brown-Henneaux type of bc's (aAdS_3):

$$\mathrm{d}s^2_{\text{aAdS}} = \mathrm{d}\rho^2 + \left(e^{2\rho}\eta_{\mu\nu} + \gamma_{\mu\nu} + \mathcal{O}(e^{-2\rho})\right)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

with $\delta\gamma =$ arbitrary

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  1. Trafos that violate bc's (forbidden)
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- Canonical boundary charges (á la Regge–Teitelboim) generate asympotic symmetries
- Consistency means they are finite, integrable, non-trivial and conserved (in time)

Outline

Chern–Simons formulation of Einstein gravity recap

- Bulk action:

$$S = I_{\text{CS}}[A^+] - I_{\text{CS}}[A^-]$$

with $A^\pm \in \mathsf{sl}(2,\,\mathbb{R})$ and

$$I_{\text{CS}}[A^\pm] = \frac{k}{4\pi}\,\int \langle A^\pm \wedge \mathrm{d}A^\pm + \tfrac{2}{3}\,A^\pm \wedge A^\pm \wedge A^\pm \rangle$$

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- For imposing bc's separate radial dependence

$$A = b^{-1}\big(\,\mathrm{d}{+}a(t,\,\varphi)\big)b$$

with suitable group element b, assuming $\delta b = 0$ and $\partial_\rho b \neq 0$

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- Essence of bc's contained in choices of a and δa

Loosest choice of bc's (see 1608.01308)

- No restriction on a

$$a = \sum \mu^i(t,\,\varphi) L_i \,\mathrm{d}t + \sum \mathcal{L}^i(t,\,\varphi) L_i \,\mathrm{d}\varphi$$

or its variation

$$\delta a = \sum \delta\mathcal{L}^i(t,\,\varphi) L_i \,\mathrm{d}\varphi$$

other than assuming chemical potentials are fixed, $\delta\mu^i = 0$

Algebra conventions

Generators of sl(2):

$$L_i \qquad i \in \{-1, 0, 1\}$$

Commutation relations:

$$[L_i,\, L_j] = (i-j)\, L_{i+j}$$

Bilinear form:

$$\kappa_{ij} = \langle L_i\, L_j \rangle = \mathrm{antidiag}(-1,\, \tfrac{1}{2},\, -1)_{ij}$$

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- Asymptotic symmetry algebra

$$[L^i_n,\, L^j_m] = (i-j)\, L^{i+j}_{n+m} - k\, n\, \kappa_{ij}\, \delta_{n+m,\,0}$$

sl$(2,\,\mathbb{R})_k$ current algebra in each chiral sector

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- Known bc's special cases of above!

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- Generalized Fefferman–Graham expansion

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + 2\left(e^{\rho}N_i^{(0)} + N_i^{(1)} + e^{-\rho}N_i^{(2)} + \mathcal{O}(e^{-2\rho})\right)\,\mathrm{d}\rho\,\mathrm{d}x^i \\ + \left(e^{2\rho}g_{ij}^{(0)} + e^{\rho}g_{ij}^{(1)} + g_{ij}^{(2)} + \mathcal{O}(e^{-\rho})\right)\,\mathrm{d}x^i\,\mathrm{d}x^j$$

with shift vector

$$N^{(n)} = \mu^{n+1}\,\mathrm{d}t + \mathcal{L}^{n+1}\,\mathrm{d}\varphi \qquad n = 0, 1, 2$$

and "boundary metric"

$$g_{ij}^{(n)} = \begin{pmatrix} -4\mu^1\mu^{n+4} & -2\mu^{n+4}\mathcal{L}^1 - 2\mu^1\mathcal{L}^{n+4} \\ \bullet & -4\mathcal{L}^1\mathcal{L}^{n+4} \end{pmatrix}_{ij} \qquad n = 0, 1, 2$$

with suitable redefinitions of $\mathcal{L}^6$ and μ^6

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- Metric depends on six normalizable modes/state-dependent functions ($\mathcal{L}^i$) and six non-normalizable (μ^i) modes/chemical potentials

Special cases

Possibilities in each chiral sector:

case	constraints	ASA
1	none	$\mathfrak{sl}(2)_k$
2	$\mathcal{L}^0 = 0\,, \mathcal{L}^+ = -e^{2\phi(x^+)}\,, \mathcal{L}^- = e^{-2\phi(x^+)}\mathcal{L}(x^+)$	Vir$\oplus\mathfrak{u}(1)_k$
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Previously known bc's:

- Brown–Henneaux: $3 \oplus 3$
- Compère–Song–Strominger: $3 \oplus 4$
- Troessaert: $2 \oplus 2$
- Avery–Poojary–Suryanarayana: $1 \oplus 3$
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Note: infinite set of new bc's possible by allowing $\mu^i(\mathcal{L}^j)$, subject to integrability (e.g. KdV set of bc's, see Perez–Tempo–Troncoso)

Outline

Example of double contraction (see 1704.07419)

- Starting point: case $1 \oplus 1$, but using $\mathfrak{so}(2,1)_k$ $(a, b = 0, 1, 2)$

$$[\mathcal{L}_n^{(\pm)a},\, \mathcal{L}_m^{(\pm)b}] = \epsilon^{ab}{}_c\, \mathcal{L}_{n+m}^{(\pm)c} - \tfrac{k\ell}{2}\, n\, \eta^{ab}\, \delta_{n+m,\,0}$$

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- Define Carroll-type generators $(i, j = 1, 2)$

$$P_n^i = \frac{1}{\sqrt{\ell}}\left(\mathcal{L}_n^{(+)i} + \mathcal{L}_{-n}^{(-)i}\right) \qquad G_n^i = \frac{1}{\sqrt{\ell}}\left(\mathcal{L}_n^{(+)i} - \mathcal{L}_{-n}^{(-)i}\right)$$

$$J_n = \mathcal{L}_{0,n}^{(+)} + \mathcal{L}_{0,-n}^{(-)} \qquad H_n = \frac{1}{\ell}\left(\mathcal{L}_{0,n}^{(+)} - \mathcal{L}_{0,-n}^{(-)}\right)$$

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- Double contraction yields Carroll loop algebra

$$[J_n,\, P_m^i] = \epsilon^i{}_j\, P_{n+m}^j$$

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$$[P_n^i,\, G_m^j] = -\epsilon^{ij}\, H_{n+m} - k\, n\, \delta^{ij}\, \delta_{n+m,\,0}$$

For $n = m = 0$: Carroll algebra in two spatial dimensions

Carroll gravity with fluctuating boundary metric (see 1704.07419)

Perform double contraction on gauge algebra of Chern–Simons theory

- Start with isometry group of AdS$_3$, $SO(2,1)\times SO(2,1)$

$$[L_a^{\pm},\, L_b^{\pm}] = \epsilon_{ab}{}^{c}\, L_c^{\pm}$$

and double contract

$$H = \frac{1}{\ell}\left(L_0^{+} - L_0^{-}\right) \quad J = L_0^{+} + L_0^{-} \quad G_i = \frac{1}{\sqrt{\ell}}\left(L_i^{+} - L_i^{-}\right) \quad P_i = \frac{1}{\sqrt{\ell}}\left(L_i^{+} + L_i^{-}\right)$$

Carroll trace defined by

$$\langle H\, J\rangle = -1 \qquad\qquad \langle P_i\, G_j\rangle = \delta_{ij}$$

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- Consider as non-trivial example CS connection

$$a = \mu(t,\varphi)\, H\ \mathrm{d}t + \big(\mathcal{K}(t,\,\varphi)J + \mathcal{J}(t,\,\varphi)H + \mathcal{J}^i(t,\,\varphi)(G_i + P_i)\big)\ \mathrm{d}\varphi$$

with $\delta\mu(t,\,\varphi) = 0$ and all other functions can vary freely

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- Leads to double contraction of case $1 \oplus 4$

$$[K_n,\, J_m] = k\, n\, \delta_{n+m,\,0} \qquad [J_n,\, G_m^i] = \epsilon^i{}_j\, G_{n+m}^j$$

$$[G_n^i,\, G_m^j] = -2\epsilon^{ij}\, K_{n+m} - 2n\, k\, \delta^{ij}\, \delta_{n+m,\,0}$$

Note: ASA emerges either through double contraction or direct construction

Metric-like interpretation (see 1612.02277)

For “metric”-interpretation choose $b = \exp(\rho P_2)$ in $A = b^{-1}(\mathrm{d}+a)b$

- Full connection then linear in radial coordinate ρ

$$A = a + P_2 \,\mathrm{d}\rho + \rho\,[a,\, P_2]$$

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- Carroll metric ("space") given by

$$\mathrm{d}s^2_{(2)} = \big((\rho\mathcal{K} - \mathcal{J}^1)^2 + (\mathcal{J}^2)^2\big) \, \mathrm{d}\varphi^2 + 2\mathcal{J}^2 \, \mathrm{d}\varphi \, \mathrm{d}\rho + \mathrm{d}\rho^2$$
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For "metric"-interpretation choose $b = \exp(\rho P_2)$ in $A = b^{-1}(\mathrm{d}+a)b$

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- Have looser set of bc's — perhaps more interesting physics?

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Check if Carroll gravity with looser bc's leads to entropy

Entropy of Carroll gravity with fluctuating boundary metric

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Final result for entropy: $\quad S \propto \big|\mathcal{K}\big|$

Reminiscent of universal near horizon result for entropy

$$S_{\text{NH}} = 2\pi\left(J_0^+ + J_0^-\right)$$

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Thanks for your attention!

If you really want to see the details...

Double contraction of $\mathfrak{so}(2,1)_k \oplus \mathfrak{u}(1)_k$ to Carroll gravity with fluctuating boundary metric

- ASA of case $1 \oplus 4$:

$$[J^a_n,\, J^b_m] = \epsilon^{ab}{}_c\, J^c_{n+m} - \frac{k}{2}\, n\, \eta^{ab}\, \delta_{n+m,\,0}$$
$$[\bar{J}_n,\, \bar{J}_m] = \frac{k}{2}\, n\, \delta_{n+m,\,0}$$

- Rescaled generators:

$$J_n := J_{0,\,n} + \bar{J}_{-n} \qquad G^i_n := \frac{2}{\sqrt{\ell}}\, J^i_n \qquad K_n := \frac{1}{\ell}\left(J_{0,\,n} - \bar{J}_{-n}\right)$$

- Double contraction ($\ell \to \infty$) yields ASA of Carroll gravity with fluctuating boundary metric

$$[K_n,\, J_m] = k\, n\, \delta_{n+m,\,0} \qquad [J_n,\, G^i_m] = \epsilon^i{}_j\, G^j_{n+m}$$
$$[G^i_n,\, G^j_m] = -2\epsilon^{ij}\, K_{n+m} - 2n\, k\, \delta^{ij}\, \delta_{n+m,\,0}$$